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March 26–29, 2006 Cambridge, USA

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Contents

Energy Invariants to Composition Rules for Scattering and Transfer Matrices of Propagating and Evanes- cent Waves in Dielectric Structures	10
Local Dielectric Measurement by Waveguide-type Microscopic Aperture Probe	13
Power Absorption of Near Field of Elementary Radiators in Proximity of a Composite Layer	15
The Imbedding Method in the Theory of Horn Array Antennas—Hypershort Impulses and the Near Fields	22
Near-field Response in Lossy Media with Exponential Conductivity Inhomogeneity	26
An Analysis of Coaxial Line Slot antenna for Hyperthermia Treatment by Spectral Domain Approach	31
Critical Study of DCIM, and Development of Efficient Simulation Tool for 3D Printed Structures in Multilayer Media	35
Analysis of Cylindrical Microstrip Line with Finite Thickness of Conductor	38
Detection of Small Tumors in Microwave Medical Imaging Using Level Sets and Music	43
3D Shape Reconstruction in Optical Tomography Using Spherical Harmonics and BEM	48
On the Intermittency of the Light Propagation in Disordered Optical Materials	53
A Novel Multiband and Broadband Fractal Patch Antenna	57
Improvement of Reflectarray Performances at Millimeter Waves by Reduction of the Cell Size	60
Array Patterns Synthesizing Using Genetic Algorithm	64
Space-filling Patch Antennas with CPW Feed	69
The Power Line Transmission Characteristics for an OFDM Signal	74
Relation Between Balance-unbalance Conversion Factor and Leaked Electric Field in Power Line with Branch for PLC	81
On the Application of the Radiative Transfer Approach to Scattering from a Random Medium Layer with Rough Boundaries	88
Angular Normalization of ENVISAT ASAR Data over Sahelian-grassland Using a Coherent Scattering Model	94
A Review of the Mechanisms of Interaction Between the Extremely Low Frequency Electromagnetic Fields and Human Biology	99
Calculating SAR in Two Models of the Human Head Exposed to Mobile Phones Radiations at 900 and 1800 MHz	104

Qualitative Analysis of Human Semen Using Microwaves	. 110
Effects from the Thin Metallic Substrate Sandwiched in Planar Multilayer Microstrip Lines	. 115
Extraction of Chip Power Delivery Current and Activity Variations	. 120
Principles of Synthesis of Steerable Reflect-array Antennas	. 126
Design of a Steerable Reflect-array Antenna with Semiconductor Tunable Varactor Diodes	. 130
Modeling of Low-profile Reflect Array Antenna	. 133
Waveform Prediction of a Pulse Communication Link between Antennas Modeled by a Combination of Thin-wires	. 137
Mathematical Modeling of Nonlinear Waves and Oscillations in Gyromagnetic Structures by Bifurcation Theory Methods	. 143
Mobile Wireless Communication System Antennas for 260 MHz-band	. 148
Compact Surface-mount UWB Monopole Antenna for Mobile Applications	. 150
Reliability and Availability of GPS Measures in Airport Landing Systems	. 153
Development of the Pulsed Direct Current Iontophoresis and Its Clinical Application	. 157
Computation of Resonant Frequencies of Any Shaped Dielectric Resonators by CFDTD	. 163
A Novel Transmission Line Model for Analyzing Bowtie Patch Antennas	. 168
Applying Oblique Coordinates in the Method of Lines	. 172
Nonlinear Pulse Propagation and Modulation Instability in Periodic Media with and without Defects	. 177
SAR Remote Sensing of Snow Parameters in Norwegian Areas — Current Status and Future Perspective .	. 182
Buried Cylinders Geometric Parameters Measurement by Means of GPR	. 187
Recent Advances in Bioelectromagnetics Research on Mobile Telephony and Health—An Introduction	. 192
Epidemiologic Assessment of Cancer Risk from Mobile Phone Use: Where Are We?	. 197
NSA Calculation of Anechoic Chamber Using Method of Moment	. 200
An Effective Inversion Method Based on the Padé via Lanczos Process	. 206
QN Inversion of Large-scale MT Data	. 210
2.5D Algorithm for Tomographic Imaging of the Deep Electromagnetic Geophysical Measurement	. 214
A Rigorous 3-D MT Inversion	. 219
Contrast Source Inversion of 3D Electromagnetic Data	. 223
Statistical Distribution of Field Scattered by 1-Dimensional Random Slightly Rough Surfaces	. 227
Dispersion Characteristics of Coplanar Waveguides at Subterahertz Frequencies	. 232
A Multi Conductor Transmission Line Model for the Evaluation of the Rotor Shaft Voltages in Adjustable Speed Drive Motors	. 236
Frequency-selective Power Transducers "Hexagonal Ferrite Resonator—Semiconductor Element"	. 241

