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# Empirical Studies on Electromagnetic Fields around Two Thin Wires 

Hiroshi Echigo and Katsuhiro Sato<br>Tohoku Gakuin University, Japan


#### Abstract

It is well known that two thin wires is fundamental one for the radio-frequency (RF) signal and energy transmission systems since they give the basic concept and knowledge about the RF transmission and radiation phenomena. They are the main issues of RF research field. In this paper, experimental measurements of magnetic fields around two thin wires are described. Measured data was obtained as complex numbers (amplitude and phase) so that they lead to give motion pictures of virtual wave propagations using the Phase Injection Method. The results obtained give the motion pictures showing that the propagating waves on a transmission line are changing into radiating waves and forming the plane waves.


Keywords- Two thin wires; electromagnetic near field; electromagnetic wave transmission and radiation; Phase Injection Method.

## 1. Introduction

A two thin wire system is one of the simplest and the most fundamental one to study radio-frequency (RF) signal transmission and radiation. For students and engineers, it gives basic concepts on the transmission lines and antenna system. For example, an open parallel line is treated as no radiating parts to explain the signal transmission circuits in the circuit theory. However, RF field researchers have noticed that these transmission lines can radiate EM energy. Especially in digital equipments, a gross of transmission lines are used to transfer the information signals. Their shapes are not only parallel ones but also arbitral-curved structures. The radiations from these lines are severe problems in the design and development of digital equipments. To reduce the radiation, it is very effective to know how electromagnetic waves propagate and radiate out of the lines.

In this paper,

1) The measurement system is described to collect the field data,
2) Using obtained data of complex values, the virtual wave


Fig. 1 Configuration of experiments. propagation around lines can be displayed after extra phase rotation is added to the phase term of measured complex values: Phase Injection Method. Our experiments implied that the line ends and curved portion of the line could cause the EM wave radiation.

## 2. EXPERIMENTAL SYSTEMS

## A. Two thin wires

The parameters of the thin wires used for our experiments are listed in Table 1. The wires were fixed on a flat plastic plate (Stylo Foam) to make various shapes and they were stood vertically to connect their one ends to the RF power feeding part, hanging from the ceiling of the room as shown in Fig. 1.

## B. RF power feeding system

| Table 1 Parameters of two wires |  |
| :---: | :---: |
| Radius of the wire | 0.2 mm |
| Separation of | Gradually increasing |
| The wire centers | About 1.0 m |
| Length | Copper |
| Materials | Polyvinyl Formal |
| Insulator |  |

The feeding parts were settled at the upper end of the line, which could feed RF power to each wire independently through two coaxial cables. The feeding parts and the wires are shown in Fig.1.

## C. XY positioner

To scan the electromagnetic (EM) field around the line, X-Y positioner (D3425AV1O-S) was used. This makes possible to move the arm vertically (Z-direction) and horizontally (X-direction) carrying a small loop EM sensor.

## D. EM sensors

To measure the EM fields, a small-shielded loop was used. The small-shielded loop is composed with a semi-ridged coaxial cable with its diameter 2 mm . The loop radius is 1.0 cm .

## E. Measurement equipments

EM sensor output was led to Vector Volt Meter (VVM:HP8508A) through a coaxial cable (SUCOFLEX ) of 5 m lengths and a preamplifier with its amplification gain of 25 dB .

## F. Measurement site

The measurements were accomplished in a vacant office room. To reduce the reflection from the floor, EM absorbers were settled on the floor, especially on the area near the line end.

## 3. Magnetic Field Near The Line

To confirm the current distribution on the line, the magnetic fields near the line were measured. Not to confuse the following explanation, xyz coordinate is set to the line configuration as shown in Fig.2.

To confirm the measurement, the near field of a parallel two wires was measured when RF energy of 1 GHz was supplied to the lines. The small loop sensor was scanned on the plane; $\mathrm{y}=1.5 \mathrm{~cm}, \mathrm{x}=-$ $10 \sim+10 \mathrm{~cm}, \mathrm{z}=60 \sim 200 \mathrm{~cm}$, to make the field pattern. Fig. 3 gives the results for resistive termination; ( 166 ohm resistor). Fig. 4 gives the magnetic field near one of two wires to show the current distribution on the wire. It proves that the current is flowing on the line without any irregularities.

## 4. Measurement Results Of Magnetic Near Fields Beside Two Thin Wires.

Since a complex value (amplitude and phase) is obtained at each measurement point on the measurement plane that is parallel to a


Fig. 2 The coordinate and lines.



Fig. 4 Magnetic Field Hex near the plane including two thin wires, we can show two kinds of fields (amplitude and phase). However another expression would be more effective and more attractive to understand what are going on the area.
The measured values were transformed to give real parts and imaginary part because the only real parts have essential meanings in our real world.

Figure 5 gives one of the measured fields in real parts. The left picture comes from the data obtained when the loop face was set to horizontal and the right one is for the loop face in vertical setting.

## 5. Phase Injection Method To Give Virtual Motion Pictures Showing Wave Propagations

To derive the real part from the measured complex value (amplitude and phase), phase increment of a fraction of 2 Pi were tried to give the real values at the time later by a fraction of the time period of the applied RF wave. Succeeding increment of the phase can give the real part values at the succeeding time sequentially. Consequently, this method can derive virtually the succeeding


Fig. 5 Magnetic near fields of gradually separating two wires. (feeding point :top of the wires)
fields along time sequence.
Figure 6 shows the resultant field sequence obtained by using this Phase Injection Method.

## 6. CONCLUSION

The two-wire line (a pair of thin wires) is one of the simplest and the most fundamental transmission one to study the EM wave transmission and radiation. As in the digital equipments, a lot of transmission lines are used to transfer the information signal between devices. The radiations from these lines are severe problems in design and development of digital equipments. To reduce the radiation from them, it is very important to know the phenomena (RF energy transmission and radiation) from these line structures. In this paper, the experimental results were given summarized as,

1) After confirmation of the magnetic field near the line, several field patterns were shown. 2) Using obtained data of complex values, the wave propagation could be revealed after the Phase Injection Method to give the virtual motion pictures.

According to our experiments, it would be true that the line ends and curved portion of the line could cause the EM wave radiation.

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Fig. 6 Real and Imaginary parts of near fields of gradually separating two wires. (feeding point :top of the wires)

# Transmission Properties of Metal Hole Arrays in Terahertz Region 

M. Hangyo<br>Institute of Laser Engineering, Osaka University, Japan<br>F. Miyamaru<br>Department of Physics, Shinshu University, Japan


#### Abstract

Transmission properties of metal plates periodically perforated with holes (metal hole arrays, MHAs) have been investigated in the THz region. Since metals can be considered as a nearly perfect conductor in the THz region, the surface plasmon-polariton (SPP) does not exist on a flat surface. However, the SPP-like modes exist on MHAs and contribute to the enhanced transmission of THz waves. These SPP-like modes also contribute to the resonant polarization conversion of the transmitted THz waves. We show a variety of transmission phenomena for single- and double-layer MHAs.


## 1. Introduction

Since the discovery of the extraordinary transmission phenomenon of light for metal thin films perforated with periodic subwavelength holes (metal hole arrays, MHAs), ${ }^{1}$ the localization and propagation of light on structured metals have attracted much attention. The enhanced transmission of light through the MHAs has been explained by the excitation of surface plasmon-polaritons (SPPs) localized on their surfaces. ${ }^{2}$ The extraordinary transmission has been also reported in the microwave ${ }^{3}$ and terahertz ( THz ) regions. ${ }^{4}$ The difference between the optical and THz (microwave) regions is that the SPPs on flat metal surfaces in the THz region cannot be considered as "the surface waves" since the decay length perpendicular to the surface is more than hundreds of the wavelength. This is due to the fact that the metals are nearly perfect conductor in the THz region. However, Pendry and coworkers found theoretically that the SPP-like modes localized on perfect conductor surfaces exist when the surfaces has indentations (e. g., holes, dimples, grooves, etc.). ${ }^{5}$ Their prediction has been confirmed experimentally in the microwave region. ${ }^{6}$ These structured surfaces are recognized as one of metamaterials.

In this paper, we show a variety of transmission phenomena attributed to the excitation of the SPP-like modes in the Pendry's sense (we call these modes just SPP hereafter) for MHAs in the THz region. The transmission spectra have been measured by the terahertz time-domain spectroscopy (THz-TDS) system. ${ }^{7}$ In this spectroscopy, transient THz pulses are emitted from a photoconductive antenna (PA) by exciting with a femtosecond laser and those transmitted through samples are coherently detected by another PA with a similar structure. From the wave forms measured with and without the samples, the transmittance and phase shift spectra are deduced by Fourier transformation. Polarization changes of the transmitted THz waves are measured by analyzing the polarization by wire-grid polarizers.

## 2. Basic THz Transmission Properties of Metal Hole Arrays

Figure 1 shows the schematic structure of a MHA. The MHAs are made of aluminum and have a hexagonal structure. The structure is characterized by a hole diameter $d$, a spacing between holes $s$, and a thickness of the metal slab $t$. The characteristic frequency of this MHA is the cutoff frequency of a single hole $\nu_{\mathrm{c}}=1.841 \mathrm{c} / \pi d$ and the 1st order diffraction frequency $v_{\mathrm{d}}=2 c / s \sqrt{3}$. In this paper, we deal with the cases $v_{\mathrm{c}}<v_{\mathrm{d}}$. The wave forms measured with and without the MHA with $d=0.68 \mathrm{~mm}, s=1.1 \mathrm{~mm}$, and $t=$ 0.5 mm are shown in Fig. 2 (a). ${ }^{8}$ A nearly single cycle input pulse changes to an oscillating wave form after transmitted through the sample. Figure 2 (b) shows the transmission and phase shift spectra obtained by the Fourier transformation of the wave forms in Fig. 2 (a). The transmittance shows the peak at 0.27 THz indicating that the MHA works as a bandpass filter. The transmittance at 0.27 THz is about 0.9 , which is 2.5 times higher than the porosity of the holes 0.35 . This means that the incoming THz waves are concentrated into the holes.

In order to investigate the role of the SPP in the enhanced transmission, the dependence of the transmission spectra on the in-plane wave number is measured by changing the incident angle. The results are shown in Fig. 3. The incident THz waves are p-polarized. The white dotted lines are


Figure 1: Schematic structure of a MHA.


Figure 3: Transmission spectra as a function of the in-plane wave number.
calculated assuming the dispersion of the SPP on a flat almost perfect conductor. The SPP dispersion curves agree well with the transmission peak except that the experimental transmission peak branches are slightly lower than the calculation. This discrepancy is attributed to the perturbation of the SPP on the flat surface by the hole arrays. This agreement supports the participation of the SPP in the enhanced transmission. In order to get further evidence for the participation of the SPP in the enhanced transmission, the dependence of the transmission spectra on the number of holes has been measured. ${ }^{9}$ The peak transmittance normalized by the hole area increases with increasing the number of holes indicating that the collective excitation (SPP excitation) is needed for the enhanced transmission. The surface localization of the SPP mode is confirmed by observing the shift of the frequency of the transmission peak by attaching thin dielectric films on the surface of the $\mathrm{MHA}^{10}$ and also by simulation. ${ }^{11}$

## 3. Polarization Properties of Metal Hole Arrays

Up to now, the polarization properties of the MHA have attracted less attention compared with the enhancement of the transmission. For the case of the metal gratings, Sambles and coworkers have found highly efficient resonant p-s polarization conversion phenomenon in the reflection geometry. ${ }^{12}$ They attributed this phenomenon to the resonant excitation of the SPP. We found large polarization change for the MHA when the incident angle is changed slightly from the normal as shown in Fig. 4. ${ }^{13}$ The wave forms are shown in the polarization-time space for (a) the incident wave, (b) transmitted wave for normal incidence, and (c) transmitted wave for the incident angle deviated by $3^{\circ}$ from the normal. The polarization vector and the lattice vector connecting the nearest neighbor holes makes the angle of $45^{\circ}$. As seen in Fig. 4 (c), the polarization changes with time evolution. The ellipticity and angle of rotation spectra are shown in Fig. 4 (d). Both spectra deviate from zero near the SPP frequency. This resonant frequency changes with the lattice constant as expected for the resonant SPP frequency. ${ }^{14}$ We also confirmed that the ellipticity and angle of rotation spectra are scarcely affected by the thickness of the MHA indicating that the phenomenon occurs at the surfaces. ${ }^{15}$ These results strongly indicate that the polarization conversion phenomenon is caused by the resonant excitation of the SPP.

## 4. Double-Layer Metal Hole Arrays

The double-layer MHAs show a variety of transmission properties owing to the geometrical freedom of the two layers (the structural parameters of the MHA in the following are $d=0.6 \mathrm{~mm}, s=1.13 \mathrm{~mm}$, and $t=0.25 \mathrm{~mm}$ ). ${ }^{11}$ Figure 5 shows the schematic configuration of the two layers. We measured the transmission spectra with changing the layer spacing $h$ and lateral displacement $p$. The results are shown in Fig. 6 for $p=0$ and 0.57 mm . The transmission spectra show Fabry-Perot-like interference patterns as seen in Figs. 6 (a) and (b). When $h$ is large

compared with the wavelength, the transmission spectra for $p=0$ and 0.57 mm are similar with each other as seen in Fig. 6 (d). In contrast, the spectra are quite different with each other when the spacing is less than the wavelength. For example, the peak transmittance for $p=0.57 \mathrm{~mm}$ is larger than for $p=0 \mathrm{~mm}$. This unexpected transmission property could be explained by the near-field coupling of the SPP excited on the rear surface of the first layer and that on the front surface of the second layer.

The coupling of the SPPs brings about interesting polarization changes. When the two layers are arranged keeping the hexagonal or trigonal rotational symmetry, the polarization of the incident THz wave should be kept after transmission and this is confirmed experimentally. However, when the symmetry of the configuration is low as the case shown in Fig. 7 (two-fold symmetry), the polarization conversion occurs. The polarization conversion occurs only when the spacing of the two layers is less than the wavelength. This phenomenon could be also explained by the near field coupling of the SPPs.

The above transmittance and polarization properties of the double-layer MHAs can be used to develop new optical devices controlling the transmittance and polarization ranging from microwave to visible depending on the scale of fabrication. Further, the piles of MHAs, which are metallic photonic crystals, can be considered as a new group of optical devices.

## 5. Hole Arrays made of doped Si

In the THz region, we can use various materials with a wide range of dielectric constant for hole arrays. Appropriately doped semiconductors can be considered as metals in the THz region. In order to confirm the participation of the SPP in the enhanced transmission phenomenon, we fabricated the hole array with doped Si (P-doped). The structural parameters are $d=0.37 \mathrm{~mm}, s=0.62 \mathrm{~mm}$, and $t=0.4 \mathrm{~mm}$. The doped Si changes from metallic to dielectric with decreasing temperature. Figure 8 shows the temperature dependence of the transmission spectrum. The transmission spectrum at 200 K shows a resonant peak at 0.48 THz and the height of this peak


Figure 7: Transmission spectra for polarization components (a) parallel and (b) perpendicular to that of the incident wave. (c) and (d) are spectra for $h=0.24$ and 1.00 mm , respectively.


Figure 8: Temperature dependence of the transmission spectrum of hole arrays made of doped Si normalized by the hole area..
decreases with decreasing temperature. This means that the resonant transmission due to the SPP disappears with freezing out of the carriers. This experimental fact demonstrates the role of the SPP in the enhanced transmission. The drastic increase of the transmittance at very low temperatures is due to complete freezing out of the carriers, which makes the sample the dielectric photonic crystal slab.

## 6. Summary

Transmission properties of single- and double-layer metal hole arrays (MHAs) have been investigated by using the THz-time domain spectroscopy. From the dependence of the transmission properties on the incident angle, total number of holes, thickness of thin dielectric film on the MHA surfaces, carrier density for the Si hole array etc., the enhanced transmission is attributed to the SPP-like mode in the Pendry's sense. Resonant polarization conversion has been found for the incident angle slightly deviated from the normal and the mechanism is also attributed to the SPP excitation. For the double-layer MHA, anomalous transmission and polarization conversion are found and explained by the near-field coupling of the SPPs on the two MHA surfaces.

The properties of the MHAs presented in this paper can be used to develop a new group of optical components from the microwave to optical regions such as filters and polarization converters. By introducing anisotropy and chirality to each hole, further new properties can be expected for the MHAs.

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# Parallel Computing of Electromagnetic Field Based on Domain Decomposition Method 

Tianwei LI , Jiangjun RUAN and Daochun HUANG<br>Wuhan University, China


#### Abstract

The computing velocity and memory storage of Single PC are often limited in large-scale electromagnetic simulation by finite element method (FEM), parallel processing is an important means to overcome such problems. The domain decomposition method (DDM) which decomposes the domain by nodes dominating and suits for parallel computing was illustrated first in this paper; A 2D electrostatic model was built and decomposed by the DDM; And the FEM linear system of equations was solved by using parallel CG method on the distributed parallel system composed of 6 PCs, the effective speed up reaching $97.5 \%$ was satisfying. Especially for large-scale simulation which consists of more than millions of freedoms, the parallel processing reduces computing time and increases the computing velocity greatly, it's the base on which large-scale 3D electromagnetic parallel computing.


## 1. Introduction

FEM is one of the most effective and widespread numerical methods in the electromagnetic field analysis, the coefficient matrix of the system of equation formed through FEM is sparse, symmetric and positive definite, and it is easy to resolve. However, with the precision requirement of electromagnetic calculation increasing of engineering design and scientific research, the memory storage and calculation speed for large linear equation system formed by FEM are becoming bottleneck problem. Thus, to seek proper computing mode and solution algorithm to increase memory storage and reduce solution time is an important subject matter to FEM.

Large-scale parallel computation has become a brand-new method for the science research and engineering technology, it may reduce the analog computation time greatly for most complicated analysis, and it has the potential to expand the computing capability from single processor to multiple processors.

The CPU time approximately contains two parts in solving electromagnetic field problem by FEM, one is finite element matrix formation time and one is finite element system of equations solution time. Among them, the system of equations solution time occupies over $80 \%$. Therefore, it is significant to study highly effective parallel algorithm for improving parallel efficiency.

In analyzing large-scale electromagnetic field problem by FEM, the system of equations' formation will consume large computing time due to the memory limitation of single computer in computing elemental stiffness matrix and assembling total stiffness matrix, and solving the system of equation is slow due to the limited storage. DDM is a kind of large granularity preconditioning algorithm, it combines flexible domain division and balancing load assignment strategy, forming and solving distributed system of equations on parallel computer. It overcomes the limitation of storage and speed of single computer.
In this paper, the principle of DDM suited for parallel computing is expounded in section 2, parallelized conjugate gradient method is discussed in section 3 and a 2D electrostatic instance is carried in section4 on parallel computers which consists of 6 PCs connected by 1000 M Ethernet. At last a conclusion is drew.

## 2. Domain decomposition method

According to the "divide and conquer" theory of DDM, the domain $\Omega$ is divided by p sub-domains ( $\Omega_{1}, \Omega_{2}, \ldots, \Omega_{\mathrm{p}}$ ),
$\Omega_{i} \bigcap_{i \neq j} \Omega_{j}=\varnothing$, the number of nodes of each domain is approximately equal. Each sub-domain is assigned to one processor and
calculated simultaneously. Data exchanging between neighboring sub-domains in each step of iteration during solving process is carried out by calling the function MPI_Send and MPI_Recv in MPI library conveniently.

### 2.1 Initialize matrix stucture

After the nodes in a sub-domain were assigned to one processor, they are classified into three types, internal, border and external. Three kinds of nodes are depicted in Tab.1, $\mathrm{N}_{\mathrm{i}}, \mathrm{N}_{\mathrm{b}}$ and $\mathrm{N}_{\mathrm{e}}$ are the number of internal, border and external nodes respectively.

Tab. 1 Three types of nodes

| node | node vector | local serial numbers |
| :---: | :---: | :---: |
| internal | need update | $0 \rightarrow \mathrm{~N}_{\mathrm{i}}-1$ |
| border | need update | $\mathrm{N}_{\mathrm{i}} \rightarrow \mathrm{N}_{\mathrm{i}}+\mathrm{N}_{\mathrm{b}}-1$ |
| external | no update | $\mathrm{N}_{\mathrm{i}}+\mathrm{N}_{\mathrm{b}} \rightarrow \mathrm{N}_{\mathrm{i}}+\mathrm{N}_{\mathrm{b}}+\mathrm{N}_{\mathrm{e}}-1$ |


(a)

(b)

Fig. 1 Organization of vector components
For Proc0 in Fig. 1 (a), node 0 is internal; node 1 and 2 are border; node 3, 4 and 5 are external. $N_{i}=1, N_{b}=2, N_{e}=3$. Each node corresponds one row of the matrix, nodes 0,1 and 2 in Proc 0 correspond three rows, and the nonzero columns number are as follow:

| row | nonzero columns |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 1 | 2 |  |  |  |
| 1 | 1 | 0 | 2 | 5 |  |  |
| 2 | 2 | 0 | 1 | 5 | 3 | 4 |

Further, there are 6 nodes in Fig. 1 (b), a matrix of $6 \times 6$ can be formed due to each node corresponding to one row. The distributed matrix structure is as follow. A $3 \times 6$ matrix is on Proc 0 , a $1 \times 6$ matrix is on Proc 1 and a $2 \times 6$ matrix is on Proc 2 .

|  | row | nonzero columns |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| proc0 | 0 | 0 | 1 | 3 | 4 |  |  |
|  | 1 | 1 | 0 | 3 |  |  |  |
|  | 3 | 3 | 1 | 0 | 4 | 2 | 5 |
| proc1 | 4 | 4 | 0 | 3 | 2 |  |  |
| proc2 | 2 | 2 | 4 | 3 | 5 |  |  |
|  | 5 | 5 | 3 | 2 |  |  |  |

For Proc $0: N_{i}=1, N_{b}=2, N_{e}=3$;For Proc1: $N_{i}=0, N_{b}=1, N_{e}=3$; For Proc2: $N_{i}=1, N_{b}=1, N_{e}=2$.
For distributed parallel computing, the global serial numbers of rows and nonzero columns were transformed into local ones. After transformation, the result is as follows:

|  | row | nonzero columns |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| proc0 | 0 | 0 | 1 | 3 | 4 |  |  |
|  | 1 | 1 | 0 | 2 |  |  |  |
| proc1 | 2 | 2 | 1 | 0 | 4 | 3 | 5 |
| proc2 | 0 | 0 | 1 | 3 | 2 |  |  |
|  | 0 | 0 | 3 | 2 | 1 |  |  |
|  | 1 | 1 | 2 | 0 |  |  |  |

### 2.2 Calculate matrix components

With regard to the distributed matrix data structure above, the component values of each nonzero column can be calculated through FEM according to the node coordinates and element structure information included in the processor. The component values then were assigned to the corresponding positions to form distributed sub-matrix $A_{i}$, which will make preparations for parallel computing.

## 3. Parallelized CG method

Using parallelized CG method to solve linear equation:

$$
\begin{equation*}
\mathrm{Ax}=\mathrm{b} \tag{1}
\end{equation*}
$$

The iteration formula is $\mathrm{x}_{\mathrm{k}+1}=\mathrm{x}_{\mathrm{k}}+\mathrm{a}_{\mathrm{k}} \mathrm{p}_{\mathrm{k}}$, when the residual $\left\|X_{k+1}-x_{k}\right\|_{2}$ is smaller than a supposed standard $\mathrm{r}_{0}$, then $\mathrm{x}_{\mathrm{k}}$ is the solution. CG method converges quickly, requires relative small memory, but when condition number cond(A) of matrix $A$ is bigger than $10^{2}$, the CG iteration is extremely slow. So, to incompletely Cholesky factorize matrix A , then $\mathrm{A} \approx \mathrm{CC}^{\mathrm{T}}$. C is sparse the same as A, does not need extra memory. Transforms linear equation (1) into $\left[C^{-1} A\left(C^{T}\right)^{-1}\right] C^{T} x=C^{-1} b$, thus, the condition number of matrix $A$ can be improved, the iteration speed will enhance greatly. The flowchart of parallelized ICCG method is in Fig.2.


Fig. 2 Flow chart of parallelized CG method

## 4. Result and discussion

### 4.1 Distributed parallel environment

A parallel system connected with 6 high performance PCs through 1000M Ethernet was in Fig.3, and the configuration of PCs are in Tab.2.

Tab. 2 PC cluster configuration

| Processor | Pentium4 3.0 GHz |
| :--- | :--- |
| Memory | $1 \mathrm{G} \quad(\mathrm{p} 02 \mathrm{G})$ |
| Network | 1000M Ethernet |
| OS | Fedora core 2 |
| MPI | Mpich 1.2.6 |



Fig. 3 Distributed parallel computer

### 4.2 Instance

A fixed capacity, $\varepsilon=8.85 \times 10 \mathrm{e}-12 \mathrm{~F} / \mathrm{m}$, the potential is 0 V inside, and 1 V outside. The aim is to simulate the potential distribution in the domain $\Omega$.The partial differential equation is:

$$
\begin{equation*}
-\nabla \cdot(\varepsilon * \nabla \mathrm{u})=\mathrm{eq} \quad \text { in } \Omega \quad(\mathrm{eq}=0.0) \tag{2}
\end{equation*}
$$



Fig. 4 2D FE model of fixed capacity



Fig. 5 Partitioned domains of the FE model


Fig. 6 Time elapsed of medium and large-scale parallel computation


Fig. 7 Efficiency speedup ratios


Fig. 8 Computation time comparison between quadrilateral and triangle mesh ( 1043000 nodes)

The domain $\Omega$ is divided into five parts in Fig.5, calculated by six processors, and the result is in Fig.4.Fig. 6 and Fig. 7 show that, as the numerical computation scale growing, parallel computation time grows, the efficiency speedup increases gradually; when the computation scale is invariable, the parallel computing time decreases as the number of processors increasing; Fig. 8 shows that, the computing time of triangle element mesh is smaller than quadrilateral element mesh for large-scale computation; the parallel system gets satisfied speedup, and it has good extension.

## 5. Conclusion

A parallel hardware and software environment was put up in this paper, and a 2D electrostatic problem was simulated in parallel based on it. The results showed that DDM combined with ICCG fitted for parallel computation well, the computation speed increased greatly. Satisfied speedup ratios and parallel efficiency obtained, and it has the capability to simulate large-scale 3D electromagnetic problem in parallel quickly and precisely.

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# Novel Phase Noise Reduction and Harmonic Improvement Methods of an Oscillator Utilizing a CCPW EBG Structure 

Cheol Gyu Hwang, and Noh Hoon Myung<br>Department of Electrical Engineering and Computer Science<br>Korea Advanced Institute of Science and Technology (KAIST), Republic of Korea


#### Abstract

This paper presents new phase noise reduction and harmonics improvement methods of a microwave oscillator utilizing a corrugated coplanar waveguide (CCPW) electromagnetic bandgap (EBG) structure. Utilizing the inherent high-Q and unique spurious-free characteristics of a CCPW structure, we achieved a phase noise reduction of 8 dB at 1 MHz offset and an increased second harmonic suppression of 10.16 dB when compared to those of a conventional CPW oscillator without the CCPW structure.


## 1. Introduction

In the last several yeas, the EBG structure, which has its origin in optics, has been successfully applied to various microwave components such as power amplifiers [1], filters [2], and antennas [3]. There has also been strong interest in application of this structure in microwave oscillator applications. In this regard, several novel oscillator circuits utilizing the advantages of the EBG structure have been proposed, where planar integration of the chip is the primary concern. H.W. Liu et al. reported an efficiency improved VCO using a defected ground structure (DGS) as a harmonic tuner of the oscillator circuit at the expense of additional chip area [4]. Y.T. Lee et al. showed the phase noise enhancement of an oscillator using a DGS structure as a resonator component of a conventional oscillator circuit [5]. They used a DGS as a harmonic suppressor or a novel phase noise reduction component of a conventional oscillator circuit.

However, DGS based oscillators inherently have the disadvantages of requiring an additional air gap between the perforated backside metal plane and a metallic shielding enclosure package [6],[7]. These problems in turn lead to reliability problems and difficulty in miniaturization of the chip. Also, the inherently required five or six periods of the unit cell in the implementation of such a structure may lead to a size problem. Additionally, the extra processing of the backside metal plane increases the complexity in a fully monolithic application of the chip.

In this paper, a novel compact EBG based oscillator based on CCPW technology is presented for the first time. The CCPW structure, originally suggested as a low pass filter, is modified to the microwave resonator component of the oscillator circuit maintaining its advantages of uniplanar structure and compactness. The higher quality factor of the structure relative to that of the conventional CPW resonator plays a role of phase noise reduction of the oscillator circuit. In addition, the unique harmonic characteristic of the CCPW plays a role of harmonic suppression of the oscillator circuit, which leads to enhancement in DC-AC power efficiency of the circuit.

## 2. Design of a CCPW Resonant Cell



Fig. 1. Corrugated CPW EBG structure.

Recently, a one-quarter wavelength deep high impedance CCPW structure [8] was proposed as a planar version of Sievenpiper's high-impedance surface originally proposed in [9]. As can be seen in Fig. 1, the CCPW structure consists of a center strip separated by a narrow gap from two ground planes and a numerous high impedance slots running down into the ground planes of the structure. The width of the slot is much shorter than the wavelength and the depth of the slot is one-quarter wavelength. This one-quarter wavelength of each slot transforms the zero impedance of the ground plane to infinite, and forbids the propagation of transverse magnetic surface waves along the CPW line. Consequently, a deep stop-band corresponding to the bandgap of any other EBG structure is generated. Its inherent uniplanar characteristic and compactness in size solve the aforementioned problems of conventional DGS based oscillators and affords greater possibility of monolithic application of EBG structures.


Fig. 2. Simulated and measured S-parameter of the CCPW EBG resonant cell.
Fig. 2 shows the simulated and measured S-parameter of the CCPW structure fabricated on a RT/Duroid 6010 substrate having a dielectric constant of 10.2 and thickness of 25 mil . The widths of the line and the gap of the 50 ohm CPW transmission line were calculated as 0.9 mm and 0.55 mm , respectively. The depth of the slot was set to 5 mm , which corresponds to one quarter wavelength at the offset frequency of 6 GHz .

In 1969, K. Kurokawa derived following equation for the frequency spectra of phase perturbation $\delta \phi(\omega)$ of oscillating signal that is directly applicable to the phase noise estimation.

$$
\begin{align*}
|\delta \phi(\omega)|^{2}= & \frac{2|e|^{2}}{\omega^{2} A_{0}^{2}} \frac{\omega^{2}\left|Z_{c}{ }^{\prime}\left(\omega_{0}\right)\right|^{2}+A_{0}{ }^{2}\left[\left(\frac{\partial R_{d}}{\partial A}\right)^{2}+\left(\frac{\partial X_{d}}{\partial A}\right)^{2}\right]}{\omega^{2}\left|Z_{c}{ }^{\prime}\left(\omega_{0}\right)\right|^{4}+A_{0}{ }^{2} S^{2}}  \tag{1}\\
& , \text { where } S=\frac{\partial R_{d}}{\partial A} X_{c}{ }^{\prime}(\omega)-\frac{\partial X_{d}}{\partial A} R_{c}{ }^{\prime}(\omega)
\end{align*}
$$

In his oscillator model, the active device for negative resistance generation was represented by $Z_{d}=R_{d}+j X_{d}$ and the impedance of the resonator was represented by $Z_{c}(\omega)=R_{c}+j X_{c}$. The entire circuit was expressed as a series connection of $Z_{d}$ and $Z_{c}(\omega)$ where $e(t)$ represents noise that may be present.

From this equation, we can notice that the reduction of phase noise can be achieved by increasing the magnitude of $Z_{c}{ }^{\prime}\left(\omega_{0}\right)$, which means the drastic impedance variation of a resonator with respect to frequency at the point of resonance.

In our CCPW resonator circuit, the cutoff frequency of 6 GHz near the resonator application frequency of 5.5 GHz causes the input impedance of the CCPW structure to vary rapidly from $50 \Omega$ as the frequency deviates from the resonant frequency, which means the drastic increase of $Z_{c}{ }^{\prime}\left(\omega_{0}\right)$ in equation (1). This increase of $Z_{c}{ }^{\prime}\left(\omega_{0}\right)$ directly leads to the reduction of the frequency spectra of phase perturbation $\delta \phi(\omega)$ and finally contributes to the reduction of phase noise of an oscillator circuit. The resonance frequency and subsequent oscillation frequency can be easily tuned by changing the depth of the CCPW structure.

Another advantage of the CCPW resonator is its harmonic tuning characteristic. The harmonics of the
circuit can be suppressed by controlling the reflection phase of terminating resonance, as suggested in [11]. The length between the CCPW structure and the transistor was tuned to 9.09 mm in order to negatively feedback the second harmonic component signal of the oscillator circuit. In addition to this methodology, we utilized the inherent absence of any nth-harmonic in the $\mathrm{S}(1,1)$ characteristic of the 5.5 GHz CCPW resonator so as to reduce the harmonics of the final oscillator circuit. From this suppression of harmonics, an increase in the fundamental output power and dc-ac power efficiency can be achieved.

The total size of the CCPW EBG structure was only $12 \mathrm{~mm} \times 9.5 \mathrm{~mm}$, which corresponds to $0.57 \lambda_{\mathrm{g}} \times$ $0.45 \lambda_{\mathrm{g}}$, where $\lambda_{\mathrm{g}}$ is the wavelength of resonance frequency.

## 3. Design of Oscillators



Fig. 3. (a) Layout and (b) photograph of the oscillator with CCPW resonant cell.

Fig. 3 shows the layout and fabricated result of a 5.5 GHz oscillator circuit. Negative resistance to compensate for the loss in the resonator was generated using a short stub in the source terminal of the transistor, which can be easily fabricated in CPW technology. Output matching stubs were tuned to meet the small signal oscillation condition, and the designed CCPW cell was implemented as a fundamental frequency selection component of an oscillator circuit. For comparison, a conventional CPW oscillator without the CCPW resonator structure was also designed and fabricated. The other components, i.e. except the CCPW structure, were set to be identical including the transistor, an Agilent ATF-36077 pHEMT. The fabrication processes of the oscillators were extremely simple without any via-hole process, pattern on the backside metal layer, or any lumped element soldering process.

## 4. Measurement Results



Fig. 4. Measured output spectrums of the CCPW based-oscillator.(a) Fundamental output power spectrum. (b) Harmonic characteristic.

Fig. 4 shows the photograph of the measured fundamental output spectrum and harmonic performance of the fabricated CCPW oscillator. The oscillator exhibits a measured oscillation frequency of 5.41 GHz with a measured peak output of 3.50 dBm at a bias condition of $\mathrm{V}_{\mathrm{ds}}=1.5 \mathrm{~V}, \mathrm{I}_{\mathrm{ds}}=10 \mathrm{~mA}$, and $\mathrm{V}_{\mathrm{gs}}=-0.2 \mathrm{~V}$. The second and third harmonic suppressions were measured as -42.67 dB , and -27.00 dB , respectively. And, the phase noise is measured as $-115.3 \mathrm{dBc} / \mathrm{Hz}$ at an offset of 1 MHz .

(a)

Fig. 5. Measured output spectrums of the reference oscillator without the CCPW structure.(a) Fundamental output power spectrum. (b) Harmonic characteristic.

In Fig 5, for comparison, we show the measured fundamental output spectrum and harmonic performance of the reference CPW oscillator with the CCPW in Fig. 3 replaced by a conventional CPW line without any corrugation. The output power of the reference oscillator oscillating at 5.58 GHz was measured as 0.381 dBm with only a 32.51 dB rejection of the second harmonic and phase noise of $-107 \mathrm{dBc} / \mathrm{Hz}$ at 1 MHz offset at the same bias conditions of $\mathrm{V}_{\mathrm{ds}}=1.5 \mathrm{~V}, \mathrm{I}_{\mathrm{ds}}=10 \mathrm{~mA}$, and $\mathrm{V}_{\mathrm{gs}}=-0.2 \mathrm{~V}$.

These results constitute a 10.16 dB reduction in second harmonic suppression and resulting DC to AC power efficiency improvement of $7.2 \%$ of the newly developed CCPW based-oscillator when compared to those of a conventional CPW oscillator without the CCPW structure. The phase noise improvement due to the higher phase slope of the CCPW-based oscillator is measured as 8 dB at 1 MHz offset from the carrier frequency.

## 5. Conclusion

In this paper, a novel oscillator that incorporates a uniplanar CCPW EBG structure as a resonator component
of the conventional CPW oscillator circuit was presented. The introduction of the CCPW EBG structure was verified to be effective in reducing the phase noise and enhancing the harmonic performance and dc-ac power efficiency of the oscillator circuit in a very small chip size increment. The small size and uniplanar structure characteristic of the circuit can be easily applied to MMIC applications of the circuit while avoiding the drawbacks of the conventional DGS based EBG oscillators.

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# A Novel Compact CPW Bandpass Filter with Super-wide Stopband Suppression 

Shry-Sann Liao, Pou-Tou Sun, Chin-Yen Tao, Hsien-Wen Liu, Chia-Yi Chien RF\&MW Circuits Design Laboratory, Department of Communication Engineering, Feng-Chia University, 100, Wen-Hua Rd., Taichung, Taiwan 407, R.O.C.


#### Abstract

A novel miniaturized CPW bandpass filter with super-wide stopband suppression was proposed in this study. The folded and bended techniques have been considered. The advantages of this filters reveals a super-wide stopband suppression. The measured result of the $\mathbf{S}_{\mathbf{2 1}}$ is less than -20 dB up to 7.5 GH . The dimension of this novel bandpass filter is only $\mathbf{2 0 \%}$ of the circuit area compared to the conventional design. It is quite useful in the MICs and MMICs applications.


## Index Terms - Compact-size, coplanar-waveguide (CPW) structures, bandpass filter.

## 1. Introduction

The advantages of the coplanar waveguide (CPW) structures in the design of microwave and millimeter wave circuits are ease in series and shunt connections, no via hole, insensitive to the substrate thickness, and low dispersion effect. Recently, in the RF front-end of a modern communication system, bandpass filters with wide stopband and high selectivity are usually to enhance the overall circuit performance. There were some methods to design a bandpass filter with supper-wide stopband suppression.

In [1], it used the corrugated structure to equalize the phase velocities of the two eigen-models in the propagation direction, and the designs bandpass filter had a wide upper stopband with satisfactory attenuation levels. [2] used parallel-coupled stepped impedance resonators (SIR) to design bandpass filters with an optimal rejection bandwidth. And the filters with SIRs of lower impedance ratios were found to have higher spurious resonant frequency and better rejection levels at $2 f_{0}$, twice the passband frequency. The design was based on a theory that the even-mode and odd-mode phase velocities of suspended coupled microstrips can be equalized on a substrate with a proper suspension height. This property was applied to design the coupled stages of a parallel-coupled line filter so that the spurious response at $2 f_{0}$ can be completely suppressed [3]. There are some advantages, like high selectivity, wide stop-band and low insertion loss ( $<3 \mathrm{db}$ ) by comparing with above architectures each others. Disadvantage was to use substrate with high cost to achieve circuits, and the circuit areas were too large.

Up to now, BPFs with low cost, small size and lightweight characteristic are the fundamental requirements for the components of communication system. In this report, we proposed a CPW bandpass filter occupied not only small circuit area, but also had super-wide stopband suppression. It is very useful in MICs and MMICs applications.

## 2. Design Description

In general, a series inductor was represented by a $\lambda / 8$ short-end series stub. At the same length, a shunt open-end stub was equivalent to a shunt capacitor [4]. Ignoring $R_{T}$ in the series resonant circuits,
the transition band and the selectivity of the filter were influenced by the different values of inductor and capacitor. Traditionally, a bandpass filter was designed by using a series short-end stub and two shunt open-end stubs. However, this filter always exist the bad selectivity, and the transition band is not sharp enough. Adding a novel bandstop filter can overcome this problem [5]-[6]. All the structures obviously occupy too many circuit dimensions. Therefore, bending and folding techniques have been considered to reduce the circuit area further.

The final circuit and its corresponding equivalent circuit are shown in Fig. 1(a)-(b), respectively. The length of the stub line which is shown in Fig. 1(a) is summarized in table I(a), and the values of the capacitors and inductors which are shown in Fig. 1(b) is summarized in table I(b). The transmission zero is controlled by the total length $4 \mathrm{~d} 1+4 \mathrm{~g} 1+\mathrm{w} 2$. The second and third harmonic frequencies can be suppressed by adjusting the shunt capacitor $\mathrm{C}^{\prime}$.


Fig. 1 (a) A novel compact-size CPW bandpass filter with super-wide stopband suppression. (b) The equivalent circuit of the (a).

Table. I (a) Parameters of filter. (b) Values of the equivalent circuit.

| $\omega_{1}$ | $\omega_{2}$ | $\omega_{3}$ | $\omega_{4}$ | $\omega_{5}$ | $\omega_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 mm | 0.25 mm | 1.5 mm | 1 mm | 1.5 mm | 0.75 mm |
| $\omega_{7}$ | $g_{1}$ | $g_{2}$ | $d_{1}$ | $d_{2}$ | $d_{3}$ |
| 2.875 mm | 0.25 mm | 0.5 mm | 4 mm | 1.75 mm | 0.5 mm |

(a)

| $L$ | $L^{\prime}$ | $C$ | $C^{\prime}$ | $R_{T}$ | $R_{T}{ }^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 26.4 nH | 11.5 nH | 0.18 pF | 0.46 pF | $8 \Omega$ | $2.3 \Omega$ |

(b)

## 3. Simulation and Measured Results

This new structure is applied to design a reduced-size CPW bandpass filter, and that is proposed and examined. The implemented circuits in this report were fabricated on the FR4 substrate $\left(\varepsilon_{\mathrm{r}}=\right.$ $4.7 \mathrm{~mm}, \tan \delta=0.022$, thickness $=0.8 \mathrm{~mm}$, and metal thickness $=0.02 \mathrm{~mm}$ ). A full-wave Sonnet $\boldsymbol{e m}$ simulator was used for all simulations, and an Agilent 8510C Vector Network Analyzer (VNA) was used for all measurements. The simulated and measured results are shown in Fig. 2(a)-(b). The simulated and measured results are summarized in Table II. It reveals that $S_{21}$ is less than -20 dB up to 7.5 GHz .


Fig. 2. (a) The simulated result of the novel compact CPW bandpass filter at 2.4 GHz with super-wide stopband suppression. (b) The measured result of the novel compact CPW bandpass filter at 2.4 GHz with super-wide stopband suppression.

Table. II. Summarized the measured and simulated results.

|  | Center <br> Frequence <br> $(\mathrm{GHz})$ | Bandwidth <br> $(\mathrm{MHz})$ | $\mathrm{S} 21(\mathrm{~dB})$ | $\mathrm{S} 11(\mathrm{~dB})$ | Stopband <br> Suppression <br> $(3.1 \sim 6 \mathrm{GHz})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Measurement | 2.4 | 310 | -2.86 | -16.08 | Yes |
| Circuit Size |  |  |  |  |  |
| $\left(\mathrm{mm}^{2}\right)$ |  |  |  |  |  |

## 4. Conclusions

In this study, we have proposed a novel miniaturized CPW bandpass filter with super-wide stopband suppression. We are successful using the concept of frequency combiner with high Q factor which reveals supper wide harmonic suppression at least two and half times passband frequency. The proposed filter has the performances better than that of the conventional $\lambda / 4$ bandpass filter, and the dimension of this novel bandpass filter is only $20 \%$ of the circuit area compared to the conventional design. The element size of the component can easily be fabricated by using standard printed-circuit etching processes. It is quite useful in the wireless communication systems.

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# Intelligent Use of the Non Uniform Transmission Lines To Design Active and Passive Microwave Circuits 

M. BOUSSALEM ${ }^{1,2}$, H.GAHA $^{2}$, F. CHOUBANI ${ }^{1}$, J. DAVID ${ }^{2}$, R. CRAMPAGNE ${ }^{2}$<br>${ }^{1}$ 6'TEL - SUP'COM -TUNISIA<br>${ }^{2}$ LEN7 - ENSEEIHT - FRANCE


#### Abstract

In this paper, we have used the non uniform transmission lines (NUTL) to design the output matching network of a non linear power amplifier. The immediate result, we have noticed, is that harmonics are sharply reduced by such use of NUTL. On the other hand, we have applied cascaded NUTL to design microstrip filters in order to attenuate drastically harmonics in outbands. Hence, we have optimised the profiles of the different elements of non uniform filter in such way that each element removes harmonics of its adjacent elements.


## 1. Introduction

To improve the performance of microwave circuits and particularly the non linear active or passive circuits, we have suggested the use of non uniform transmission lines due to their frequency behavior. This fundamental property allows the elimination of undesirable frequencies within the spectrum of interest.

The analysis of such structures is given by a numerical calculation program based on the work of Hill[3], which consists of determining the general solution of the propagation distribution equation of the electric and magnetic field and deducing the accurate model of the transmission line. Therefore, several non uniform transmission lines with various profiles (linear, exponential, and hyperbolic) have been analyzed. Their contribution to control, reduce and eliminate the harmonic frequency, generated by the non linearity of some active and passive microwave circuits has been experimentally validated.

## 2. Analysis of non uniform transmission lines

In most propagation problems occurring in non uniform structures, the propagation equation can be put, handling some transformations, in the form of a Hill's equation without a first derivative term [1].

$$
\begin{equation*}
\frac{d^{2} U(\xi)}{d \xi^{2}}+g(\xi) U(\xi)=0 \tag{1}
\end{equation*}
$$

$\mathrm{U}(\xi)$, represents a voltage or one component of electric or magnetic filed, $\mathrm{g}(\xi)$ describes the non uniformity profile and $\xi$ denotes the longitudinal coordinate.
According to the Floquet theorem, the general solution $U(\xi)$ is a combination of two linearly independent particular solutions $U_{1}(\xi)$ and $U_{2}(\xi)$ written as:

$$
\begin{equation*}
U_{1}(\xi)=e^{\mu l \xi} \cdot u_{1}(\xi) \tag{2}
\end{equation*}
$$

$$
\begin{align*}
& U_{2}(\xi)=e^{\mu 2 \xi} \cdot u_{2}(\xi)  \tag{3}\\
& U(\xi)=A \cdot U_{1}(\xi)+B \cdot U_{2}(\xi) \tag{4}
\end{align*}
$$

A and B are determined by the boundary conditions, $\mu_{1}$ and $\mu_{2=-} \mu_{1}$ are the Floquet exponents. $u_{1}(\xi), u_{2}(\xi)$ are $\pi$-periodical functions expressed by infinite sums of this form:

$$
\begin{equation*}
u_{1}(\xi)=\sum_{-\infty}^{+\infty} C_{1, n} e^{i n n^{i n g}} \quad \text { and } \quad u_{2}(\xi)=\sum_{-\infty}^{+\infty} c_{2, n \cdot} e^{i n e^{i n \xi}} \tag{5}
\end{equation*}
$$

Where $\mathrm{C}_{1, \mathrm{~N}}$ and $\mathrm{C}_{2, \mathrm{~N}}$ are the coefficients of Fourier series expansion of $\mathrm{u}_{1}(\xi)$ and $\mathrm{u}_{2}(\xi)$, respectively.

The above equation (eq.1) can be solved in a systematic fashion by:
a) First expanding $g(\xi)$ in Fourier series :

$$
\begin{equation*}
g(\xi)=\sum_{-\infty}^{+\infty} \theta_{n} e^{j 2 n \xi} \tag{6}
\end{equation*}
$$

b) Second, truncating the infinite set of linear and inhomogeneous equations to solve for Floquet' exponents.
c) Finally, expressing the general solution in terms of calculated coefficients and exponents as follows:

$$
\begin{equation*}
U(\xi)=A \cdot e^{\mu_{1}, \xi^{+N}} \sum_{-N} C_{1, n} e^{i 2 n \xi}+B \cdot e^{\mu_{2} \xi} \sum_{-N}^{+N} C_{2, n} e^{j 2 n \xi} \tag{7}
\end{equation*}
$$

For that, the $\mathrm{g}(\xi)$ expansion combined with a particular solution $\left(U_{1}(\xi)\right.$ or $\left.U_{2}(\xi)\right)$ are inserted in equation (1) to obtain the resulting infinite set of equations:

$$
\begin{equation*}
(\mu+j 2 n)^{2} \cdot C_{n}+\sum_{p=-\infty}^{p=\infty} \theta_{n-p} \cdot C_{p}=0 \quad, n \in Z \tag{8}
\end{equation*}
$$

It is noteworthy that, Fourier coefficients $\theta_{\mathrm{n}}$ of $\mathrm{g}(\xi)$ decay rapidly to zero, allowing hence the truncation of this series to a finite and low number of harmonics ensuring sufficient precision. According to H. Pointcarré and of L. Ince investigations [5], the determinant of the truncated system converges and may be written in the closed-form expression:

$$
\begin{equation*}
\Delta(\mu)=\frac{\left(e^{\pi \mu}-e^{\pi \mu_{1}}\right)\left(e^{\pi \mu}-e^{-\pi \mu_{1}}\right)}{\left(e^{\pi \mu}-e^{\pi \xi_{1}}\right)\left(e^{\pi \mu}-e^{-\pi \xi_{1}}\right)} \quad \text { With: } \quad \xi_{1}=j \sqrt{\theta_{0}} \tag{9}
\end{equation*}
$$

Floquet exponents, solutions of this equation, are found iteratively by canceling the determinant $\Delta\left(\mu_{1,2}\right)=0$, while the $\mathrm{C}_{\mathrm{i}, \mathrm{n}}$ are assumed different from zero.

Different NUTLs with several profiles have been analyzed using the Hill's method. Their behavior in terms of reflection and transmission coefficients has been observed over a wide band of frequency. The effects of geometrical shapes have been assessed [7]. Results obtained of simple lines and exponential ones optimized to resonate at a fundamental frequency equal to 1 GHz (Figure 1) are shown in Figure 2.


Figure 1: Profiles of simple and non uniform transmission lines


Figure 2: Frequency behaviour of simple and non uniform transmission lines
Actually, the non uniform transmission lines have a frequency behaviour which strictly depends upon their forms and their profiles of non homogeneity. While the transmission structures resonate in a regular multiple of fundamental frequencies, the non uniform lines resonate on frequencies which are different from integer multiples of fundamental.

## 3. Applications of Non Uniform Transmission Lines

### 3.1 Design of matching networks

Figure 3 depicts a power amplifier example where input and output matching structures typically uniform, are replaced by non uniform transmission lines with appropriate profiles.


Figure 3: Enhanced matching networks of a power amplifier.

The two circuits configurations are absolutely the same except. In the first case (A) transmission lines are simple and uniform whereas in the second one, transmission lines and stubs are chosen non uniform with appropriate profiles. The aim of using non uniform lines in this scheme is to reduce the amplitudes of harmonics at a convenient level while keeping the same behaviour at the fundamental frequency. Of course, this feature must not mismatch the power amplifier at the input nor at the output.

### 3.2 Design of a rejection filter

In a Stop band filter designed by simple uniform lines (Figure 4), we have replaced open circuit stubs with non uniform transmission lines resonating at the same fundamental frequency, but obviously at different harmonics (Figures5).


Figure 4: Stop Band filter designed by simple lines


Figure 5: Stop Band filter designed by non uniform stubs.


Figure 6: Transmission Coefficient $\left(\mathrm{S}_{21}\right)$ of a Stop Band filter.

Figure 6 depicts Transmission coefficient of both filters with simple and non uniform stubs after optimization of dimensions $\left(W_{1}, W_{2}, W_{3}, W_{4}\right.$ and $\left.L 1, L_{2}, L_{3}, L_{4}\right)$. With such choice, it is possible to achieve a better harmonic suppression in the rejection band while the response elsewhere is practically unaltered.

## 4. Conclusion

The analysis of non uniform transmission lines (NUTLs) using Hill's equation is achieved using an efficient iterative method based on Floquet' exponents determination. Once voltages and currents are defined over each point x along the transmission structure, S-parameters and other pertinent features can be easily derived.

The NUTLs have a frequency behavior which depends on their geometric profiles. This fundamental property was used for harmonic control in active and passive microwave circuits. Applications for matching as well as filtering structures have exhibited attractive results and showed good agreement with experience.

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# Laboratory Evidence of Electric Reconnection Model by a Universal Electric-Cusp Type Plasma Reactor and Possible Applications to Diamond and New Material Production 

H. Kikuchi<br>Institute for Environmental Electromagnetics<br>3-8-18, Komagome, Toshima-ku, Tokyo 170, Japan


#### Abstract

Successful results of laboratory experiment of novel electric discharge by a universal electric-cusp type plasma reactor under the atmospheric pressure are presented for the first time. This has also led to direct laboratory evidence of electric cusp-mirror and reconnection model based on electrohydrodynamics (EHD) developed recently by the author, apart from another evidence obtained from field experiment during winter thunderstorms in a coastal region of the Sea of Japan in 1985-89 which showed that the stroke points fall most likely in an electric cusp region. The universal electric-cusp type plasma reactor is composed of positive and negative spherical lead electrodes with a diameter of 15 mm and an interval of 5 cm suspended horizontally 2.75 cm high above a copper plane on which an object (semispherical lead with a diameter of 12 mm ) is placed. Typical electrode voltages for discharges were $\pm 22 \sim 27 \mathrm{KV}$ and discharge currents were $100 \sim 700 \mu \mathrm{~A}$. It is shown that electric discharge from both electrodes to object on the copper plane occurred only when it is placed inside an electric cusp and that as soon as the object goes outside a cusp region, it turned to discharge between positive and negative electrodes only (no discharge to object). In addition, it has been observed during electric discharge that dust grains are moving or EHD wind flows inside the device and dust collection by an object and the electrodes took place. This is thought to be due to electric forces exerted on induced or polarized charges of dust grains. In particular, dust collection by an object seems to be much stronger than the electrodes, indicating possible application to gas cleaning as an electric precipitator and plasma processing. Moreover, it has been observed that sharp discharge shots to object may cause its oxidization, indicating that for instance, some seeds of diamond as an object might lead to possible diamond production.


## 1. Introduction

It is now recognized that there are three kinds of merging-reconnection phenomena, fluid vortex, magnetic, and electric reconnection in the regime of hydrodynamics (HD), magnetohydrodynamics (MHD), and electrohydrodynamics (EHD), respectively. While fluid vortex and magnetic reconnection are known, in particular the latter is so familiar to plasma physicists as well as geo-astro-physicists and has extensively been investigated for the past four decades on the basis of MHD [1], there have never appeared any ideas on an electric version of field line merging reconnection surprisingly, except the author's report [2]. This does not mean no existence or importance of electric reconnection, but simply means no attention of scientists to that so far. Analogously, electric reconnection concept should be evolved on the basis of EHD for unconventional plasmas including dusty and dirty plasmas and/or aerosols, that has been developed recently.

When an object and/or dust grain is placed in an electric cusp formed by an electric quadrupole, electric field line merging towards the object or grain from the four poles and subsequent electric reconnection occur and, if the background gas pressure is beyond the breakdown threshold, are followed by electric discharge. Such a laboratory experiment was performed by using a universal electric-cusp type plasma rector [3] for the first time. This paper presents successful results that a catastrophe from zero to very high electric field by placing an object in an electric cusp and consequent object-related electric reconnection fol- lowed by electric discharge occurs, and indicates its possible applications to diamond and new material production. Specifically, when an object (semispherical lead) is placed at the electric cusp center on a copper plane, electric discharge from both electrodes to an object only occurred as a result of object-related electric reconnection. When an object is shifted 1 cm left from the cusp center on a copper plane, electric discharge from both electrodes to an object still occurred as a result of object-related
electric reconnection. This indicates that the object is still inside a cusp region. When an object is placed under the positive or negative electrode on a copper plane, it turned to discharge between positive and negative electrodes only. Discharge from electrode(s) to the object never occurred, indicating that the object is no longer in a cusp region. These observations are just laboratory evidence of the electric cusp-mirror and reconnection model proposed more than a decade ago. In addition, it has been observed during electric discharge that dust grains are moving or EHD wind flows inside the device and dust collection by object and electrodes took place. This is thought to be due to electric forces exerted on induced or polarized charges of dust grains. In particular, dust collection by the object seems to be much stronger than the electrodes. Sharp discharge to the object placed in an cusp region appeared to oxidize it in contrast to conventional uniform RF discharge for thin diamond diaphragm formation. When some seeds of diamond or new material are placed in an cusp region on a copper plane, one might expect riper one. Along this line, a trial is in progress.

## 2. Observations of Novel Electric Discharge by a Universal Electric-Cusp Type Plasma Reactor

Novel electric discharge under the atmospheric pressure has been observed by a universa electric-cusp type plasma reactor for the first time. This has also led to direct laboratory evidence of electric cusp-mirror and reconnection model based on EHD developed recently by the author. The universal electric-cusp type plasma reactor is composed of positive and negative spherical lead electrodes with a diameter of 15 mm and an interval of 5 cm suspended horizontally 2.75 cm high above a copper plane on which an object (semispherical lead with a diameter of 12 mm ) is placed as shown in Figs. $1 \sim 6$.
2.1 Case of object placed at the cusp center


Fig. 1.A universal electric-cusp type plasma reactor in DC operation. Diameter of lead electrodes is 15 mm . Semispherical lead placed at the cusp center (diameter: 12 mm ). Back and both sides of the reactor are coved with a black paper to avoid unnecessary reflected light.


Fig.2. Electric discharge from positive and negative electrodes to an object in the case of Fig.1. The voltages of both electrodes are $\pm 22 \mathrm{KV}$. Above strong light is a real discharge, while cm (refer to Fig.1).
2.2 Case of object shifted left 1 cm from the cusp centre (Others are the same as Fig.1)


Fig.3. Case of object shifted left 1 cm from the cusp center. Others are the same as Fig.1.


Fig.4. Electric discharge from positive and negative electrodes to an object shifted left 1 cm from the cusp center on a copper plane. The voltages of both electrodes are $\pm 20.5 \mathrm{KV}$. Above strong light is a real discharge, while lower weak light is its image and discharge current was about $350 \mu \mathrm{~A}$. The object, height of both electrodes and their interval are the same as Fig. 1 (refer to Fig.3).

### 2.3 Case of object placed under the positive electrode (Others are the same as Fig.1)



Fig.5. Case of object placed under the positive electrode (Others are the same as Fig.1).


Fig.6. Electric discharge between positive and negative electrodes for the case when an object is placed on a copper plane just under the positive electrode. The voltages of both electrodes are +21 KV and -20.5 KV , respectively. Above strong light is a real discharge, while lower weak light is its image and discharge current was about $400 \mu \mathrm{~A}$. The object, height of both electrodes, and their interval are the same as Fig. 1 (refer to Fig.5).

### 2.4 Observational results

The above observations conclude that electric discharge from both electrodes to object on the copper plane occurs only when it is placed inside an electric cusp and that as soon as the object goes outside a cusp region, it turns to discharge between positive and negative electrodes only (no discharge to object). These observations are just laboratory evidence of the electric cuspmirror and reconnection model predicted more than a decade ago.

## 3. Summary of Electric Cusp-Mirror and Reconnection Model

Electric discharge observations described in the preceding section are well explained by the theory of electric cusp-mirror and reconnection that are summarized hereafter.

### 3.1. Electric cusp

Electric cusp is an electrically neutral point, line or sheet, typically formed by an electric quadrupole or electric dipole above a metallic ground, across which electric field reversal occurs and which is a bifurcation point with a marginal stability as well as a saddle point in terms of electric potential and field, respectively, as schematically illustrated in Figs.6~7. At the same time, the quadrupole constitutes electric mirrors.
When an uncharged or charged dust grain, conducting or dielectric is placed in the cusp center, electric field line merging toward the grain occurs from the four poles, inducing or polarizing electric charges on its surface or in its volume, negative or positive facing positive or negative poles. respectively.


Fig.6. Electric cusp (shaded area) formed by an electric


Fig.7. Electric potential and field profile at an electric cusp quadrupole (left) or electric dipole above a metallic ground (right)
3.2 Object- and/or dust-related electric reconnection and electric discharge

When an object and/or dust is placed in an electric cusp, field merging towards it occurs, and the electric fields around it become very high. If the background gas pressure is beyond the breakdown threshold, there occurs a local surface discharge and ionization around the object
and/or dust, initiating critical ionization flows or EHD shocks towards each cloud in the form of streamer and leader, resulting in discharge channel formation, and being followed by an eventual main discharge or a return stroke in collisional or atmospheric gases, though rarely retaining a plasma layer only around the grain when critical ionization flows are forced to stop. Consequently, some of the electrostatic energy accumulated in the environment is converted into ionization and flow energy. In many cases of gas discharge, both electric reconnection and critical velocity effects are thought to be involved jointly. In fact, a number of new discharge and ionization phenomena which are difficult to understand otherwise, could be explained by such joint effects that are thought to be new aspects of discharge and ionization physics.

## 4. Applications

There seems to be a variety of applications of a universal electric-cusp type plasma reactor:
(1) basic studies of EHD, discharge or ionization physics;
(2) laboratory simulation of atmospheric and space electricity phenomena including natural and triggered lightning and tomadic thunderstorms;
(3) pollution control and gas cleaning for environments, industrial applications to plasma processing and new material production, in particular diamond production.

### 4.1 Diamond production

Sharp electric discharge shots to an object and its oxidization indicates that some seeds of diamond as an object might lead to possible riper diamond production and a trial along this line is in progress.

## 5. Conclusion

Successful results of first laboratory experiment by a universal electric-cusp type plasma reactor with positive and negative spherical lead electrodes and an object (semispherical lead) placed on a copper plane are summarized as follows:
(1) When an object is placed at the electric cusp centre on a copper plane, electric discharge from both electrodes to an object only occurred as a result of object-related electric reconnection.
(2) When an object is shifted 1 cm left from the cusp center on a copper plane, electric discharge from both electrodes to the object still occurred like the above case as a result of object-related electric reconmection. This indicates that the object still inside a cusp region.
(3) When an object is placed under the positive electrode on a copper plane, it turned to electric discharge between positive and negative electrode only. Discharge from electrode(s) to the object never occurred, indicating that the object is no longer in a cusp region.
(4) Above observations (1) (3) are just laboratory evidence of the electric cusp-mirror and reconnection model proposed more than a decade ago.
(5) In addition, it has been observed during electric discharge that dust grains are moving or EHD wind flows inside the device and dust collection by an object and the electrodes took place. This is thought to be due to electric forces exerted on induced or polarized charges of dust grains. In particular, dust collection by an object seems to be much stronger than the electrodes.
(6) These observations indicate possible application to pollution control and gas cleaning as an electric precipitator and industrial applications to plasma processing and new material production.
(7) In particular, sharp electric discharge shots to object and its oxidization indicates that some seeds of diamond as an object might lead to possible riper diamond production and a trial along this line is in progress.

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# Similarities and Differences among Parametrically Amplifying Traveling-Wave Antennas, Traveling-Wave Tubes, and Esaki Diodes 

H. Kikuchi<br>Institute for Environmental Electromagnetics<br>3-8-18, Komagome, Toshima-ku, Tokyo 170, Japan


#### Abstract

Parametrically amplifying traveling-wave antennas invented by the present author are somewhat analogous to traveling-wave tubes and Esaki diodes in function and characteristics, though there are essential differences. It is shown in this paper that similarities and differences among parametrically amplifying traveling-wave antennas, traveling-wave tubes, and Esaki diodes are elucidated in detail. Parametrical amplification of traveling-wave antennas corresponds to amplification of traveling wave tubes and the incident waves along the wire(s) play a role of electron beams. There are, however, essential differences between them. The role of electron beams in traveling wave tubes is an amplification only when an external microwave is passing through a helical circuit. In contrast, there are twofold roles of an incident wave in present traveling antemas. One is to induce a current wave along the wire. The other is to amplify its induced current wave at the same time. Further, there is a difference that the coupling to a helical circuit of electron beams is capacitive for traveling-wave tubes but the coupling to the wire(s) of incident waves is inductive for present antennas. While the present antennas are expressed by an equivalent active distributed parameter line whose shunt conductance G is negative in the transition region, the resistance in an equivalent lamped circuit of Esaki diodes is also negative. In this respect, there is some correspondence or analogy is seen between them, though those negative effects are different. The negative shunt conductance of the present antennas leads to new results that the attenuation constant of the induced current wave decreases with increasing frequency and the wave-mode becomes a fast wave, while the negative resistance of Esaki diodes causes current decrease with voltage increase as a result of quantum-mechanical tumnel effects.


## 1. Introduction


$k_{2}$
Fig.1. Wire above ground. $k_{1}, k_{2}$ :wave number


Fig.2. Attenuation characteristics of a copper wire above ground: theory and experiment ( $h:$ height of wire $=7.5 \mathrm{~m}, a$ : wire radius $=$ $7.5 \mathrm{~mm}, \sigma_{2}=$ earth conductivity $=10^{-2} \mathrm{Sm}^{-1}$ )


FRECUENCY (MAIZ)

Fig. 3. Attenuation characteristics of a polyethylenecoated copper wire ( $a=1.15 \mathrm{~mm} ; a^{\prime}=4.2 \mathrm{~mm} ; \varepsilon_{1}=$ $2.3 \varepsilon_{0}, \tan \delta_{1}=3 \times 10^{-4}, \sigma_{\mathrm{c}}=5.8 \times 10^{7} \mathrm{Sm}^{-1}, \mu_{2}=$ $\mu_{c}=\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} \mathrm{m}^{-1}$ ) in the air ( $\varepsilon_{1}=\varepsilon_{0}=$ $8.854 \times 10^{-12} \mathrm{~F} \mathrm{~m}^{-1}, \mu_{1}=\mu_{0}, \sigma_{1}=0$ ) above ground. The first approximation indicated in a dashed curve almost coincides with a final solution, so that their differences are unreadable on the plot. Case when $h$ $=60 \mathrm{~cm}, \sigma_{2}=8 \times 10^{-2} \mathrm{Sm}^{-1}, \varepsilon_{2}=5 c_{0}$.

The first wave reception by wire(s) above ground as the wave antenna for long wave communications beyond 3 km in wavelength was performed by Beverage in 1923 at long Island in the stuburb of New York for a transmitting wave from England [1]. Its propagation characteristics are described by Carson-Pollaczek's theory $[2,3]$ that considers conduction current only regarding the earth as a conductor. It has been shown, however, that the increase of attenuation constant along a wire above ground tended to decrease with increasing frequency, specifically around 10 MHz at field experiments carried out by using a test line (Fig. 1) at Electrotechnical Laboratories [4]. To explain this result, a new theory was soon developed, taking into consideration the dielectric constant of the earth, approaching that of Sommerfeld ssurface-wave line beyond that frequency [7] and found that the wave became a fast wave in this transition region and was always a slow wave in other frequency region $[5,6,18 \sim 17]$. The existence of maxmum and minimum attenuation was also demonstrated for a dielectric-coated conductor [10, 13], indicating that a ground return circuit transfers to Goubau's surface-wave line [11, 12] as shown in Fig.3. Comparing Figs. $2-3$, maximum and minimum attenuation, consequently the transition region is shifted to higher frequencies when the height of wire becomes lower. The first approximation indicated in a dashed curve almost coincides with a final solution, so that their differences are unreadable on the plot, while some examples of phase characteristics are shown in Fig. 4 [13].

(a) Cinge when $e_{2}=c_{0}$

Fig.4. Phase characteristics of a polyethylene-coated copper wire.

## 2. From Electromagnetic Theory to Generalized Transmission Line Theory on Propagation along a Wire above Ground for an Entire Range of Frequencies with and without External Eectromagnetic Fields. Its Equivalent Distributed

Parameter Line Representation (Fig.5 [5~16])


Fig.5. Equivalent circuit representation of active distributefparameter line above a lossy ground in an external electric field. The current, charge, and field are supposed to contain the common factor $\exp (-\Gamma z+j \omega t)$, which, however, will be omitted for convenience in the formulas The propagation constant $I=\alpha+j \beta(\alpha=$ attenuation constant, $\beta=$ phase constant or $v=$ phase velocity to be determined are expressed as [13]

$$
\begin{gather*}
\alpha=\frac{\omega}{c} \frac{\frac{\delta_{a}}{4 a}+P-P^{r}}{\log _{s} \frac{2 h-a}{a}}\left(\delta_{c}=\sqrt{\frac{2}{\omega \sigma_{d} e_{c}}}\right)  \tag{1}\\
\beta=\left(\begin{array}{c}
\frac{\omega}{c} \\
1+\frac{\delta_{c}}{4 a}+Q-Q^{\prime} \\
\log _{a} \frac{2 h-a}{a}
\end{array}\right)  \tag{4}\\
\cdots  \tag{3}\\
y_{p}=c\left(\frac{\frac{\delta_{c}}{4 a}+Q-Q^{\prime}}{\log _{a} \frac{2 h-a}{a}}\right)
\end{gather*}
$$

(2)


$$
\begin{align*}
& Z=Z_{i}+Z_{e}=\frac{(1+j)}{2 \sqrt{2} \pi a} \sqrt{\frac{\alpha \mu_{c}}{\sigma_{c}}}+\frac{j \omega \mu_{1}}{2 \pi}\left\{\log _{a} \frac{2 j}{\gamma \lambda_{1} a}-\right. \\
& \left.-j \frac{\pi}{2} H_{0}^{(0)}\left(\lambda_{1} 2 h_{1}-a\right)+2(Q-j p)\right\}, \\
& Y=\frac{j 2 \pi D \varepsilon_{1_{1}}}{\log _{f} \frac{2 j}{\gamma \gamma_{1} a}-j \frac{\pi}{2} H H^{\mathrm{D}}\left(\hat{l}_{1} 2 \hbar-a\right)+2\left(Q^{\prime}-j P^{\prime}\right)}  \tag{5}\\
& Q_{0}-j P_{0}=p \int_{0}^{\infty} \frac{e^{-(2 h-a)} \sqrt{u^{2}-x^{2}}}{p \sqrt{u^{2}-\lambda_{2}^{2}}+q \sqrt{u^{2}-\lambda_{1}^{2}}} d u= \\
& =\int_{0}^{\infty} \frac{e^{-(2 t-c)} \sqrt{u^{2}-x^{2}}}{\sqrt{u^{2}-\lambda_{2}^{2}}+\eta \sqrt{u^{2}-\lambda_{1}^{2}}} d \boldsymbol{d}= \\
& =\left\{\begin{array}{l}
Q-j P: p=q=1 \text { or } \eta=1, \\
Q^{\prime}-j P^{\prime} ; p=k_{1}^{2}, q=k_{2}^{2} \text { or } \eta=\frac{k_{2}^{2}}{k_{1}^{2}} .
\end{array}\right. \tag{6}
\end{align*}
$$

Fig.6. Graphs of the function $P, Q, P^{\prime}$, and $Q^{\prime} \cdot *^{-3} .5=2 \times 10^{-3}[5,13]$ (notice $P^{\prime}>0, G$ $<0$ in the transition region)

## 3. Similarities and Differences between Parametrically Amplifying Traveling-

Wave Antennas (PATA) and Esaki Diodes (ED)


Fig.7. V-I characteristics of Esaki diodes (c). (a) Negative bias; (b) Negative resistance state for positive bias [19].

Shunt conductance $G<0$ of PATA in the transition region corresponds to $R<0$ of an equivalent lumped circuit of ED.
Phase Characteristics of bare wire above ground in the transition region (fast wave: phase velocity $\geq$ velocity of light) and its correspondence to quantum-mechanical tunnel effect of ED. Quantum-mechanically, wave functions of electrons tend to penetrate into a barrier and when the barrier is thin enough, electrons can run through the barrier with some probability and arrive at the opposite electrode [19].

## 4. Experimental Evidence of a Fast Wave of PATA



Fig.8. Experimental evidence of a fast wave in the transition region for an iron tube (H: height; $a=12.5 \mathrm{~mm}$ : radius of iron pipe; base plate: polyethylene: $\sigma=0.27 \mathrm{Sm}^{-1}$ for $2 \mathrm{GHz} ; \varepsilon_{\mathrm{s}}=2.2, \sigma=0.40 \mathrm{~S} \mathrm{~m}^{-1}$ for 3 GHz [左~17]

## 5. Conclusion: Similarities and Differences

 among PATA, TWT, and EDThe paper is summarized and concluded by the following Table [17~19].

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PARAMETRICALLY AMPLIFYING TRAVELING-ANTENNAS (PATA), TRAVELING-WAVE TUBES (TWT) AND ESAKI DIODES (ED) (SIMILARITIES AND DIFFERENCES)

|  | PATA | TWT | ED |
| :---: | :---: | :---: | :---: |
| STRUCTURE | WIRE(S) ABOVE GROUND (NATURAL OR MAN-MADE) | HELICAL CIRCUTT AND ELECTRON BEAM GUN | ABRUPT, HEAVILY DOPED P-N JUNCTION |
| FUNCTION | WAVE RECEPTION WITH AMPLIFICATION AND HIGH DIRECTIVITY | WAVE AMPLIFICATION | QUANTUM-MECHANICAL TUNNEL EFFECT <br> TRIGGERING PULSE FORMING SAMPLING MEMORY, <br> ANALOG-TO-DIGITAL CONVERSION, LOGIC, OSCILLATIONS, TUNNEL PHOTO DIODES |
| FREQUENCY RANGE | $100 \mathrm{MHZ}-10 \mathrm{GHZ}$ | $1 \mathrm{GHZ}-10 \mathrm{GHZ}$ | RESONANT TUNNELING DIODES EHF UP TO-700 GHZ |
| MEANS OF AMPLYFIED | BY INCIDENT SKY WAVE | BY ELECTRON BEAM |  |
| WAVES TO BE AMPLIFIED | INDUCED WAVE BY INCIDENT SKY WAVE | MICROWAVE TRANSMITTED TO HELICAL CIRCUIT |  |
| ROLES OF SKY WAVE OR ELECTRON BEAM | EIGEN-WAVE INDUCTION AND ITS AMPLIFICATION (TWO ROLES) | AMPLIFICATION ONLY |  |
| WAVE COUPLING | MAGNETIC COUPLING | CAPACITIVE COUPLING |  |
| WAVE MODE | FAST HYBRID WAVE | SLOW TM WAVE |  |
| EQUIVALENT CIRCUTT | ACTIVE DISTRIBUTED PARAMETER LINE | ACTINE DISTRIBUTED PARAMETER LINE | ACTIVE LUMPED CIRCUIT |
| CIRCUIT PARAMETER | SHUNT CONDUCTANCE $G<0$ | SHUNT CONDCTGANCE $G<0$ | SERIES RESISTANCE $\mathrm{R}<0$ |
| $\frac{\text { PARAMETER }}{\text { CHARACTERISTICS FIGURE }}$ | LETTER N-LIKE SHAPE OF ATTENUATION CHARACTERISTICS |  | LETTER N-LIKE SHAPE OF CURRENT-VOLTAGE CHARACTERISTICS |

# Spectral considerations about 2D paraxial and non-paraxial beam solutions 

R. Mahillo-Isla, M. J. González-Morales, C. Dehesa-Martínez<br>University of Valladolid, Spain


#### Abstract

This work presents the spectral equivalence for the two steps given in the real propagation space to obtain Gaussian beams from Complex beams. The Plane Wave Spectrum of both solutions is obtained and a discussion about their differences is also presented.


## Introduction

The fundamental steps to get Gaussian beams from Complex beams are known for both 2D and 3 D scenarios [1], [2] and [3]. This work deals with the 2D problem. Such a work has been extensively investigated in the real space [3], [2] and [4]. Two approximations are made so as to obtain 2D Gaussian beams from Complex beams: the complex High Frequency-Far Field (HF-FF) condition, and the paraxial condition [4]. In [4], both approximations are deeply studied giving the zones where Gaussian beams are close to Complex beams under certain error criteria. Thus, with those given, the paraxial regions are defined.

Up to the authors' knowledge, the relations, in a spectral domain, between Complex beams and Gaussian beams have not been discussed until now, although spectral techniques have been used in scattering problems where Complex beams are involved [5]. So, the translation of both approximations in the spectral domain can achieve a deeper understanding of the solutions to problems in which Complex beams are used as the illumination.

There are also scattering problems dealing with Gaussian beams where their spectral decomposition is made [6]. Solutions obtained by these means are also discussed.

The paper organization is the following. The spectral decomposition of 2D Complex beams is obtained first. Second, 2D Gaussian beams are treated in order to get their Plane Wave Spectrum (PWS). After doing this, the differences found between their spectra are discussed. Finally, the conclusions of this work and the future research lines are summarized.

## 1. Plane Wave Spectrum of Complex beams

2D complex beam solutions are obtained by displacing a point source located at $\left(x_{s}, y_{s}\right)$ a complex quantity in the form:

$$
\left.\begin{array}{r}
\mathbf{x}_{\mathbf{s}}=x_{s}-i b \cos \phi  \tag{1}\\
\mathbf{y}_{\mathbf{s}}=y_{s}-i b \sin \phi
\end{array}\right\}, \quad b>0
$$

The parameter $b$ is related to the beam width at its waist and $\phi$ is the angle between the beam and $x$ axes (see Fig. 1). The fields obtained by means of this source translation are the same as the ones calculated for the line current source along z axis carrying $I_{0}$, replacing the real source coordinates with the complex ones [3]. Assuming that time harmonic dependence is $e^{-i \omega t}$, the electric field has only one component, $\vec{E}=\hat{z} E_{\mathrm{CB}}[3]$ and [4]:

$$
\begin{equation*}
E_{\mathrm{CB}}=-I_{0} \frac{\omega \mu_{0}}{4} H_{0}^{(1)}\left(k_{0} \mathbf{R}\right), \quad \mathbf{R}=\sqrt{\left(x-\mathbf{x}_{\mathbf{s}}\right)^{2}+\left(y-\mathbf{y}_{\mathbf{s}}\right)^{2}} . \tag{2}
\end{equation*}
$$

Two quantities arise in this radiation problem: the complex distance, $\mathbf{R}$, and the complex angle, $\boldsymbol{\theta}$. These complex polar coordinates are related to the complex cartesian coordinates in the usual way,

$$
\left.\begin{array}{l}
\mathbf{x}=x-\mathbf{x}_{\mathbf{s}}  \tag{3}\\
\mathbf{y}=y-\mathbf{y}_{\mathbf{s}}
\end{array}\right\} \rightarrow\left\{\begin{array}{l}
\mathbf{x}=\mathbf{R} \cos \boldsymbol{\theta} \\
\mathbf{y}=\mathbf{R} \sin \boldsymbol{\theta}
\end{array}\right.
$$

The definition of the PWS $P(\alpha)$ for 2D waves can be found in [7]. For a 2D TE polarized wave, the electric field can be expressed in the following terms,

$$
\begin{equation*}
E(r, \theta)=I_{0} \int_{C} P(\alpha) e^{i k_{0} r \cos (\theta-\alpha)} d \alpha \tag{4}
\end{equation*}
$$

being $(r, \theta)$ polar coordinates $(x=r \cos \theta, y=r \sin \theta), P(\alpha)$ the PWS of the $\vec{E}$ field, and $C$ the path in Fig. 2 for the contour integral. The aim of this section is to find the PWS of a Complex beam expressed in (2).


Fig. 1: Complex beam radiation problem.


Fig. 2: Contour $C$ used in equation (4).

For that, the integral representation of cylindrical harmonics of index $\nu$ is needed [8]:

$$
\begin{equation*}
\Psi_{\nu}(r, \theta)=H_{\nu}^{(1)}\left(k_{0} r\right) e^{i \nu \theta}=\frac{e^{-i \nu \frac{\pi}{2}}}{\pi} \int_{C} e^{i k_{0} r \cos (\theta-\alpha)+i \nu \alpha} d \alpha \tag{5}
\end{equation*}
$$

Accordingly, the Hankel function in (2), in which $(\mathbf{R}, \boldsymbol{\theta})$ replace $(r, \theta)$, is represented by

$$
\begin{equation*}
H_{0}^{(1)}\left(k_{0} \mathbf{R}\right)=\frac{1}{\pi} \int_{C} e^{i k_{0} \mathbf{R} \cos (\boldsymbol{\theta}-\alpha)} d \alpha \tag{6}
\end{equation*}
$$

After some calculation, one finds that:

$$
\begin{equation*}
H_{0}^{(1)}\left(k_{0} \mathbf{R}\right)=\frac{1}{\pi} \int_{C} e^{-i k_{0}\left(\mathbf{x}_{\mathbf{s}} \cos \alpha+\mathbf{y}_{\mathbf{s}} \sin \alpha\right)} e^{i k_{0}(x \cos \alpha+y \sin \alpha)} d \alpha \tag{7}
\end{equation*}
$$

which can be rewritten using real polar coordinates $(r, \theta)$, as:

$$
\begin{equation*}
H_{0}^{(1)}\left(k_{0} \mathbf{R}\right)=\int_{C} \frac{1}{\pi} e^{-i k_{0}\left(\mathbf{x}_{\mathbf{s}} \cos \alpha+\mathbf{y}_{\mathbf{s}} \sin \alpha\right)} e^{i k_{0} r \cos (\theta-\alpha)} d \alpha \tag{8}
\end{equation*}
$$

By substituting this result in (2), the definition given in (4) leads to the PWS of field $E_{C B}$, which may be written in terms of real quantities by using (1), as

$$
\begin{equation*}
P_{\mathrm{CB}}(\alpha)=A e^{-i k_{0}\left(x_{s} \cos \alpha+y_{s} \sin \alpha\right)} e^{k_{0} b \cos (\alpha-\phi)}, \quad A=A=-I_{0} \frac{\omega \mu_{0}}{4 \pi} \tag{9}
\end{equation*}
$$

The term $e^{-i k_{0}\left(x_{s} \cos \alpha+y_{s} \sin \alpha\right)}$ gives account for the real displacement of the source. This term will appear also whether a line source is displaced from the origin to a real point $\left(x_{s}, y_{s}\right)$. Then, for the study of the PWS of Complex beams, without any loss of generality, both $x_{s}$ and $y_{s}$ can be taken as 0 . If a new complex angle $\Theta$ is defined as $\Theta=\boldsymbol{\theta}-\phi$, what amounts to rotate the coordinate system an angle $\phi$, the PWS is

$$
\begin{equation*}
P_{\mathrm{CB}}(\alpha)=A e^{k_{0} b \cos \alpha} \tag{10}
\end{equation*}
$$

which is the PWS of a Complex beam related to a new coordinate system $(\xi, \eta)$ adapted to the beam: the beam axis is the $\xi$ axis (Fig. 1). In the whole following discussion, the beam will be referred to the $(\xi, \eta)$ cartesian coordinate system.

### 1.1 Spectral aspects of Far Field Complex beams

At this point, it is going to be discussed the first of the two approximations applied to Complex beams in order to get Gaussian beams. The result of applying the complex (HF-FF) to Complex beams is called Far Field Complex beams. From a spectral point of view, it can be shown that Far Field Complex beams are obtained by asymptotic evaluation of (6) using the Steepest Descent Path (SDP) method. The integral in (6) has the form:

$$
\begin{equation*}
\int_{C} e^{k q(\alpha)} d \alpha \tag{11}
\end{equation*}
$$

where

$$
\begin{equation*}
q(\alpha)=i \mathbf{R} \cos (\Theta-\alpha) \tag{12}
\end{equation*}
$$

The saddle point is $\alpha_{S P}=\Theta$. It is simple as $q^{\prime \prime}(\alpha)=-q(\alpha) \neq 0$. The SDP method approximation consists in the use of a second order Taylor series approximation for $q(\alpha)$ around the saddle point,

$$
\begin{equation*}
q(\alpha) \approx i \mathbf{R}\left[1-\frac{1}{2}(\Theta-\alpha)^{2}\right] \tag{13}
\end{equation*}
$$

Since no further approximations are made, the translation onto the angular spectral domain is that for each complex angle $\Theta$, the function $q$ can be quite well approximated with its second order Taylor series. This means that $q$ is a smooth function around $\Theta$, as $\Theta$ is determined through $\tan \Theta=\frac{\eta}{\xi-i b}$.