Wave Propagation in Grounded Dielectric Slabs with Double Negative Metamaterials	6
Detection of Groundwater by Ground Penetrating Radar	1
GPR Ground Bounce Removal Methods Based on Blind Source Separation	6
A High-order Finite Element Method for Electrical Impedance Tomography	0
Arbitrary Lagrangian Eulerian Electromechanics in 3D 265	5
Hybrid Numerical Simulation of Micro Electro Mechanical Systems	0
EM Field Induced in Inhomogeneous Dielectric Spheres by External Sources	5
Advanced Design of Phased Array Beam-forming Networks	9
Slotline Leaky Wave Antenna with a Stacked Substrate	3
Light Scattering on 2D Nanostructured Resonant Gratings	8
Modified Equivalent Circuit Model of Microwave Filter with LTCC Technique	2
Design of a Metafilm-composite Dielectric Shielding Structure Using a Genetic Algorithm	7
Statistical and Adaptive Signal Processing for UXO Discrimination for Next-generation Sensor Data 302	2
New GL Method and Its Advantages for Resolving Historical Difficulties	6
MATLAB SIMULINK Based DQ Modeling and Dynamic Characteristics of Three Phase Self Excited Induction Generator	2
Estimation of Higher Order Correlation between Electromagnetic and Sound Waves Leaked from VDT Environment Based on Fuzzy Probability and the Prediction of Probability Distribution	7
A Single Phase Single Stage AC/DC Converter with High Input Power Factor and Tight Output Voltage Regulation	2
A New Generalized Space Vector Modulation Algorithm for Neutral-point-clamped Multilevel Converters . 330	0
Improved Mesh Conforming Boundaries for the TLM Numerical Method	6
The Spectral Expansion on the Entire Real Line of Green's Function for a Three-layer Medium in the Fundamental Functions of a Nonself-adjoint Sturm-Liouville Operator	4
Some Applications of the High-mode-merging Method	7
Design of a Non-uniform High Impedance Surface for a Low Profile Antenna	2
Survey on Interference Mitigation via Adaptive Array Processing in GPS	7
A Simulation Tool for Space-time Adaptive Processing in GPS	3
Analysis and a Novel Design of the Beamspace Broadband Adaptive Array	8
A Subspace-based Robust Adaptive Capon Beamforming	4
Amplitude Estimation of Multichannel Signal in Spatially and Temporally Correlated Noise	0
A Neural Network Approach for Electromagnetic Diagnostic Applications	5
2.5D AGILD Electromagnetic Modeling and Inversion	0
Measured Electromagnetic Pulses Verify Asymptotics and Analysis for Linear, Dispersive Media	5