By doing the required calculations in order to follow the SDP method, one retrieves the expression of Far Field Complex beams [3] and [4]:

$$
\begin{equation*}
E_{\mathrm{FFCB}}=-I_{0} \frac{\omega \mu_{0}}{4} \sqrt{\frac{2}{\pi}} e^{-i \frac{\pi}{4}} \frac{e^{i k_{0} \mathbf{R}}}{\sqrt{k_{0} \mathbf{R}}}, \quad\left|k_{0} \mathbf{R}\right| \gg 1 \tag{14}
\end{equation*}
$$

## 2. Plane Wave Spectrum of Gaussian beams

Gaussian beams are obtained when applying the paraxial condition to (14) [3].

$$
\begin{equation*}
E_{\mathrm{GB}}(\xi, \eta)=-I_{0} \frac{\omega \mu_{0}}{2 \sqrt{2 \pi}} e^{-i \frac{\pi}{4}} \frac{e^{k_{0} b}}{\sqrt{k_{0}(\xi-i b)}} \exp \left(\frac{-\eta^{2}}{\mathcal{W}^{2}(\xi, b)}\right) \exp \left[i k_{0}\left(\xi+\frac{\eta^{2}}{2 \mathcal{R}(\xi, b)}\right)\right] \tag{15}
\end{equation*}
$$

where $\mathcal{W}=\left[\frac{2 b}{k_{0}}\left(1+\frac{\xi^{2}}{b^{2}}\right)\right]^{1 / 2}$ is the beam width and $\mathcal{R}=\frac{b^{2}}{\xi}\left(1+\frac{\xi^{2}}{b^{2}}\right)$ is the curvature radius of the beam phase front.

Once the expression for the PWS of Complex beams has been obtained, it is mandatory to get the corresponding expression for Gaussian beams. Before doing this, it is quite convenient to remark some topics. Gaussian beams are not solutions of the 2D Helmholtz wave equation; they are solutions of the 2D paraxial wave equation, indeed. So, the definition of PWS of Gaussian beams has not sense strictly speaking, since only solutions of Helmholtz wave equation are subject of this decomposition.

Gaussian beams profile at their waists is used in order to define a expression for theier PWS. Notice that if the PWS obtained is introduced in (4) for the purpose of retrieving the field at any space point, the field obtained is a valid solution of 2D Helmholtz wave equation.

As it has been seen, to obtain the PWS of Gaussian beams requires the evaluation of is expression (15) at $\xi=0^{+}$, for instance,

$$
\begin{equation*}
\left.E_{G B}\right|_{\xi=0^{+}}=-I_{0} \frac{\omega \mu_{0}}{4} \sqrt{\frac{2}{\pi}} \frac{e^{k_{0} b}}{\sqrt{k_{0} b}} \exp \left(-\frac{\eta^{2}}{2 b / k_{0}}\right) . \tag{16}
\end{equation*}
$$

As $\left.E_{G B}\right|_{\xi=0^{+}}$is a Gaussian function of $\eta$, (16) can be expressed in the form of an inverse Fourier transform of a Gaussian function as well. In fact, the expression of the Fourier transform of $\left.E_{G B}\right|_{\xi=0^{+}}$ is:

$$
\begin{equation*}
\mathcal{F}\left\{\left.E_{G B}\right|_{\xi=0^{+}}\right\} \doteq F_{E}(\kappa)=-I_{0} \frac{\omega \mu_{0}}{2} \frac{e^{k_{0} b}}{k_{0}} \exp \left(-\frac{\kappa^{2}}{2 k_{0} / b}\right) \tag{17}
\end{equation*}
$$

so that,

$$
\begin{equation*}
\left.E_{\mathrm{GB}}\right|_{\xi=0^{+}}=\int_{-\infty}^{+\infty} \frac{1}{2 \pi} F_{E}(\kappa) e^{i z \kappa} d \kappa \tag{18}
\end{equation*}
$$

and after performing a change of variable $\kappa=k_{0} \sin \alpha$, (18) becomes:

$$
\begin{equation*}
\left.E_{\mathrm{GB}}\right|_{\xi=0^{+}}=\int_{C^{*}} \frac{k_{0}}{2 \pi} \cos \alpha F_{E}(\alpha) e^{i k_{0} \eta \sin \alpha} d \alpha \tag{19}
\end{equation*}
$$

The path $C^{*}$ is the contour $C$ in Fig. 2 displaced $\frac{\pi}{2}$ towards left. The functions involved in the integral are regular functions (there are neither poles nor branch points in the integral). Thus, $C^{*}$ can be deformed into contour $C$ without more ado. What it is wanted to find is the PWS of Gaussian beams, i.e., $P_{G B}(\alpha)$ in

$$
\begin{equation*}
E_{\mathrm{GB}}=\int_{C} P_{\mathrm{GB}}(\alpha) e^{i k_{0}(\xi \cos \alpha+\eta \sin \alpha)} d \alpha \tag{20}
\end{equation*}
$$

By identifying the evaluation of (20) at $\xi=0^{+}$with (19), $P_{\mathrm{GB}}(\alpha)$ is obtained:

$$
\begin{equation*}
P_{\mathrm{GB}}(\alpha)=A \cos \alpha e^{k_{0} b\left(1-\frac{1}{2} \sin ^{2} \alpha\right)} \tag{21}
\end{equation*}
$$

where $A=-I_{0} \frac{\omega \mu_{0}}{4 \pi}$ as in (9). In [6], a very similar procedure is made in order to get a spectral decomposition of Gaussian beams. In order to give a deeper understanding of this result, the differences observed between expressions (10) and (21) are going to be discussed.

As the trigonometric functions can be expressed in terms of coordinates, one can see that, when $\eta \ll|\xi-i b|$ (this fact is used to apply paraxial condition in the real space domain [3] and [4]), $\alpha \approx 0$. The paraxial condition, from an angular spectral point of view, consists in the use of Taylor series at $\alpha=0$ of the trigonometric functions. For the cosine function,

$$
\begin{equation*}
\cos \alpha=1-\frac{1}{2} \alpha^{2}+O\left(\alpha^{4}\right) . \tag{22}
\end{equation*}
$$

It is useful to maintain a trigonometric function so as to change into wavenumber spectral domain. So, one can use the paraxial approximation of the sine function $\left(\sin \alpha=\alpha+O\left(\alpha^{3}\right)\right)$ to substitute $\alpha$ in (22):

$$
\begin{equation*}
\cos \alpha=1-\frac{1}{2}\left[\sin ^{2} \alpha+O\left(\alpha^{6}\right)+2 \sin \alpha O\left(\alpha^{3}\right)\right]+O\left(\alpha^{4}\right)=1-\frac{1}{2} \sin ^{2} \alpha+O\left(\alpha^{4}\right) \tag{23}
\end{equation*}
$$

which explains the exponential dependence of the Gaussian beam PWS (21).
The term $\cos \alpha$ in (21) means an angular limit in the propagation, since at $\alpha= \pm \frac{\pi}{2}$ equals zero; so, it gives account of the lack of propagative behavior along the $\eta$ axis of Gaussian beams.

Going back to the discussion presented at the beginning of this section, the comparison between the solution of the 2D Helmholtz wave equation with PWS given in (21) and 2D Gaussian beam can be deeper studied in order to check the space regions where the Gaussian beam is a good approximation to the 2D Helmholtz wave equation under a certain criterion. That is to compare the field provided by a Gaussian illumination,

$$
\begin{equation*}
E_{G I}=\int_{C} P_{G B}(\alpha) e^{i k_{0} r \cos (\theta-\alpha)} d \alpha \tag{24}
\end{equation*}
$$

with the Gaussian beam expression (15).

## Conclusions

The complex HF-FF condition approximation has been revealed as the Steepest Descent Path evaluation of the Plane Wave Spectrum integral. The SDP method, in this case, means to substitute $q(\alpha)$ with (13) in the integral (6). This result checks the previous work [4], that states the regions in which the complex HF-FF condition is held. For example, if $k_{0} b \gg 1$, almost the whole space fulfills the complex HF-FF condition. But points electrically near the branch points of $\mathbf{R}, \eta= \pm b$, will never meet this condition, as $q(\alpha)$ is not regular at such points.

The paraxial condition applied in the real propagation space leads to gives conditions in the angular spectral domain. The first one is a truly angular condition at the beam axis, and the other condition defines the lack of propagative behavior along the $\eta$ axis. In a future work, the comparison between the solution of 2D Helmholtz wave equation with Gaussian illuminations and Gaussian beams with the same profile at its waist will be done.

The steps followed in order to obtain Complex beams PWS also show how translations in real propagation space are translated into phase changes in the angular spectral domain, and rotations in the real propagation space give translations in the angular spectral domain. This could be anticipated, since the PWS is closely related to the Fourier Transform, as seen when obtaining the PWS of Gaussian beams.

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# Computation of Scattering from Randomly Distributed Dielectric Circular Cylinders 

N. Nakashima and M. Tateiba<br>\{norimasa, tateiba\}@csce.kyushu-u.ac.jp<br>Faculty of Computer Science and Communication Engineering, Kyushu University<br>6-10-1 Hakozaki, Higashi-ku, Fukuoka, 812-8581 Japan


#### Abstract

A Monte Carlo simulation is done for electromagnetic (EM) wave scattering from randomly distributed 4225 cylinders. The scattered field is computed by means of the boundary element method (BEM) with our fast techniques: a multilevel fast multipole algorithm (FMA) and a generalized minimal residual (GMRES) iterative solver with two-step preconditioning (TSP). Numerical examples show the normalized power densities for scattered far and near fields for regularly and randomly distributed cylinders. We discuss the characteristics of scattered fields by random medium.


## 1 Introduction

Multiple scattering by many random particles has been theoretically studied by many researchers and applied to communication and remote sensing technology. One of authors proposed a new multiple scattering theory [1] and a method for the estimation of effective medium parameters [2]. However, there are some limits to the particles of random medium. In order to deal with random media composed of several kinds of particles in shape, size and material, we consider computational analysis as a suitable method and use a Monte Carlo simulation.

The BEM is widely used for numerically calculating EM wave scattering. In computation of EM wave scattering, the most time-consuming part rises from solving a linear system of $L$ equations derived by the BEM. Our previous works presented a multilevel FMA [3] and a GMRES method [4] with TSP [5]. They reduce both the complexities and the net quantities of computation and memory drastically.

This paper treats EM wave scattering from randomly distributed cylinders. We carry out a Monte Carlo simulation and numerically estimate the normalized average power density for the coherent component and incoherent component of scattered waves. Aforementioned fast techniques are applied to the computation of scattered near and far fields from a realization of random medium. In numerical examples, we assume a random medium containing 4225 cylinders.

## 2 Formulation

Let us consider the two-dimensional problem of EM wave scattering by $N$ infinitely long cylinders in a vacuum. The relative permittivity and permeability of the $i$ th cylinder are $\varepsilon_{r}^{(i)}$ and $\mu_{r}^{(i)}$, respectively. The wave numbers of the vacuum and $i$-th cylinder are given by $k_{0}$ and $k_{i}=k_{0} \sqrt{\varepsilon_{r}^{(i)} \mu_{r}^{(i)}}$, respectively. Each axis of the cylinders is parallel to the $z$-axis of the cylindrical coordinate system. We formulate this problem in the electrical field integral equations (EFIE) for TM wave and in the magnetic field integral equations (MFIE) for TE wave. The $z$-components of unknown fields $\psi_{z}$ and $\partial \psi_{z} / \partial n$ are given by

$$
\begin{align*}
& \psi_{z}^{\mathrm{inc}}\left(\boldsymbol{\rho}_{i}\right)=\frac{1}{2} \psi_{z}\left(\boldsymbol{\rho}_{i}\right)-\frac{1}{4 \mathrm{j}} \sum_{n=1}^{N}\left[\int_{C_{n}}\left\{\psi_{z}\left(\boldsymbol{\rho}_{n}^{\prime}\right) \frac{\partial \mathrm{H}_{0}^{(2)}\left(k_{0}\left|\boldsymbol{\rho}_{i}-\boldsymbol{\rho}_{n}^{\prime}\right|\right)}{\partial n_{n}^{\prime}}-\mathrm{H}_{0}^{(2)}\left(k_{0}\left|\boldsymbol{\rho}_{i}-\boldsymbol{\rho}_{n}^{\prime}\right|\right) \frac{\partial \psi_{z}\left(\boldsymbol{\rho}_{n}^{\prime}\right)}{\partial n_{n}^{\prime}}\right\} \mathrm{d} l_{n}^{\prime}\right] \\
& 0=\frac{1}{2} \psi_{z}\left(\boldsymbol{\rho}_{i}\right)+\frac{1}{4 \mathrm{j}} \int_{C_{i}}\left\{\psi_{z}\left(\boldsymbol{\rho}_{i}^{\prime}\right) \frac{\partial \mathrm{H}_{0}^{(2)}\left(k_{i}\left|\boldsymbol{\rho}_{i}-\boldsymbol{\rho}_{i}^{\prime}\right|\right)}{\partial n_{i}^{\prime}}-\mathrm{H}_{0}^{(2)}\left(k_{i}\left|\boldsymbol{\rho}_{i}-\boldsymbol{\rho}_{i}^{\prime}\right|\right) \alpha_{r}^{(i)} \frac{\partial \psi_{z}\left(\boldsymbol{\rho}_{i}^{\prime}\right)}{\partial n_{i}^{\prime}}\right\} \mathrm{d} l_{i}^{\prime}  \tag{1}\\
&(i=1,2, \ldots, N) . \tag{2}
\end{align*}
$$

Here, $C_{i}$ is the boundary of the $i$ th cylinder, and $\boldsymbol{\rho}_{i}$ and $\boldsymbol{\rho}_{i}^{\prime}$ are the observation and integration points on $C_{i}$, respectively. The $\mathrm{H}_{0}^{(2)}$ is the zero order Hankel function of the second kind, and $\partial / \partial n_{i}$ is the outward normal derivative on $C_{i}$. The $\alpha_{r}^{(i)}$ is a relative medium constant of the $i$ th cylinder. We fixed at $\psi_{z}=E_{z}$ and $\alpha_{r}^{(i)}=\varepsilon_{r}^{(i)}$ for TM wave and $\psi_{z}=H_{z}$ and $\alpha_{r}^{(i)}=\mu_{r}^{(i)}$ for TE wave. The $\psi_{z}^{\text {inc }}$ is an incident wave.

The integral equations can be discretized through the BEM. Dividing each boundary into $M_{i}$ boundary elements and using the point matching method, we obtain a dense linear system of $L$ equations $\mathcal{A} \boldsymbol{x}=\boldsymbol{b}$, where $L=2\left(M_{1}+M_{2} \cdots+M_{N}\right)$. The square coefficient matrix $\mathcal{A}$ is composed of four blocks of $L / 2$ order. These four blocks are further composed of subblocks:

$$
\mathcal{A}=\left[\begin{array}{cc}
\mathcal{A}_{\text {out }} & \mathcal{B}_{\text {out }}  \tag{3}\\
\mathcal{A}_{\text {in }} & \mathcal{B}_{\text {in }}
\end{array}\right]=\left[\begin{array}{ccc|ccc}
a_{11}^{\text {out }} & \cdots & a_{1 N}^{\text {out }} & b_{11}^{\text {out }} & \cdots & b_{1 N}^{\text {out }} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
a_{N 1}^{\text {out }} & \cdots & a_{N N}^{\text {out }} & b_{N 1}^{\text {out }} & \cdots & b_{N N}^{\text {out }} \\
\hline a_{11}^{\text {in }} & & 0 & b_{11}^{\text {in }} & & 0 \\
0 & \ddots & a_{N N}^{\text {in }} & 0 & \ddots & b_{N N}^{\text {in }}
\end{array}\right]
$$

Here, the notation 0 is zero matrix. The subblocks $a_{i j}, b_{i j}$ are the matrix of $M_{i}$ by $M_{j}$. The notations "out" and "in" mean outer and internal fields of cylinders, respectively. The right-hand side vector $\boldsymbol{b}$ is composed of $2 N$ subvectors:

$$
\begin{equation*}
\boldsymbol{b}=\left(\boldsymbol{b}_{1}, \boldsymbol{b}_{2}, \cdots, \boldsymbol{b}_{N}, \mathbf{0}_{1}, \mathbf{0}_{2}, \cdots, \mathbf{0}_{N}\right) \tag{4}
\end{equation*}
$$

The notation $\boldsymbol{b}_{i}$ and $\mathbf{0}_{i}$ are the subvector of $M_{i}$ order and the zero vector of $M_{i}$ order, respectively.

## 3 Computation of scattered wave

Solving the linear system by an iterative method needs much computation time and memory. As shown in equation (3), the upper half of coefficient matrix is dense and the memory complexity is $O\left(L^{2}\right)$. In solving the linear system by using an iterative method, the computational complexity is estimated at $O\left(L^{2}\right)$ per one iteration due to the product of the upper half of coefficient matrix and a vector. We apply our multilevel FMA [3] to the computation of the product and reduce the computational and memory complexities. On the other hand, the lower half of coefficient matrix is sparse and the memory complexity is $O(L)$. The computational complexity is estimated at $O(L)$ for the product of the lower half of coefficient matrix and a vector. Because both the complexities are very low, we directly compute the product.

Our multilevel FMA is similar to the Multilevel Fast Multipole Algorithm (MLFMA) [6], proposed by W. C. Chew et al., but differs in the computation stages. Our FMA is based on Graf's addition theorem while the MLFMA on the integral representation of the Bessel function. The addition theorem becomes inefficient for some computations of such a high frequency region that a cell size is larger than a certain value. Then we apply the Fast Fourier Transform (FFT) to reduce the inefficiency. The computational complexity of our algorithm is theoretically estimated at $O\left(L \log ^{2} L\right)$ and slightly larger than that of the MLFMA [7]. The memory complexity is $O(L \log L)$ both for the MLFMA and our algorithm. However, we treat a volume scattering problem in this paper, and the computational and memory complexities are estimated at $O(L)$ which is the same as the MLFMA.

Our FMA can expedite the computation of scattered far and near fields. The far field is computed at the azimuthal directions $\theta_{i}$ for the $x$-axis. The $\theta_{i}$ is given by

$$
\begin{equation*}
\theta_{i}=\frac{2 \pi}{P} \quad(i=0,1, \cdots, P-1), \quad P=2^{c}>4 p+1>2^{c-1} \tag{5}
\end{equation*}
$$

where the $c$ is a natural number and $p$ is the truncation number. In the near field computation, we determine an analytical region and divide it into the same size $4^{l}$ cells. The centers of the $4^{l}$ cells correspond to the observation. Here $l$ is related to the resolution for the image of the near field and given by $2^{l}>4 p+1>2^{l-1}$ in this paper. The truncation number $p$ depends on the size of the analytical region.

Next we consider the choice of an efficient iterative method. We can see from equation (3) that the coefficient matrix $\mathcal{A}$ is non-Hermitian. Many iterative methods for non-Hermitian have been proposed but the fastest and the most efficient iterative method is not found yet. Our previous works showed the comparison among well-known 11 iterative methods in terms of iteration number, computation time and the amount of used memory up to convergence [5][8]. It is found from some numerical results that GMRES method [4] converges fastest though the number of iteration may be more than other iterative methods. This is caused by the fact that the number of executions of matrix-vector products in an
iteration is one for GMRES method and two for many other iterative methods for non-Hermitian. The frequency of matrix-vector products up to convergence becomes smaller for GMRES method than for other methods; thus the computation time is shorter for GMRES method. Consequently we conclude that GMRES method is currently the most effective iterative method for fast computation of scattering from many cylinders.

Another advantage of GMRES method over other iterative methods is to be free from breakdowns in iterative process. When we assume non-singular coefficient matrix and ignore the round-off error, the iterative process of GMRES method converges to a numerical solution with required accuracy. However, the amount of used memory may be much more for GMRES method than for other methods because it increases with the number of iterations. To minimize memory increment, we apply efficient preconditionings to iterative process. We present TSP combined with half reduction and block Jacobi [5].

We propose the following two preconditioners:

$$
\mathcal{K}_{1}=\left[\begin{array}{cc}
-\mathcal{A}_{\text {in }}^{-1} & 0  \tag{6}\\
\mathcal{B}_{\text {in }}^{-1} & \mathcal{B}_{\text {in }}^{-1}
\end{array}\right]^{-1} \quad \text { and } \quad \mathcal{K}_{2}=-\tilde{\mathcal{A}}_{\text {out }} \mathcal{A}_{\text {in }}^{-1}+\tilde{\mathcal{B}}_{\text {out }} \mathcal{B}_{\text {in }}^{-1}
$$

Here, $\tilde{\mathcal{A}}_{\text {out }}$ and $\tilde{\mathcal{B}}_{\text {out }}$ are the block diagonal matrices composed of $a_{11}^{\text {out }}$ to $a_{N N}^{\text {out }}$ and $b_{11}^{\text {out }}$ to $b_{N N}^{\text {out }}$, respectively. Before solving the linear system $\mathcal{A} \boldsymbol{x}=\boldsymbol{b}$, we apply $\mathcal{K}_{1}$ as a right preconditioner. Then we can obtain the preconditioned linear system whose number of unknowns is reduced in half of the original one. In solving the preconditioned linear system by GMRES method, we apply a standard block Jacobi preconditioning to the iterative process. The block Jacobi preconditioner is given by $\mathcal{K}_{2}$.

The feature of TSP is to apply two preconditioner. The aim of the first preconditioner $\mathcal{K}_{1}$ is just to reduce the number of unknowns. We do not take account of the improvement of the convergence rate for GMRES method. The convergence is improved by the second preconditioner $\mathcal{K}_{2}$. We use a standard Block Jacobi preconditioner in this paper. The reduction of the number of unknowns leads to suppress the increment of memory use. However, the net computation time is not reduced because we have to compute matrix-vector product for two blocks $\mathcal{A}_{\text {out }}$ and $\mathcal{B}_{\text {out }}$ in spite of the use of $\mathcal{K}_{1}$.

## 4 Coherent and Incoherent components of scattered field

The characteristics of scattering from random medium containing many scatterers are expressed in terms of statistical quantities. Thus we prepare $N_{s}$ realizations of random medium containing $N$ cylinders. The method to generate a realization of random medium is shown in [9]. Briefly speaking, $N$ cylinders regularly placed in advance are randomly moved in order that all the cylinders may not overlap each other.

The scattered field from a random medium is constituted of the coherent component and the incoherent component. They are estimated as the statistical quantities. If the scattered fields for $N_{s}$ realizations of random medium $\operatorname{are} \psi_{z}^{\mathrm{s}(1)}, \psi_{z}^{\mathrm{s}(2)}, \ldots, \psi_{z}^{\mathrm{s}\left(N_{s}\right)}$, then the coherent component of scattered fields from the $N_{s}$ realizations $\psi_{z}^{\mathrm{co}}$ and the incoherent component of scattered field from the $i$-th realization $\psi_{z}^{\text {inco }(i)}$, respectively, are given by

$$
\begin{equation*}
<\psi_{z}^{\mathrm{co}}>=\frac{1}{N_{s}} \sum_{i=1}^{N_{s}} \psi_{z}^{\mathrm{s}(i)} \quad \text { and } \quad \psi_{z}^{\mathrm{inco}(i)}=\psi_{z}^{\mathrm{s}(i)}-\psi_{z}^{\mathrm{co}} \tag{7}
\end{equation*}
$$

The average power density of each component normalized by incident field is given by these equations:

$$
\begin{equation*}
P_{\mathrm{co}}=\left|\left\langle\psi_{z}^{\mathrm{co}}\right\rangle\right|^{2} /\left|\psi_{z}^{\mathrm{inc}}\right|^{2} \quad \text { and } \quad P_{\mathrm{inco}}=\left(\frac{1}{N_{s}} \sum_{i=1}^{N_{s}}\left|\psi_{z}^{\mathrm{inco}(i)}\right|^{2}\right) /\left|\psi_{z}^{\mathrm{inc}}\right|^{2} \tag{8}
\end{equation*}
$$

## 5 Numerical Examples

We consider scattering from regularly and randomly distributed 4225 cylinders in a square region whose fractional volume is 0.1 . Here the scattered field has only coherent component for the regular distribution because the combination of distribution of cylinders is only one. The radii, relative permittivities and
relative permiabilities of cylinders are the same for all cylinders and assumed as $k a=1.0, \varepsilon_{r}=2.0$ and $\mu_{r}=1.0$. The number of realizations of random medium $N_{s}$ is 100 . We assume TM plane wave incidence and put $l$ at 10 in the computation of scattered near field from each realization. For scattered far field, we check the energy conservation by using the optical theorem and assure ourself that it is satisfied at 95 percent. The computation is performed on Compaq Alpha 21264 with 667 MHz CPU and 4 GB main memory.

The normalized power densities of scattered far fields are shown in figure 1 (a) for regular distribution and figure 2 for random one. We can find that the peaks of the coherent component for the regular distribution are suppressed except for the forward scattering direction. The incoherent component shows the maximum and minimum peaks around the backward and forward directions, respectively.

Numerical results for near fields are shown in figure 1 (b) and figure 3. For the regular distribution, strong peaks are found around $k y=0, \pm 60, \pm 120$ in forward scattering direction: $k x<0$. These strong peaks may depend on the low-order Floquet modes. It can be found from figure 3 (a) that the regular peaks in $y$ direction are disappeared for the random distribution. The incoherent component is large in the region of $100<k x<200$ and $|k y|<100$. In this area, a peak of coherent component is also found. We imagine that the effect of the random distribution of cylinders is significant around the backward scattering direction rather than the forward one.

## 6 Conclusion

EM wave scattering from randomly distributed $N$ cylinders is considered, and the normalized average power densities for coherent and incoherent components of scattered fields are estimated by means of the Monte Carlo simulation. In the computation of scattered field, our multilevel FMA is applied to GMRES iterative solver with TSP. Numerical examples show the case of 4225 dielectric cylinders. The peaks of coherent component of scattered field are suppressed except for forward scattering direction. The incoherent component of scattered field becomes maximum and minimum around the backward and forward scattering directions, respectively. We suppose from these results that the effects of the random distribution of cylinders is significant around the backward scattering direction.

As future works, we calculate the coefficient of coherence attenuation. We will simulate scattering from random medium contained by over 10,000 cylinders with arbitrary cross section.

## Acknowledgement

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Figure 1: Normalized power densities for scattered near (right) and far (left) fields from regularly placed cylinders.


Figure 2: Normalized average power densities for the coherent (left) and incoherent (right) components of scattered far field from randomly distributed cylinders.


Figure 3: As in figure 2, but for scattered near field.

# Static stability \& plate spacing for diamagnetic levitating magnets 

Joe Nhut Ho \& Wei-Chih Wang<br>Mechanical Engineering Department<br>University Of Washington, Seattle WA 98195, USA


#### Abstract

This paper analyzes the static stability of a diamagnetic levitating rotor. Balancing forces is necessary but does not guarantee stable levitation. Restoring forces (minimizing potential energy) of a levitating magnetic around the zero force point is also required. An equivalent statement is that any small displacement from the zero force point causes the magnet to return to the zero force point. A derivation using this concept gives a stability criterion for cylindrical levitating magnets. The criterion controls the spacing of the diamagnetic plates needed to stably levitate a magnet. This was verified by experiments using regular graphite and pyrolitic graphite plates. When adjustment for plate thickness is included, theoretical stability requirements and experimental results match well.


## Introduction

For an object to stably levitate the potential energy must be a local minimum at the levitation point. According to Earnshaw's theorem no configuration of bodies interacting with $1 / r^{2}$ forces can be statically suspended [1-2]. Thus stably levitating a magnet is impossible because no local energy minimums exist. However, this theorem does not include dynamic and quantum mechanical systems thus planets can stably orbit stars and electrons in atoms can remain in their energy levels around an atomic nucleus.

Diamagnetic levitation is a combination of dynamic and quantum mechanical effects and are excluded from Earnshaw's theorem. When a magnet approaches a diamagnetic material, the repulsive force grows stronger and vice versa. The repulsion force its self is generated by quantum mechanical interaction between the magnet's field and the atoms of diamagnetic materials [3-4]. This paper will examine how a magnet is stably levitated for the system described in [5].

## Stability Analysis

The levitation points can be found by balancing forces. There two possible levitating positions shown in figure 1 , one above and below the suspending magnet. These points can be estimated with fairly good accuracy by applying the Lorentz force law and then using a multipole expansion for the suspending magnet's field. If one assumes a thin, cylindrical magnet, the levitation points can be found from this expression:
 (attractive)

Figure 1: The lower and upper stability points. The arrows show the directions of the forces the magnets exert on each other.


Figure 2: FBD for levitating magnet suspended between two horizontal diamagnetic plates
$1 \quad z= \pm\left(\frac{3 \mu_{0} M R^{2} H\left|M_{L}\right| R_{L}^{2} H_{L} \pi}{2 m g}\right)^{1 / 4}\left(+\right.$ if $M_{L}$ is negative, - if $M_{L}$ is positive $)$
where $R_{L}, M_{L}$, and $H_{L}$ are the radius, magnetization, and thickness of the levitating magnet and $R, M$, and $H$ are the radius, magnetization, and thickness of the suspending magnet. For the lower levitation point, $M_{L}$ is positive (attractive) and for the upper one $M_{L}$ is negative (repulsive).

The required diamagnetic strength and maximum plate spacing are governed by how difficult the equilibrium points are to stabilize. Stability is often found by examining potential energy directly [1, 2], however, this can be done indirectly by examining forces affecting a levitating magnet. Net conservative forces and potential energy of an object are related by [6]:
$2-\nabla E_{p}=\sum \boldsymbol{F}$
For stability, the potential energy must be minimized around the levitation point and can be expressed in terms of cylindrical coordinates by

3

$$
\text { a) } \frac{\partial^{2} E_{p}}{\partial z^{2}}>0 \text { and b) } \quad \frac{\partial^{2} E_{p}}{\partial r^{2}}>0
$$

for an axis symmetric system like this one. Expressing these requirements in terms of net forces gives
4
a) $\frac{\partial}{\partial z} \boldsymbol{\sum}<0$ and b) $\frac{\partial}{\partial r} \boldsymbol{\sum} \boldsymbol{F}<0$

Equations of 4 are more useful than 3 because forces can be calculated more easily from the Lorentz force law than potential energy. Equations 4 expresses that stability requires opposing (restoring) forces for any small displacement from the equilibrium position.

The forces acting on the levitating rotor are weight, magnetic attraction/repulsion from the suspending magnet, and stabilizing forces from the diamagnetic plates (see figure 2 for a free body diagram). One possible way to model the forces from the diamagnetic plates near the levitation point is to model them as linear springs by adding terms with the form -constant $\times$ displacement. The reasoning for this is that the potential energy well around a levitation point should be parabolic shaped (as least for a first order approximation) [1, 2]. Adding two additional term representing the effects of the diamagnetic plates gives the following force balance:

5

$$
m g \boldsymbol{e}_{z}-M_{L} \frac{\partial}{\partial r} \int_{0}^{2 \pi} \int_{z_{1}^{\prime}}^{z_{2}^{\prime}}\left(R_{L} d \theta d z \boldsymbol{e}_{\theta}\right) \times\left(B_{r} \boldsymbol{e}_{\boldsymbol{r}}+B_{z} \boldsymbol{e}_{z}\right)-\text { ด }_{z} z^{\prime} \boldsymbol{e}_{\boldsymbol{r}}-\text { ด }_{r} r \boldsymbol{e}_{\boldsymbol{r}}<0
$$

where the first term is the magnet's weight, the second is the magnetic forces, and the last two terms are from the diamagnetic plates which oppose any displacements (where $z^{\prime}$ and $r$ are displacements from the levitation point $z_{0}$ $z^{\prime}=z-z_{0}$ ). Since the suspending magnet is cylindrical, $\boldsymbol{B}$ has only has radial and vertical components. The next step is to find how large (or how "stiff") $ด_{z}$ and $ด_{r}$ need to be in order maintain stability. Ultimately this sets a maximum limit about how far away a specified diamagnetic material can be to trap a levitating magnet (given a material with magnetic susceptibility $\chi$, what is the maximum plate spacing that will still stably levitate a magnet).

First, the analysis will be done for the vertical direction. Applying 4a to 5, performing the cross product, integrating along the radial direction, moving $ด_{z}$ to the other side, and keeping only the vertical components produces:

6

$$
2 \pi R_{L} M_{L} \frac{\partial}{\partial z} \int_{z_{1}}^{z_{2}} B_{r} d z<ด_{z}
$$

where $z_{1}=z_{0}-H_{L} / 2$ and $z_{2}=z_{0}+H_{L} / 2$ are the top and bottom of the levitating magnet centered at the levitating point (see figure 1). Using a multipole expansion
$7 \quad B_{r}=\frac{3 \mu_{0} M H R^{2} z r}{4\left(r^{2}+z^{2}\right)^{5 / 2}}$
and $r=R_{L}$ gives (since levitating magnet is at center axis.

8

$$
\frac{3}{2} \mu_{0} M H R^{2} M_{L} R_{L}^{2} \pi\left[\frac{\left(z_{0}-H_{L} / 2\right)}{\left(R_{L}^{2}+\left(z_{0}-H_{L} / 2\right)^{2}\right)^{5 / 2}}-\frac{\left(z_{0}+H_{L} / 2\right)}{\left(R_{L}^{2}+\left(z_{0}+H_{L} / 2\right)^{2}\right)^{5 / 2}}\right]<ด_{z}
$$

or assuming a thin cylindrical magnet gives:
$9 \quad-\frac{3}{2} \mu_{0} M H R^{2} M_{L} R_{L}^{2} H_{L} \pi\left[\frac{R_{L}^{2}-4 z_{0}^{2}}{\left(R_{L}^{2}+z_{0}^{2}\right)^{7 / 2}}\right]<$ 日 $_{z}$
Equation 8 is easier to examine the stability of the upper and lower levitation points. For small magnets, $z_{0}$ is much larger than $R_{L}$ so for the lower levitation point the left side of equation 8 is always positive. This means the lower point is unstable unless $ด_{z}$ is greater than the positive quantity on the left side. Thus, diamagnetic plates are required for stabilization. For the upper point, however, $M_{L}$ is negative which makes the left side negative. So $ด_{z}$ could be zero and the upper point would still be negative. Thus, no diamagnetic plates are needed to vertically stabilize the upper point. Note that equation 8 can be evaluated at the levitation point $z_{0}$ to find the required diamagnetic stiffness to stabilize the lower point. This will be done in the next section to find plate spacing.

For the radial direction, applying equation 4 b to 5 , one gets:

$$
\begin{equation*}
M_{L} \frac{\partial}{\partial r} \int_{0}^{2 \pi} \int_{z_{1}^{\prime}}^{z_{2}^{\prime}}\left(R_{L} d \theta d z \boldsymbol{e}_{\theta}\right) \times\left(B_{r} \boldsymbol{e}_{\boldsymbol{r}}+B_{z} \boldsymbol{e}_{z}\right)-\text { ด }_{r} \boldsymbol{e}_{\boldsymbol{r}}<0 \tag{10}
\end{equation*}
$$

and doing the cross product and integrating the circumference produces (and ignoring the vertical component):

$$
\begin{equation*}
2 \pi R_{L} M_{L} \frac{\partial}{\partial r} \int_{z_{1}}^{z_{2}} B_{z} \boldsymbol{e}_{\boldsymbol{r}}<ด_{r} \boldsymbol{e}_{\boldsymbol{r}} \tag{11}
\end{equation*}
$$

The term $\partial B_{z} / d r$ is always negative around the vertical axis (because the $z$ component decreases away from the axis) so the integral is always negative. Continuing the analysis, using a multipole expansion for $B_{z}$ :
$12 \quad B_{z}=\frac{\mu_{0} M H R^{2}}{4}\left[\frac{2 z^{2}-r^{2}}{\left(r^{2}+z^{2}\right)^{5 / 2}}\right]$
and substituting this gives a pair of expressions
13

$$
\frac{3}{2} \mu_{0} M H R^{2} M_{L} R_{L}^{2} \pi\left[\frac{z_{0}+H_{L} / 2}{\left(R_{L}^{2}+\left(z_{0}+H_{L} / 2\right)^{2}\right)^{5 / 2}}-\frac{z_{0}-H_{L} / 2}{\left(R_{L}^{2}+\left(z_{0}-H_{L} / 2\right)^{2}\right)^{5 / 2}}\right]<\text { ด }_{r}
$$

or for small magnet approximation:
$14 \quad \cdot \frac{3}{2} \mu_{0} M H R^{2} M_{L} R_{L}^{2} H_{L} \pi\left[\frac{R_{L}^{2}-4 z_{0}^{2}}{\left(R_{L}^{2}+z_{0}\right)^{7 / 2}}\right]<ด_{r}$
Again the second expression is easier to interpret. The upper point is unstable since $M_{L}$ is negative (must be oriented the opposite direction of the suspending magnet for repulsion) and the left side is positive. A diamagnetic cylinder is required stabilize a magnet radially [1]. The lower point is stable and no diamagnetic cylinder is needed.

## Analysis for plate spacing

The minimum value for $ด_{z}$ has been found in the previous section. But the value of $ด_{z, \min }$ is not a very useful engineering tool. One would rather know (or estimate) the diamagnetic plate spacing around the levitating magnet given the susceptibility $\chi$ of the plates. This requires a model of the magnetic field distribution from the diamagnetic plates around the levitating magnet. One can use a multipole expansion and the method of images [2, 7] to find the stabilization from diamagnetic plates. As noted before, the radial field is responsible for the vertical force between two horizontal magnets. The multipole expansion for the radial field of a cylindrical image magnet is:

$$
B_{r}=\frac{3 \mu_{0} H_{L} M_{i} R^{2} z r}{4\left(r^{2}+z^{2}\right)^{5 / 2}}
$$

while setting $H=H_{L}$ and $R=R_{L}$ for the geometry of the image magnet and $M=M_{i}$ is the magnetization in the diamagnetic material. In order to find the force on the magnet from diamagnetic material placed below the magnet, one needs to apply the Lorentz force law:

15

$$
\boldsymbol{F}_{\text {diamag }}=\int \boldsymbol{M}_{L} d A \times \boldsymbol{B}=\frac{3 H_{L} M_{L} M_{i} R_{L}^{3} \mu_{0}}{4} \int_{0}^{2 \pi} \int_{z_{1}}^{z_{2}} \frac{z R_{L} d \theta d z}{\left(R_{L}^{2}+z^{2}\right)^{5 / 2}} \boldsymbol{e}_{z}=\frac{3 H_{L} M_{L} M_{i} R_{L}^{4} \mu_{0} \pi}{2} \int_{z_{1}}^{z_{2}} \frac{z d z}{\left(R_{L}^{2}+z^{2}\right)^{5 / 2}} \boldsymbol{e}_{z}
$$

Differentiating with respect to $z$ and applying the integration limits gives:

16

$$
\frac{\partial \boldsymbol{F}_{\text {diamag }}}{\partial z}=\frac{3 H_{L} M_{i} M_{L} R_{L}^{4} \mu_{0} \pi}{2} \frac{\partial}{\partial z} \int_{z_{1}}^{z_{2}} \frac{z d z}{\left(R_{L}^{2}+z^{2}\right)^{5 / 2}} \boldsymbol{e}_{z}=\frac{3 H_{L} M_{i} M_{L} R^{4} \mu_{0} \pi}{2}\left[\frac{z_{2}}{\left(R_{L}^{2}+z_{2}^{2}\right)^{5 / 2}}-\frac{z_{1}}{\left(R_{L}^{2}+z_{1}^{2}\right)^{5 / 2}}\right] \boldsymbol{e}_{z}
$$

A similar result can be found for diamagnetic material above the levitating magnet. Combining the results for both plates and using a substitution for the local coordinates in terms of plate spacing $s$ (see figure 3), $M_{i}=\chi M_{L} / 2$ for the image magnetization [7], and $H_{L}=t$ (magnet thickness), gives an equation for $ด_{z}$ :

$$
\begin{equation*}
\text { ด }_{z, \text { plates }}=-\frac{3 \chi M_{L}^{2} R_{L}^{4} H_{L} \mu_{0} \pi}{4}\left[\frac{2 s+H_{L}}{\left(R_{L}^{2}+\left(s+H_{L} / 2\right)^{2}\right)^{5 / 2}}-\frac{2 s-H_{L}}{\left(R_{L}^{2}+\left(s-H_{L} / 2\right)^{2}\right)^{5 / 2}}\right] \tag{17}
\end{equation*}
$$

This result assumes the levitating magnet is centered between the two plates (so $z^{\prime}=0$ ) as shown in figure 3 and the negative sign is from $ด_{z}$ was defined to be negative.

## Results

Experiments show that magnets can be stably levitated between two diamagnetic plates at the lower point without any diamagnetic material to constrain the magnets radially (using a diamagnetic cylinder). The experiments described here used a suspending NeFeB magnet with $H=38 \mathrm{~mm}$, and $R=19 \mathrm{~mm}$ and a levitating magnet with $R_{L}=$ 3.15 mm and $H_{L}=1.6 \mathrm{~mm}$, and $\rho=7800$ [9]. Furthermore, magnets slightly displaced radially automatically return to the center axis and self center themselves showing radial stability around the central axis.

Using the above magnet parameters, assuming a magnetization for both magnets $M=978 \mathrm{kA} / \mathrm{m}$ [9], and using the levitation point $z_{0}=13.4 \mathrm{~cm}$ (measured from experiments) gives a minimum value for stability around $ด_{z, \text { min }}>0.114 \mathrm{~N} / \mathrm{m}$. The largest spacing to stably levitate the levitating magnet is 6.0 mm using pyrolitic graphite plates 3.0 mm thick $\left(\chi \approx-4.5 \times 10^{-4}[1]\right)$ with the magnet was directly centered between the plates $\left(z^{\prime}=0\right)$. With $s=$ 6.0 mm , equation 18 gives a value of $0.136 \mathrm{~N} / \mathrm{m}$ which is greater than the minimum value. Using an adjustment from figure 8 in [5] to account for the finite thickness of the plates (values for parameters needed to use the charts: $T / H_{L}=$


$$
\begin{aligned}
& s_{b}=s+2 z^{\prime} \\
& s_{t}=s-2 z^{\prime} \\
& z_{1}=s_{b}-H_{L^{\prime}} / 2 \Rightarrow z_{1}=s+2 z^{\prime}-H_{L} / 2 \\
& z_{2}=s_{b}+H_{L} / 2 \Rightarrow z_{2}=s+2 z^{\prime}+H_{L} / 2 \\
& z_{3}=s_{t}-H_{L} / 2 \Rightarrow z_{3}=-s+2 z^{\prime}-H_{L} / 2 \\
& z_{4}=s_{t}+H_{L} / 2 \Rightarrow z_{4}=-s+2 z^{\prime}+H_{L}^{\prime} 2
\end{aligned}
$$

Note that $s_{t}$ is negative since levitating magnet is below the image

Figure 3: The geometry and variables used to find the stabilization coefficient. Vertical coordinates are positive pointing upward.
3.0/1.6 $=1.875, R_{L} / H_{L}=1.97, s / 2=z=3.0 \mathrm{~mm}$, thus $z / H_{L}=1.67$ ) gives a thickness adjustment around 0.85 . Applying this value gives $ด_{z, p l a t e}=0.115 \mathrm{~N} / \mathrm{m}, 0.8 \%$ above $ด_{z, \text { min }}$.

Using ordinary pressed graphite plates $\left(\chi \approx-1.6 \times 10^{-4}[1]\right)$ that were 10 mm thick with the same magnets gave a gap spacing of 4.4 mm . Using 4.4 mm gives a value of $0.115 \mathrm{~N} / \mathrm{m}$, the same as above. Since the ordinary graphite plates are so thick $(T / z=10 / 1.6=6.25)$ so they are practically semi infinite with an adjustment of 1.0 . Thus the experimental value for the ordinary graphite plates agrees within $0.8 \%$ as well. The differences for both cases could be due asymmetries in the experimental set up, uncertainties about magnetization and susceptibility, or the multipole approximation. Figure 6 plots values of $ด_{z}$ for both ordinary and pyrolitic graphite (without adjustment for plate thickness) as a function of plate spacing. Whenever the stability coefficient is greater than $ด_{z, \min }$ (smaller plate spacing) the magnet would always levitate easily.

## Conclusions

Criteria for stable levitation were presented and agree well with experiments, verifying that equations 1,8 (or 9) and 17 can be used together for estimating plate spacing which is necessary for designing a levitating rotor. Adding more terms to the multipole would produce more accurate results, especially for tighter plate spacing but would make the formulas inconvenient.

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Figure 4: Plot for $ด_{z}$ as a function of plate spacing for two different materials (with no adjustment factors) for the experimental set up. The dashed curve lines are where equation the multipole expansion is no longer valid because the truncated terms of the expansion become significant there. The straight dashed line is the value of $ด_{z, \text { min }}$.

# Effect of the interface separating a homogeneous medium and a photonic crystal 

R. Pierre, B. Gralak, T. Decoopman, G. Tayeb, S. Enoch and D. Maystre<br>Institut Fresnel-CNRS (UMR 6133), 13397 Marseille cedex 20, France


#### Abstract

Effective properties of photonic crystals are investigated using a new theoretical method. In the one-dimensional case, analytic expressions of permittivity and permeability of an anisotropic and homogeneous equivalent are obtained. From this analysis, we show that the effective permittivity and permeability can have very small variations in a wide range of wavevectors including propagating and evanescent waves. However, the one-dimensional configuration and its model are limited. We then chose to adapt our theory to two-dimensional structures, with a two-dimensional crystal flat lens with optimized impedance adaptation as an illustration.


## 1. Introduction

The concept of negative index materials has been introduced by V. G. Veselago [1]. J. B. Pendry remarked that such materials could permit the realization of new flat lenses with, a priori, unlimited resolution [2]. Although there is some controversy concerning negative index materials, it is clear that the amplification of the evanescent waves induced by this class of new materials will have important applications to subwavelength imaging, as well as to light trapping [3]. The challenge is now to design composite structures presenting properties as close as possible to those of negative index materials.

Periodic structures made of metallic wires or "double C resonators" possess some properties similar to the desired ones. However, the relevance of these metallic structures should decrease if the "visible domain" (when the wavelength is smaller than $1.5 \mu m$ ) is considered. Indeed, at these wavelengths, absorption takes place in metals and fabrication becomes difficult since these structures are used in the "homogenization domain" (when the dimensions of the unit cell of the periodic structure are much smaller than the wavelength). Purely dielectric periodic structures, the second kind of structures expected to mimic negative index materials, do not suffer from absorption and present interesting properties in the "resonance domain" (when the dimensions of the unit cell of the periodic structure are similar to the wavelength). Since important applications are expected in the visible domain, this communication is devoted to dielectric periodic structures.

Determination of effective properties of dielectric structures operating in the resonance domain is a difficult task. It is convenient to split the general concept of negative index into three phenomena: negative refraction, the effect of the truncation of the boundary layer, and evanescent wave amplification. Negative refraction with photonic crystals is now a well-known phenomenon: it has been predicted [4,5] and then demonstrated in several experiments [6]. On the other hand, to our knowledge, the effect of the truncation of the boundary layer has only been occasionally studied, and then mostly in an empiric manner [7-9]. Also to our knowledge, amplification of evanescent waves has not been studied in the photonic crystal context.

The new theoretical method is based on $r_{\#}$ [10], the reflection coefficient on a photonic crystal but, for the moment, a one-dimensional structure made of $p$ identical unit cells is considered. Each unit cell is in practice a stack of $m$ homogeneous layers (see figure 1). Surrounded by two homogeneous and isotropic media, this structure is illuminated by an incident electromagnetic beam field $U^{i}$. The choice has been made to consider it harmonic [with time dependence in $\exp (-i \omega t)$ ] and $s$-polarized. The response of this finite-thickness structure is the reflected beam $U_{p}^{r}$ which can be deduced from $U^{i}$ by the usual reflection coefficient $r_{p}$ associated with a plane wave.


Figure 1: Left: one-dimensional structure made of a stack of $p$ identical unit cells. Right: unit cell made of a stack of $m$ homogeneous layers with constant permittivity $\varepsilon_{j}$, constant permeability $\mu_{j}$ and thickness $h_{j}(j=1,2, \ldots, m)$.


Figure 2: Left: reflection coefficient $r_{\#}$ on a photonic crystal. Right: reflection coefficient $r_{a}$ on an anisotropic and homogeneous medium. The identification $r_{\#}=r_{a}$ provides the effective properties of the crystal.

In order to reach intrinsic properties of the crystal, the crystal effect has to be enhanced and $p$ has to tend toward infinity. Unfortunately $r_{p}$ does not always have a limit while $p$ grows but $U_{p}^{r}$ does. This limit, denoted
by $U_{\#}^{r}$ can be expressed similarly to $U_{p}^{r}$ using a new reflection coefficient $r_{\#}$ deduced from the coefficients of the transfer matrix $T$ corresponding to a unit cell (see figure 1):

$$
T=\left[\begin{array}{ll}
T_{11} & T_{12}  \tag{1}\\
T_{21} & T_{22}
\end{array}\right]=T_{1} T_{2} \cdots T_{m}, \quad T_{j}=\left[\begin{array}{lr}
\cos \left(\kappa_{j} h_{j}\right) & \mu_{j} \kappa_{j}^{-1} \sin \left(\kappa_{j} h_{j}\right) \\
-\mu_{j}^{-1} \kappa_{j} \sin \left(\kappa_{j} h_{j}\right) & \cos \left(\kappa_{j} h_{j}\right)
\end{array}\right], \quad \kappa_{j}^{2}=\omega^{2} \varepsilon_{j} \mu_{j}-k_{1}^{2}
$$

where $T_{j}$ is the transfer matrix corresponding to the homogeneous layer $j(j=1,2, \ldots, m)$ and $k_{1}$ is the tangential component of the wavevector of each plane wave forming the beam field.

The extraction of effective properties from an inhomogeneous medium like a photonic crystal leads to model theoretically this medium by a new one, homogeneous this time, revealing similar optical characteristics. If, for given domains of temporal and spatial frequencies, the structure and its homogeneous equivalent have identical reflection properties, then these two media will be considered as equivalent.

From a dispersion law point of view, good candidates for simulating photonic crystals are anisotropic media. This communication is dedicated to the simulation of photonic crystals by anisotropic and homogeneous media of which electric permittivity and magnetic permeability tensors are denoted $\varepsilon_{a}$ and $\mu_{a}$ respectively.

$$
\varepsilon_{a}=\left[\begin{array}{ccc}
\varepsilon_{\|} & 0 & 0  \tag{2}\\
0 & \varepsilon_{\|} & 0 \\
0 & 0 & \varepsilon_{\perp}
\end{array}\right], \quad \mu_{a}=\left[\begin{array}{ccc}
\mu_{\|} & 0 & 0 \\
0 & \mu_{\|} & 0 \\
0 & 0 & \mu_{\perp}
\end{array}\right]
$$

Parameters $\varepsilon_{a}$ and $\mu_{a}$ are chosen such that they provide a dispersion law as close as possible to the one inside the crystal; they can take a priori any complex value. Such a medium, taken as semi-infinite, has a reflection coefficient $r_{a}$ (see figure 2).

Finally, the identification $r_{a}=r_{\#}$ will provide an analytic expression for both $\varepsilon_{a}$ and $\mu_{a}$. From this analytic expression, it is shown in [11] that effective permittivity and permeability take in general complex values. Moreover, it is proved that these quantities are purely real in the case where the unit cell of the one-dimensional crystal is symmetric with respect to a horizontal plane. This condition determines the adequate truncation of the crystal. Then this communication will be devoted to crystals with symmetric unit cell.

The symmetry of the unit cell allows another property for effective quantities like $\varepsilon_{a}$ and $\mu_{a}$. In fact, effective permittivity and permeability can be almost constant for a wide range of wavevectors including propagating and evanescent waves. That is to say that optical properties for propagating and evanescent waves are nearly the same.

Nevertheless, the modeling of a one-dimensional crystal reveals instructive limitations that have to be enlightened. In particular, it is shown that a negative effective index of refraction cannot be obtained with one-dimensional photonic crystals. Finally, we present a numerical method to study the impedance adaptation in the two-dimensional case with a flat lens example.

## 2. Noteworthy cell configuration

As notified before [11], a solution to obtain a purely real effective permittivity is to consider a crystal with a unit cell symmetric with respect to a horizontal plane (see figure 4). In other words, if a one-dimensional crystal with a unit cell made of two homogeneous layers is considered, then a solution is to choose the truncation at the middle of the first or the second layer. For example, we can consider a resulting unit cell made of $m=3$ homogeneous layers: $h_{1}=h_{3}=h_{\#} / 4, h_{2}=h_{\#} / 2, \sqrt{\varepsilon_{1} / \varepsilon_{0}}=\sqrt{\varepsilon_{3} / \varepsilon_{0}}=1.5\left(\right.$ for $\left.\mathrm{SiO}_{2}\right), \sqrt{\varepsilon_{2} / \varepsilon_{0}}=3.4$ (for Si$)$ and $\mu_{1}=\mu_{2}=\mu_{3}=\mu_{0}$. The dispersion law inside this structure is represented on figure 3 .

## 3. Analytic expressions for effective permittivity and permeability



Figure 3: Representation of the dispersion law inside the crystal for s-polarization where $k_{1}$ is the horizontal component of the wavevector. The two horizontal lines correspond to normalized frequencies $\omega h_{\#} /(2 \pi c)=0.462$ and 0.255 .

Figure 4: Example of a unit cell invariant under a mirror reflection with respect to the horizontal plane represented by the dashed line. This symmetry implies $T_{11}=T_{22}$ for the transfer matrix, and then $s=0$.

### 3.1 Simple dispersion law

At suitable frequencies the dispersion law of the crystal becomes elliptical and, denoting by $K_{1}$ and $K_{3}$ the length of the semi-axes of this ellipse, it can be modeled by $k_{1}^{2} / K_{1}^{2}+k_{\#}^{2} / K_{3}^{2}=1$ ( $k_{\#}$ is the vertical component of the wavevector inside the crystal). One can compare it to that of an anisotropic medium characterized by constants (2):

$$
\begin{equation*}
k_{a}^{2}+\frac{\mu_{\|}}{\mu_{\perp}} k_{1}^{2}=\omega^{2} \varepsilon_{\|} \mu_{\|}, \tag{3}
\end{equation*}
$$

where $k_{a}$ denotes the vertical component of the wavevector inside the anisotropic medium. For instance, at normalized frequency $\omega h_{\#} /(2 \pi c)=0.462$, the dispersion diagram inside the crystal is very close to an ellipse in the range $k_{1} h_{\#} /(2 \pi) \in[-0.5,0.5]$ (figure 5). It can be well-approached by the dispersion law (3) inside the homogeneous and anisotropic medium. Because of the polarization choice for the incident beam field, one can see that $\varepsilon_{\perp}$ does not appear in (3). Performing the identification of the two dispersion laws and $k_{a}=k_{\#}$, one obtains permeabilities $\mu_{\perp}$ and $\mu_{\|}$as functions of $\varepsilon_{\|}$:

$$
\begin{equation*}
\mu_{\perp}=K_{1}^{2} /\left(\omega^{2} \varepsilon_{\|}\right), \quad \mu_{\|}=K_{3}^{2} /\left(\omega^{2} \varepsilon_{\|}\right) \tag{4}
\end{equation*}
$$

At this stage, we need the third equation that will permit us to find an analytic expression for $\varepsilon_{\|}$, i.e. $r_{a}=r_{\#}$. As a result, there comes an analytic expression of the effective permittivity for the considered structure:

$$
\begin{array}{ll}
\varepsilon_{\|}=\left(K_{3} / \omega^{2}\right) \beta_{\#}\left[1-k_{1}^{2} / K_{1}^{2}\right]^{-1 / 2} & k_{1}^{2}<K_{1}^{2},  \tag{5}\\
\varepsilon_{\|}=\left(K_{3} / \omega^{2}\right)\left(\beta_{\#} / i\right)\left[k_{1}^{2} / K_{1}^{2}-1\right]^{-1 / 2} & k_{1}^{2} \geq K_{1}^{2},
\end{array}
$$

where $\beta_{\#}$ is defined as follow:

$$
\begin{array}{ll}
\beta_{\#}=\frac{\operatorname{Re}\left[T_{12}\right]}{\left|\operatorname{Re}\left[T_{12}\right]\right|} T_{12}^{-1} \sqrt{1-g^{2}}+i T_{12}^{-1} s & \text { if } g^{2}<1, \\
\beta_{\#}=T_{12}^{-1} \sqrt{1-g^{2}}+i T_{12}^{-1} s & \text { otherwise, }  \tag{6}\\
g=\left(T_{11}+T_{22}\right) / 2, \quad s=\left(T_{11}-T_{22}\right) / 2, &
\end{array}
$$

where the argument of the complex square root $\sqrt{1-g^{2}}$ is in $\left[0, \pi\left[\right.\right.$ and $\operatorname{Re}\left[T_{12}\right]$ is the real part of the coefficient $T_{12}$. Resulting values for the effective permittivity and permeability are represented on figure 6 . Effective permittivity



Figure 5: Dispersion diagram at normalized frequency $\omega h_{\#} /(2 \pi c)=0.462$ corresponding to the horizontal line on figure 4. Values of the semi-axes lengths are given by $K_{1} h_{\#} /(2 \pi) \approx 0.107, \quad K_{3} h_{\#} /(2 \pi) \approx 0.0455$.
and permeabilities are purely real and present relative variations under $10 \%$ for this $k_{1}$-range around five times wider than the one restricted to propagating waves.

### 3.2 Limits of the model

The results we illustrated until now are valid as the dispersion law of the crystal is simple enough to be faked by that of an anisotropic and homogeneous medium. Now, in order to show the limits of our one-dimensional model, let us consider the frequency $\omega h_{\#} /(2 \pi c)=0.255$ where the dispersion law becomes more complicated (see figure 7). At this frequency, the dispersion law is close to an ellipse centered at $k_{\#} h_{\#} /(2 \pi)= \pm 1 / 2$, it can be modeled by $k_{1}^{2} / K_{1}^{2}+\left(k_{3} \mp 1 / 2\right)^{2} / K_{3}^{2}=1$, and the most natural way is to model it by an hyperbole. Unfortunately, the fact that this hyperbole is defined for all $k_{1} \in \mathbb{R}$ implies that considering evanescent waves has no sense. As a matter
of fact, it seems that there is no simple model for frequencies taken in band gaps or for which the dispersion law is an ellipse centered on $k_{\#} h_{\#} /(2 \pi)= \pm 1 / 2$. Nevertheless we can remark that the dispersion law is only determining the direction of the energy flow via the group velocity. Since the latter remains invariant if the dispersion law is shifted, we tried the following.

Let us consider that the dispersion law centered on $k_{\#} h_{\#} /(2 \pi)=1 / 2$ (figure 7 ) is in fact centered at the origin. Using the same notations as those of the example described in the precedent section, the two dispersion laws that have to match are again

$$
\begin{equation*}
\frac{k_{1}^{2}}{K_{1}^{2}}+\frac{k_{\#}^{2}}{K_{3}^{2}}=1 \quad \text { and } \quad k_{a}^{2}+\frac{\mu_{\|}}{\mu_{\perp}} k_{1}^{2}=\omega^{2} \varepsilon_{\|} \mu_{\|} \tag{7}
\end{equation*}
$$

Since the unit cell of the crystal is made of ordinary materials (with all $\mu_{j}$ real and positive) then the horizontal component $P_{\|}$of the Poynting vector $\boldsymbol{P}=(\boldsymbol{E} \times \overline{\boldsymbol{H}}+\overline{\boldsymbol{E}} \times \boldsymbol{H}) / 4$ has always the same sign inside and outside the crystal: there is no negative refraction. Indeed, for $\mu>0$, the horizontal component of the Poynting vector is roughly

$$
\begin{equation*}
P_{\|}=(2 \omega \mu)^{-1}|E|^{2} k_{1} . \tag{8}
\end{equation*}
$$

This absence of negative refraction inside one-dimensional photonic crystals imposes the restriction

$$
\begin{equation*}
\mu_{\perp}=\operatorname{Re}\left[\mu_{\perp}\right]>0 \tag{9}
\end{equation*}
$$

in the model for $s$-polarization. Obtained values for $\varepsilon_{\|}, \mu_{\|}$and $\mu_{\perp}$ are depicted in figure 7. In this case, condition (9)



Figure 7: Dispersion diagram at normalized frequency $\omega h_{\#} /(2 \pi c)=0.255$ corresponding to the horizontal line on figure 4. Values of the semi-axes lengths are given by $K_{1} h_{\#} /(2 \pi) \approx 0.168, \quad K_{3} h_{\#} /(2 \pi) \approx 0.089$.
imposes not only $\mu_{\perp}>0$ but also $\mu_{\|}>0$ and $\varepsilon_{\|}>0$ via (7). Here, we conclude that the effective index cannot take negative values for one-dimensional crystals. Moreover, figure (8) shows that the model is not adequate since the sign of $\varepsilon_{\|}$changes across the boundary delimiting evanescent and propagating wave domains at $K_{1} h_{\#} /(2 \pi) \approx 0.168$.

## 4. Two-dimensional numerical example

In this last section, the considered structure is a two-dimensional crystal consisting of a bulk of silicon $(\varepsilon=12.0)$ with drilled air holes on a hexagonal lattice (the lattice constant, i.e. the edge of the triangular unit cell, is denoted by $a$ ). The air holes have circular cross section with radius $r=0.43 a$. At the normalized frequency $\omega a /(2 \pi c)=0.336$, and for the $s$-polarization (when the electric field is parallel to the air holes), this structure has a dispersion law leading to negative refraction with the product of the effective permittivity $\varepsilon$ and permeability $\mu$ equal to unity [12]. In other words, this crystal has the same behavior as a negative index material with $n=-1$ for propagating waves.

Thanks to the one-dimensional study, this crystal has been truncated such that the first layer is symmetric with respect to an horizontal plane [12]. The effective permittivity and permeability are then real and can be determined numerically:

$$
\begin{equation*}
\varepsilon=-5.7 \quad \mu=-0.175 \tag{10}
\end{equation*}
$$

These constants are determined with the following procedure [12, 13]. The crystal is embedded in a homogeneous medium with $\mu=1 / \varepsilon$ and both negative. Then we make varying the value of the permittivity of the surrounding medium. And the value of the effective permittivity is determined when the diffracted field by the embedded crystal is minimal.


Figure 9: Map of the electric field modulus when a line source is in presence of the embedded flat lens.

Finally, in order to show an optimized efficiency of the flat lens, the crystal has been embedded in a homogeneous medium with permittivity and permeability equal to 5.7 and 0.175 respectively, such that impedance adaptation occurs. The resulting map field is shown on figure 9 .

## 5. Conclusion

We have shown that effective permittivity and permeability can present small variations in a wide range of wavevectors including propagating and evanescent waves. However, our model demonstrates that one-dimensional crystals cannot behave like negative index material. Extension of these results to two-dimensional crystals with negative refraction should have important applications in subwavelength imaging as it is proved by the twodimensional numerical example of an optimized flat lens.

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# New Numerical Experiments in Scattering by Dielectric Wedges 

Egon Marx<br>National Institute of Standards and Technology<br>Gaithersburg, MD 20899-8212, U.S.A.


#### Abstract

The behavior of the field components near the edge of a dielectric wedge for a plane monochromatic wave of arbitrary direction of incidence and polarization is summarized. The integral equations for boundary functions for a finite wedge are presented both for singular and hypersingular equations, using the single-integral-equation method. The field components are then derived from the boundary functions by integration. Results are shown for a particular case.


## 1. Introduction

When a plane wave is incident on a dielectric wedge of infinite cross section, some of the components of the fields near the edge of the wedge diverge, following a negative fractional power law of the distance from the edge. We expect the same behavior of fields near the edge of a wedge of finite cross section or of sharp edges in other scatterers. One can use singular integral equations [1] (SIEs) or hypersingular integral equations [2] (HIEs) to investigate the behavior of the field components near the edge of a dielectric wedge. The unknown boundary functions in the single-integral-equation method also diverge near the edge. If the rate of the divergence could be firmly established, this behavior could be built into the form of the function and there would be no need to concentrate patches near an edge in the numerical solution of a problem. For a dielectric wedge, power series solutions [3], [4] indicate that the behavior remains essentially that of static fields, determined by the permittivity and permeability of the wedge material, that of the surrounding medium, usually assumed to be free space, and the angle of the wedge. For oblique incidence, this means that the behavior of the fields should be that of the TE and TM modes [5], [6]. Numerical experiments with a finite dielectric wedge show disagreements between the computed and the expected behavior. For the HIE, the unknown boundary functions remain constant near the edge and the numerical difficulties come from the highly singular behavior of the kernel.

Agreement with the expected behavior occurs only for particular directions from the edge for TE and TM modes. The theory fails to indicate where the expected asymptotic behavior sets in, if at all; this usually occurs at a small fraction of the wavelength. For oblique incidence, the behavior of the fields near the edge depends on the direction of incidence, polarization, and the direction of approach to the wedge, which is not the case for the static fields. Some of the results depend on the approximations used in the numerical solution of the problem, especially for the SIE. There is also a dependence of the amplitude of the fields on the size of the finite wedge, which may be due to the overall scattering cross section of the wedge. The difficulty of matching two waves propagating at different speeds in different media on the two sides of a dielectric wedge as well as in the outside medium on the continuation of the sides suggests that there may be no general rigorous solution for this scattering problem. For a perfectly conducting wedge, where the fields on one side of the boundary vanish, there is good agreement between the predicted and computed behavior of the field components.

We consider the electromagnetic scattering of a plane monochromatic wave by an infinite wedge of angle $\beta$, permittivity $\varepsilon_{2}$ and permeability $\mu_{2}$, surrounded by a medium characterized by $\varepsilon_{1}$ and $\mu_{1}$. The fields near the edge of a dielectric wedge have been studied both theoretically and numerically. Some of the components of the fields are seen to diverge, although the computed fields do not appear to follow the theoretically derived behavior along all directions. The fields $E_{z}$ in the TE mode and $H_{z}$ in the TM mode are expected to behave like $\rho^{t}$, where $\rho$ is the distance to the edge and $t$ obeys the equation

$$
\begin{equation*}
\sin (\pi t)=r \sin [(\pi-\beta) t] \tag{1}
\end{equation*}
$$

where $r=\left|\mu_{1}-\mu_{2}\right| /\left(\mu_{1}+\mu_{2}\right)$ (TE) or $r=\left|\varepsilon_{1}-\varepsilon_{2}\right| /\left(\varepsilon_{1}+\varepsilon_{2}\right)$ (TM) [3]. The two modes are no longer independent for oblique incidence, and we seek to determine what the expected behavior of the fields will be for arbitrary direction of incidence and polarization.

## 2. General Formulas

Some details of the derivations can be found in Ref. [7], and the coordinate system is shown in Fig. 1. If the geometry of the scatterer is invariant under translations along the $z$-axis, the fields can be decomposed into longitudinal and transverse parts. The transverse components of the fields can then be expressed in terms of the longitudinal ones, $E_{z}$ and $H_{z}$. These field components obey the homogeneous Helmholtz equation in the $x y$-plane. For a dielectric wedge, the tangential fields have to be continuous across the boundary of the scatterer. The tangential components of the perpendicular fields along a curve $C$ are given by

$$
\begin{align*}
& E_{t}=-i\left(\omega \mu / k_{\perp}^{2}\right) \partial H_{z} / \partial n+i\left(k_{z} / k_{\perp}^{2}\right) \partial E_{z} / \partial s  \tag{2}\\
& H_{t}=-i\left(\omega \varepsilon / k_{\perp}^{2}\right) \partial E_{z} / \partial n+i\left(k_{z} / k_{\perp}^{2}\right) \partial H_{z} / \partial s \tag{3}
\end{align*}
$$

where $k_{\mathrm{z}}$ is the $z$-component of the propagation vector $\overrightarrow{\mathbf{k}}$ and

$$
\begin{equation*}
k^{2}=\omega^{2} \varepsilon \mu, \quad k_{\perp}^{2}=k^{2}-k_{z}^{2} \tag{4}
\end{equation*}
$$

Across the boundary $C$ separating two media, $E_{\mathrm{z}}$ and $H_{\mathrm{z}}$ and also the tangential derivatives $\partial E_{z} / \partial s$ and $\partial H_{z} / \partial s$ are continuous. The continuity conditions for the tangential components of the perpendicular fields give a relationship between the normal derivatives,

$$
\begin{align*}
& \left(\partial E_{z} / \partial n\right)_{-}=\alpha\left(\partial E_{z} / \partial n\right)_{+}+x \partial H_{z} / \partial s  \tag{5}\\
& \left(\partial H_{z} / \partial n\right)_{-}=\alpha^{\prime}\left(\partial H_{z} / \partial n\right)_{+}+x^{\prime} \partial E_{z} / \partial s \tag{6}
\end{align*}
$$

where

$$
\begin{gather*}
k^{\prime 2}=\omega^{2} \varepsilon_{2} \mu_{2}, \quad k_{\perp}^{\prime 2}=k^{\prime 2}-k_{z}^{2}, \quad \kappa=k_{\perp}^{\prime} / k_{\perp}^{2}, \quad \alpha=\kappa \varepsilon_{1} / \varepsilon_{2} \\
\alpha^{\prime}=\kappa \mu_{1} / \mu_{2}, \quad x=-\left(k_{z} / \omega \varepsilon_{2}\right)(1-\kappa), \quad x^{\prime}=\left(k_{z} / \omega \mu_{2}\right)(1-\kappa) \tag{7}
\end{gather*}
$$

These conditions can then be used to obtain integral equations or the behavior of the static fields.

## 3. Integral equations

A minimal set of singular or hypersingular integral equations can be derived following the methods explained in Refs. 1 and 2. The singular equations are

$$
\begin{gather*}
{\left[\left(\frac{1}{2}+G_{2}\right)+\alpha N_{2}\left(\frac{1}{2}+N_{1}^{\prime}\right)\right]\{\eta\}+x N_{1}^{\prime \prime}\left\{\eta^{\prime}\right\}} \\
=-\left(\frac{1}{2}+G_{2}\right)\left\{E_{z}^{i n}\right\}-\alpha N_{2}\left\{\partial E_{z}^{i n} / \partial n\right\}-x N_{2}\left\{\partial H_{z}^{i n} / \partial s\right\}, \quad \overrightarrow{\mathbf{x}} \in C, \tag{8}
\end{gather*}
$$

$$
\begin{gather*}
x^{\prime} N^{\prime \prime}\{\eta\}+\left[\left(\frac{1}{2}+G_{2}\right)+\alpha^{\prime} N_{2}\left(\frac{1}{2}+N_{1}^{\prime}\right)\right]\left\{\eta^{\prime}\right\} \\
=-\left(\frac{1}{2}+G_{2}\right)\left\{H_{z}^{i n}\right\}-\alpha^{\prime} N_{2}\left\{\partial H_{z}^{i n} / \partial s\right\}-x^{\prime} N_{2}\left\{\partial E_{z}^{i n} / \partial n\right\}, \quad \overrightarrow{\mathrm{x}} \in C, \tag{9}
\end{gather*}
$$

and the hypersingular equations are

$$
\begin{gather*}
{\left[\left(\frac{1}{2}+G_{2}\right)\left(\frac{1}{2}+N_{1}\right)+\alpha N_{2} M_{1}^{\prime}\right]\{\varphi\}+x M_{1}^{\prime \prime}\left\{\varphi^{\prime}\right\}} \\
=-\left(\frac{1}{2}+G_{2}\right)\left\{E_{z}^{i n}\right\}+\alpha N_{2}\left\{\partial E_{z}^{i n} / \partial n\right\}-x N_{2}\left\{\partial H_{z}^{i n} / \partial s\right\}, \quad \overrightarrow{\mathrm{x}} \in C,  \tag{10}\\
\\
x^{\prime} M_{1}^{\prime \prime}\{\varphi\}+\left[\left(\frac{1}{2}+G_{2}\right)\left(\frac{1}{2}+N_{1}\right)+\alpha^{\prime} N_{2} M_{1}^{\prime}\right]\left\{\varphi^{\prime}\right\}  \tag{11}\\
=-\left(\frac{1}{2}+G_{2}\right)\left\{H_{z}^{i n}\right\}-\alpha^{\prime} N_{2}\left\{\partial H_{z}^{i n} / \partial n\right\}-x^{\prime} N_{2}\left\{\partial E_{z}^{i n} / \partial s\right\}, \quad \overrightarrow{\mathrm{x}} \in C,
\end{gather*}
$$

where, in terms of Hankel functions of $k_{\perp} R=k_{\perp}\left|\overrightarrow{\mathbf{X}}-\overrightarrow{\mathbf{x}}^{\prime}\right|$
, we have

$$
\begin{gather*}
G\{\eta\}(\overrightarrow{\mathrm{x}})=-\frac{i}{4} \int_{C} d s^{\prime} \eta\left(s^{\prime}\right) H_{0}^{(1)}(k R),  \tag{12}\\
N\{\varphi\}(\overrightarrow{\mathrm{x}})=\frac{i}{4} k \int_{C} d s^{\prime} \varphi\left(s^{\prime}\right) H_{1}^{(1)}(k R) \hat{\mathrm{n}}^{\prime} \cdot \hat{\mathrm{R}},  \tag{13}\\
N^{\prime}=\hat{\mathrm{n}} \cdot \nabla G, \quad N^{\prime \prime}=\hat{\mathrm{t}} \cdot \nabla G, \quad M^{\prime}=\hat{\mathrm{n}} \cdot \nabla N, \quad M^{\prime \prime}=\hat{\mathrm{t}} \cdot \nabla N . \tag{14}
\end{gather*}
$$

The functional $\mathrm{M}^{\prime}$ is hypersingular. These sets of coupled equations can then be used to determine the behavior of the fields near the edge of the wedge.

## 4. Static fields

A number of authors [3-6] agree that the behavior of the total fields near the edge of a wedge in a scattering problem should be that of static fields. Both $E_{\mathrm{z}}$ and $H_{\mathrm{z}}$ obey the Laplace equation, which can be separated into an angular and a radial part. Since we have to match the fields along the sides of the wedge, we assume that all fields have the same dependence on $\rho$. We set

$$
\begin{array}{ll}
E_{1 z}(\rho, \varphi)=R(\rho)[A \sin (t \varphi)+B \cos (t \varphi)], & \beta \leq \varphi \leq 2 \pi \\
E_{2 z}(\rho, \varphi)=R(\rho)\left[A^{\prime} \sin (t \varphi)+B^{\prime} \cos (t \varphi)\right], & 0 \leq \varphi \leq \beta \\
H_{1 z}(\rho, \varphi)=R(\rho)\left[A^{\prime \prime} \sin (t \varphi)+B^{\prime \prime} \cos (t \varphi)\right], & \beta \leq \varphi \leq 2 \pi, \\
H_{2 z}(\rho, \varphi)=R(\rho)\left[A^{\prime \prime \prime} \sin (t \varphi)+B^{\prime \prime \prime} \cos (t \varphi)\right], & 0 \leq \varphi \leq \beta, \tag{18}
\end{array}
$$

and, for $R(\rho)=\rho^{\mathrm{t}}$, the continuity conditions give eight homogeneous equations for the unknowns $A, A^{\prime}, A^{\prime \prime}, A^{\prime \prime \prime}, B$, $B^{\prime}, B^{\prime \prime}$, and $\mathrm{B}^{\prime \prime \prime}$. The solution of this set of linear equations is the trivial solution unless the determinant of the coefficient vanishes. The vanishing of this $8 \times 8$ determinant gives a transcendental equation for $t$ that allows us to find the
behavior of the components of the fields near the edge. This equation reduces to

$$
\begin{equation*}
\left\{q^{2}\left[(\alpha+1)\left(\alpha^{\prime}+1\right)+x x^{\prime}\right]-\left[(\alpha-1)\left(\alpha^{\prime}-1\right)+x x^{\prime}\right]\right\}^{2}-4 q^{2}\left(\alpha-\alpha^{\prime}\right)^{2}=0 \tag{19}
\end{equation*}
$$

where we have substituted $\sin [(\pi-\beta) t]=q(t) \sin (\pi t)$.
We can reduce Eq. (19) to the pair of

$$
\begin{equation*}
q^{2}\left[(\alpha+1)\left(\alpha^{\prime}+1\right)+x x^{\prime}\right] \pm 2 q\left(\alpha-\alpha^{\prime}\right)-\left[(\alpha-1)\left(\alpha^{\prime}-1\right)+x x^{\prime}\right]=0 \tag{20}
\end{equation*}
$$

whose solutions differ only in sign. Then Eq. (20) gives the corresponding value of $t$, which depends on the wedge angle $\beta$. These equations have the same form as the ones obtained in the TE and TM modes. Once $t$ is known, the constants such as $A$ are determined from the boundary conditions and then substituted into Eqs. (15) through (18).

## 5. Example

We show results for the direction of incidence at $\theta=60^{\circ}$ and $\varphi=90^{\circ}$ both for the HIE method and the SIE method. We choose the relative constants of the media to be $\varepsilon_{1}=1, \mu_{1}=1, \varepsilon_{2}=5$, and $\mu_{2}=3$ and the wavelength $\lambda=1$ $\mu \mathrm{m}$. These parameters give two possible behaviors for the field components near the edge of the wedge.

In Fig. 2 we show the computed field components in a log-log plot that emphasizes the behavior near the edge of the wedge. A power law for the fields translates into dashed straight lines, proportional to $\rho^{t-1}$, on this plot. We see that some components follow one line and some follow the other. The field components obtained using HIEs show one or the other asymptotic $(\rho \rightarrow 0)$ behavior for the polarizations that correspond to TE and TM modes, but the plot for a oblique polarization falls somewhere in between. The field components obtained using SIEs, shown in Fig. 3, have a steep ascent near the edge, a behavior that has been noted before in TE and TM modes, and which is probably due to errors in the divergent boundary functions or in the integration for such a function.

The boundary functions, shown in Fig. 4, for the HIE follow mostly the expected constant behavior near the edge, but some of them grow as one gets away from the edge of the wedge. Those for the SIE show the expected divergent behavior, but it is not clear whether they follow a power law that would be indicated by a straight line.

## 6. Conclusions

Perpendicular components of the static fields near the edge of a dielectric wedge diverge with power laws determined by a transcendental equation in the configuration that corresponds to oblique incidence. There are essentially two field modes that reduce to the TE and TM modes for normal incidence.

## ACKNOWLEDGEMENTS

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Fig. 2. Magnetic field component and expected asymptotic behavior for HIE.


Fig. 3. Magnetic field component and expected asymptotic behavior for SIE.


Fig. 4. Typical boundary functions for HIE and SIE showing behavior near the edge of the wedge.

# Scattering by a Finite Grating on a Substrate 

Egon Marx<br>National Institute of Standards and Technology<br>Gaithersburg, MD 20899-8212, U.S.A.


#### Abstract

Computer simulation of field components and of microscope images of finite gratings are performed using singular integral equations for boundary functions. Some of the equations that are obtained in the single-integral-equation method are shown, as well as an integral of the boundary function that gives a field component. Image formation in a microscope is discussed. Field components and images at different focus heights are shown for a finite grating of Si lines on a Si substrate.


## 1. Introduction

The semiconductor industry uses lines and other structures on a semiconductor to overlay different layers of a device. Accurate measurements of these features are important to the alignment, as well as to predict the behavior of lines in integrated circuits. We reduce the problem to a two-dimensional scattering process by assuming that the lines are infinitely long and that the incident field is a monochromatic plane wave. Measurements of the transverse dimensions of these lines or strips using optical images or fields are also of great interest to the industry. A number of these lines placed on a substrate form a finite grating, which sometimes can be approximated by an infinite grating. For critical dimensions significantly smaller than the wavelength of the light used in a microscope, the images do not have the same shape as the features and simulation has to be used to interpret the images and perform measurements [1].

## 2. Integral Equations

We use Maxwell's equations for two-dimensional scatterers in integral form [2-4] to determine the fields scattered by the cylinders that represent the lines on the substrate. The presence of a semi-infinite substrate causes a reflected wave to be present in the region $\mathrm{V}_{1}$ in addition to the incident wave and a refracted wave in the region $\mathrm{V}_{2}$, as shown in Fig. 1. These are all plane monochromatic waves with no sources in the finite region and are called homogeneous fields. We define scattered fields as the difference between the total fields and these homogeneous fields. The scattered fields are cylindrical waves that obey the radiation condition at infinity. The region $V_{3}$, of finite cross section, can represent a single line or a finite number of such lines in the form of a grating. The $\varepsilon_{i}$ are the permittivities and the $\mu_{i}$ the permeabilities of the $V_{i}$. The coordinate system is chosen with the $z$-axis along the generator of the cylinder, the $x$-axis along the interface $\mathrm{C}_{1}$,


Fig. 1. Scattering by a strip on a substrate. and the $y$-axis perpendicular to the interface. We use the single-integral equation method [5] to find the unknown boundary functions $\eta_{i j}$ and then determine the fields by integration.