A New Formulation for Scattering by Impedant 3D Bodies
Chasmas Including Magnetic Effects
Generation of Solar Magnetic Fields Using a Quadripolar Seed Field 406
Higher Order Fourier Analysis for Multiple Species Plasma
Accuracy of Air Ion Field Measurement
Numerical Simulation of Nonlinear and Parametric Oscillations in a Semiconductor Resonator Structure 416
A Parallel Computer Implementation of Fast Low-rank QR Approximation of the Biot-Savart Law 420
Full Wave Analysis of RF Signal Attenuation in a Lossy Rough Surface Cave Using a High Order Time Domain Vector Finite Element Method
A Matlab-Based Virtual Propagation Tool: Surface Wave Mixed-path Calculator
Optical Binding in Air
Towards Efficient Modelling of Optical Micromanipulation of Complex Structures
Analysis of Evanescent Waves Scattering by a Single Particle in Total Internal Reflection Microscopy 447
T-matrix Simulation of Plasmon Resonances of Particles on or Near a Surface
Magnetic Nanostructure Hysteresis Loop Calculation for Modified Thin Film Multi-layer by Ion Irradiation455
Energetical Model Interpretation of Thermal Stability by Changing Direction of the Magnetization of Nano Magnetic Structure
Innovation Use of Nano Technology in Magnetic Storage Devices and Nano Computers
Innovation Use of Nano Technology in Magnetic Storage Devices and Nano Computers
Innovation Use of Nano Technology in Magnetic Storage Devices and Nano Computers
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Innovation Use of Nano Technology in Magnetic Storage Devices and Nano Computers
Innovation Use of Nano Technology in Magnetic Storage Devices and Nano Computers
Innovation Use of Nano Technology in Magnetic Storage Devices and Nano Computers 461 Application of Finite Network Theory to the Transient Process of Electromagnetic Forming 461 Exact Expressions of the Orbit-curvature and Curvature-radius of the Toroidal/Helical Orbits 476 Dual-band/Broadband Circular Polarizers Designed with Cascaded Dielectric Septum Loadings 477 High-accuracy Approximation to the Integrated Length of Toroidal/Helical Orbits 478 Low Cost 60 GHz Gb/s Radio Development 488 Nonlinear Inversion of Multi-frequency Microwave Fresnel Data Using the Multiplicative Regularized 489 Geometric Optics and Electromagnetic Models for Cylindrical Obstacles 499 3D and 2.5D AGLID EMS Stirring Modeling in the Cylindrical Coordinate System 500 Reduction of FDTD Simulation Time with Modal Methods 510 Approximate Decomposition for the Solution of Boundary Value Problems for Elliptic Systems Arising in 514 Some Elliptic Traveling Wave Solutions to the Novikov-Veselov Equation 514
Innovation Use of Nano Technology in Magnetic Storage Devices and Nano Computers 461 Application of Finite Network Theory to the Transient Process of Electromagnetic Forming 461 Exact Expressions of the Orbit-curvature and Curvature-radius of the Toroidal/Helical Orbits 476 Dual-band/Broadband Circular Polarizers Designed with Cascaded Dielectric Septum Loadings 477 High-accuracy Approximation to the Integrated Length of Toroidal/Helical Orbits 476 Low Cost 60 GHz Gb/s Radio Development 483 Nonlinear Inversion of Multi-frequency Microwave Fresnel Data Using the Multiplicative Regularized 486 New Stochastic AGLID EM Modeling and Inversion 490 Geometric Optics and Electromagnetic Models for Cylindrical Obstacles 493 3D and 2.5D AGLID EMS Stirring Modeling in the Cylindrical Coordinate System 506 Reduction of FDTD Simulation Time with Modal Methods 510 Approximate Decomposition for the Solution of Boundary Value Problems for Elliptic Systems Arising in 514 Some Elliptic Traveling Wave Solutions to the Novikov-Veselov Equation 514 Effects of the Resonant Scattering of Intensive Fields by Weakly Nonlinear Dielectric Layer. 514

Optimized Satellite System-like Data Fitting on a Spherical Shell	533
A Study of RF Absorber for Anechoic Chambers Used in the Frequency Range for Power Line Commu- nication System	538
Author Index	544

Energy Invariants to Composition Rules for Scattering and Transfer Matrices of Propagating and Evanescent Waves in Dielectric Structures