For oblique incidence and arbitrary polarization, three typical integral equations out of eight are

$$
\begin{gather*}
G_{11}^{1}\left\{\eta_{11}\right\}+G_{12}^{1}\left\{\eta_{12}\right\}-G_{21}^{1}\left\{\eta_{21}\right\}-G_{23}^{1}\left\{\eta_{23}\right\}=0  \tag{1}\\
\frac{1}{2} \eta_{11}+{N^{\prime}}_{12}^{\prime}\left\{\eta_{12}\right\}+\frac{1}{2} \bar{b}_{1} \eta_{21}-\bar{\beta}_{1} N^{\prime \prime}{ }_{11}^{1}\left\{\eta_{11}^{\prime}\right\}-\bar{\beta}_{1} N^{\prime \prime}{ }_{12}^{1}\left\{\eta_{12}^{\prime}\right\}=0 \tag{2}
\end{gather*}
$$

$$
\begin{align*}
& {\left[\left(\frac{1}{2}+N_{32}^{2}\right) G_{11}^{2}+b_{2} G_{32}^{2} N_{11}^{\prime 2}\right]\left\{\eta_{11}\right\}+\left[\left(\frac{1}{2}+N_{32}^{2}\right) G_{12}^{2}+b_{2} G_{32}^{2}\left(\frac{1}{2}+N_{12}^{\prime 2}\right)\right]\left\{\eta_{12}\right\}} \\
& -N_{33}^{2} G_{21}^{3}\left\{\eta_{21}\right\}+\left(\frac{1}{2} \bar{b}_{3} G_{33}^{2}-N_{33}^{2} G_{23}^{3}\right)\left\{\eta_{23}\right\} \\
& +\beta_{2} G_{32}^{2} N^{\prime \prime 2}{ }_{11}^{2}\left\{\eta_{11}^{\prime}\right\}+\beta_{2} G_{32}^{2} N^{\prime \prime 2}{ }_{12}\left\{\eta_{12}^{\prime}\right\}-\bar{\beta}_{3} G_{33}^{2} N^{\prime \prime}{ }_{21}\left\{\eta^{\prime}{ }_{21}\right\}-\bar{\beta}_{3} G_{33}^{2} N^{\prime \prime 3}{ }_{23}\left\{\eta_{23}^{\prime}\right\} \\
& =-\left(\frac{1}{2}+N_{32}^{2}\right)\left\{E_{z}^{h l}\right\}-b_{2} G_{32}^{2}\left\{\partial E_{z}^{h l} / \partial n\right\}+N_{33}^{2}\left\{E_{z}^{h 2}\right\}+\bar{b}_{3} G_{33}^{2}\left\{\partial E_{z}^{h 2} / \partial n\right\} \\
& -\beta_{2} G_{32}^{2}\left\{\partial H_{z}^{h l} \partial s\right\}+\bar{\beta}_{3} G_{33}^{2}\left\{\partial H_{z}^{h 2} / \partial s\right\}, \tag{3}
\end{align*}
$$

where $G, N, N^{\prime}$, and $N^{\prime \prime}$ are functionals related to the Green function of the two-dimensional Helmholtz equation as shown in Ref. 2 (the first subindex refers to the constants of the medium, the second one to the curve over which the integration is carried out, and the superindex to the curve where the value of the functional is evaluated),

$$
\begin{gather*}
b_{i}=\frac{\varepsilon_{j}\left(\omega^{2} \varepsilon_{\ell} \mu_{\ell}-k_{z}^{2}\right)}{\varepsilon_{\ell}\left(\omega^{2} \varepsilon_{j} \mu_{j}-k_{z}^{2}\right)}, \quad b_{i}^{\prime}=\frac{\mu_{j}\left(\omega^{2} \varepsilon_{\ell} \mu_{\ell}-k_{z}^{2}\right)}{\mu_{\ell}\left(\omega^{2} \varepsilon_{j} \mu_{j}-k_{z}^{2}\right)}, \quad \beta_{i}=\frac{\omega k_{z}\left(\varepsilon_{j} \mu_{j}-\varepsilon_{\ell} \mu_{\ell}\right)}{\varepsilon_{\ell}\left(\omega^{2} \varepsilon_{j} \mu_{j}-k_{z}^{2}\right)}, \\
{\beta_{i}^{\prime}=}_{=\frac{\omega k_{z}\left(\varepsilon_{\ell} \mu_{\ell}-\varepsilon_{j} \mu_{j}\right)}{\mu_{\ell}\left(\omega^{2} \varepsilon_{j} \mu_{j}-k_{z}^{2}\right)}, \quad \text { where for } i=\left\{\begin{array}{l}
1 \\
2, \\
3
\end{array} \quad j=\left\{\begin{array}{l}
1 \\
1 \\
3
\end{array} \text { and } \ell=\left\{\begin{array}{l}
2 \\
3 \\
2
\end{array}\right.\right.\right.}^{\bar{b}_{i}=1 / b_{i}, \quad \bar{\beta}_{i}=-\beta_{i} b_{i}, \quad \bar{b}_{i}^{\prime}=1 / b_{i}^{\prime}, \quad{\overline{\beta^{\prime}}}_{i}^{\prime}=-\beta_{i}^{\prime} / b_{i}^{\prime} .} \tag{4}
\end{gather*}
$$

and $E_{z}^{h 1} H_{z}^{h 1} E_{z}^{h 2} \quad, H_{z}^{h 2}$, and are the homogeneous field components in $V_{1}$ and $V_{2}$. From Eq. (2) we can solve for $\eta_{21}$ and, from a similar equation, for $\eta^{\prime}{ }_{21}$, which can be substituted back into the remaining six equations. A typical equation used to determine the field components is

$$
\begin{equation*}
E_{x}^{s c}(\vec{\xi})=-\frac{1}{4 k} \sum_{j=1}^{2}\left[\omega \mu_{1} \int_{C_{j}} d s^{\prime} \eta^{\prime}{ }_{1 j}\left(s^{\prime}\right) H_{1}^{(1)}\left(k_{\perp} \rho\right) \frac{\left(y-y^{\prime}\right)}{\rho}+k_{z} \int_{C_{j}} d s^{\prime} \eta_{1 j}\left(s^{\prime}\right) H_{1}^{(1)}\left(k_{\perp} \rho\right) \frac{\left(x-x^{\prime}\right)}{\rho}\right], \tag{6}
\end{equation*}
$$

where $\vec{\xi}=x \hat{e}_{x}+y \hat{e}_{y} \rho=|\vec{\xi}| \quad$,
A similar method based on integral equations can be used to obtain the efficiencies of infinite gratings [6] or they can be determined by the rigorous coupled-wave analysis method [7]. The fields of infinite gratings are independent of the distance to the substrate.

## 3. Microscope Images

To simulate the image formation in a microscope, we consider that the illumination is a superposition of a number of plane waves defined as Köhler illumination. We compute the scattered fields for each incident wave and add the intensities. The aperture angle from the axis to the edge, shown in Fig. 2, is $\Theta$, defined by $\sin \Theta=1 / n$, where 1 is the illumination numerical aperture and $n$ is the refractive index. The scattered field components can be computed at a given height above the substrate and the lines by solving the integral equations and integrating. The Fourier components are determined numerically and we keep those that fall into the col-


Fig. 2. Illumination and coordinate system. lection numerical aperture, eliminating evanescent fields and components that propagate at too shallow an angle. We choose a focus height and propagate these components using the homogeneous Helmholtz equation. The field components are further modified by the effect of the
magnification of the microscope [8], which for most applications is large and can be approximated by infinity. After the inverse Fourier transform is computed, the intensities of the scattered fields for each incident plane wave are added up to obtain the image.

## 4. Grating Efficiencies

For infinite gratings, the outgoing flow of energy is limited to a finite number of directions associated with the diffraction orders. They are determined by the diffraction equation

$$
\begin{equation*}
(\Lambda / \lambda)\left(\sin \bar{\theta}+\sin \bar{\theta}_{i}\right)=n \tag{7}
\end{equation*}
$$

where $\Lambda$ is the period of the grating, $\lambda$ is the period of the light, $\bar{\theta}$ is the angle with respect to the normal of the substrate, and $\overline{\boldsymbol{\theta}}_{\boldsymbol{i}}$ that of the incident direction. For a finite grating, the amplitude of the scattered far fields form loops [9] about the same direction, and the area in the loop is proportional to the flow of energy in that direction. The intensity of the far field in a direction $\varphi$ is given by

$$
\begin{equation*}
I(\varphi)=\left|\int_{C_{1}} d s^{\prime} \eta_{11}\left(s^{\prime}\right) \exp \left[-i k \rho^{\prime} \cos \left(\varphi-\varphi^{\prime}\right)\right]+\int_{C_{2}} d s^{\prime} \eta_{12}\left(s^{\prime}\right) \exp \left[-i k \rho^{\prime} \cos \left(\varphi-\varphi^{\prime}\right)\right]\right|^{2} \tag{8}
\end{equation*}
$$

The zeroth-order diffraction peak presents a problem because it has to be added to the reflected wave, which propagates in the same direction. This is not possible for the far fields, which correspond to a cylindrical wave at large distances from the scatterer and decrease as $1 / \sqrt{ } \rho$, while the reflected fields are those of a plane wave. Alternatively, one has to determine fields that are close to the finite grating, which should correspond to an approximation of the fields of infinite gratings, at least in the center portion where the edge effects are small. On the other hand, the near fields found at distances from the scatterer smaller than the wavelength do not need to have the periodic behavior one expects.

## 5. Example

We consider a finite grating of 21 Si lines, each 230 nm high, 163 nm wide, and positioned at a pitch of 541 nm , on a Si substrate. The dimensions of the lines are somewhat smaller than the wavelength of the incident light, which is 546 nm . For this configuration, only the zeroth-order diffraction occurs for infinite gratings. In Fig. 3 we show the magnitude of the z-components of the scattered plus reflected fields at heights of $5 \mu \mathrm{~m}$ and $0.3 \mu \mathrm{~m}$ above the substrate. At $5 \mu \mathrm{~m}$, there is a considerable influence of the reflected field while at $0.3 \mu \mathrm{~m}$ the fields look more periodic, at least in the center. We have chosen normal incidence and TM or TE mode, which correspond to $\mathrm{E}_{\mathrm{z}}=0$ and $\mathrm{H}_{\mathrm{z}}=0$, respectively. The whole array has a width of about $10 \mu \mathrm{~m}$, which suggests that at a height of $5 \mu \mathrm{~m}$ one gets significant contributions from both the scattered and the reflected fields. At $0.3 \mu \mathrm{~m}$ one gets the expected behavior in the TM mode but a more complicated one in the TE mode.

In Fig. 4 we show microscope images for the same finite grating at different focus heights. These images are normalized to 1 at large distances from the grating. The actual focus height of the microscope is unknown, so that the focus height is adjusted automatically to a maximum of a function of the image profile called a focus metric [10]. Ideally one compares a series of measured and computed profiles to establish the correspondence between them. In this figure we notice that the contrast increases with focus height, then decreases until it essentially vanishes at 500 nm above the substrate, and then increases again, giving rise to at least two maxima. We also notice that the images have a minimum at the center of the profile for heights less than 500 nm and a maximum above that height.

## 6. Conclusions

The method of integral equations for unknown boundary functions has been used to determine field components and microscope images for finite gratings with features of a size comparable to the wavelength of the incident light. An important issue when comparing measured and simulated microscope images is the focus height, which is given for the simulations but unknown for the measured images. A series of images can be compared as the focus height is varied, and the focus metric helps to match the images in the sequences.

In the example presented above, there is only zeroth-order diffraction present for infinite gratings or for far fields. Far fields from a cylindrical wave that decrease like the inverse to the distance from the scatterer cannot be added to the constant reflected plane wave. The fields near the substrate present a more periodic structure, as seen in Fig. 3, that can be used as an approximation to those obtained for infinite gratings.

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Field Hz at a height of $5 \mu \mathrm{~m}$ above the substrate


Field Hz at a height of $0.3 \mu \mathrm{~m}$ above the substrate



Fig. 3. Computed field components for TM and TE modes at different heights above the substrate.


Fig. 4 Computed intensities of the electric fields at different heights above the substrate.

# Open Shells of Revolution: Method of Analytical Regularisation 

Sergey B. Panin ${ }^{1}$, Paul D. Smith ${ }^{1}$, Elena D. Vinogradova ${ }^{1}$, and Sergey S. Vinogradov ${ }^{2}$<br>${ }^{1}$ Department of Mathematics, Macquarie University, NSW 2109, Sydney, Australia,<br>E-mail: serpanin@maths.mq.edu.au; pdsmith@maths.mq.edu.au; evinogra@maths.mq.edu.au<br>${ }^{2}$ School of Physics, University of Sydney, NSW 2006, Sydney, Australia, E-mail: svinogra@physics.usyd.edu.au


#### Abstract

Based on the idea of an analytical regularisation a mathematically rigorous and numerically efficient approach is proposed to solve the potential problem for open arbitrary shaped shell of revolution with Dirichlet boundary condition. The initial integral equation is reduced to a form admitting decomposition of the integral kernel into the sum of the Green's function for a sphere, which includes all the singularities of the reformulated problem, and a smooth remainder. An effective calculation technique for the coefficients of the Fourier expansion of the remainder was obtained. Using the analytical regularisation, the problem is equivalently reduced to an infinite system of linear algebraic equations of the second kind. Such equations can be effectively and efficiently solved by standard numerical methods. The convergence improvement of the series describing the surface charge is implemented.


## 1. Introduction

At present, diffraction by open shells of revolution is studied mostly by direct numerical techniques. Frequently, codes based on these techniques require powerful computer resources, but usually no guarantee on the accuracy of computations is available. This arises because the customary approach is based on the numerical solution of the first kind Fredholm equation which is deducible from the single-layer potential representation. It is well known that this equation is ill-posed, and as a consequence, the computational scheme is neither stable nor convergent. Thus it is highly desirable to transform this equation into a second kind Fredholm equation, which provides a stable and fast converging computational algorithm that enables us to reach any required accuracy of computation [1]. A transformation technique for canonically shaped open shells was developed in $[2,3]$ by applying a method of analytical regularisation.

The first step in the generalisation of this method $[2,3]$ to diffraction from arbitrarily shaped open shells of revolution is to solve the corresponding electrostatic potential problem. Our proposed method for solving the boundary value problem with the Dirichlet boundary condition has several noteworthy features. The solution, in its integral equation formulation, depends upon an analysis of the singularities of the Green's function describing the body of revolution. Thus guided, the integral equation is transformed so that the specially scaled kernel may be decomposed as a sum of the Green's function for a sphere and a remainder term. It is essential for our method that all singularities are confined to the spherical Green's function, and that the remainder is smooth. This reformulation was first suggested by Tuchkin [4].

The reason for performing such a reformulation is to ensure regularity of the extracted remainder, and this allows us to use a standard and effective calculation technique for its Fourier coefficients. Another motivation is so that we may apply a well-known regularisation method which employs analytical transformations connected with Abel's integral equation. When these transformations are applied to the canonical Green's function for a sphere, the problem is reduced to the second kind Fredholm equation as desired, in the form of an infinite system of linear algebraic equations. This system is very effectively solved by a truncation method; fast convergence of this technique is proven theoretically.

## 2. Problem formulation

Let $l$ be a plane, smooth, and non-self-crossing curve specified by a given single-valued continuous function $\rho(\theta) \in(0, \infty)$, where $\theta \in[0, \pi]$ is a zonal angle of spherical coordinates; furthermore the element of arc length should be positive: $\lambda(\theta)=\sqrt{\{\rho(\theta)\}^{2}+\left\{\rho^{\prime}(\theta)\right\}^{2}}>0$, and also $\rho^{\prime}(0)=\rho^{\prime}(\pi)=0$. When $l$ is rotated about the vertical $O Z$ - axis, a closed body of revolution is obtained. Fix some $\theta_{0} \in(0, \pi)$. Let $S_{0}$ be the open shell of revolution formed by rotation of the curve $l_{0}=\left\{l=l(\theta): \theta \in\left[0, \theta_{0}\right]\right\}$. The surface $S_{1}$ generated by the curve $l_{1}=\left\{l=l(\theta): \theta \in\left(\theta_{0}, \pi\right]\right\}$ forms the "aperture" of $S_{0}$ (see Fig.1). A more general geometry of the shell, obtained by allowing the function $\rho(\theta)$ to be multivalued, may be described by using a parameterisation in which the arc-length of the contour $l$ is the parameter. In this case the shell of revolution is defined by two given functions. Our approach permits generalisation for such geometry.


Fig.1. (a) The problem geometry. (b) The generating curve.
Let the shell $S_{0}$ be charged to some electrostatic potential, described by the given function $\Psi(p), p \in S_{0}$. The scalar function $U=U(q)$ defining the electrostatic potential in the space observation point $q \in R^{3}$ is to be found. As is well known the solution to this mixed boundary value problem for Laplace's equation with Dirichlet boundary condition is equivalent to the following Fredholm integral equation of the first kind,

$$
\begin{equation*}
-\iint_{S_{0}} \sigma_{D}(p) G(q, p) d s_{p}=\Psi(q), \quad q, p \in S_{0} \tag{1}
\end{equation*}
$$

where $\sigma_{D}(p)$ denotes unknown surface charge density, $G(q, p)=(4 \pi R(q, p))^{-1}$ is the free space Green's function for Laplace's equation, and $R(q, p)=|q-p|$ is the distance function; the point $p \in S_{0}$ denotes the variable of integration.

## 3. Solution method

We introduce the unknown function $X(p)$ to describe a scaled version of the charge density distribution, and extend it to the full interval of zonal angle $\theta_{p}$ in spherical coordinates:

$$
X\left(\theta_{p}, \varphi_{p}\right)=\left\{\begin{array}{l}
-\sqrt{\rho\left(\theta_{p}\right)} \lambda\left(\theta_{p}\right) \times \sigma_{D}\left(\theta_{p}, \varphi_{p}\right), \quad \theta_{p} \in\left[0, \theta_{0}\right], \quad \varphi_{p} \in[0,2 \pi] \\
0, \quad \theta_{p} \in\left(\theta_{0}, \pi\right], \quad \varphi_{p} \in[0,2 \pi]
\end{array}\right.
$$

Thus the initial equation (1) can be rewritten in the form

$$
\begin{equation*}
\int_{0}^{2 \pi} d \varphi_{p} \int_{0}^{\pi} d \theta_{p} \sqrt{\rho\left(\theta_{p}\right)} \sin \theta_{p} X\left(\theta_{p}, \varphi_{p}\right) G\left(\theta_{q}, \theta_{p}, \varphi_{q}, \varphi_{p}\right)=\Psi\left(\theta_{q}, \varphi_{q}\right), \quad q \in S \tag{2}
\end{equation*}
$$

Expanding the functions of azimuth angle in Fourier series, it is possible to implement the integration over variable $\varphi_{p}$ in (2), and then separate the surface integral equation into an infinite set of single integral equations over zonal angle, where each equation corresponds to a value $m$ of azimuth index:

$$
\begin{equation*}
\int_{0}^{\pi} d \theta_{p} \sqrt{\rho\left(\theta_{p}\right)} \sin \theta_{p} X_{m}\left(\theta_{p}\right) G_{m}\left(\theta_{q}, \theta_{p}\right)=\frac{1}{2 \pi} \Psi_{m}\left(\theta_{q}\right), \quad m=0, \pm 1, \pm 2, \ldots, \quad q \in S . \tag{3}
\end{equation*}
$$

Asymptotic estimates show that when the point $p$ tends to the point $q$ fixed somewhere on the shell, the azimuth angle Fourier coefficients of Green's function $G_{m}\left(\theta_{q}, \theta_{p}\right)$ exhibit two kinds of singularities: a rational fraction singularity on the "top" or "bottom" of the shell $\left(\theta_{q} \in O Z\right)$, and otherwise a logarithmic one:

$$
G_{m}\left(\theta_{q}, \theta_{p}\right)=\left\{\begin{array}{l}
\left(4 \pi \rho\left(\theta_{p}\right) \sin \theta_{p}\right)^{-1} \delta_{m}^{0}, \quad \text { when } \theta_{q} \in O Z, \\
-\left(8 \pi^{2} \sqrt{\rho\left(\theta_{q}\right) \rho\left(\theta_{p}\right) \sin \theta_{q} \sin \theta_{p}}\right)^{-1} \ln \left|\theta_{q}-\theta_{p}\right|+F_{m}\left(\theta_{q}, \theta_{p}\right), \text { when } \theta_{q} \notin O Z,
\end{array}\right.
$$

when $\left|\theta_{p}-\theta_{q}\right| \ll 1$; here $F_{m}\left(\theta_{q}, \theta_{p}\right)$ is a smooth function, and $\delta_{m}^{0}$ is the Kronecker symbol. Introducing the remainder

$$
D_{m}\left(\theta_{q}, \theta_{p}\right)=\xi\left(\theta_{q}\right) \xi\left(\theta_{p}\right) \times G_{m}\left(\theta_{q}, \theta_{p}\right)-G_{m}^{0}\left(\theta_{q}, \theta_{p}\right), \quad m=0, \pm 1, \pm 2, \ldots
$$

where $\xi(\theta)=\sqrt{\rho(\theta)}$ is a scale factor, it is simple to check that the difference $D_{m}\left(\theta_{q}, \theta_{p}\right)$ between the rescaled Green's function for the shell of revolution and the Green's function for the unit sphere $G_{m}^{0}\left(\theta_{q}, \theta_{p}\right)$ is a smooth function.

Now we can rewrite the integral equation (3) in the form

$$
\begin{equation*}
\int_{0}^{\pi} d \theta_{p} \sin \theta_{p} X_{m}\left(\theta_{p}\right)\left\{G_{m}^{0}\left(\theta_{q}, \theta_{p}\right)+D_{m}\left(\theta_{q}, \theta_{p}\right)\right\}=\widetilde{\Psi}_{m}\left(\theta_{q}\right), \quad m=0, \pm 1, \pm 2, \ldots, q \in S \tag{4}
\end{equation*}
$$

where $\tilde{\Psi}_{m}\left(\theta_{q}\right)=\xi\left(\theta_{q}\right) \Psi_{m}\left(\theta_{q}\right) / 2 \pi$. Thus the kernel of (4) is separated into $G_{m}^{0}\left(\theta_{q}, \theta_{p}\right)$ - a spherical canonical part that involves all the singularities and can be effectively treated - and the continuous remainder $D_{m}\left(\theta_{q}, \theta_{p}\right)$. The idea of such a transformation of the initial equation (1) was first suggested by Yu.A. Tuchkin [4].

The functions $G_{m}^{0}\left(\theta_{q}, \theta_{p}\right)$ and $D_{m}\left(\theta_{q}, \theta_{p}\right)$ are represented in the radial coordinate $r$ "discontinuous form" [2], as

$$
G_{m}^{0}\left(\theta_{q}, \theta_{p}\right)=\frac{1}{4 \pi} \sum_{n=m}^{\infty} \frac{2}{2 n+1} \hat{P}_{n}^{m}\left(\cos \theta_{q}\right) \hat{P}_{n}^{m}\left(\cos \theta_{p}\right), D_{m}\left(\theta_{q}, \theta_{p}\right)=\frac{1}{4 \pi} \sum_{k, l=m}^{\infty} A_{k l}^{m} \hat{P}_{k}^{m}\left(\cos \theta_{q}\right) \hat{P}_{l}^{m}\left(\cos \theta_{p}\right), \theta_{q, p} \in[0, \pi],
$$

where $\hat{P}_{n}^{m}(x)$ is the normalised associated Legendre polynomial. For calculation of the Fourier-Legendre coefficients of the smooth remainder it is possible to obtain the following expressions when $m=0$ :

$$
\begin{gathered}
A_{k l}^{0}=4 \pi \int_{0}^{\pi} \int_{0}^{\pi} d \theta_{q} d \theta_{p} \sin \theta_{q} \sin \theta_{p} \hat{P}_{k}\left(\cos \theta_{q}\right) \hat{P}_{l}\left(\cos \theta_{p}\right) \times D_{0}\left(\theta_{q}, \theta_{p}\right), \\
D_{0}\left(\theta_{q}, \theta_{p}\right)=\frac{1}{2 \pi^{2}}\left\{\xi\left(\theta_{q}\right) \xi\left(\theta_{p}\right) \alpha^{S}\left(\theta_{q}, \theta_{p}\right) K\left(\beta^{S}\left(\theta_{q}, \theta_{p}\right)\right)-\alpha^{0}\left(\theta_{q}, \theta_{p}\right) K\left(\beta^{0}\left(\theta_{q}, \theta_{p}\right)\right)\right\} .
\end{gathered}
$$

Here $K(x)$ is an elliptic integral of the first kind, and the following values are introduced:

$$
\begin{aligned}
& \alpha^{S}\left(\theta_{q}, \theta_{p}\right)=\left[\left\{\rho\left(\theta_{q}\right)\right\}^{2}+\left\{\rho\left(\theta_{p}\right)\right\}^{2}-2 \rho\left(\theta_{q}\right) \rho\left(\theta_{p}\right) \cos \left(\theta_{q}+\theta_{p}\right)\right]^{-1 / 2}, \beta^{S}\left(\theta_{q}, \theta_{p}\right)=4 \rho\left(\theta_{q}\right) \rho\left(\theta_{p}\right) \sin \theta_{q} \sin \theta_{p}\left[\alpha^{S}\left(\theta_{q}, \theta_{p}\right)\right]^{2} \\
& \alpha^{0}\left(\theta_{q}, \theta_{p}\right)=\left[2 \sin \frac{\theta_{q}+\theta_{p}}{2}\right]^{-1}, \beta^{0}\left(\theta_{q}, \theta_{p}\right)=1-\left[\left(\sin \frac{\theta_{q}-\theta_{p}}{2}\right)\left(\sin \frac{\theta_{q}+\theta_{p}}{2}\right)^{-1}\right] 2, \theta_{q}, \theta_{p} \in[0, \pi]
\end{aligned}
$$

When the points $q$ and $p$ are close together, we can obtain the following expansion in $\delta=\left(\theta_{q}-\theta_{p}\right) / 2 \ll 1$ :

$$
D_{0}\left(\theta_{q}, \theta_{p}\right)=\frac{1}{4 \pi^{2} \sin \theta_{q}} \ln \left(\frac{\rho\left(\theta_{q}\right)}{\lambda\left(\theta_{q}\right)}\right)+\frac{\cos \theta_{q}}{4 \pi^{2} \sin ^{2} \theta_{q}}\left\{\left[\rho\left(\theta_{q}\right)-1\right] \ln \left(\frac{4 \sin \theta_{q}}{|\delta|}\right)+\rho\left(\theta_{q}\right) \ln \left(\frac{\rho\left(\theta_{q}\right)}{\lambda\left(\theta_{q}\right)}\right)\right\} \times \delta+o\left(\delta^{2}\right)
$$

Finally expanding the unknown function $X_{m}$, and the right hand side of (4) in the Fourier-Legendre series,

$$
\begin{equation*}
X_{m}\left(\theta_{p}\right)=\sum_{n=m}^{\infty} x_{n}^{m} \hat{P}_{n}^{m}\left(\cos \theta_{p}\right), \quad \tilde{\Psi}_{m}\left(\theta_{q}\right)=\frac{1}{4 \pi} \sum_{n=m}^{\infty} a_{n}^{m} \hat{P}_{n}^{m}\left(\cos \theta_{q}\right), \quad a_{n}^{m}=2 \int_{0}^{\pi} d \theta \sin \theta \hat{P}_{n}^{m}(\cos \theta) \times \xi(\theta) \Psi_{m}(\theta) \tag{5}
\end{equation*}
$$

the integral equation (4) can be reduced to the following system of dual series equations for unknowns $x_{n}^{m}$. This system is equivalent to an operator equation of the first kind in the Hilbert space given by the Meixner condition [1]:

$$
\begin{cases}\sum_{n=m}^{\infty} \frac{2}{2 n+1} x_{n}^{m} \hat{P}_{n}^{m}(\cos \theta)=\sum_{n=m}^{\infty}\left(a_{n}^{m}-\sum_{l=m}^{\infty} A_{n l}^{m} x_{l}^{m}\right) \hat{P}_{n}^{m}(\cos \theta), \quad \theta \in\left[0, \theta_{0}\right]  \tag{6}\\ \sum_{n=m}^{\infty} x_{n}^{m} P_{n}^{m}(\cos \theta)=0, & \theta \in\left(\theta_{0}, \pi\right] .\end{cases}
$$

The potential on the shell $\Psi(q)$ is assumed to depend only on zonal angle $\theta$, and also due to the symmetry of the shell, the unknown function $X(p)$ is not a function of azimuth angle $\varphi$; thus $x_{n}^{m}=0$ when $m \neq 0$. The system (6) is ill-conditioned, therefore the standard truncation technique is generally inappropriate. The form of the obtained system allows us to apply the standard analytical regularisation procedure described in [2], based on the transformations connected with Abel's integral equation, which helps us to get rid of this ill-conditioning and arrive at the algebraic system

$$
\begin{equation*}
\hat{x}_{m}+\sum_{n=0}^{\infty} W_{m n}\left(\theta_{0}\right) \hat{x}_{n}=V_{m}\left(\theta_{0}\right), \tag{7}
\end{equation*}
$$

where we introduce the following notations: $\hat{x}_{m}=(m+1 / 2)^{-1 / 2} x_{m}^{0}, W_{m n}\left(\theta_{0}\right)=\sqrt{n+1 / 2} \sum_{l=0}^{\infty} \sqrt{l+1 / 2} A_{l n}^{0} Q_{m l}^{+}\left(\theta_{0}\right)$,

$$
Q_{m n}^{+}\left(\theta_{0}\right)=\frac{1}{\pi}\left\{\frac{\sin (m-n) \theta_{0}}{m-n}+\frac{\sin (m+n+1) \theta_{0}}{m+n+1}\right\}, V_{m}\left(\theta_{0}\right)=\sum_{l=0}^{\infty} \sqrt{l+1 / 2} a_{l}^{0} Q_{m l}^{+}\left(\theta_{0}\right) .
$$

The analysis of the matrix $W_{m n}\left(\theta_{0}\right)$ shows that the system (7) is equivalent to the Fredholm operator equation of the second kind: thus the coefficients of the expansion of charge density on the open shell can be effectively calculated with any prescribed accuracy by means of the truncation procedure.


Fig.2. Potential distribution $U$ and relative error $\delta U=\left|U-U^{0}\right| /\left|U^{0}\right|$, where $U^{0}$ is calculated according to [2]:
a)- along line segment oA as a function of distance $r_{O A}$; b)- along circle arc $B C$ of radius 0.75 as a function of $\theta$.

The series $X(\theta)$ in (5), describing the charge density, has to exhibit at the shell edge a singularity prescribed by the Meixner condition, and must vanish on the aperture. To see this behaviour directly requires a large number of terms in the sum (5). However, utilizing a known discontinuous series [2], the charge density calculation can be transformed to

$$
\begin{equation*}
\sigma_{D}\left(\theta, \theta_{0}\right)=-\frac{1}{\Lambda(\theta)}\left\{\frac{\sqrt{2} H\left(\theta_{0}-\theta\right)}{\pi \sqrt{\cos \theta-\cos \theta_{0}}} \sum_{m=0}^{\infty} \chi_{m} \cos (m+1 / 2) \theta_{0}-\sum_{m=0}^{\infty} \chi_{m}(m+1 / 2) S_{m}\left(\theta, \theta_{0}\right)\right\} \tag{8}
\end{equation*}
$$

where $H\left(\theta_{0}-\theta\right)$ is the Heaviside function, and the elements of the vectors $\chi_{m}$ and $S_{m}\left(\theta, \theta_{0}\right)$ are defined as

$$
\begin{aligned}
& \chi_{m}=\sqrt{m+1 / 2} a_{m}-\sqrt{m+1 / 2} \sum_{n=0}^{\infty} A_{m n} x_{n}, \quad S_{0}\left(\theta, \theta_{0}\right)=1-\frac{2}{\pi} \arcsin \left(\frac{\cos \left(\theta_{0} / 2\right)}{\cos (\theta / 2)}\right), \\
& S_{m}\left(\theta, \theta_{0}\right)=\frac{2}{\pi} \frac{\cos (m-1 / 2) \theta_{0}}{m} \sqrt{2\left(\cos \theta-\cos \theta_{0}\right)}+\frac{2 m-1}{m} \cos \theta S_{m-1}\left(\theta, \theta_{0}\right)-\frac{m-1}{m} \cos \theta S_{m-2}\left(\theta, \theta_{0}\right) .
\end{aligned}
$$

In the representation (8) the singularity is extracted in an explicit form. The swift convergence of the calculation process for $\chi_{m}$ and for $S_{m}\left(\theta, \theta_{0}\right)$ from its recurrence relation makes the calculation of $\sigma_{D}\left(\theta, \theta_{0}\right)$ very effective.

## 4. Numerical results

The approach was validated by comparison against the results obtained for canonical geometry structures, for example, the perfectly conducting open prolate spheroid: $\rho(\theta)=a b / \sqrt{a^{2}-\left(a^{2}-b^{2}\right) \cos ^{2} \theta}, a \geq b$ (see Fig.2).


Fig.3. (a) Surface charge density $\sigma_{D}$ versus $\theta$ for different shell shapes. (b) Potential space distribution $U$ for $a=0.53$.
More, and wider, possibilities of our approach may be demonstrated by the numerical investigation of the open noncanonical geometry shell obtained by rotation of the "Pascal Limacon": $\rho(\theta)=a+b \cos (\theta-\pi), a \geq b$ (see Fig.3).

## 5. Conclusion

A mathematically rigorous and numerically efficient approach for solving the electrostatic potential problem with a Dirichlet boundary condition on the arbitrary shaped open shell of revolution is described. By considering the singularities of the Green's function, we transformed the initial integral equation to a form that permits separation of the integral kernel into a singular part, which is the Green's function for the unit sphere, and a continuous remainder. The smoothness of the remainder ensures that the calculation of its Fourier coefficients is efficient and effective.

We transformed the integral equation to a system of dual series equations, and then applying a standard analytical regularisation procedure, transformed it to an algebraic system of the second kind Fredholm type, which can be solved numerically by means of the truncation method, with any prescribed accuracy. Series convergence was improved by utilising a certain discontinuous series enabling us to extract explicitly the charge density singularity at the shell edge.

This general approach effectively parallels that developed previously for canonical shells. The accuracy and possibilities of this approach were demonstrated by the examples of the prolate spheroid and the non-canonical geometry body obtained by rotation of the "Pascal Limacon". Although important in itself, this method provides the first, and perhaps the major, step in a comparable analysis of full wave scattering problems for open shells of revolution.

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# Analysis of Common Mode Propagation Based on Single Conductor Line 

T. Hisakado and K. Yoshimura<br>Department of Electrical Engineering, Kyoto University, Japan<br>K. Okumura<br>Department of Information Science, Hiroshima Institute of Technology, Japan


#### Abstract

The common mode radiation is an important problem for reducing EMI. In order to clarify the mechanism of the common mode propagation which has no explicit return path, this paper analyzes a propagation wave on a single conductor line which also has no explicit return path. We derive an integral equation in the complex frequency domain using boundary conditions on the conductor surface and obtain the time domain current waveform using the numerical Laplace transform. This method figures out the source of electric field on the conductor surface, and makes clear the mechanism of the current propagation of the common mode.


## 1. Introduction

The common mode radiation is an important problem for reducing EMI [1]. In the case of two conductor transmission lines, the common mode has no explicit return path whereas the differential mode has the obvious return path. The purpose of this paper is to clarify the mechanism of common mode propagation which has no explicit return path. In order to simplify the problem, we consider a single conductor line which also has no explicit return path. A. Sommerfeld originally studied the modes on single conductor lines using Hankel function [2] and G. Goubau analyzed the surface waves in detail [3]. However, because these researches deal with an infinite single conductor line, it is not clear how the modes are generated and propagate on the single conductor line with a current source. We reveal the mechanism using a semi-infinite single conductor line.

First, by a real simple experiment we observe the generation of a common mode on two conductor lines with dielectric coat. This experiment clarifies the generation of the common mode in a simple circuit. In order to reveal the mechanism of the generation and propagation of the common mode, we consider a semi-infinite cylindrical perfect single conductor line and apply current. Using the electric field generated by the current and the charge on the central axis of the single conductor line, we derive an integral equation based on the boundary condition on the surface of the cylindrical perfect conductor. We solve the integral equation numerically in the complex frequency domain and obtain the waveforms in time domain using numerical inversion of Laplace transform [4].

The proposed method figures out the source of the electric field on the conductor surface. Using the property of the proposed method, we reveal the cause-and-effect relationship of current propagation based on the model that the source current causes the electric field and then the electric field generates induced current. The recursively induced current leads to the wave propagation without the explicit return path.

## 2. Observation of Common Mode by Simple Experiment

We observe the generation of a common mode on two conductor lines. The setup of the experiment considered here is shown in Fig.1. A step-like voltage is applied to the lines ( $l=1.41 \mathrm{~m}$ ) and we observe the voltage at the near end $\left(V_{i}\right)$ and the far end $\left(V_{o}\right)$. It must be noted that $V_{o}$ is the voltage between terminal 2 and the ground of the oscilloscope. The cross section of the two conductor lines, which are cylindrical conductors without and with $\operatorname{dielectric}\left(\epsilon_{r}=2.4, \mu_{r}=1.0\right)$, are shown in Fig.2.

Figure 3 illustrates the observed waveform of $V_{i}$ and $V_{o}$. In the case without dielectric, the wave propagates close to the light speed $c$. On the other hand, in the case of the conductor coated by the dielectric, the leading edge of the pulse at the far end ( $V_{o}$ ) consists of two components; one rises at 485 nsec and the other rises at 487 nsec . Because the first component propagates close to the light speed despite of the dielectric coat, this component is considered to be the common mode. The second component corresponds to the differential mode whose propagation velocity is about $2.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$ which almost agrees with $c / \sqrt{\epsilon_{r} \mu_{r}}$. Thus, we can confirm the generation of the common mode using. this simple experiment.


Figure 1: Measurement setup for common mode observation.


Figure 2: Cross section of two conductor lines without and with dielectric.


Figure 3: Observed voltage $V_{i}$ and $V_{o}$ for conductor without and with dielectric.

## 3. Analysis of Single Conductor Line by Laplace Transform

The common mode observed on the two conductor lines is equivalent to the mode on a single conductor line in the sense that the both modes propagate without explicit return paths. In order to reveal the mechanism of the wave propagation without the explicit return path, we analyze the single conductor line.

We consider a semi-infinite cylindrical perfect conductor line with radius $R$ shown in Figure 4 and apply current $I_{0}$ at the origin.


Figure 4: Semi-infinite conductor line.

In order to simplify the analysis, we apply the thin wire approximation, i.e., using electric field generated by the current $I(z, t)$ and charge $\lambda(z, t)$ on the central axis of the single conductor line, we derive an integral equation based on the boundary condition on the surface of cylindrical perfect conductor.

The z-component of the vector potential $A_{z}$ and scalar potential $\phi$ at $z=z_{0}$ on the surface in complex frequency domain are represented by

$$
\begin{equation*}
A_{z}\left(z_{0}, s\right)=\frac{\mu_{0}}{4 \pi} \int_{0}^{\infty} \frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}} I(\xi, s) \mathrm{d} \xi, \quad \phi\left(z_{0}, s\right)=\frac{1}{4 \pi \varepsilon_{0}} \int_{0}^{\infty} \frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}} \lambda(\xi, s) \mathrm{d} \xi, \tag{1}
\end{equation*}
$$

where $s$ denotes complex frequency and $\hat{r}=\sqrt{\left(z_{0}-\xi\right)^{2}+R^{2}}$. The z-component of the electric field $E_{z}$ is represented by

$$
\begin{equation*}
E_{z}\left(z_{0}, s\right)=-\left.\frac{\partial \phi(z, s)}{\partial z}\right|_{z=z_{0}}-c\left\{s A_{z}\left(z_{0}, s\right)-\left.A_{z}\left(z_{0}, \tau\right)\right|_{\tau=0}\right\} \tag{2}
\end{equation*}
$$

where the initial condition $\left.A\left(z_{0}, \tau\right)\right|_{\tau=0}=0$ is satisfied. The current $I(z, s)$ and the charge $\lambda(z, s)$ satisfy the relation

$$
\begin{equation*}
\left.\frac{\partial I(z, s)}{\partial z}\right|_{z=z_{0}}+c\left\{s \lambda\left(z_{0}, s\right)-\left.\lambda\left(z_{0}, \tau\right)\right|_{\tau=0}\right\}=0 \tag{3}
\end{equation*}
$$

where the condition $\left.\lambda\left(z_{0}, \tau\right)\right|_{\tau=0}=0$ is satisfied on $z_{0}=0$. Substituting Eq.(3) into the $\lambda$ in Eq.(1), we obtain

$$
\begin{align*}
-\left.\frac{\partial \phi(z, s)}{\partial z}\right|_{z=z_{0}} & =-\frac{1}{4 \pi \varepsilon_{0}} \frac{\partial}{\partial z} \int_{0}^{\infty} \frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}\left\{-\frac{1}{c s} \frac{\partial I(\xi, s)}{\partial \xi}\right\} \mathrm{d} \xi \\
& =\frac{1}{4 \pi \varepsilon_{0} c s} \int_{0}^{\infty}\left(\frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}\right)^{\prime}\left\{\frac{\partial I(\xi, s)}{\partial \xi}\right\} \mathrm{d} \xi \tag{4}
\end{align*}
$$

where

$$
\begin{equation*}
\left(\frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}\right)^{\prime} \equiv \frac{\partial}{\partial z}\left(\frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}\right)=-\frac{(1+\hat{r} s)\left(z_{0}-\xi\right)}{\hat{r}^{3}} \mathrm{e}^{-\hat{r} s} \tag{5}
\end{equation*}
$$

Using partial integration of Eq.(4), we can eliminate the differentiation by $\xi$;

$$
\begin{equation*}
-\left.\frac{\partial \phi(z, s)}{\partial z}\right|_{z=z_{0}}=\frac{1}{4 \pi \varepsilon_{0} c s}\left[\left(\frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}\right)^{\prime} I(\xi, s)\right]_{0}^{\infty}-\frac{1}{4 \pi \varepsilon_{0} c s} \int_{0}^{\infty} \frac{\partial}{\partial \xi}\left(\frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}\right)^{\prime} I(\xi, s) \mathrm{d} \xi . \tag{6}
\end{equation*}
$$

The boundary conditions are $I(0, s)=I_{0}(s) \equiv \mathcal{L}\left\{I_{0}(t)\right\}$ and $\left.I(z, s)\right|_{z=\infty}=0$, where $\mathcal{L}$ denotes the Laplace transform. Substituting Eq.(6) and $A_{z}$ in Eq.(1) into Eq.(2) and using the boundary condition $E_{z}=0$ on the surface of the perfect conductor, we obtain an integral equation

$$
\begin{equation*}
\int_{0}^{\infty}\left\{s \frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}+\frac{1}{s} \frac{\partial}{\partial \xi}\left(\frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}\right)^{\prime}\right\} I(\xi, s) \mathrm{d} \xi=-\left.\left.\frac{1}{s}\left(\frac{\mathrm{e}^{-\hat{r} s}}{\hat{r}}\right)^{\prime}\right|_{\xi=0} I(\xi, s)\right|_{\xi=0} \tag{7}
\end{equation*}
$$

We solve the integral equation numerically in the complex frequency domain and obtain the waveforms in time domain using numerical inversion of Laplace transform[4].

In order to confirm the above method, we compare the computational results by the proposed method and those by FDTD method. Figure 5 represents the current $I$ and the charge $\lambda$ in time domain on $z=1.5 \times 10^{-2} \mathrm{~m}$ when a step-like current is applied at the origin of the single conductor line with radius $R=0.75 \times 10^{-3} \mathrm{~m}$. We can confirm that the results by the proposed method almost agree with the results by FDTD method.


Figure 5: Comparison of the time domain current and charge between the proposed method (Laplace) and FDTD method on $z=1.5 \times 10^{-2} \mathrm{~m}$.

## 4. Mechanism of Wave Propagation without Explicit Return Path

In order to clarify the property of the wave propagation on the single conductor line, we calculate the current and charge waveform by applying the rigorous step current $I_{0}=1 \mathrm{~A}$ at the origin. The results calculated by the proposed method are shown in Figures 6 and 7. From these figures we can confirm that the step current propagates with distortion. The charge accumulation in the vicinity of the origin in Fig. 7 is linked to the distortion of the current waveform.


Figure 6: Current waveform by step input.


Figure 7: Charge waveform by step input.

In the proposed method, the electric field $E_{z}=0$ is satisfied on the surface. That is, the $E_{z}$ generated in one region balances the $E_{z}$ generated by the other regions. In other word, the proposed method is based on the model that the current and charge generate the electric field and then the electric field causes the induced current.

In order to clarify the source of $E_{z}$ on a region, we divide the single conductor line into 5 regions shown in Fig.8. Let us consider the electric field $E_{z}$ on the small region (4) which is caused by the current and charge in each region. The electric field $E_{z}$ on the region (4) generated by the each region is shown in Fig.9. The "by $A$ " and "by $\phi$ " in the figure denote the component of the $E_{z}$ generated by the vector potential $A$ due to the current $I$ and the component of the $E_{z}$ generated by the scalar potential $\phi$ due to the charge $\lambda$, respectively. The "by $A+\phi$ " represents the total $E_{z}$. The peak of the waveform corresponds to the pass of the rising edge of the wave in each region.

In the case of the regions from (1) to (3), the scalar potential $\phi$ generates positive $E_{z}$ on the region (4), although the components of $E_{z}$ due to $A$ and $\phi$ almost cancel each other. This means that the accumulated charge generates the positive $E_{z}$ on the region (4). On the other hand, the current and charge in the region (4) generates negative $E_{z}$ when the rising edge comes in the region (4). The region (5) generates negative $E_{z}$ after the rising edge passes the region (4) and cancel the positive $E_{z}$ generated by the regions from (1) to (3). Thus, the electric field generated by the accumulated charge mainly induce the current on the rising edge and the recursive induction causes the current propagation.


Figure 8: The divided 5 regions of the single conductor line.


Figure 9: The electric field $E_{z}$ in the region (4) caused by each region.

## 5. Conclusion

First we observed the common mode propagation in a simple experiment. Next, in order to clarify the mechanism of the wave propagation without the explicit return path, we propose a method which gives the transient waveform using the numerical inversion of Laplace transform. Using the method we analyzed the current propagation on a single conductor line. Because the method figures out the source of the electric field on each region, the method reveals the cause-and-effect relationship of the current induction. On the other hand, the FDTD method is based on only the action through medium and does not give the above cause-and-effect relationship. Using the cause-and-effect relationship, we reveal the mechanism of the wave propagation based on the model in which the current and charge causes the electric field, and the electric field causes the induced current. Especially, we clarified that the electric field caused by the accumulated charge plays a large part for the induction.

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# Analysis on 20H Rule Applied Printed Circuit Board 

Shinichi Ikami, Akihisa Sakurai<br>Technology Development Center, IBM Japan, Yamato, Kanagawa, Japan


#### Abstract

The effect of so-called " 20 H rule" was studied with EM solver using MoM. The rule recommends that the power plane should be shrunk against the ground plane by 20 times of the distance between the planes to realize lower EMI radiation. Bypass capacitors were populated between power and ground planes to simulate a realistic printed circuit board. From the evaluation of the electric and magnetic dipole moment, the contributions of common and differential modes current were calculated. It was found that there arises common mode current in the ground plane when the 20 H rule was applied. At the resonance frequencies, the common and differential mode radiations are observed in relation to its resonance mode, the common mode radiation due to y - or x -directional electric dipole moment for TM01 or TM10 respectively. Considering the common mode current occurrence is a fundamental mechanism of the electromagnetic radiation, it is concluded that the application of the rule increases the radiation.


## I. Introduction

Recently the high-speed and large power devices have become common for IT devices. Due to the usage of differential signaling for high speed connection, the EMI radiation from the signal trace becomes not a major issue of the radiate EMI, while the power noise from the high-speed, high-power LSI becomes a major one. The high speed switching in the LSI causes a large and high-frequency power noise. The noise flows out to the power and ground planes of the printed circuit board. As the planes are usually the largest patterns in the card, it can be the most effective radiator or antenna in the system. Many works have been done on the radiation from the power planes. From the EMI design point of view, the placement of the bypass capacitors between power and ground planes are the key for the suppression of EMI radiation. 20 H rule, a unique methodology was proposed on the size of the planes. The rule was introduced in an EMC textbook [1]. It states that the physical size of power plane of a multilayered PCB should be smaller than the ground plane as much as 20 times the distance between the both planes, H , to reduce electromagnetic emission from the fringe. The rule is attractive because it is so simple to realize in the actual design, its effectiveness has often been discussed [2,3]. The power and ground planes build a parallel plane structure. The structure shows the cavity resonance. An analysis on the internal-field in the cavity, Q-value and radiated EMI was done on the simple structure of planes and voltage source [4], no effectiveness can be found about the rule. In this paper, the EMI-radiation will be discussed on the more realistic case which has bypass capacitors between the planes using a 3D electromagnetic solver [5]. The solver employs method of moments and it can handle dielectrics. The current distribution is analyzed in detail from the electromagnetic radiation point of view.

## II. Simulation Model and Parameters:

Total of 4 models were studied; (1) The same size power and ground planes excited by a source, (2) bypass capacitors were populated evenly on it, (3) 20 H ruled power and ground planes without bypass capacitors and (4) 20 H -ruled planes with bypass capacitors. The calculation models for the cases (2) and (4) were shown in Figure 1.


Figure 1: Calculation models

The printed circuit board has 5 cm by 9 cm large power and ground planes. The planes were separated by 0.4 mm thick substrate. The dielectric constant of the substrate was set to 4.2 and its loss tangent was set to 0.01 as simulate the typical FR4. The model has a voltage source between power and ground planes with output resistance of 1ohm. The amplitude of the source voltage is 1 V . The 24 bypass capacitors of 10000 pF are distributed evenly with 2 cm separations. The ESR and ESL of the capacitors are set to be 1 mOhm and 1 nH respectively. For 20 H -ruled models, the same size of power plane are used to eliminate the effect of resonance frequency shift result from the power plane shrink.

## III. Simulation Results - Radiated E-Field

The calculation was done for the 200 points of frequency of 10 MHz to 2 GHz with 10 MHz step. The radiated E-Field for 512points of 3 m distance was calculated. The maximum value was selected for the radiated EMI for the frequency. Figure 2 shows the result for the effect of bypass capacitors.


Figure 2: The effectiveness of bypass capacitors
For frequencies less than about 1 GHz , the bypass-capacitors suppress the radiated-EMI as expected. However above the frequency, the resonance plays a main roll and no obvious advantage can be found. As an EMI design rule, distribution of the bypass capacitor with density of $1 / 10$ of the wavelength of target frequency. The 2 cm corresponds to 730 MHz in the FR4 with dielectric constant of 4.2 , and the rule can be confirmed.

The effectiveness of 20 H rule is shown in Figure 3. The radiated EMI is equal or larger for the 20 H -ruled models regardless of application of bypass capacitors. For lower frequencies where no resonance happens, the radiation is almost the same. For higher frequencies where the resonance occurs, the 20 H rule rather increases the radiation.


Figure 3: The effectiveness of 20 H rule
The power and ground planes in the printed circuit board build a parallel plane resonator. The resonance frequencies are summarized in the table 1. The resonance mode is determined from the current distribution of the planes. It is found that there is a circuit resonance with bypass capacitors. The resonance frequencies increased with bypass capacitors as expected. However,
there still exist the resonances. Mathematically the resonances are determined by the boundary condition of the system. The point connections between the power and ground planes are not so "strong" so that the frequency change is not big for today's highfrequency applications.
On the effect of the 20 H rule, the radiation increases about 5 dB even with the bypass capacitors for parallel plane resonances. It can be noticed that the less increase for TM02 mode. A little change was observed for circuit resonance frequency with 20H rule.

Table 1: Resonance Data

| Resonance <br> Mode | 20 H <br> Rule | No Bypass <br> Capacitors | With Bypass <br> Capacitors |
| :---: | :---: | :---: | :---: |
|  | 0 H | $940 \mathrm{MHz} / 102.7 \mathrm{~dB}$ | $1420 \mathrm{MHz} / 106.2 \mathrm{~dB}$ |
|  | 20 H | $/ 108.1 \mathrm{~dB}$ | $/ 111.1 \mathrm{~dB}$ |
| TM10 | 0 H | $1490 \mathrm{MHz} / 110.9 \mathrm{~dB}$ |  |
|  | 20 H | $1870 \mathrm{MHz} / 110.8 \mathrm{~dB}$ |  |
| TM02 | 0 H | $1640 \mathrm{MHz} / 101.2 \mathrm{~dB}$ | $/ 116.4 \mathrm{~dB}$ |
|  | 20 H | $1650 \mathrm{MHz} / 103.6 \mathrm{~dB}$ | $1990 \mathrm{MHz} / 107.0 \mathrm{~dB}$ |
| TM11 | 0 H | $1840 \mathrm{MHz} / 107.6 \mathrm{~dB}$ |  |
|  | 20 H | $/ 113.1 \mathrm{~dB}$ | Above 2 GHz |
|  | Circuit | 0 H | None |
| Resonance | 20 H |  |  |
|  |  |  |  |

## IV. Radiation Mode Analysis

The above result is a direct observation of the EM simulation. Strictly speaking, the result is only valid for the special case of the models. To extend the results or findings to the general cases, more detailed analysis is required for the mechanism of the EM radiation. For this purpose, the current distribution is analyzed in detail with radiation mode analysis [6].
The result of the radiation mode analysis is shown in Figure 4 and 5 for the models without and with bypass capacitors respectively. The plot "emisim3m" shows the result from the EM solver where the radiated field was calculated directly from the current distribution with free-field Green function. The "CmnX, Y, Z" stands for the far field re-calculation from the x, y and z directional electric dipole moment of the current distribution. Similarly the "LpYZ, XZ, XY" represents the radiated field recalculated from $\mathrm{x}, \mathrm{y}$ and z directional components of the magnetic dipole moments of the current distribution. For example, the "CmnZ" means the radiated field due to the z-directional common mode current and "LpYZ" does the one due to the loop current in YZ plane. The coordinate system is shown in Figure 1. For example, it can be seen from the Figure 4 (a) that the frequency range below 250 MHz , the radiation occurs from the common mode current in z direction. This means that without the bypass capacitor, there is only z-directional current in the voltage source and this current act as a common mode current. As the common mode radiation is the first term of the multi-pole expansion of the electromagnetic radiation, only the 0.4 mm travel-distance is enough to dominate the total radiation of the system.


Figure 4: Radiation mode analysis - No bypass capacitors


Figure 5: Radiation mode analysis - With bypass capacitors
For each frequency range classified as the lower resonance-free region, the cavity resonances and circuit resonances, the main contributors of the total radiation are summarized in Table 2. For the resonance frequencies, the component of each mode is described also in the parentheses.

Table 2: Radiation mode analysis

| Classification | 20 H <br> Rule | No Bypass <br> Capacitors | With Bypass <br> Capacitors |
| :---: | :---: | :---: | :---: |
| Less 300MHz | Same | CommonZ | LoopYZ, LoopXZ |
|  | 20H | Same | LoopYZ(101) |$|$ LoopYZ(101)

As for the effect of bypass capacitors, the radiation mode is the same for each classification. Of course the radiation frequencies are changed as in Table 1, but the basic radiation mechanism keeps. It is natural that the LoopYZ contributes mainly for the TM01 and LoopXZ for the TM10 resonances. In TM02, there are two resonance loop currents canceling each other as shown in Figure 6, the LoopYZ cannot happen and the remaining z-directional common mode current determines the radiation. For the circuit resonance with bypass capacitors, there is in-phase current on all of the capacitors as shown in Figure 7. This means that the planes behave as a large single capacitor. As for the effect of 20 H rule, there arises common mode current in the direction of current distribution of each resonance mode. There arises y-directional common mode current for TM01, and x-directional one for TM10. Figure 8 shows the current distribution on the power and ground planes for 20 H ruled configuration for TM01 at the instance that the maximum common mode current flow. The model is deformed as each mesh has same size. Basically electric current on the power and ground planes at the same XY position cancels each other. As a result, there remains the current on the ground plane outside the power plane covered area, i.e. extended area due to 20 H rule. This current behaves as a common mode current and makes contribution to the total radiation additional to the original radiation due to LoopYZ.
As the mechanism is independent of the resonance, the effect occurs regardless of resonance. There is no radiation increase in lower frequencies. In this frequency range, the loop currents flowing bypass capacitors are large enough and hide the effect of the common mode current radiation of the 20 H rule. It is interesting that the additional common mode contribution has almost the same magnitude as the original contribution of loop current.


Figure 6: The loop current in TM02


Figure 7: Circuit Resonance Current

Common Mode Current


Figure 8: Common Mode Current for 20H ruled board

## V. Conclusion

The simulation analysis was done for the radiation from the power planes. Additional to the usual printed circuit board where the power and ground planes have the same size, so-call 20 H ruled power plane was analyzed. From the typical 4 cases of model configuration and detailed analysis on the current distribution about the radiation mode, the radiation characteristics were studied. The each contribution of common and loop current obtained by the mode analysis is qualitatively natural for each resonance mode. Moreover the analysis suggests quantitative results. As an example, the 20 H rule was analyzed and it was found that the common mode currents on the extended part of the ground plane by which the radiation increased over all frequencies. Also with the realistic condition of distributed bypass capacitors, there arises a circuit resonance where the planes act as a large capacitor, where the in-phase current flow on all of the capacitors. As the result of radiation mechanism is independent of model size parameter, the conclusion can be general enough for EMI design rule definition. As the direct conclusion from the radiation mechanism, it can be concluded that the power plane EMI radiation can be reduced with smaller space between power and ground planes by which the differential mode radiation will be reduced with loop area reduction, and common mode radiation will be reduced according to its travel distance reduction.

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# Common- and Differential-mode Components at Asymmetric Pattern-Layout Lines on PCB 

Fengchao Xiao ${ }^{1} \quad$ Yoshio Kami ${ }^{1}$<br>Department of Information and Communication Engineering, University of Electro-Communications ${ }^{1}$


#### Abstract

A procedure for determining mode of currents flowing in two parallel trace lines on a printed circuit board is discussed. Common- and differential-mode currents are well known as orthogonal mode currents when the line system has symmetrical cross-sectional dimensions. The network function for a line system with asymmetrical dimensions is discussed, and the in-phase and out-of-phase current components are discussed for the case in which terminal conditions are applied to the function.


## 1 Introduction

Currents in trace lines are an important issue in electromagnetic compatibility (EMC). The currents flowing in parallel transmission lines with a symmetrical configuration can be decomposed into two orthogonal modes: differential and common. To estimate the currents, a network function for asymmetric trace lines is obtained using a modal analysis technique. Applying terminal conditions to the network function, we derived the in-phase and out-of-phase currents in the asymmetric trace lines.

## 2 Analysis of Multiconductor Transmission Line

When currents in parallel transmission lines are in a transverse electromagnetic (TEM) mode, the telegrapher's equations describing line voltages and currents hold. Let those lines be in the $x$-direction, and let $\boldsymbol{L}$ and $\boldsymbol{c}$ be the line inductance and capacitance matrices, respectively. Let the line voltage and current vectors, $\boldsymbol{V}$ and $\boldsymbol{I}$, be decomposed into the mode voltage and current vectors, $\boldsymbol{V}_{m}$ and $\boldsymbol{I}_{m}$, as

$$
\begin{equation*}
\boldsymbol{V}=\boldsymbol{T}_{v} \boldsymbol{V}_{m}, \quad \text { and } \quad \boldsymbol{I}=\boldsymbol{T}_{i} \boldsymbol{I}_{m} \tag{1}
\end{equation*}
$$

where $\boldsymbol{T}_{v}$ and $\boldsymbol{T}_{i}$ are the voltage and current conversion matrices, respectively. Then, the telegrapher's equations describing the mode voltages and currents are written as [1]

$$
-\frac{d}{d x}\left[\begin{array}{c}
\boldsymbol{V}_{m}(x)  \tag{2}\\
\boldsymbol{I}_{m}(x)
\end{array}\right]=\left[\begin{array}{cc}
\boldsymbol{O} & j \omega \boldsymbol{T}_{v}^{-1} \boldsymbol{L} \boldsymbol{T}_{i} \\
j \omega \boldsymbol{T}_{i}^{-1} \boldsymbol{c} \boldsymbol{T}_{v} & \boldsymbol{O}
\end{array}\right]\left[\begin{array}{c}
\boldsymbol{V}_{m}(x) \\
\boldsymbol{I}_{m}(x)
\end{array}\right] \equiv\left[\begin{array}{cc}
\boldsymbol{O} & j \omega \boldsymbol{L}_{m} \\
j \omega \boldsymbol{C}_{m} & \boldsymbol{O}
\end{array}\right]\left[\begin{array}{c}
\boldsymbol{V}_{m}(x) \\
\boldsymbol{I}_{m}(x)
\end{array}\right]
$$

If $\boldsymbol{L}_{m}$ and $\boldsymbol{C}_{m}$ in (2) are orthogonal, they correspond to the inductance and capacitance of mode $m$. This means that (2) can be obtained using a procedure similar to that for an ordinary transmission line, for example, in the form of an ABCD matrix:

$$
\left[\begin{array}{c}
V_{m}(0)  \tag{3}\\
I_{m}(0)
\end{array}\right]=\left[\begin{array}{cc}
\cos \beta_{m} x & j Z_{m} \sin \beta_{m} x \\
j \frac{1}{Z_{m}} \sin \beta_{m} x & \cos \beta_{m} x
\end{array}\right]\left[\begin{array}{c}
V_{m}(x) \\
I_{m}(x)
\end{array}\right]=\left[\begin{array}{cc}
A_{m}(x) & B_{m}(x) \\
C_{m}(x) & D_{m}(x)
\end{array}\right]\left[\begin{array}{c}
V_{m}(x) \\
I_{m}(x)
\end{array}\right]
$$

where $Z_{m}=\sqrt{L_{m} / C_{m}}$ and $\beta_{m}=\omega \sqrt{L_{m} C_{m}}$ are the characteristic impedance and phase constant of mode $m$, respectively.

In this procedure, how the conversion matrices, $\boldsymbol{T}_{v}$ and $\boldsymbol{T}_{i}$, are obtained is important. Uchida [2] showed some examples for two and three lines above a ground plane, where the matrices are derived by investigating voltage sources exciting lines in a uniform dielectric material. From (2), the mode voltage and current vectors should be satisfied with

$$
\begin{equation*}
\frac{d^{2} \boldsymbol{V}_{m}}{d x^{2}}+\omega^{2}\left(\boldsymbol{T}_{v}^{-1} \boldsymbol{L} \boldsymbol{c} \boldsymbol{T}_{v}\right) \boldsymbol{V}_{m}=\boldsymbol{O}, \quad \text { and } \quad \frac{d^{2} \boldsymbol{I}_{m}}{d x^{2}}+\omega^{2}\left(\boldsymbol{T}_{i}^{-1} \boldsymbol{c} \boldsymbol{L} \boldsymbol{T}_{i}\right) \boldsymbol{I}_{m}=\boldsymbol{O} \tag{4}
\end{equation*}
$$

The mode voltage and current should propagate simultaneously, that is, at the same velocity, so that the following condition should be satisfied with

$$
\begin{equation*}
\boldsymbol{T}_{v}^{-1} \boldsymbol{L} \boldsymbol{c} \boldsymbol{T}_{v}=\boldsymbol{T}_{i}^{-1} \boldsymbol{c} \boldsymbol{L} \boldsymbol{T}_{i} \equiv\left(\boldsymbol{v}_{m}^{2}\right)^{-1} \tag{5}
\end{equation*}
$$

where $\boldsymbol{v}_{m}$ is an orthogonal matrix whose elements correspond to the mode velocities. For the identity of (5), the following condition should hold.

$$
\begin{equation*}
\boldsymbol{T}_{v}^{-1}=\boldsymbol{T}_{i}^{T} \tag{6}
\end{equation*}
$$

where superscript $T$ denotes the transpose matrix. From the above discussion, we can see that the $\boldsymbol{v}_{m}$ are determined by eigenvalues and the conversion matrices correspond to eigenvectors. In this case, the conversion matrices can be determined mathematically, but it is very hard to determine the excitation sources representing the modes.

The final network function, in the form of an ABCD matrix for line length $\ell$, is

$$
\left[\begin{array}{c}
\boldsymbol{V}(0)  \tag{7}\\
\boldsymbol{I}(0)
\end{array}\right]=\left[\begin{array}{cc}
\boldsymbol{T}_{v} & \boldsymbol{O} \\
\boldsymbol{O} & \boldsymbol{T}_{i}
\end{array}\right]\left[\begin{array}{ll}
\boldsymbol{A}_{m} & \boldsymbol{B}_{m} \\
\boldsymbol{C}_{m} & \boldsymbol{D}_{m}
\end{array}\right]\left[\begin{array}{cc}
\boldsymbol{T}_{v}^{-1} & \boldsymbol{O} \\
\boldsymbol{O} & \boldsymbol{T}_{i}^{-1}
\end{array}\right]\left[\begin{array}{c}
\boldsymbol{V}(\ell) \\
\boldsymbol{I}(\ell)
\end{array}\right] \equiv\left[\begin{array}{cc}
\boldsymbol{A} & \boldsymbol{B} \\
\boldsymbol{C} & \boldsymbol{D}
\end{array}\right]\left[\begin{array}{l}
\boldsymbol{V}(\ell) \\
\boldsymbol{I}(\ell)
\end{array}\right]
$$

where $\boldsymbol{A}_{m} \cdots \boldsymbol{D}_{m}$ consist of the elements of (3). The scattering matrix commonly used for measurement can be estimated from the above ABCD matrix.

## 3 Asymmetric Trace Lines in PCB

In a model with two conductors above a ground plane, there are two orthogonal modes. In the analytical procedure, it is not necessary to determine these modes in advance. However, it is very interesting to do this when considering other phenomena, especially crosstalk and radiation related to EMC. For two-conductor transmission lines with the same cross section, differential and common modes are generally well known. The configuration is as follows [1]: the differential-mode currents in two lines are equal in magnitude and go in opposite directions, and the common-mode currents are equal in magnitude and go in the same direction. When the cross-sectional dimensions of the two lines differ, how to determine the configuration of the modes is a problem. Consider the asymmetrical-line model shown in Fig. 1 (a); only the trace line widths differ. Consider a model where the


Fig. 1 Model of asymmetrical trace lines and modes
dielectric material surrounding the lines is uniform; that is, only one kind of material is used. Let the capacitance matrix be known using a numerical calculation technique such as a finite-element method (FEM):

$$
\boldsymbol{c}=\left[\begin{array}{ll}
c_{11} & c_{12}  \tag{8}\\
c_{21} & c_{22}
\end{array}\right]
$$

where $c_{11} \neq c_{22}$ and $c_{12}=c_{21}$. In many cases, inductance matrix $L$ is obtained from the following, assuming TEM mode holds for that case.

$$
\begin{equation*}
\boldsymbol{L}=\frac{1}{v^{2}} c^{-1} \tag{9}
\end{equation*}
$$

where $v$ is the light velocity in the dielectric material. Next, we consider the mode shown in Figs. 1 (b) and (c). We call them "balanced" and "unbalanced" modes, respectively. From the figure, we can obtain the conversion matrices:

$$
\left[\begin{array}{l}
V_{1}  \tag{10}\\
V_{2}
\end{array}\right]=\left[\begin{array}{cc}
-(1-\nu) & 1 \\
\nu & 1
\end{array}\right]\left[\begin{array}{c}
V_{b} \\
V_{u}
\end{array}\right], \quad \text { and } \quad\left[\begin{array}{l}
I_{1} \\
I_{2}
\end{array}\right]=\left[\begin{array}{cc}
-1 & \nu \\
1 & 1-\nu
\end{array}\right]\left[\begin{array}{c}
I_{b} \\
I_{u}
\end{array}\right]
$$

with

$$
\begin{equation*}
\nu=\frac{c_{12}+c_{22}}{c_{11}+c_{22}+2 c_{12}}=\frac{L_{22}-L_{12}}{L_{11}+L_{22}-2 L_{12}} \tag{11}
\end{equation*}
$$

The relationship given by (10) holds for the conversion matrices given by (5), so the combination of these modes is orthogonal. When $\nu=0.5$, that is, $c_{11}=c_{22}$, the trace lines are symmetrical, so the balanced and unbalanced modes correspond to the differential and common modes, respectively.

For two trace lines on a PCB, there are two dielectric materials surrounding the lines. In such a model, it should be determined whether a set of the above mentioned modes is orthogonal. Generally, the conversion matrices given in (10) depend on the elements of the line capacitance and inductance matrices. For TEM mode transmission, the inductance matrix is determined by applying the capacitance matrix of the lines in free space to the relationship between the velocity and the matrix. That is, the inductance matrix is estimated independently of the dielectric materials surrounding the lines. The conversion matrices determined from the line capacitance and inductance matrices generally do not agree with each other.

Consequently, we can obtain the orthogonal modes mathematically and then establish the network function in the form of an ABCD matrix. Our aim is to estimate the differential- and common-mode currents, so that we can discuss those currents by applying the terminal conditions to the network function. Here, we consider the parallel trace-line model where one line is excited, as shown in Fig. 2 (a). The terminal conditions in general are as

(a) Single-line-excitation source

(b) Unbalanced-mode source with internal resistance

Fig. 2 Source models for balanced and unbalanced modes.

$$
\begin{equation*}
\boldsymbol{V}(0)=\boldsymbol{E}_{g}-\boldsymbol{R}_{0} \boldsymbol{I}(0), \quad \text { and } \quad \boldsymbol{V}(\ell)=\boldsymbol{R}_{\ell} \boldsymbol{I}(0) \tag{12}
\end{equation*}
$$

where $\boldsymbol{R}_{0}$ and $\boldsymbol{R}_{\ell}$ are the load impedance matrices at $x=0$ and $x=\ell$, respectively, and $\boldsymbol{E}_{g}$ is the voltage source matrix. These matrices for a single-line-excitation source (Fig. 2 (a)) are

$$
\boldsymbol{E}_{g}=\left[\begin{array}{c}
E_{g}  \tag{13}\\
0
\end{array}\right], \quad \boldsymbol{R}_{0}=\left[\begin{array}{cc}
R_{g} & 0 \\
0 & R_{2 i}
\end{array}\right], \quad \boldsymbol{R}_{\ell}=\left[\begin{array}{cc}
R_{1 \ell} & 0 \\
0 & R_{2 \ell}
\end{array}\right]
$$

and for an unbalanced-mode source with internal resistance (Fig. 2 (b)) are

$$
\boldsymbol{E}_{g}=\left[\begin{array}{c}
E_{g}  \tag{14}\\
E_{g}
\end{array}\right], \quad \boldsymbol{R}_{0}=\left[\begin{array}{cc}
R_{g} & R_{g} \\
R_{g} & R_{g}
\end{array}\right], \quad \boldsymbol{R}_{\ell}=\left[\begin{array}{cc}
R_{\ell} & R_{\ell} \\
R_{\ell} & R_{\ell}
\end{array}\right]
$$

The transmission and crosstalk characteristics of the single-line-excitation model are often estimated in terms of scattering matrix elements, which are obtained from the ABCD matrix defined in (7).

Substituting (12) into (7) leads to

$$
\begin{align*}
& \boldsymbol{I}(0)=\left(\boldsymbol{C R}_{\ell}+\boldsymbol{D}\right)\left\{\boldsymbol{R}_{0}\left(\boldsymbol{C} \boldsymbol{R}_{\ell}+\boldsymbol{D}\right)+\left(\boldsymbol{A R}_{\ell}+\boldsymbol{B}\right)\right\}^{-1} \boldsymbol{E}_{g}  \tag{15}\\
& \boldsymbol{I}(\ell)=\left\{\boldsymbol{R}_{0}\left(\boldsymbol{C} \boldsymbol{R}_{\ell}+\boldsymbol{D}\right)+\left(\boldsymbol{A} \boldsymbol{R}_{\ell}+\boldsymbol{B}\right)\right\}^{-1} \boldsymbol{E}_{g}
\end{align*}
$$

From (15), the unbalanced-mode current, which is the total current flowing in the same direction, that is, in-phase components, is defined by the summation of $I_{1}$ and $I_{2}$. Generally, the in-phase components in two trace lines are different in magnitude; this differs from the definition of common-mode current. The difference is determined in terms of $\nu$, shown in (10), which corresponds to the ratio of the line current to the unbalanced-mode current, $I_{1} / I_{u}$. The differential-mode current component of the same magnitude and out-of-phase component agrees with the balanced-mode current. We can therefore estimate the mode current using

$$
\begin{equation*}
I_{C M}=I_{u}=I_{1}+I_{2}(\text { for in-phase component }), \quad I_{D M}=I_{b}=(1-\nu) I_{1}-\nu I_{2} . \tag{16}
\end{equation*}
$$

## 4 Experiments and Discussion

We discuss two models. In both, two $100-\mathrm{mm}$-long parallel trace lines are laid out 1 mm apart on a $1.6-\mathrm{mm}$-deep FR4 board. In Model 1, both trace lines are 1 mm wide, and in Model 2, one is 1 , and the other is 5 mm wide. The capacitance matrices of both models were calculated using an FEM:

$$
\boldsymbol{c}_{\text {Model } 1}=\left[\begin{array}{cc}
24.337 & -6.7199  \tag{17}\\
-6.7199 & 2.4337
\end{array}\right] \mathrm{pF} / \mathrm{m} \quad \text { and } \quad \boldsymbol{c}_{\text {Model2 }}=\left[\begin{array}{cc}
24.392 & -7.9309 \\
-7.9309 & 52.269
\end{array}\right] \mathrm{pF} / \mathrm{m}
$$

Based on the $\boldsymbol{c}$ and $\boldsymbol{L}$ matrices, the conversion matrices, or eigenvectors, were estimated using

$$
\begin{align*}
& \boldsymbol{T}_{v}=\left[\begin{array}{rr}
-0.70711 & 0.70711 \\
0.70711 & 0.70711
\end{array}\right], \quad \text { and } \quad \boldsymbol{T}_{i}=\left[\begin{array}{rr}
-0.70711 & 0.70711 \\
0.70711 & 0.70711
\end{array}\right] \quad \text { for Model 1 }  \tag{18}\\
& \boldsymbol{T}_{v}=\left[\begin{array}{rr}
0.63626 & -0.96628 \\
0.77148 & 0.25750
\end{array}\right], \quad \text { and } \quad \boldsymbol{T}_{i}=\left[\begin{array}{rr}
0.25750 & -0.77148 \\
0.96628 & 0.63626
\end{array}\right] \quad \text { for Model 2. } \tag{19}
\end{align*}
$$

For Model 1, the conversion matrices suggest the well-known mode-excitation circuits of differential and common modes, but for Model 2, it is hard to determine the mode-excitation circuits at a glance.

Figure 3 shows the measured transmission characteristics, which are in good agreement with the computed results, which are not shown here. Figure 4 shows the balanced and unbalanced currents for Models 1 and 2 when the source voltage is 1 V .


Fig. 3 Transmission characteristics measured in scattering parameters.


Fig. 4 Balanced- and unbalanced-mode currents when voltage source is 1 V .

## 5 Conclusion

We have described a model of the line currents in asymmetric parallel trace lines on a PCB. Because the independent modes in this model differ from the differential and common modes, the network function was discussed first, and then a procedure to determine the current components was introduced..

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# Extraction of Parasitic and Stray Capacitances from 1-Port Measurements 

U. Paoletti and O. Wada<br>Graduate School of Engineering, Kyoto University, Japan


#### Abstract

A de-embedding technique for the measurement of very small parasitic capacitances of package or small module interconnects is proposed. A probe with bulky ground can strongly affect measurement results. With the help of a few additional measurements with one probe tip open, the capacitances between probe tips and DUT can be estimated together with the parasitic capacitances of interest. The accuracy of the measurement can be also approximately estimated. One example of application is given. The accuracy has been verified with redundant measurements and with simulation results. A high accuracy can be obtained, even when a strong capacitive coupling between probe ground and DUT is present.


## 1 Introduction

Due to the increase in the density of interconnects in IC packages and boards, parasitic capacitive coupling increases. With the increase of clock frequency of digital circuits, the effect of small parasitic capacitances becomes significant. It is therefore important being able to calculate and measure even small capacitances. In order to verify numerical methods for capacitance calculations, it is also important to measure very small capacitances.

The major problem in this type of measurement, is that the capacitive coupling between probe and DUT can affect the results. This effect is larger when the probe presents large metallic parts near the probe tips, typically the ground tip. On the other hand, these measurements present very high repeatability.

For this reason, de-embedding techniques are needed, which allow us to eliminate the effect of the probe. In the following, a technique for extracting very small capacitances will be introduced and validated with measurement and simulation results.

## 2 De-embedding technique

Node pair capacitances can be defined in a multi-conductor system, according to [1], by 'removing' the ground or the reference node from the two-terminal capacitance equivalent circuit. The two-terminal capacitance matrix can be derived from the short-circuit capacitance matrix, which relates the charge on each conductor to the electric potentials of all the conductors, by sign inversion of the off-diagonal terms and by adding the rows for the diagonal terms. Node pair capacitances are important for building equivalent circuits, but cannot be directly measured. In particular, parasitic capacitances of passive interconnects belong usually to this class.

Total capacitance can be defined as the ratio between the charge moved from one conductor to another one, both initially not charged, and their resulting potential difference. This movement of charge is what the voltage generator inside the measurement equipment does, for example an impedance analyzer or a network analyzer. For this reason, it is simpler to extract total capacitance than node pair capacitance from measurement results. In particular, in the present work the total capacitance has been calculated from the reflection coefficient, which has been measured with a vector network analyzer.

Even though an open-short-load (OSL) calibration is conducted, the measured capacitance can be affected by the stray capacitances of the probe. Special and very expensive probes are often used, which are designed for reducing the capacitances between probe tips and between probe tips and DUT, which are usually neglected in the evaluation of measurement results. In the present work, a probe with a bulky ground has been used, whose effect cannot be neglected. The proposed de-embedding procedure allows us to give a better insight into the measurement procedure, and can be applied also to the former type of probes, for estimating their effect on the results.

The simplest calculation that can be done for taking into account the effect of the probe parasitics, is to subtract from the measured capacitance the capacitance between probe tips, or more precisely, the residual capacitance between probe tips after an OSL calibration has been conducted. The residual capacitance can be estimated with one measurement with open probe tips, as it was done also during calibration. This type of calculation will be called in the following as zeroth order approximation.

In the first order approximation, the capacitances between each probe tip and the terminals of the port under consideration are taken into account. In order to calculate these capacitances, four additional measurements are required, with one probe tip on one terminal and the second probe tip touching only the substrate. In these measurements, the capacitance between one probe tip and one port terminal is in parallel to the capacitance between probe tips, which can be measured as for the zeroth order approximation. The four capacitances between probe and DUT can be estimated for each port in this way. A necessary condition, which must be verified before any calculation, is that the measurement results with one open probe tip do not depend on the position of the probe tip on the substrate.

With the first order approximation, only the total capacitance at each port can be calculated. The second order approximation is aimed at the measurement of the node-pair capacitances, and it takes into account the series connection of two node-pair capacitances at maximum. As it will be shown, it is not necessary to consider all the possible combinations. In fact, depending on the measurement results, some of the series connections can be neglected.

Higher order approximations could be defined in the same way. For example, in the third order approximation, the series connections of three node-pair capacitances are also considered. However, the complexity of calculations increases very quickly, and the solution is practicable probably only under particular assumptions depending on the DUT. In the following example, the second order approximation was sufficient for obtaining very accurate results.

## 3 Example of application

The DUT is a four layer module of dimensions $25 \mathrm{~mm} \times 25 \mathrm{~mm}$. The nodes and measurement positions of interest on the first layer are shown in Figure 1. On layers 2 and 4 two ground planes are present, and on layer 3 a power plane. All the layers with vias and interconnects are shown in Figure 1. On the second and the third layers, conductors are indicated in white, whereas on the first and fourth layers they are shadowed. All vias are through vias. The thickness of the FR4 substrate is $0.28 \mathrm{~mm}-1.0 \mathrm{~mm}-0.28 \mathrm{~mm}$.


Figure 1: Module under test with measurement nodes

In the measurements, an ICM TDR probe $\mathrm{P} / \mathrm{N}$ A0113866B with a Cascade Microtech EZ probe positioner has been used. For the calibration the Agilent N1020A-K05 standard substrate has been used. The reflection coefficient has been measured with a network analyzer HP8753D in the frequency range $10 \mathrm{MHz}-1.5 \mathrm{GHz}$. All the measured ports presented a capacitive behavior within an accuracy of 0.05 pF or less, up to a frequency of at least a few hundreds MHz .

The total capacitances calculated from measurement results are listed in Table 1. The position of the probe tips during measurements is indicated by the port nodes. The first node in the name refers to the node connected to the ground tip of the probe, and the second one to the signal tip, respectively. In bold face the measurements that would be conducted by traditional measurement techniques with zeroth order approximation are indicated. Some of the additional measurements are necessary for the present de-embedding technique, and some are redundant and will be used for verifying the results.

Table 1: Capacitances extracted from measurements

| Pos. <br> air | $\begin{aligned} & \hline \text { Cap. } \\ & \hline 0.34 \mathrm{pF} \\ & \hline \end{aligned}$ | Pos. | Cap. | Pos. | Cap. |  | Cap. | $\frac{\text { Pos. }}{\text { G1G2 }}$ | $\frac{\text { Cap. }}{4.25 \mathrm{pF}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
|  |  | G1V1 | 1.40 pF | G2V2 | 25.40 pF | G3V3 | 25.40 pF | G2G1 | 1.43 pF |
|  | 1.40 pF | G1open | 0.36 pF | G2open | 0.34 pF | G3open | 0.36 pF | V1V2 | 3.75 pF |
| G0open | 0.35 pF | openV1 | 1.10 pF | openV2 | 2.90 pF | openV3 | 2.90 pF | V2V1 | 1.38 pF |
| open | 1.10 pF | V1G1 | 1.4 pF | V2G2 | 28.20 pF | V3G3 | 28.10 pF | G1V2 | 3.75 pF |
| V0 | 1.4 p | V1open | 0.41 pF | V2open | 0.35 pF | V3open | 0.35 pF | V2G1 | 1.39 pF |
| V0open | 0.35 pF | openG1 | 1.10 pF | openG2 | 3.25 pF | openG3 | 3.20 pF | G2V1 | 1.42 pF |
| openG0 | 1.10 |  |  |  |  |  |  | V1G2 | 4.30 pF |

It can be noticed, that by inverting the polarity of the probe, different results are obtained in most cases. This is due to the stray capacitances between probe and DUT. The stray capacitances of the probe affecting measurement at G2V2 are shown in figure 2. The effect of nearby nodes, indicated with dots in the figure, are symbolically represented by means of the capacitances to one of the nodes $\left(N_{N}\right)$.

(a) Complete

(b) Simplified

Figure 2: Equivalent circuits for parasitic node-pair capacitances of probe and module
The measurement in air of Table 1 corresponds to the residual capacitance $c_{P s P g}$ between the probe signal tip $\left(P_{s}\right)$ and the probe ground tip $\left(P_{g}\right)$ after calibration, and the measured value was 0.34 pF .

During measurements with signal tip of the probe not connected (e.g. G2open), the results show the parallel connection between $c_{P s P g}$, the capacitance $c_{G 2 P s}$ from measurement node $G_{2}$ to the signal probe tip, and all the series combinations of capacitances from signal tip to node $N_{N}$ and from node $N_{N}$ to probe ground tip or node $G_{2}$. Since it is clear from Table 1 and from the geometry of the module, that the capacitance between ground planes (nodes $G_{2}, G_{3}$ ) and power plane (nodes $V_{2}$ or $V_{3}$ ) is much larger than all the others, only the series connections with the ground/power planes capacitance will be considered.

$$
\begin{equation*}
c_{V 2 P s}+c_{P s P g}+\frac{c_{V 2 G 2} c_{G 2 P s}}{c_{V 2 G 2}+c_{G 2 P s}} \simeq 0.35 p F, \quad c_{G 2 P s}+c_{P s P g}+\frac{c_{V 2 G 2} c_{V 2 P s}}{c_{V 2 G 2}+c_{V 2 P s}} \simeq 0.34 p F \tag{1}
\end{equation*}
$$

These results show that the capacitances between these nodes and signal tip of the probe can be neglected. The simplified equivalent circuit is shown in Figure 2. Similar results can be obtained also for the nodes $G_{3}$, $V_{3}, G_{0}, V_{0}, G_{1}$ and $V_{1}$.

Similarly, during measurements with ground tip not connected (e.g. openV2), the results show the parallel connection between $c_{P s P g}$, the capacitance $c_{V 2 P g}$ from node $V_{2}$ to the ground tip, and all the series connections of capacitances from probe ground to node $N_{N}$ and from node $N_{N}$ to $V_{2}$. Again, it can be assumed, that the series connection containing the capacitance between power and ground planes will dominate. In particular, for the measurements on the nodes $G_{2}$ and $V_{2}$ :

$$
\begin{equation*}
c_{V 2 P g}+c_{P s P g}+\frac{c_{V 2 G 2} c_{G 2 P g}}{c_{V 2 G 2}+c_{G 2 P g}} \simeq 2.9 p F, \quad c_{G 2 P g}+c_{P s P g}+\frac{c_{V 2 G 2} c_{V 2 P g}}{c_{V 2 G 2}+c_{V 2 P g}} \simeq 3.25 p F \tag{2}
\end{equation*}
$$

In order to solve these equations, the assumption that $c_{G 2 V 2} \gg c_{V 2 P g}$ can be used. This is based on the fact that the power plane is on the third layer and is completely covered by the larger ground planes on the second and the fourth layer.

$$
\begin{equation*}
c_{G 2 P g}+c_{P s P g}+c_{V 2 P g} \simeq 3.25 p F \tag{3}
\end{equation*}
$$

These relations can be used for calculating $c_{V 2 G 2}$. In fact, the different measurement results obtained by inverting the polarity of the probe can now be explained.

$$
\begin{equation*}
c_{G 2 V 2}+c_{V 2 P g}+c_{P s P g} \simeq 25.4 p F, \quad c_{V 2 G 2}+c_{G 2 P g}+c_{P s P g} \simeq 28.2 p F \tag{4}
\end{equation*}
$$

By combining Eqs. (3) and (4), the following results are obtained. This solution is consistent with the previous assumption, and within an accuracy of 0.05 pF also with the first of Eq. (2), which has not been used in the calculations.

$$
\begin{equation*}
c_{G 2 P g} \simeq 2.85 p F, \quad c_{V 2 P g} \simeq 0.05 p F, \quad c_{G 2 V 2} \simeq 25.0 p F \tag{5}
\end{equation*}
$$

Similar calculations can be made for the nodes G3 and V3. It is interesting to observe, that these results could have been obtained more easily with the measurements on G2V2 and G3V3, and by taking into account only the capacitance between the probe tips $c_{P g P s}$. Indeed, it was more obvious to connect ground tip of the probe to the large and nearby ground plane than vice versa. However, nothing could have been said in this case about the accuracy of the results. Moreover, for other nodes it is not always obvious to decide where to connect the probe ground, as in the case, for example, of nodes V2V1, which are connected to the power plane on layer 3 and to a trace on layer 1, respectively. For this measurement in particular, the only assumption is that the series between power/ground plane capacitance and two other small capacitances can be neglected, or in other words, that the second order approximation is used.

$$
\begin{align*}
c_{V 1 P g}+c_{P s P g}+\frac{c_{V 1 G 2} c_{G 2 P g}}{c_{V 1 G 2}+c_{G 2 P g}} & \simeq 1.1 p F  \tag{6}\\
c_{V 1 V 2}+c_{V 2 P g}+c_{P s P g}+\frac{c_{V 2 G 2}\left(c_{G 2 P g}+c_{G 2 V 1}\right)}{c_{V 2 G 2}+c_{G 2 P g}+c_{G 2 V 1}} & \simeq 3.75 p F \\
c_{V 2 V 1}+c_{V 1 P g}+c_{P s P g}+\frac{c_{G 2 V 1}\left(c_{G 2 P g}+c_{G 2 V 2}\right)}{c_{V 2 G 2}+c_{G 2 P g}+c_{G 2 V 1}} & \simeq 1.4 p F
\end{align*}
$$

With the use of Eq.(5), the last equation can be rearranged into a second order algebraic equation. Its solutions can be used for calculating the other capacitances. Since one of the solutions brings to negative capacitances, it must be excluded. The module presents two identical traces between the nodes G0G1 and V0V1. This explains why the measurement results relative to G2V1 are very similar to those relative to G2G1. For this reason, the calculations related to the ports G2G1 and G1V2 have the same results.

$$
\begin{equation*}
c_{G 1 G 2} \simeq c_{V 1 G 2} \simeq 0.85 p F, \quad c_{G 1 P g} \simeq c_{V 1 P g} \simeq 0.1 p F, \quad c_{G 1 V 2} \simeq c_{V 1 V 2} \simeq 0.15 p F \tag{7}
\end{equation*}
$$

These results are completely different from those obtained with the zeroth order or the first order approximation. Since these results contain also information about the nodes V1G2, it can be interesting to verify them by using other results in Table 1.

$$
\begin{align*}
c_{V 1 G 2}+c_{G 2 P g}+c_{P s P g}+\frac{c_{V 2 G 2}\left(c_{V 2 P g}+c_{V 2 V 1}\right)}{c_{V 2 G 2}+c_{V 2 P g}+c_{V 2 V 1}} & \simeq 4.3 p F  \tag{8}\\
c_{G 2 V 1}+c_{V 1 P g}+c_{P s P g}+\frac{c_{V 2 V 1}\left(c_{V 2 P g}+c_{G 2 V 2}\right)}{c_{V 2 G 2}+c_{V 2 P g}+c_{V 2 V 1}} & \simeq 1.4 p F
\end{align*}
$$

After substituting Eqs. (5) and (7) into the left-hand side of the last equations, the values $4.25 p F$ and $1.45 p F$ are obtained, which agree within an accuracy of $0.05 p F$ with the values on the right-hand side. Further confirmations of these results can be found in the measurement results in Table 1 relative to the nodes G1V2 and G1G2, which agree with those relative to V1V2 and V1G2 respectively. Finally, the values of $c_{V 1 G 2}$ and $c_{G 1 G 2}$ have been confirmed within an accuracy of 0.05 pF by simulation results as well, which were based on
a combination of pseudo-analytical techniques [2], a two-dimensional finite element method tool (ANSOFT MAXWELL SV) and a three-dimensional boundary element method tool [3].

Some of the measurement results in Table 1 are not affected by the large capacitance between the power and ground planes, because this is in series with two other small capacitances. In particular, this is the case for measurements with none of the probe tips on the ground or power planes, such as G1V1 or G0V0. However, since these capacitances are very small, they can be affected by the capacitance between a ground plane and probe ground tip $\left(c_{G 2 P g}\right)$.

$$
\begin{equation*}
c_{V 1 P g}+c_{P s P g}+\frac{c_{V 1 G 1} c_{G 1 P g}}{c_{V 1 G 1}+c_{G 1 P g}}+\frac{c_{V 1 G 2} c_{G 2 P g}}{c_{V 1 G 2}+c_{G 2 P g}} \simeq 1.1 p F, c_{V 1 G 1}+c_{P s P g}+\frac{\left(c_{V 1 G 2}+c_{G 2 P g}\right) c_{G 1 G 2}}{c_{V 1 G 2}+c_{G 1 G 2}+c_{G 2 P g}} \simeq 1.4 p F \tag{9}
\end{equation*}
$$

With the use of the results in Eqs. (5) and (7), last equation can be used for calculating $c_{G 1 V 1}$, whose value, $0.35 p F$, can be introduced later in the first equation for verifying the accuracy. The results are equal to $1.18 p F$, which agrees with the results on the right-hand side within an accuracy of $0.1 p F$.

With similar arguments, and based on the fact that nodes $G_{1}$ and $G_{0}$ are connected, as well as $V_{1}$ and $V_{0}$, it is possible to extend these results to G0V0. These results have been also confirmed by simulation results within an accuracy of 0.05 pF .

All the results are summarized in Table 2. With the second order approximations, the accuracy is greatly improved. It can be observed, that some of the results can be obtained also with zeroth or first order approximations. However, this is not systematic, and no control on the accuracy is possible.

Table 2: Capacitances in pF after de-embedding

| Pos. | $\begin{gathered} \hline 2^{n d} \\ \text { order } \end{gathered}$ | First order |  | Zeroth order |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Pg Ps | PsPg | PgPs | PsPg |
| G0V0 | 0.35 | 0.3 | 0.3 | 1.05 | 0.75 |
| G1V1 | 0.35 | 0.3 | 0.3 | 1.05 | 0.75 |
| G2V2 | 25.0 | 22.50 | 24.95 | 25.05 | 27.85 |
| G3V3 | 25.0 | 22.50 | 24.90 | 25.05 | 27.75 |


| Pos. | $\begin{gathered} \hline 2^{n d} \\ \text { order } \end{gathered}$ | First order |  | Zeroth order |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | PgPs | PsPg | Pg Ps | PsPg |
| G2G1 | 0.85 | 0.35 | 1.0 | 1.1 | 3.9 |
| G2V1 | 0.85 | 0.30 | 1.05 | 1.05 | 3.95 |
| V1V2 | 0.15 | 0.85 | 0.28 | 3.4 | 1.05 |
| G1V2 | 0.15 | 0.85 | 0.30 | 3.4 | 1.05 |

## 4 Conclusions

With zeroth order approximation, only the residual capacitance between probe tips ( $c_{P s P g}$ ) after calibration is considered. The residual capacitance can be measured with both probe tips open, after that calibration has been conducted. In order to extract the node-pair capacitance from measurement results, the use of additional measurements with one of the probe tip open has been proposed. In the first order approximation, only the capacitances between probe tips and nodes are considered. In this case only the total capacitance at the nodes can be the goal of the calculations. However, the first order approximation is not enough in the present study. In the second order approximation, the series of two capacitances are considered. In the present study it was enough to consider only series of one capacitance with the power-plane capacitance in most cases, in order to obtain consistent results within an accuracy of 0.1 pF .

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# Excitation of Electromagnetic Modes by a Signal Transmission Line Overpassing a Slit of Return Plane 

Tohlu Matsushima, Youhei Sakai, Kengo Iokibe, Yoshitaka Toyota, Ryuji Koga<br>Graduate School of Natural Science and Technology, Okayama University, Japan<br>Tetsushi Watanabe<br>Industrial Technology Center of Okayama Prefecture, Japan<br>Osami Wada<br>Graduate School of Engineering, Kyoto University, Japan


#### Abstract

Printed circuit boards (PCBs) sometimes have a separated power or ground plane. Difierential mode having been propagated along the microstrip line is divided into multiple modes when the difierential mode arrives at the intersection of the signal line and the slit. They are difierential mode that re ects backward and runs through, slot line mode and parallel plane mode. In order to evaluate excitation of undesirable electromagnetic modes, "transmission factor" is deflned as a power ratio of excited mode to incident mode and is expressed in a matrix form like $S$ matrix. The transmission factor is obtained by the electromagnetic calculation near the slit, since it is determined only by the structure of the intersection between the signal line and the slit.


## 1. Introduction

Printed circuit boards in digital devices have multiple power/ground planes and become multi-layer structures in order to supply stable power/ground voltages. The power/ground planes sometimes should be divided, by slits, into several parts in order to isolate the voltage levels between the difierent segments on the same layer. However, the signal transmission line is arranged above the separated power/ground planes. The slit on the power/ground planes breaks signal current through either power plane or ground plane as return path, which causes undesirable efiects such as re ection and excitation of harmful electromagnetic mode which cause EMI and deterioration of signal integrity[1], [2], [3].

This paper clearly explains the electromagnetic phenomenon that arises at the intersection. Slot-line mode and parallel-plane mode are found to be dominant after FDTD simulation carried only in the local domain around the intersection. Coupling between the incident signal as the transmitted mode and excited mode are given in term of "transmission factor". This factor is available to the same local structures sitting at any place of a PCB.

Numerical examples are shown and easiness of calculation is appealed. With this approach, further development for quicker calculation is expected to decrease the required calculation resource and the working time of PCB designers for better signal integrity and common-mode control.


Figure 1: A 3-layers PCB with slit on return plane.

## 2. Excitation of Slot-line Mode and Parallel-Plane Mode

To investigate the excited unintentional electromagnetic mode, this paper deals with a slit on the middle layer as shown in Fig. 1. The top layer has a signal line and the bottom layer is large enough to cover the slit in the middle layer. According to FDTD calculation, difierential mode having been propagated along the microstrip line is split in to multiple modes: difierential mode that re ects backward and runs through, slot-line mode that propagated along the slit on middle layer, and parallel-plane mode that spreads cylindrically between the middle and bottom layer, when the difierential mode arrives at the intersection of the signal line and the slit.

The test PCB using for FDTD calculation is shown in Fig. 2 and TABLE 1. Parameters employed in the calculation are shown in TABLE 2 and whole area is flled with air. An isolated pulse with Gaussian proflle is fed into the microstrip line at the feed point, and propagates along the microstrip line. The re ection at the end of the signal line does not take place, because the signal line and any edge of the test board are connected to the Perfectly Matched Layer (PML). The voltage waveform was observed at points \#1-3 shown in Fig.2.


Figure 2: Test PCB using for FDTD.

Table 1: Parameters of the test board.

| parameter | value $[\mathrm{mm}]$ |
| :---: | :---: |
| $W_{\mathrm{MSL}}$ | 0.75 |
| $W_{\mathrm{SLT}}$ | 0.75 |
| $h_{\mathrm{SR}}$ | 0.4 |
| $h_{\mathrm{RG}}$ | 0.4 |

Table 2: Parameters of FDTD calculation.

| Cell size $(\Delta x, \Delta y, \Delta z)[\mathrm{mm}]$ | $(0.075,0.075,0.050)$ |
| :---: | :---: |
| Number of cells $\left(N_{\mathrm{x}}, N_{\mathrm{y}}, N_{\mathrm{z}}\right)$ | $(610,610,32)$ |
| Total cells | $11,907,200$ |
| Time steps $[\mathrm{ps}]$ | 0.121 |
| Absorbing boundary condition | PML (4 layers) |
| Feed type | delta-gap feed |

Figure 3(a) shows the voltage waveform of the incident signal and that re ected by the slit observed at the point $\# 1$, which is convinced by the fact that the round trip propagation time between the observation point $\# 1$ and the slit is 80 ps ; whole the area is flled with air. The waveform of the transmitted over the slit at the point \#2 is shown Fig. 3(a) too. The amplitude of re ected wave has about $20 \%$ of the input wave, and that of transmitted wave has about $80 \%$ of the input wave.

The voltage waveform in Fig. 3(b) is observed at the point $\# 3$ on the slit. The waveform is excited by the signal around the microstrip line and it spills into the slit as the slot-line mode. The waveform in Fig. 3(c) is the voltage propagates along the pair of the return plane and ground plane. The peak voltage is inversely proportional to the square root of $r$, which is the distance from the intersection of the signal line and the slit to the observation point (Fig. 3(d)). Thus, the power which propagates along the pair of the return and ground planes is inversely proportional to the distance $r$, which is the evidence that the fleld distribution is a cylindrical wave as a parallel-plane mode with its center at the intersection.


Figure 3: Voltage waveforms observed at each observation point and that between return and ground plane by FDTD calculation.

## 3. Transmission Factor for the Evaluation of Each Mode Excited

Re ection into the microstrip line as well as the slot-line mode and parallel-plane mode are excited by the transmission line overpassing the slit on return plane. A transmission factor is deflned as the ratio of power of excited modes to that of incident for evaluation of each mode excitation. Power of signal which propagates along each transmission line or waveguide is obtained by integrating total power passing through the closed surface deflned beforehand. Power of slot-line mode and parallel-plane mode are observed together in the observation point \#3. Parallel-plane mode was separated with the dependence on distance $r$.

TABLE 3 shows examples of transmission factors of the structure shown in Fig. 2. Transmission factors show that the re ection from and to the microstrip line is 12 dB . Slot-line mode was excited at the rate of 16 dB and parallel-plane mode with 18 dB . This value is determined only by the local structure around intersection of the signal line and the slit. These factors are obtained by the analysis carried on the local area around the intersection, and are available to the same local structure flnding the same PCB.

Figure 4 shows the transition of the transmission factors as the slit width. $S_{\mathrm{A}, \mathrm{B}}$ is transmission factor, which means the ratio of power of excited mode A to that of the incident, B. For example, $S_{\text {SLt, MSL1 }}$ means the ratio of power of slot-line mode to that of difierential mode on microstrip line. When the $W_{\text {SLT }}$ becomes large, $S_{\mathrm{MSL} 1, \mathrm{MSL} 1}$ and $S_{\mathrm{SLT}, \mathrm{MSL} 1}$ tend to increase. On the other hand, $S_{\mathrm{MSL} 2, \mathrm{MSL} 1}$ and $S_{\mathrm{PP}, \mathrm{MSL} 1}$ tend to degrease.

Table 3: Example of transmission factors of the structure shown in Fig. 2.

|  | MSL1 | MSL2 | SLT1 | SLT2 | PP1 | PP2 |
| :---: | :---: | :---: | :---: | :---: | ---: | ---: |
| MSL1 | 12 | 2.7 | 16 | 16 | 18 | 18 |
| MSL2 | 2.7 | 12 | 16 | 16 | 18 | 18 |



Figure 4: Transition of the transmission factors as the slit width.

## 4. Conclusion

The slot-line mode and parallel plane mode excited at the intersection of the signal line and the slit. In order to evaluate these modes, " transmission factor" is deflned as the ratio of power of excited mode to that of incident. Magnitudes of these modes were calculated with FDTD method in the local domain around the intersection to give the transmission factor. This procedure lends us the scheme to know the magnitude of the exited undesirable electromagnetic modes which deteriorate of the signal integrity or gives rise to the common-mode current known as the ground bounce, with the minimum cost of resource.

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# Crosstalk Analysis of Two Bent Lines Above a Ground Plane 

Sang Wook Park ${ }^{1} \quad$ Fengchao Xiao ${ }^{1} \quad$ Dong Chul Park ${ }^{2} \quad$ Yoshio Kami ${ }^{1}$<br>Department of Information and Communication Engineering, University of Electro-Communications ${ }^{1}$ Department of Radio Science and Engineering, Chungnam National University ${ }^{2}$


#### Abstract

A model is described for analyzing the crosstalk between two parallel lines above a ground plane and bent parallel to each other. It is based on a circuit concept approach, which incorporates both transmission line and field theories. Experiments demonstrated that the proposed model accurately predicts the crosstalk.


## 1 Introduction

In a printed circuit board (PCB), there are various layouts of trace lines. Even if a trace line is designed independently, it affects nearby trace lines and vice versa. That is, electromagnetic interference, or crosstalk, occurs there. Crosstalk is a serious problem in the field of electromagnetic compatibility because high densities of trace lines are demanded in small packages and for high-speed operations. The crosstalk between parallel trace lines can be analyzed using the telegrapher's equations for line-voltage and line-current vectors under assumptions of transverse electromagnetic (TEM) mode propagation or at least quasi-TEM mode propagation. In a PCB, there are straight lines and bent lines. To analyze these lines, a cascading technique may be used for the straight parts, but it is unclear whether the technique is adequate for high-frequency operation at high-density wiring. The telegrapher's equations are commonly used because they are so concise and simple. Therefore, the limitations on how they are applied should be clarified. The telegrapher's equations for multiconductor lines are based fundamentally on a concept of circuit theory using capacitive and inductive couplings, which correspond to the electric- and magnetic-field couplings in electromagnetic field theory, respectively.

When a transmission line is excited by external electromagnetic fields, a current is induced in the line. That is, the coupling phenomenon occurs there. Application of Maxwell's equations to this phenomenon produced modified telegrapher's equations obtained in a form where the external fields play roles of distributed voltage and current sources along the line, corresponding to magnetic- and electric-field couplings. This technique is called the "circuit-concept approach".

In this paper we use this approach to analyze the crosstalk between two bent lines. Taking account of electromagnetic fields caused by the currents flowing in various line sections, we derive the network functions in the form of an ABCD matrix. By comparing the computed and the measured results, we verify the proposed approach.

## 2 Circuit Concept Approach for Non-parallel-line Coupling

When a transmission line is in an electromagnetic field, an induced current flows in the line. This phenomenon can be expressed in the form of telegrapher's equations describing line voltage $V$ and current $I$ [1]-[4]. The equations are derived from Maxwell's equations under the assumption that the induced current flows of TEM mode. In these equations, the effects due to external fields are assumed to be distributed voltage and current sources along the line, signifying magnetic- and electric-field couplings, respectively. A set of solutions to the telegrapher's equations for a line length of $\ell$ is given as follows in the form of an ABCD matrix using a state variable [4],[5]:

$$
\left[\begin{array}{c}
V(0)  \tag{1}\\
I(0)
\end{array}\right]=\boldsymbol{F}(\ell)\left[\begin{array}{c}
V(\ell) \\
I(\ell)
\end{array}\right]+\int_{0}^{\ell} \boldsymbol{F}\left(x^{\prime}\right)\left[\begin{array}{c}
V_{f}\left(x^{\prime}\right) \\
I_{f}\left(x^{\prime}\right)
\end{array}\right] d x^{\prime}
$$

with

$$
\boldsymbol{F}\left(x^{\prime}\right)=\left[\begin{array}{ll}
A\left(x^{\prime}\right) & B\left(x^{\prime}\right)  \tag{2}\\
C\left(x^{\prime}\right) & D\left(x^{\prime}\right)
\end{array}\right]=\left[\begin{array}{cc}
\cos \beta x^{\prime} & j Z_{0} \sin \beta x^{\prime} \\
j \frac{1}{Z_{0}} \sin \beta x^{\prime} & \cos \beta x^{\prime}
\end{array}\right]
$$

and

$$
\left[\begin{array}{c}
V_{f}\left(x^{\prime}\right)  \tag{3}\\
I_{f}\left(x^{\prime}\right)
\end{array}\right]=\left[\begin{array}{c}
-j \omega \int_{0}^{h} B_{z} d y^{\prime} \\
j \omega C \int_{0}^{h} E_{y} d y^{\prime}
\end{array}\right]=\left[\begin{array}{c}
-j \omega\left\{\int_{0}^{h}\left(\frac{\partial A_{y}}{\partial x}-\frac{\partial A_{x}}{\partial y}\right)\right\} d y^{\prime} \\
j \omega C_{1}\left\{\left.-j \omega \int_{0}^{h} A_{y} d y^{\prime}+\frac{\nabla \cdot \boldsymbol{A}}{j \omega \mu_{0} \epsilon_{0}} \right\rvert\, \begin{array}{l}
y^{\prime}=h \\
y^{\prime}=0
\end{array}\right\}
\end{array}\right]
$$

where the line is in the $x$-direction at height of $y=h$ and is excited by external electric field $\boldsymbol{E}\left(E_{x}, E_{y}, E_{z}\right)$ and magnetic field $\boldsymbol{H}\left(H_{x}, H_{y}, H_{x}\right)$. We also assume a lossless transmission line of line inductance $L$ and capacitance $C$ in free space with permeability $\mu_{0}$ and permittivity $\epsilon_{0}$. Terms $Z_{0}$ and $\beta$ are the characteristic impedance and the phase constant of the line, respectively.

Term $V_{f}$ is the distributed voltage source along the line corresponding to the magnetic-field coupling interpreted by Faraday's law, and $I_{f}$ the current source of the electric-field coupling. In the second term on the right side of (3), the terms are written in terms of the vector potential of the field.

Even if transmission lines were laid out in arbitrary directions, coupling between them would occur. When the transmission lines are parallel, the coupling can be estimated by using a multiconductor transmission line theory based on the telegrapher's equations. However, when two parallel transmission lines are bent in the same direction, the transmission line theory would not be applicable because the bent corner effect is not taken into account by the theory. We focus on this configuration and investigate whether the theory is applicable. A current flowing in a transmission line generates electromagnetic fields, and the fields affect nearby transmission lines. This effect is a physical phenomenon similar to the coupling of electromagnetic fields to transmission lines. The coupling between non-parallel transmission lines has been studied using this concept [6],[7],[8]. In these studies, two transmission lines are straight, and terminal or riser effects are taken into account for fields affecting another line. Here, we consider the crosstalk phenomenon between two parallel lines bent in the same direction, as shown in Fig. 1. We assume the coupling level is so weak that the characteristic impedance is similar to that of an isolated line.

(a) Coordinate system

(b) Top view of experimental model

Fig. 1 A model of two parallel lines bent in same direction
First, consider the electromagnetic fields created by a finite-length line bent in the $z-x$ plane and with the same height $(y=h)$ as another line, as shown in Fig. 1. Let the line system consist of two parts of a finite line length, $\ell_{1}$ and $\ell_{2}$, and let the line ends have risers. For a straight line, the electromagnetic fields are generated by currents flowing in the line section and its image in the $x$ direction, and at the risers at both line ends in the $y$ direction. Then the components of the vector potential are written as

$$
\begin{equation*}
A_{x}=\frac{\mu_{0}}{4 \pi} \int_{0}^{\ell}\left\{\frac{e^{-j \beta R_{x 1}}}{R_{x 1}} I\left(x^{\prime}\right)-\frac{e^{-j \beta R_{x 2}}}{R_{x 2}} I\left(x^{\prime}\right)\right\} d x^{\prime}, \quad A_{y}=\frac{\mu_{0}}{4 \pi} \int_{-h}^{h}\left\{\frac{e^{-j \beta R_{y 0}}}{R_{y 0}} I(0)-\frac{e^{-j \beta R_{y \ell}}}{R_{y \ell}} I(\ell)\right\} d y^{\prime} \tag{4}
\end{equation*}
$$

with

$$
\begin{array}{ll}
R_{x 1}=\sqrt{\left(x-x^{\prime}\right)^{2}+(y-h)^{2}+z^{2}} & R_{x 2}=\sqrt{\left(x-x^{\prime}\right)^{2}+(y+h)^{2}+z^{2}} \\
R_{y 0}=\sqrt{x^{2}+\left(y-y^{\prime}\right)^{2}+z^{2}} & R_{y \ell}=\sqrt{x^{2}+\left(y+y^{\prime}\right)^{2}+z^{2}} \tag{5}
\end{array}
$$

and current $I\left(x^{\prime}\right)$ at arbitrary point $x^{\prime}$ can be expressed in terms of the line-terminal voltage and current as

$$
\begin{equation*}
I\left(x^{\prime}\right)=-j \frac{V(0)}{Z_{0}} \sin \beta\left(x^{\prime}\right)+I(0) \cos \beta\left(x^{\prime}\right)=j \frac{V(\ell)}{Z_{0}} \sin \beta\left(\ell-x^{\prime}\right)+I(\ell) \cos \beta\left(\ell-x^{\prime}\right) \tag{6}
\end{equation*}
$$ second term on the right side of (1) is written as

$$
\int_{0}^{\ell} \boldsymbol{F}\left(x^{\prime}\right)\left[\begin{array}{c}
V_{f}\left(x^{\prime}\right)  \tag{7}\\
I_{f}\left(x^{\prime}\right)
\end{array}\right] d x^{\prime}=\boldsymbol{f}(0)\left[\begin{array}{c}
V_{t}(0) \\
I_{t}(0)
\end{array}\right]=\boldsymbol{f}(\ell)\left[\begin{array}{c}
V_{t}(\ell) \\
I_{t}(\ell)
\end{array}\right]
$$

For a bent line consisting of two line sections ( $\ell_{1}$ and $\ell_{2}$ ), the network function can be written in the following form:

$$
\left[\begin{array}{c}
V_{1}(0)  \tag{8}\\
I_{1}(0)
\end{array}\right]=\boldsymbol{F}_{1}\left(\ell_{1}\right)\left[\begin{array}{c}
V_{1}\left(\ell_{1}\right) \\
I_{1}\left(\ell_{1}\right)
\end{array}\right]+\boldsymbol{f}_{12}\left(\ell_{2}\right)\left[\begin{array}{c}
V_{2}\left(\ell_{2}\right) \\
I_{2}\left(\ell_{2}\right)
\end{array}\right] \quad, \quad\left[\begin{array}{c}
V_{2}(0) \\
I_{2}(0)
\end{array}\right]=\boldsymbol{F}_{2}\left(\ell_{2}\right)\left[\begin{array}{c}
V_{2}\left(\ell_{2}\right) \\
I_{2}\left(\ell_{2}\right)
\end{array}\right]+\boldsymbol{f}_{21}(0)\left[\begin{array}{c}
V_{1}(0) \\
I_{1}(0)
\end{array}\right] .
$$

Here, applying the continuity condition, $V_{1}\left(\ell_{1}\right)=V_{2}(0)$ and $I_{1}(\ell)=I_{2}(0)$, at a connecting point, such as point A in Fig. 1 (a), gives the final network function:

$$
\left[\begin{array}{c}
V_{1}(0)  \tag{9}\\
I_{1}(0)
\end{array}\right]=\left\{\boldsymbol{U}-\boldsymbol{F}_{1}\left(\ell_{1}\right) \boldsymbol{f}_{21}(0)\right\}^{-1}\left\{\boldsymbol{F}_{1}\left(\ell_{1}+\ell_{2}\right)+\boldsymbol{f}_{12}\left(\ell_{2}\right)\right\}\left[\begin{array}{c}
V_{2}\left(\ell_{2}\right) \\
I_{2}\left(\ell_{2}\right)
\end{array}\right]
$$

where we assume $\boldsymbol{F}_{1}(x)=\boldsymbol{F}_{2}(x)$ for the same cross-sectional dimensions in both line sections.
Next, consider the application of the above concept to the model shown in Fig. 1. The equations corresponding to (8) are written by adding all other effects due to the line sections and then rewritten as

$$
\begin{align*}
& {\left[\begin{array}{cc}
\boldsymbol{U}-\boldsymbol{F}_{1}\left(\ell_{1}\right) \boldsymbol{f}_{21}(0) & -\boldsymbol{F}_{1}\left(\ell_{1}\right) \boldsymbol{f}_{23}(0)-\boldsymbol{f}_{13}(0) \\
-\boldsymbol{F}_{3}\left(\ell_{3}\right) \boldsymbol{f}_{41}(0)-\boldsymbol{f}_{31}(0) & \boldsymbol{U}-\boldsymbol{F}_{3}\left(\ell_{3}\right) \boldsymbol{f}_{43}(0)
\end{array}\right]\left[\begin{array}{c}
{\left[\begin{array}{c}
V_{1}(0) \\
I_{1}(0)
\end{array}\right]} \\
{\left[\begin{array}{c}
V_{3}(0) \\
I_{3}(0)
\end{array}\right]}
\end{array}\right]=} \\
& {\left[\begin{array}{cc}
\boldsymbol{F}_{1}\left(\ell_{1}+\ell_{2}\right)+\boldsymbol{f}_{12}\left(\ell_{2}\right) & \boldsymbol{F}_{1}\left(\ell_{1}\right) \boldsymbol{f}_{24}\left(\ell_{4}\right)+\boldsymbol{f}_{14}\left(\ell_{4}\right) \\
\boldsymbol{F}_{3}\left(\ell_{3}\right) \boldsymbol{f}_{42}\left(\ell_{2}\right)+\boldsymbol{f}_{32}\left(\ell_{2}\right) & \boldsymbol{F}_{3}\left(\ell_{3}+\ell_{4}\right)+\boldsymbol{f}_{34}\left(\ell_{4}\right)
\end{array}\right]\left[\begin{array}{c}
{\left[\begin{array}{c}
V_{2}\left(\ell_{2}\right) \\
I_{2}\left(\ell_{2}\right)
\end{array}\right]} \\
{\left[\begin{array}{l}
V_{4}\left(\ell_{4}\right) \\
I_{4}\left(\ell_{4}\right)
\end{array}\right]}
\end{array}\right] .} \tag{10}
\end{align*}
$$

From the above equation, the final ABCD matrix can be obtained, and, from that, the scattering matrix can be derived.

## 3 Experiment and Discussion

We did an experiment using the model shown in Fig. 1. Wire lines of 0.5 mm in diameter and 200 mm in length were set 4 mm above an aluminum ground plane. The characteristic impedance for the isolated line was about $208 \Omega$. Ports 1 and 3 were the line ends of one line, and ports 2 and 4 those of the other line. The near-end and far-end crosstalks, in terms of scattering matrix parameters $S_{21}$ and $S_{41}$, were estimated when the line ends were terminated with $50-\Omega$ loads. Figure 2 is for $S_{21}$, and Fig. 3 for $S_{41}$.


Fig. 2 Comparison between calculated and measured results of near-end crosstalk $S_{21}$ : (a) magnitude and (b) phase.


Fig. 3 Comparison between calculated and measured results of far-end crosstalk $S_{41}$ : (a) magnitude and (b) phase.

In Fig. 2, the calculated, the measured, and the results simulated by a commercical solver are shown together. These magnitude and phase results are in good agreement.

In Fig.3, the results are also in good agreement. The agreement between the measured and calculated results shows that the proposed circuit-concept approach can be used for analysis of crosstalk on various types of lines.

## 4 Conclusion

We proposed a circuit-concept approach to analyze the crosstalk between nearby transmission lines of finite length. Electromagnetic fields caused by line sections and risers at the line ends are taken into account in applying the approach. The agreement between calculated results and results measured from crosstalk between two lines bent parallel to each other above a ground plane shows that our approach can be used.

## Acknowledgements

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# Determination of Absorbing Materials' Complex EM-parameters via Scalar Reflectometer 

Chun-Ping Chen ${ }^{1}$, Zhewang $\mathrm{Ma}^{2}$, Tetsuo Anada ${ }^{1}$, Jui-Pang Hsu ${ }^{1}$<br>${ }^{1}$ High-Tech Research Center, Kanagawa University, Japan, email: chen@kanagawa-u.ac.jp<br>${ }^{2}$ Department of Electrical and Electronic Systems, Saitama University, Japan


#### Abstract

A scalar reflectometer based, low cost, method, named "multi-thickness method"(MTM), is proposed for nondestructively and simultaneously charactering complex permittivity and complex permeability via an open-ended coaxial probe. The measurement system is established, while the sample-loaded open-ended coaxial probe is modeled by Spectral domain immitance method. A discussion about how to select the multi thicknesses of test sample is also included. A typical absorbing material is measured under different thickness combination conditions. The results agree well with the reference data, which validates the feasibility and effectiveness of this technique.


Key Words: Multi-thickness method(MTM), Scalar reflectometer, Permittivity, Permeability

## I. Introduction

Recently, with the ever-growing application of the high-loss materials in electromagnetic compatibility (EMC), radar stealth technology etc., simultaneous complex EM parameters $(\dot{\varepsilon}, \dot{\mu})$ ' characterization techniques, especially nondestructive ones, are becoming more and more important for materials' development, detection and application[1][2]. Since most of the resonance methods need carefully sample preparation, transmission/reflection method with open-ended sensors is more suitable for nondestructive test[2]. Compared with transmission method, reflection method requires less in sample preparations(only a reflectometer, one sensor and more simple test fixtures)[2]. Accordingly, it is more suitable for practical application. Furthermore, because of its intrinsic merits such as broadband-capability, opened-structure and wide-compatibility, etc., open-ended coaxial reflection probe(OECRP) is essentially applicable for nondestructive and broadband testing of materials' EM parameters at microwave frequency[1]-[7]. So far, some OECRP-based, complex-EM-parameters characterization method have been proposed, e.g. "two-thickness method"(TTM) [1]-[5], "frequency-varadation Method" (FVM)[1][2][6], "combination method"[2][5], etc.. All the above-mentioned methods are based on measuring the complex reflection coefficients of the sample loaded OECRP by the vector reflectometer.

Obviously, for vector reflection, accurate phase detection, which relies strongly on the test sample's surface condition, requires more carefulness in sample preparation and more cost for test instrumentation [2][7]. Especially, the price of vector reflectometer is generally more than twice higher than scalar one. Accordingly, to cater for the always-requirement of costdown, in this paper, an improved, lower cost(scalar reflectometer based) method for complex EM parameters will be studied.

As well known, in reflection method, EM parameters are indirectly determined from the information of reflection coefficients. To simultaneously obtain two complex EM-parameters $(\dot{\varepsilon}, \dot{\mu})$, which embody 4 scalars, the key point is to find at least 4 scalar reflections under different test conditions. Here, for a primary research, the method to get multi-reflections under different thickness of sample-----"multi-thickness method"(MTM) will be studied, with an amphasis on the validation of
its feasibility.

## II. Theoretical Modeling

In MTM, the tested samples' two complex EM-parameters $\dot{\varepsilon}$ and $\dot{\mu}$ are simultaneously determined via measuring the reflection coefficients' magnitude of a sample-loaded OERCP. Theoretically, MTM could be applied in either the half space infinite sample case, or the air-backed case, or the metal-backed case[1]. But, because the latest case is most suitable and easiest in practical application, in this paper, we will choose such case as an example. Fig. 1 depicts the measurement configuration of a short-circuited microwave absorbing coating with a flanged open-ended coaxial probe, assuming infinitesimal dimension of flange, short-circuited plane and test sample. Correspondingly, SDI (Spectral Domain Impedance) method could be introduced to construct the full-wave spectral domain model [6]. The matrix equation that specifies the input admittance could be established as:

$$
\begin{equation*}
\sum_{i=0}^{N \rightarrow \infty}\left[\dot{\Gamma}_{i} \cdot\left(A_{i j}+Y_{i} \cdot \delta_{i j}\right)\right]=Y_{0} \cdot \delta_{0 j}-A_{0 j} \tag{1}
\end{equation*}
$$

where $i, j=0,1,2, \cdots$ are the mode indices. $i=j=0$ gives dominate TEM mode; $\dot{\Gamma}_{i}$ denotes the vector voltage reflection coefficient at $z=0$ while $Y_{i}$ is the mode admittance in coaxial line. $\dot{A}_{i j}$ can be treated as equivalent input admittance in space domain, for we shall obtain $A_{00}=Y_{0} \cdot\left(1-\dot{\Gamma}_{0}\right) /\left(1+\dot{\Gamma}_{0}\right)$ in considering merely the dominant $T E M$ mode in the coaxial line. For interested readers, please refer to [6] for the detail.

## II. Measurement System



Fig. 1 Measurement configuration

The measurement system for MTM is the same as TTM, which has been shown in Fig.2[3]. In MTM, four needed scalar $\left|\dot{\Gamma}\left(\dot{\varepsilon}_{r}, \dot{\mu}_{r}, f, d, \cdots\right)\right|$ are recorded by measuring a sample with four different thicknesses $d_{1}, d_{2}, d_{3}$ and $d_{4}$, respectively. Then, a right $\dot{\varepsilon}_{r}$ and $\dot{\mu}_{r}$ pair should lead the calculated reflection coefficients as little difference as possible to the measured ones:

$$
\left\{\begin{array}{l}
\left|\dot{\Gamma}_{m 1}\right|=\left|\dot{\Gamma}_{c 1}\left(\varepsilon_{r}^{\prime}, \varepsilon_{r}^{\prime \prime}, \mu_{r}^{\prime}, \mu_{r}^{\prime \prime}, f, d_{1} \cdots\right)\right| \\
\left|\dot{\Gamma}_{m 2}\right|=\left|\dot{\Gamma}_{c 2}\left(\varepsilon_{r}^{\prime}, \varepsilon_{r}^{\prime \prime}, \mu_{r}^{\prime}, \mu_{r}^{\prime \prime}, f, d_{2} \cdots\right)\right|  \tag{2}\\
\left|\dot{\Gamma}_{m 3}\right|=\left|\dot{\Gamma}_{c 3}\left(\varepsilon_{r}^{\prime}, \varepsilon_{r}^{\prime \prime}, \mu_{r}^{\prime}, \mu_{r}^{\prime \prime}, f, d_{3} \cdots\right)\right| \\
\left|\dot{\Gamma}_{m 4}\right|=\left|\dot{\Gamma}_{c 4}\left(\varepsilon_{r}^{\prime}, \varepsilon_{r}^{\prime \prime}, \mu_{r}^{\prime}, \mu_{r}^{\prime \prime}, f, d_{4} \cdots\right)\right|
\end{array}\right.
$$

where the subscripts $m$ denotes "measured", $c$ denotes "calculated". Then, from (2) we are able to work out through numerical iterations the both $\dot{\varepsilon}_{r}=\varepsilon_{r}^{\prime} \cdot\left(1-j \tan \delta_{\varepsilon}\right)$ and $\dot{\mu}_{r}=\mu_{r}^{\prime} \cdot\left(1-j \tan \delta_{\mu}\right)$.


Fig. 2 The flow-graph of measurement system

Obviously, in MTM, the key technique is to
reasonably choose 4 thicknesses of the test sample. In our experiment, we found the proper thickness selection is strongly relative to the properties of the sample itself. In the following, we will make a discussion about the this technique in MTM, with an emphasis on the material with very high loss. A typical absorbing material Sj 1 will be studied as an example. Fig. 3 shows the variation of the calculated $|\dot{\Gamma}|$ of Sj 1 along with its (electrical) thickness in X-band modeled by Fig.1. The shape of curves corresponding to different frequency seem alike and are very similar to a damped evanescent
sinusoidal wave. Firstly, in region I of Fig. 3, $|\dot{\Gamma}|$ declines very fast with the increase of $d / \lambda_{g}$, i.e. a small uncertainty in measured thickness $d$ will possibly introduce a big uncertainty of $|\dot{\Gamma}|$. For example, for Sj 1 (at $10 \mathrm{GHz}, d=0.56 \pm 0.1 \mathrm{~mm}$ ), the corresponding $\Delta|\dot{\Gamma}| \approx 0.05$. Accordingly, one must pay great attention to the accuracy of measured sample's thicknesses if they are selected in region I. Secondly, in region III, $|\dot{\Gamma}|$ varies very slowly with $d / \lambda_{g}$. As well known, the sensitivity of general reflectometer for $|\dot{\Gamma}|$ is about 0.01 . Practically, if we choose more than one thicknesses in region III, the corresponding equations in (2) will be correlated and sometimes result in an unacceptable errors in solutions. In other words, from physical meaning point of view, in III, $|\dot{\Gamma}|$ is no longer the function of $d$, which contradicts with the assumption of MTM. Thirdly, based on above discussion, for Sj 1 , region II seems the best proper region for selecting the measured thicknesses. However, one point should be mentioned: the more thicknesses to be selected in II, the smaller intervals between neighboring thicknesses will be got. This will sometimes deteriorate accuracy of measured EM-parameters because of the errors in direct measurement quantities. So, based on the author's experience, to choose one or two thicknesses in region I and III will sometimes get more accurate results, which could be found in next session.

## III. EXPERIMENTAL VERIFICATION

A measurement system has been set up as Fig. 2. An ANA(Agilent8722ES) is used as the


Fig. 3 Variation of magnitude of reflection coefficients along with the electric thickness of sample(Refer to Fig. 4 for the EM properties of Sj 1 ) flanged open-ended coaxial probe was fabricated to measure the absorbing coatings modeled by Fig. 1. An X-band frequency-swept measurement was carried out on a typical absorbing material Sj1. The fitted measurement results by both MTM under different thicknesses combinations and TTM are shown in Fig.4. Then we can get some results: 1). The results measured by MTM under all 3 selections of thicknesses combinations have good agreements with the reference data by TTM. The maximum deviations of all the results are limited to $10 \%$ for $\varepsilon_{r}, 0.03$ for $\tan \delta_{\varepsilon}, 5 \%$ for $\mu_{r}$ and $10 \%$ for $\tan \delta_{\mu}$, which verified the effectiveness of MTM; 2). In comparison, for MTM, the test results of C (thickness combination) 1 and C2 seem agree more well with the reference data than those of C3. This is most probably because of the influence of the smaller intervals of the adjacent test thicknesses, which makes equations in (2) correlated and increased the measurement uncertainties. On the other hand, these results also verified the critical influence of sample's thicknesses selection on the accuracy of measured results.

## IV. CONCLUSIONS

An improved method, named "multi-thickness method"(MTM) was proposed for nondestructively and simultaneously charactering complex permittivity and complex permeability. Compared with two-thickness method(TTM)(need vector reflectometer), MTM requires only scalar reflectometer, which simplifies the test system and lowers the cost. (Although it requires more samples with 4 different thickness, practically, we can try to get more sample thicknesses by piling-up or folding the samples.) In this paper, the basic measurement system was set up while the key point of how to


Fig. 4 Broadband frequency-swept measurements for sample Sj 1 (Refer to Fig. 3 for the value of $d_{l} \sim d_{6}$ )
reasonably select sample's test thicknesses has been discussed. The experiments on a typical absorbing material under different thicknesses combinations have been conducted. The good comparison between MTM results with reference data(by TTM) validated the feasibility of this improved technique.

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# A Stable and Fast 3-D Imaging Algorithm for UWB Pulse Radars with Fractional Boundary Scattering Transform 

T. Sakamoto and T. Sato<br>Graduate School of Informatics, Kyoto University<br>Yoshida-Honmachi, Sakyo-ku, Kyoto, 606-8501 Japan


#### Abstract

Radar imaging for a nearby target is known as an ill-posed inverse problem, on which various studies have been done. However, conventional algorithms require too long computational times. In order to resolve this difficulty, SEABED algorithm was developed. This algorithm is based on a reversible transform between the real and data spaces. In a noisy environment, the performance of the SEABED algorithm is severely degraded. In this paper, we newly introduce a fractional IBST, which is obtained by expanding the conventional IBST, which enables us to deal with the intermediate space between the real and data spaces, and propose a stable 3-D imaging algorithm by using the FIBST.


## 1 Introduction

The UWB (ultra-wideband) pulse radar is a promising candidate as an environment measurement method for robots. Radar imaging for a nearby target is known as an ill-posed inverse problem, on which various studies have been done. However, conventional algorithms require long computational time, which makes it difficult to apply them to real-time operations of robots. We have proposed a fast radar imaging algorithm, the SEABED algorithm, for UWB pulse radars [1, 2, 3]. This algorithm is based on a reversible transform, IBST (Inverse Boundary Scattering Transform), between the target shape and the observed data. This transform enables us to estimate target shapes quickly and accurately in a noiseless environment. However, in a noisy environment the image estimated by the SEABED algorithm is degraded because IBST utilizes differential operations. In this paper, we newly introduce a FIBST (Fractional IBST), which is obtained by expanding the conventional IBST, which enables us to deal with the intermediate space between a real and data spaces, and propose a stable 3-D imaging algorithm by using the FIBST. We investigate the estimation errors for the conventional algorithms and the proposed one with numerical simulations.

## 2 System Model

In our system model, UWB mono-cycle pulses are transmitted at a fixed interval and received by the same omni-directional antenna. We express a real space with the parameters $(x, y, z)$. The antenna is scanned on the $x-y$ plane in the real space. We define $s(X, Y, Z)$ as the electric field received at the antenna location $(x, y, z)=(X, Y, 0)$, where we define $Z$ with time $t$ and the speed of the radiowave $c$ as $Z=c t /(2 \lambda)$. It should be noted that the received data is expressed with $(X, Y, Z)$, and the target shapes is expressed with $(x, y, z)$. We define a data space as the space expressed by $(X, Y, Z)$. The transform from the data space $(X, Y, Z)$ to the real space $(x, y, z)$ corresponds to the imaging we deal with in this paper. We normalize $x, y, z, X, Y$ and $Z$ by $\lambda$, the center wavelength.

## 3 Conventional SEABED Algorithm

In the SEABED algorithm, quasi-wavefronts $(X, Y, Z)$ are easily extracted from the received data $s(X, Y, Z)$. We apply IBST to the quasi-wavefront to obtain the final image as

$$
\left\{\begin{array}{ccc}
x= & X-Z \partial Z / \partial X  \tag{1}\\
y= & Y-Z \partial Z / \partial Y \\
z= & Z \sqrt{1-(\partial Z / \partial X)^{2}-(\partial Z / \partial Y)^{2}}
\end{array}\right.
$$

This technique is very simple, but it works well only for noiseless environments. Eq. (2) contains the derivative operations, which make the obtained image degraded with random components. Therefore, we have to utilize smoothing algorithm to stabilize the estimated image.

We utilized the following simple smoothing technique [4]. In the conventional smoothing technique, we apply smoothing to the quasi-wavefront to suppress the noise. Next, we apply the IBST to the smoothed quasiwavefront to obtain the final stabilized image. We have clarified that this simple technique can stabilize the image to some extent. However, the smoothing process can distort the final image because the quasi wavefronts are not necessarily smooth even if the true target is smooth. On the other hand, the target shape itself is not guaranteed to be smooth, which implies that the smoothing of the final image is neither suitable. Consequently, the both of the smoothing processes in the data and real spaces are inappropriate, which is a fundamental problem for the stabilization.

In order to solve the problem, we have developed the new smoothing technique in the intermediate space between the real and data spaces for 2-dimensional imaging. The data in the intermediate space is guaranteed to be smooth regardless of the target shape. We need the transform FIBST in order to deal with the intermediate space. In the next section, we expand the 2-dimensional FIBST to the 3-dimensional one, to stabilize the 3 -dimensional radar imaging.

## 4 Proposed Extended-SEABED Algorithm

The simple smoothing effectively works for convex targets because the quasi-wavefront is smooth for a convex shape. However, for general cases the quasi-wavefront is not necessarily smooth, so the image resolution can be degraded by unsuitable smoothing. To resolve this problem, we introduce FIBST by expanding the conventional IBST, and transform the data to an intermediate space between the real and data spaces, where the smoothing process hardly degrades the resolution. FIBST is expressed as

$$
\left\{\begin{align*}
{\left[\begin{array}{c}
x_{\theta, \alpha, \beta} \\
y_{\theta, \alpha, \beta}
\end{array}\right] } & =\left[\begin{array}{c}
X \\
Y
\end{array}\right]-Z R(-\theta)\left[\begin{array}{cc}
\alpha & 0 \\
0 & \beta
\end{array}\right] R(\theta)\left[\begin{array}{l}
\partial Z / \partial X \\
\partial Z / \partial Y
\end{array}\right]  \tag{2}\\
z_{\theta, \alpha, \beta} & =Z \sqrt{1-\left[\begin{array}{ll}
\partial Z / \partial X & \partial Z / \partial Y
\end{array}\right] R(-\theta)\left[\begin{array}{cc}
\alpha & 0 \\
0 & \beta
\end{array}\right] R(\theta)\left[\begin{array}{c}
\partial Z / \partial X \\
\partial Z / \partial Y
\end{array}\right]}
\end{align*}\right.
$$

In our proposed stable imaging algorithm, we select suitable parameters $(\theta, \alpha, \beta)$ depending on the roughly estimated target shape, and apply the smoothing process to ( $x_{\theta, \alpha, \beta}, y_{\theta, \alpha, \beta}, z_{\theta, \alpha, \beta}$ ) and finally apply FIBST again to obtain the final image. The procedure of the proposed algorithm is shown in Fig. 1 in contrast with the conventional one. First, the proposed algorithm estimates the rough image by utilizing the conventional algorithm. In this step, the image is severely distorted by the inappropriate smoothing process. Next, we calculate the Hesse matrix of the rough image for each point on that. Then, we obtain the eigenvalues and eigen vectors of the Hesse matrix. We determine the parameters $\alpha, \beta$ and $\theta$ based on the eigenvalues and the eigen vectors. We apply the FIBST with these parameters to the original quasi wavefront. Then, we apply a smoothing to the obtained FIBST. Finally, we apply the residue FIBST with $1-\alpha, 1-\beta$ and $-\theta$ to obtain the final image.

## 5 Numerical Simulations

We show some results of the numerical simulation to investigate the performance of the conventional and proposed algorithms. We assume the true targets shape in Fig. 2, which has saddle points. We adopt this


Figure 1: Procedures of the conventional and proposed algorithm


Figure 2: True target shape used in our numerical simulation.
shape because the saddle point is unique for 3-dimensional system compared to the 2-dimensional shape. The quasi-wavefront for a convex target is smooth, and the quasi-wavefront for a concave target is sharp and not smooth. The saddle point contains both of these effects, which is very difficult to adequately deal with.

The quasi-wavefront for the true target shape is shown in Fig. 3. We see that the quasi-wavefront is sharp in the direction of $Y$, which is caused by the concave target shape in the $y$ direction. On the other hand, the quasi-wavefront is smooth in the direction of $X$, which corresponds to the convex target shape in the $x$ direction. It is obvious that the conventional simple smoothing distorts the quasi-wavefront for $Y$ direction.

In this paper, we omit the process of calculation of eigenvalues and eigen vectors for simplicity. We assume that the suitable parameters $\alpha, \beta$ and $\theta$ are chosen before the application of the FIBST. We adopt $\alpha=0.1$, $\beta=0.9$ and $\theta=0$ here. We assume noiseless environment in order to evaluate the distortion of image without noise. The performance evaluation with noise is an important future task. We set the correlation length of the smoothing is equal to the center wavelength. The smoothing process of the conventional and proposed methods are displayed in the $\alpha-\beta$ diagram as in Fig. 4. Here, we assume $\theta=0$ is fixed for simplicity. The parameters of FIBST $\alpha$ and $\beta$ determine the space of the processed data. The point $(0,0)$ is the data space, where the data is an original quasi-wavefront extracted with the received data. The point $(1,1)$ is the real space, where the data directly express the real target shape. Other points $(\alpha, \beta)$ for $0<\alpha<1,0<\beta<1$ correspond to the fractional data spaces. Especially, we apply the smoothing in the fractional data space $(\alpha, \beta)=(0.1,0.9)$ as described above.

The estimated target shape by the proposed method is shown in Fig. 5. The estimation errors of the images with the conventional algorithm and the proposed algorithm are shown in Fig. 6 and Fig. 7. The estimation


Figure 3: Quasi wavefront for the assumed target shape.


Figure 4: $\alpha-\beta$ diagram and three spaces.
accuracy for the proposed algorithm is higher than that of the conventional one by more than 2 times.

## 6 Conclusion

In this paper, we newly introduced the 3-dimensional fractional inverse boundary scattering transform (3D FIBST), which enables us to deal with the intermediate space between the real and data spaces for 3-D problem. By utilizing the 3-D FIBST, we have clarified that we can apply a smoothing process with distortion suppressed.

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Figure 5: Estimated target image by the proposed algorithm.


Figure 6: Estimation error for the smoothing in the data space (conventional).


Figure 7: Estimation error for the smoothing in the fractional data space (proposed).

# A Robust and Fast Imaging Algorithm with an Envelope of Circles for UWB Pulse Radars 

Shouhei KIDERA, Takuya SAKAMOTO and Toru SATO<br>Graduate School of Informatics, Kyoto University<br>Abstract

UWB pulse radar systems are promising as high-resolution imaging techniques for household or rescue robots. We have already proposed a fast imaging algorithm called SEABED based on a reversible transform BST(Boundary Scattering Transform) between the received signals and the target shape. However, the image obtained with SEABED deteriorates in a noisy environment because it utilizes derivative of received data. In this paper, we propose a robust and fast imaging method with an envelope of circles.

## 1 Introduction

UWB pulse radar systems have a great potential for a high-resolution imaging, which is suitable and e-cient for measuring techniques of house-hold and rescue robots. While many imaging algorithm for radar systems have been proposed, they require intensive computations. To solve this problem, we have already proposed a fast imaging algorithm called SEABED (Shape Estimation Algorithm based on BST and Extraction of Directly scattered waves) for UWB pulse radars based on a reversible transform BST between the received signals and the target shape [1, 2]. However, the estimated image with SEABED is not stable in a noisy environment because it utilizes derivatives of the received data. In this paper, we propose a robust and fast imaging algorithm without derivative operation. We utilize circles with the estimated delay for each antenna location. We follow the principle that these circles circumscribe the target boundary [3]. By utilizing this principle and the inverse transform of BST, we prove that the boundary of a convex and a part of concave targets are expressed as a boundary of an union set of these circles. This method does not depend on a derivative of a received data, and enables us to realize a robust imaging even in a noisy environment. We show application examples of the proposed method with a numerical simulation.

## 2 System Model

We show the system model in Fig. 1. We deal with 2-dimensional problems and TE mode waves. We assume that a target has a uniform permittivity, and surrounded by a clear boundary which is composed of smooth curves concatenated at discrete edges. We also assume that the propagation speed of the radio wave is constant and known. We utilize a mono-static radar system. The induced current at the transmitting antenna is a mono-cycle pulse. We deflne r-space as the real space, where targets and the antenna are located. We express r-space with the parameters $(x, y)$. An omni-directional antenna is scanned along $x$ axis. Both $x$ and $y$ are normalized by , which is the center wavelength of the transmitted pulse. We assume $y>0$ for simplicity. We deflne $s^{\prime}(X, Y)$ as the received electric fleld at the antenna location $(x, y)=(X, 0)$, where we deflne $Y$ with time $t$ and speed of the radio wave c as $Y=c t /(2)$. We apply the matched flter with the transmitted waveform to $s^{\prime}(X, Y)$. We deflne $s(X, Y)$ as the output of the fllter. We deflne d-space as the space expressed by $(X, Y)$,


Figure 1: System model.


Figure 2: Quasi wavefront with noise (Left side), and an estimated image with the SEABED (Right side).
and call it a quasi wavefront. The transform from d-space to r-space corresponds to imaging which we deal with in this paper.

## 3 Conventional method

We have already developed a non-parametric shape estimation algorithm called SEABED [1]. This method utilizes a reversible transform BST between the point of r-space $(x, y)$ and the point of d-space $(X, Y)$, which is extracted by the received signal $s(X, Y)$. IBST (Inverse BST) is expressed as

$$
\begin{align*}
x & = \\
y & =Y \sqrt{1 \quad(\mathrm{~d} Y / \mathrm{d} X)^{2}} \tag{1}
\end{align*}
$$

where $|\mathrm{d} Y / \mathrm{d} X| \leq 1$ holds. This transform gives us a complete solution for an inverse problem. SEABED has an advantage that it can directly estimate target boundaries with IBST, and achieves a fast and high resolution imaging. However, the estimated image with the SEABED easily deteriorates in a noisy environment because IBST utilizes the derivatives of the quasi wavefront as $\mathrm{d} Y / \mathrm{d} X$. The left side of Fig. 2 shows a quasi wavefront with random error whose standard deviation is 0.01 . Here, we smooth the quasi wavefront with a Gaussian fllter, whose standard deviation is 0.05 . The right side of Fig. 2 shows the estimated points with IBST. The estimated points with SEABED have large errors, and the maximum error is over 0.5 , which is not acceptable. We conflrm that the estimated point with IBST should exist on the circle whose center is $(X, 0)$ and radius is $Y$. The angle shown in Fig. 1 is determined with $\mathrm{d} Y / \mathrm{d} X$, which is sensitive to a noise. The accuracy of strongly depends on that of $\mathrm{d} Y / \mathrm{d} X$. Therefore, the estimated point readily moves around this circle in a noisy environment. To suppress the deterioration of the estimated image with SEABED, the methods for stabilizing images have been proposed [4]. However, they cannot completely remove the error occurred by the derivative operations because they utilize an inverse transform with $\mathrm{d} Y / \mathrm{d} X$.

## 4 Proposed method

To solve the instability of SEABED, we propose a new imaging algorithm based on an envelope of circles. First, we clarify the relationship between the group of points on a target boundary and the points on an envelope of circles. We assume that the target boundary $\partial T$ is expressed as a single-valued and difierentiable function. $(X, Y)$ is a point on the quasi wavefront of $\partial T$, and we deflne $\partial D$ as the quasi wavefront. We deflne $\Gamma$ as the domain of $X$ for $\partial D$. We deflne $g(X, Y)=X \quad Y \mathrm{~d} Y / \mathrm{d} X$, and $\gamma$ as a domain of $g(X, Y)$. We deflne $S_{(X, Y)}$ as an open set which is an interior of the circle, which satisfles $\left(\begin{array}{ll}x & X\end{array}\right)^{2}+y^{2}=Y^{2}$. Fig. 3 shows the relationship between d-space and set of circles in r-space. If $\partial D$ is a single-valued and continuous function, we deflne $S_{+}$ as $S_{+}=(x, y) \mid(x, y) \in \bigcup_{X \in \Gamma} S_{(X, Y)}, x \in \gamma$. We deflne $\partial S_{+}$as the boundary points of $S_{+}$. Here the next proposition holds

Proposition 1. If $\partial g(X, Y) / \partial X>0$ satisfies, the next equation holds

$$
\begin{equation*}
\partial T=\partial S_{+} \tag{2}
\end{equation*}
$$



Figure 3: Quasi wavefront in d-space (Left side) and a set of circles in r-space (Right side).

A proof of the proposition 1 is given in the appendix A. Proposition 1 says that $\partial S_{+}$expresses a part of the target boundary. Under the condition $\partial g(X, Y) / \partial X>0$, all $\partial S_{(X, Y)}$ in $(X, Y) \in \partial D$ circumscribe $\partial T$. Also $\partial g(X, Y) / \partial X>0$ satisfles at the case of a general convex target and a part of concave targets as shown in Fig. 3. We also conflrm that the edge can be estimated as the intersection point of circles $\partial S_{(X, Y)}$ when a target boundary includes an edge, where $(X, Y)$ is transformed into the edge point $(x, y)$ with the IBST. Therefore, the target boundary $\partial T$ with edges can be expressed as $\partial S_{+}$. In our proposed method, we estimate the target boundary with an envelope of circles by utilizing these relationships. This method enables us to transform the group of the points $(X, Y)$ to the group of the points $(x, y)$ without the derivative operation.

We explain the actual procedures of the proposed method as follows. Here we deflne $R\left(X, X^{\prime}\right)$ as $x$ coordinates of the intersection point of $\partial S_{(X, Y)}$ and $\partial S_{\left(X^{\prime}, Y^{\prime}\right)}$. We deflne $X_{\max }$ and $X_{\min }$ as maximum and minimum $X \in \Gamma$. We also deflne $\Delta X$ as the sampling interval of the antenna.

Step 1). Apply the matched fllter to the received signals $s^{\prime}(X, Y)$ and obtain the output $s(X, Y)$.
Step 2). For each $X$, determine $Y$ as $Y=\max _{Y^{\prime}} s\left(X, Y^{\prime}\right)$, and extracts $(X, Y)$ as quasi wavefront $\partial D$.
Step 3). Extract boundary points $(x, y)$ on $\partial S_{+}$as $y=\max _{X \in \Gamma} \sqrt{Y^{2} \quad(x \quad X)^{2}}$ where $x$ is sampled at an equal interval in the domain $\Gamma$.

Step 4). Determine $\partial T=\partial S_{+},\left(x_{\min } \leq x \leq x_{\max }\right)$, where $x_{\min }=R\left(X_{\min }, X_{\min }+\Delta X\right)$ and $x_{\max }=$ $R\left(X_{\max }, X_{\max } \quad \Delta X\right)$.

## 5 Performance Evaluation

We show an application example of SEABED and the proposed method as follows. The signals are received at 101 locations for $2.5 \leq x \leq 2.5$. We add a white noise to the received data $s^{\prime}(X, Y)$ calculated with the FDTD method. In this case, $\mathrm{S} / \mathrm{N}$ is about 7.0 dB . The left and right side of Fig. 4 show the estimated image with SEABED and the proposed method, respectively. We set the standard deviation of Gaussian fllter as 0.05 . We conflrm that the image obtained with SEABED is not accurate, especially around the edges of the target. On the contrary, the image obtained by the proposed method is stable and expresses an almost accurate target boundary. This is because the proposed method does not spoil the information of the inclination of the target shape. The calculation time of the algorithm is within 0.1 sec with a Xeon 3.2 GHz processor, which is short enough for a realtime imaging.

To evaluate a limitation of the imaging stability v.s. S/N, we introduce two evaluation values, which are mean value of the error deflned as $=\frac{1}{N} \sum_{i=0}^{N}\left|e\left(x_{i}\right)\right|$, and standard deviation of the errors deflned as $\sigma=\sqrt{\frac{1}{N} \sum_{i=0}^{N}\left(\left|e\left(x_{i}\right)\right| \quad\right)^{2}}$ where $e(x)=y_{\mathrm{e}}(x) \quad y_{\text {true }}(x)$. Here, $y_{\text {true }}(x)$ is the true target boundary, and $y_{\mathrm{e}}(x)$ is the estimated image, and $N$ is number of the estimated points. The left and right side of Fig. 5 shows that and $\sigma$ for $\mathrm{S} / \mathrm{N}$ in the case of the target as shown in Fig. 2. We obtain 5 times improvement for , and 2 times improvement for $\sigma$ compared to those of SEABED when $\mathrm{S} / \mathrm{N}$ is over 7 dB . These improvements are obtained regardless of the noise power. We should notice that this method achieves a fast and robust imaging, which cannot be obtained with the conventional algorithms. It is important future work to extend this method for a


Figure 4: Estimated image with the SEABED (Left side) and the proposed method (Right side).


Figure 5: $\quad$ (Left side) and $\sigma$ (Right side) for $\mathrm{S} / \mathrm{N}$ of an each method.
general target shape including the concave with a large curvature.

## 6 Conclusion

We proposed a robust and fast imaging method with an envelope of circles. We clarifled that a general convex target and a kind of a concave target boundary can be expressed as a boundary of an union set of circles with time delays. We also showed that the application range of the proposed method. In the numerical simulation, we clarifled that the proposed method achieves a stable imaging compared with the SEABED. Besides, we conflrmed that the proposed method achieves a fast imaging like SEABED algorithm.

## Acknowledgment

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Figure 6: Relationship between $\partial T$ and $P, Q, R$.
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## A Proof of Proposition 1

First, let us prove that if $\partial g(X, Y) / \partial X>0$ holds at $(X, Y) \in \partial D, \partial S_{(X, Y)}$ circumscribes $\partial T$. With $(x, y) \in \partial T$, the curvature on $\partial T$ is expressed as

$$
\begin{align*}
& =\frac{\mathrm{d}^{2} y / \mathrm{d} x^{2}}{\left(1+(\mathrm{d} y / \mathrm{d} x)^{2}\right)^{3 / 2}}  \tag{3}\\
& =\frac{\ddot{Y}}{1 \quad Y \ddot{Y} \quad \dot{Y}^{2}} . \tag{4}
\end{align*}
$$

where we deflne $\dot{Y}=\mathrm{d} Y / \mathrm{d} X, \ddot{Y}=\mathrm{d}^{2} Y / \mathrm{d} X^{2}$, and utilize $\mathrm{d} y / \mathrm{d} x=\dot{Y} / \sqrt{1 \quad \dot{Y}^{2}}, \mathrm{~d}^{2} y / \mathrm{d} x^{2}=\frac{\ddot{Y}}{\left(1 \quad \dot{Y}^{2}\right)^{3 / 2}\left(1 \quad Y \ddot{Y} \quad \dot{Y}^{2}\right)}$, which are derived [2]. Here, the condition that $\partial S_{(X, Y)}$ circumscribes $\partial T$ is that $>1 / Y$ holds because a curvature of $\partial S_{(X, Y)}$ should be minus at $y>0$. If $\partial g(X, Y) / \partial X>0$ holds, this condition is expressed as $1(\mathrm{~d} Y / \mathrm{d} X)^{2}>0$. This equation should hold because $y$ is a real number in IBST. Therefore, the previous proposition is proved. We utilize this proposition to prove the proposition 1 as follows.
(a) Proof of $\partial S_{+} \subset \partial T$.

We assume that a point $P=\left(x_{\mathrm{p}}, y_{\mathrm{p}}\right),\left(x_{\mathrm{p}} \in \gamma\right)$ exists, which satisfles $P \in \partial S_{+}, P \cap \partial T=$, where $\quad$ is a null set. We deflne $Q=\left(x_{\mathrm{q}}, y_{\mathrm{q}}\right),\left(x_{\mathrm{q}} \in \gamma\right)$ on $\partial T$ which satisfles that its distance from $P$ is minimum of all points on $\partial T$. We deflne $\left(X_{\mathrm{q}}, Y_{\mathrm{q}}\right)$ which is transformed from $\left(x_{\mathrm{q}}, y_{\mathrm{q}}\right)$ with the BST, and $R=\left(X_{\mathrm{q}}, 0\right)$. Fig. 6 shows the relationship between $\partial T$ and $P, Q, R$.

We deflne $x_{\min }=g\left(X_{\min }, Y_{\min }\right)$ and $x_{\max }=g\left(X_{\max }, Y_{\max }\right)$ on $\partial T$. Here $X_{\min }<X_{\mathrm{q}}<X_{\max }$ holds because $\partial g(X, Y) / \partial X>0$ and $x_{\min }<x_{\mathrm{q}}<x_{\text {max }}$ satisfles. Here, all $\partial S_{(X, Y)}$ on $\partial D$ circumscribes $\partial T$ because $\partial g(X, Y) / \partial X>0$ holds. Therefore, $P \in S_{\mathrm{q}}$ satisfles because $\overline{P R}<\overline{Q R}$ holds as shown in Fig. 6. $P \in S_{+}$holds because of $S_{\mathrm{q}} \subset S_{+}$. Accordingly, $P \cap \partial S_{+}=$holds because $\partial S_{+} \cap S_{+}=$satisfles. However, this equation contradicts to the previous assumption. Therefore $\partial S_{+} \subset \partial T$ should hold.
(b) Proof of $\partial T \subset \partial S_{+}$.

We assume that $P=\left(x_{\mathrm{p}}, y_{\mathrm{p}}\right)$ will exist where $P \in \partial T, P \cap \partial S_{+}=$holds. With the $\operatorname{IBST},\left(x_{\mathrm{p}}, y_{\mathrm{p}}\right)$ is transformed to $\left(X_{\mathrm{p}}, Y_{\mathrm{p}}\right)$ where

$$
\begin{equation*}
\left(x_{\mathrm{p}} \quad X_{\mathrm{p}}\right)^{2}+y_{\mathrm{p}}^{2}=Y_{\mathrm{p}}^{2}, \tag{5}
\end{equation*}
$$

satisfles. For all $(X, Y) \in \partial D,\left(x_{\mathrm{p}} \quad X\right)^{2}+y_{\mathrm{p}}^{2} \geq Y^{2}$ holds because all $\partial S_{(X, Y)}$ on $\partial D$ circumscribe $\partial T$. Therefore, $P \cap S_{+}=$satisfles. If $P \cap \partial S_{+}=$holds, $\left(x_{\mathrm{p}} \quad X\right)^{2}+y_{\mathrm{p}}^{2}>Y^{2}$ should hold in any $(X, Y) \in \partial D$. However, this fact contradicts Eq. (5). Therefore the assumption is not true, and $\partial T \subset \partial S_{+}$is proved. According to the fact (a), (b), proposition 1 is proved.

# A Precise Electromagnetic Field Estimation in Elevators Considering Implantable Cardiac Pacemaker EMI from Cellular Radios 

Louis-Ray HARRIS, Takashi HIKAGE, Toshio NOJIMA, Manabu OMIYA<br>Hokkaido University, Japan,<br>Ally SIMBA, Soichi WATANABE and Takashi SHINOZUKA<br>National Institute of Information and Communications Technology, Japan.


#### Abstract

The purpose of this study was to estimate the Electromagnetic Field (EMF) distributions in environments surrounded by conductive surfaces, e.g. train carriages or elevators. Analysis of the results would then enable us to determine possible effects of EMI on Implantable Cardiac Pacemakers in these kinds of environment, commonly referred to as a semi-echoic environments (SEE).


## 1. Introduction

In the environments in question, precise measurements in the actual environment are difficult because of disturbing of fields due to the presence of measuring equipment or human bodies. In this paper, precise numerical simulations were carried out in order to examine the EMF in actual elevators. Additionally, a simplified histogram estimation method for electric field strength was developed to deal with the complicated EMF distributions. Here, the relative field strength normalized to a certain reference level determined from the experimentally obtained maximum interference distance of implantable cardiac pacemakers was used [1]. This allows the EMI risk to pacemakers by cellular radio transmission to be quantitatively evaluated. By way of example, this is a case in which one user is present in the elevator. In this paper, we used 1-cm cubic cells for FDTD modeling. In the actual environment, the effects of losses due to the human body cannot be disregarded. Then, in order to examine the realistic and complicated situations where humans are present in the elevator, we applied the homogeneous human phantom model in the FDTD analysis. They have realistic shapes and homogeneous electric parameters. Here we use a half-wavelength dipole antenna to represent a cellular radio operating in the 2 GHz band. The antenna is set 20 mm from the human phantom's head and 158 cm from the floor.

## 2. Methodology

The modeling was carried out using 2 GHz dipole antennas and a varying number of human phantom models. An elevator having a PEC body was used and the numerical model of the elevator is shown in Figure 1. The cases considered ranged from having an empty elevator scenario where there was only a single antenna, to up to 4 phantom models in varying positions within the elevator. Modeling the scenario having only a single antenna enabled us to determine the background electric field distribution in the absence of any materials whose properties can result in a disturbance in the electric field. The case of 1 phantom model present in the center of the elevator holding a single cellular radio 20 mm from the head was first modeled, followed by additional cases with 1 phantom model in a corner of the elevator, then several phantom models present at different locations inside the elevator each holding a single cellular radio in the same position. For all of the cellular radios simulated in the model, the signal frequency, amplitude, phase and output power were identical. In each of these scenarios, field histograms were used to estimate the percentage of areas having the same field strength in the inside plane of the elevator [2],[3]. Histograms are useful for carrying out complete estimations of the whole area in a predetermined plane within the environment. The reference value used was the EMF strength at the maximum interference distance mentioned in the objective above. Also, we chose to use the horizontal plane 130 cm from the floor for the estimation of histograms. The percentage of the area in the chosen plane whose electric field strength exceeds this reference value gives an indication of the possibility of EMI negatively impacting the pacemaker. We can determine whether or not a pacemaker malfunction is likely to occur by identifying those areas where the EMF exceeds that reference value.


Figure 1: FDTD model for elevator

| Cell size (cubic) | $\mathrm{d}=10 \mathrm{~mm}$ |
| :---: | :---: |
| Total Problem Space | $255 \times 185 \times 255$ (cells) |
| Absorbing Boundary Conditions | Perfectly Matched Layers (8 layers) |
| Frequency | 2142 MHz |
| Number of Iterations | 1,000 |
| Required Memory | 700 MB |
| Elevator Model | Body: PEC |
|  | Rectangular Opening in roof: $300 \times 600 \mathrm{~mm}$ <br> Diameter of Circle in roof: 300 mm |
| Human phantom model | Homogeneous ( $\varepsilon$ r $=50, \sigma=1.3$ ); realistic shape |
| Cellular Radio | Half-wavelength Dipole Antenna |

Table 1: Computation Parameters

## 3. Numerical Analysis

The technique employed was the FDTD method which allows the electric field strength in every cell within the defined environment to be accurately determined. Each cell is cubic shaped with dimensions of $1 \times 1 \times$ 1 cm . The smaller the cell size used, the more accurate the results will be however the greater will be the memory requirements for the simulation. Each cell's edge is an electric field location and the material for each edge can be specified independently of the material for its adjacent edge. The FDTD method allows Maxwell's equations to be solved in the following sequence: 1) Calculate the electric field in all cells at a given instant in time, 2) Calculate the magnetic field in all cells at the next instant in time, and 3) Repeat the process [4]. The dimensions of the total problem space are shown in Table 1 and are: $255 \times 185 \times 255$ cells. However, the elevator's dimensions were $220 \times 162 \times 216 \mathrm{~cm}$. The phantom model defined has a height of 165 cm and the distance of the antenna from the floor when placed at the head of the phantom models was 158 cm .

## 4. Results

In Figure 2, there is 1 phantom model in the center of the elevator with one mobile radio in operation, while in Figure 4 there are 2 phantom models with only one mobile radio in operation. Increased absorption due to the presence of the additional human model in the elevator results in the reduction in the maximum value of the field strength in the plane of the pacemaker $(130 \mathrm{~cm})$. This is observed in the corresponding histograms shown in Figures 3 and 5. Increasing the number of phantoms and cellular radios present in the model allows us to more accurately depict a real-life scenario where there are more signal sources as well as additional humans which may further increase absorption of signals. Figure 6 and Figure 7 show the position of cases where there are 3 and 4 humans in the model each using an antenna 2 cm from their heads. In Figure 8, we are able to see the corresponding effect on the relative values of electric field strength. The combined results in Figure 8


Figure 2: E-Field: 1 antenna; 1 phantom


Figure 4: E-Field: 1 antenna; 2 phantoms


Figure 6: E-Field: 3 antennas; 3 phantoms

Figure 3: Histogram: 1 antenna; 1 phantom


Figure 5: Histogram: 1 antenna; 2 phantoms


Figure 7: E-field: 4 antennas; 4 phantoms
suggests that there is an disproportional increase in the field strength as the number of humans in the model was increased from one to four. The maximum field strength with 4 persons present is 3 dB higher than that for 1 person. The difference would be expected to be more than 3 dB considering that there are 3 additional antennas but the additional 3 homogeneous phantoms also increase the absorption of electric fields in the elevator. In a real-world setting however, mobile phones in use do not all have identical settings and there are many possible combinations of signal frequency, amplitude, phase and power output. The parameters chosen give results which are very conservative hence it is expected that any modifications which are made will result in electric field strength values which are lower than those achieved in the cases presented in this paper.

The case of 2 persons using mobile phones but for different purposes was also examined since functions such as text messaging or browsing the internet is also a common usage for mobile phones. In the scenario depicted in Figure 9, one of the phones is held above the reference plane while the other is held below the reference plane. From the histogram generated, we observe in Figure 10 that the maximum EMF strength in this case was -10 dB . When compared with the case of 2 persons when only 1 of them is using a mobile phones (Figures 4 5), the maximum field strength in Figure 10 has increased as expected but only by 1dB.

## 5. Conclusion

With an increase in the number of passengers using cellular radios in the elevator the results consistently show that there is an increase in the Electric field strength, however there was not a linear increase in the values


Figure 8: Histogram showing Field Strengths for cases with 2, 3 and 4 persons in the elevator


Figure 9: Mobile phone use includes both speaking and sending text messages


Figure 10: Histogram for scenario in Figure 9
due to increased attenuation of EMF energy as a result of more phantom models being present. In the cases studied, there was no result in which the maximum EMF strength approached or exceeded the threshold limit which can adversely affect pacemaker operation. It remains to be seen however, the resulting outcome when there are more phantom models included in the simulation with an increased number of mobile phones, having different specifications (power, amplitude, phase, frequency). It is expected that there will eventually be a maximum number of persons with mobile phones which will result in the electric field strength becoming closer to the threshold value for safe pacemaker operation. Practically however, there are limits to the number of persons who can travel in an elevator. It is our intention to remain within these practical limits as we continue to conduct further research in this field.

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# Study on Dielectric Properties of Phantom Material for SAR Test in a Human Body 

A.K. Lee, J.I. Choi, D.U. Sim, and H.D. Choi<br>Digital Broadcasting Research Division, Electronics and Telecommunications Research Institute, Korea


#### Abstract

Human body equivalent material would be needed to determine the specific absorption rate of body-mounted devices such as a personal digital assistant. In this paper, one-dimensional multi-layered models extracted from a Korean and a European voxel models of different physique were used to investigate the SAR level in the human trunk region exposed by a plane wave. The multi-layered models over one thousand cover various thicknesses and compositions of chest and abdomen parts of the two anatomical models. Peak 1- and $10-\mathrm{g}$ SARs in each model were calculated and compared to those in the head-simulating tissue. The results show that a gap between the maximum SARs of the multi-layered models from the trunk region and the head-simulating tissue at most frequencies was very wide (the maximum 5.5 dB ) and the maximum SAR level in the human trunk was too high to allow the equivalent SAR level in any single material. Therefore, a proper scaling factor which does not excessively overestimate the test result should be discussed and arranged from the SAR level in the head simulating tissue for SAR test for EM exposure at the human body region.


## 1. Introduction

The present standards such as IEEE Std 1528 [1] and IEC 62209-1 [2] for compliance test with radiation safety limits generally deal with hand held devices to be used close to the human head. Dielectric parameters of tissue simulating liquids for the compliance test in these standards are defined based on the worst-case tissue layer composition and thickness for the user group including adults and children in the region of the ear and the temporal bone with respect to absorption at each frequency.

Meanwhile the wireless communication devices mounted on the human body such as a personal digital assistant (PDA) have been rapidly appeared. The specific absorption rate (SAR) determining procedure for local exposure from bodymounted devices is under consideration in IEC TC106. The body-mounted devices radiate RF field to waist, abdomen, chest, etc of the human body. The compositions and thicknesses of the trunk are significantly different from those of the head.

In this paper, the peak $1-$ and $10-\mathrm{g}$ SARs have been analyzed in each one-dimensional multi-layered model for plane wave incidence in the frequency range of $300-6000 \mathrm{MHz}$. These models were extracted from two adult trunk parts of Korean and European male voxel models. The multi-layered models are extremely various in composition and thickness of tissues. The SAR values were analyzed statistically and the $90^{\text {th }}$ and $100^{\text {th }}$ percentile SARs at each frequency were compared with those in the head-equivalent tissue and the published data.

## 2. Material and Method

The used two anatomical models for implementation of multi-layered tissue models are from Korean and European voxel models [3], [4]. These anatomical models have different voxel sizes, partially different tissue types, and different body physique. The anatomical data of the European model has originated from the US National Library of Medicine and Brooks Air Force Base has converted it to the voxel model. The Korean male model is close to the Korean average physique but the European model has a big body and the thick layer of fat tissue compared to the Korean model. The dielectric properties of the tissues in the frequency range of $300-6000 \mathrm{MHz}$ were obtained from [5].

Table 1 shows the physiques of the bodies, which were obtained mostly at the cross-section views of the voxel models. From the data, we can see the wide gaps in physique between the two models. Figure 1 compares the example of tissue layered models extracted near the omphalos. The thicknesses of the two models are 186 and 261 mm and the skin and fat types are different each other. The relative permittivities and conductivities of the skin and fat of the Korean model are 46.08 and
$0.84 \mathrm{~S} / \mathrm{m}$ (wet skin) and 11.33 and $0.11 \mathrm{~S} / \mathrm{m}$ (fat), respectively at 900 MHz and those of the European model are 41.41 and $0.87 \mathrm{~S} / \mathrm{m}$ (dry skin), and 5.46 and $0.05 \mathrm{~S} / \mathrm{m}$ (mean fat).

The 306 and 728 points evenly at the front surfaces covering the chest and abdomen of the Korean and European models were selected for planar multi-layered models. Analysis method to calculate SARs in the multi-layered models is similar to that in [6], which considered the various thicknesses according to the age for the tissue composition in the vicinity of the ear and used the infinite half-space layered models terminated with the brain tissue. In this paper, the variation of tissue thickness according to age was not considered but the various tissue compositions and thicknesses over one thousand for the two adult male trunks were analyzed. And each layered model was terminated with the outer air because the real compositions of tissues passing through from the front to the rear of the human body.


Figure 1: Example of planar multi-layered models.

Table 1: Comparison of the physiques.
(unit: mm)

| Items | Korean model | European model |
| :---: | :---: | :---: |
| Stature | 1761 | 1874 |
| Chest breadth | 354 | 384 |
| Waist breadth | 324 | 358 |
| Hip breadth | 366 | 406 |
| Chest depth | 216 | 266 |
| Waist depth | 174 | 240 |
| Buttock depth | 186 | 238 |

## 3. Results and Discussion

Effect on SAR result of different tissue types between the two anatomical models was investigated. Figure 2 compares
peak SARs according to the frequency variation when the superficial tissue structure is (dry skin-mean fat-muscle) and (wet skin-fat-muscle), respectively. The (dry skin-mean fat-muscle) structure roughly produced higher SAR at low frequencies than in the (wet skin-fat-muscle) structure and the behavior was reverse at high frequencies. This characteristic was appeared in the separately analyzed results of the multi-layered models obtained from the European and Korean male models (MLEM and MLKM, respectively); the MLEM contributed dominantly to the peak 1-and $10-\mathrm{g}$ SARs at frequencies below about 2.5 GHz and the MLKM was dominant at upper frequencies.


Figure 2: Effect on SAR of dielectric properties of skin and fat tissues. The incident power density is $1 \mathrm{~W} / \mathrm{m}^{2}$.

The mean and standard deviation of peak 1- and $10-\mathrm{g}$ SAR results in the whole layered tissue models (MLEM and MLKM) were calculated at each frequency. Figure 3 represents $90^{\text {th }}$ and $100^{\text {th }}$ percentile values of peak 1 -g SARs in the multi-layered models. The statistically expected value was obtained from the mean and standard deviation at each frequency. The graphs show that the statistically expected and the real values fit in well with each other for the $90^{\text {th }}$ percentile value, but those for 100 percentile one do not. And the peak SAR in the head simulating tissue is lower than even the $90^{\text {th }}$ SAR value at most of the considered frequencies.

The increase in SAR in the human trunk was observed compared to the head region; the maximum 5.5 dB at 900 MHz between the statistically expected $100^{\text {th }}$ percentile and the head simulating values. It might be because various compositions of tissues at many points of the human trunks were considered. The wide gap of $1.9-3.2 \mathrm{~dB}$ even between $90^{\text {th }}$ and $100^{\text {th }}$ percentile body SARs was observed.


Figure 3: SARs in multi-layered models covering the human trunk region and in the head simulating tissue.

## 4. Conclusion

First of all, the dielectric properties of the tissue-simulating material for the SAR test ought to be one that the evaluated result should not underestimate the real-exposed level. However, the worst case concept such as the maximum SAR level in the above graphs could lead to excessively overestimate the exposed level in a general situation. And what is more, the maximum SAR levels in this paper were too high at many frequencies considered to be obtained in any real-single material.

More investigations of SAR levels considering sex, age, etc are desirable but it might just record the higher value in the maximum SAR level. Lively discussion among researchers on the proper mark than the renewed maximum level in the SAR evaluation seems to be required.

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