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Abstract—We present as a basis to modern wave multiple scettering theory an extended unitarity for the S-scattering matrix and an extended pseudo-unitarity for the transfer matrix of propagating and evanescent (near field) electromagnetic waves in a volume or surface lossless dielectric structure with spatial inhomogeneities of any dimension. The formalism of angular spectrum wave amplitudes is used. The presented extended unitarity and pseudo-unitarity are shown to be energy invariants to composition rules for the S-matrix and the transfer matrix, respectively. From composition rules, we derive a complete system of nonlinear differential equations for blocks of the S-matrix, with Riccati equation being a main one, and a linear equation for the transfer matrix.

Section 1.

During the last one and half decade the wave multiple scattering theory based on composition rule [1] for scattering operator (T-matrix) was reformulated in terms of virtual splitting the volume or surface inhomogeneous dielectric structure into a stack of elementary layers (slices), with slices being perpendicular to an embedding parameter and separated by splits, which may be vanishingly thin. In result using the Sommerfeld-Weyl angular-spectrum decomposition of wave amplitudes, a system of exact equations (transfer relation) [2] was obtained for the operator wave reflection and transmission coefficients of the structure and the operator wave amplitudes of waves in splits between slices (local fields).

The report aims to show that the recently derived, at study the effect of energy emission from an evanescent wave, extended unitarity of the 2×2 block S-scattering matrix [3] is an energy invariant to a specific composition rule for S-matrix, which is a consequence from the transfer relations. This composition rule describes the incremental change of S-matrix of subsystem of slices upon attachment an additional subsystem of slices. In the case of infinitesimally thin attached slice, we obtain a complete system of nonlinear differential equations for blocks of the S-matrix, with Riccati equation being a main one and taking into account a strong singularity of the electric field Green tensor function in a background. The S-matrix is closely related to the transfer matrix, for which we derive a linear equation with an energy invariant in the form of an extended pseudo-unitarity of the transfer matrix.

Section 2.

Let a volume or surface dielectric structure with scalar dielectric permittivity $\epsilon(\vec{r})$ occupies a region between planes z = 0 and z = L of Cartesian coordinate system x, y, z. The electric field of monochromatic electromagnetic wave to be incident onto the left boundary plane z = 0 is written as (see details in [2] and [3]) $(2\pi)^{-2} \int d\vec{k}_{\perp} \exp(i\vec{k}_{\perp}\vec{r}_{\perp})E^{\circ}_{\alpha}(\vec{k}_{\perp})\exp(i\gamma_{k}z)$. Here \vec{k}_{\perp} is the transverse to the z axis component of a wave vector \vec{k} , and the angular spectrum amplitude $E^{\circ}_{\alpha}(\vec{k}_{\perp})$ of the incident electric field describes either propagating or evanescent wave, depending on $k_{\perp} < k_{\circ}$ and $\gamma_{k} = \sqrt{k_{\circ}^{2} - k_{\perp}^{2}}$ is real or $k_{\perp} > k_{\circ}$ and $\gamma_{k} = i\sqrt{k_{\perp}^{2} - k_{\circ}^{2}}$ is purely imaginary quantity, respectively. The quantity k_{\circ} is the wave number in a background with dielectric permittivity ϵ_{\circ} . The angular spectrum amplitudes of electric field, transmitted through and reflected from the structure, are written in terms of the tensor operator transmission $A_{\alpha\beta}(\vec{k}_{\perp}, \vec{k'}_{\perp}) E^{\circ}_{\beta}(\vec{k'}_{\perp})$ and $(2\pi)^{-2} \int d\vec{k}_{\perp} B_{\alpha\beta}(\vec{k}_{\perp}, \vec{k'}_{\perp}) E^{\circ}_{\beta}(\vec{k'}_{\perp})$, respectively. An electromagnetic wave may be incident upon the right boundary plane z = L with angular spectrum amplitude $\tilde{E}^{\circ}_{\alpha}(\vec{k}_{\perp})$. In this case the angular spectrum amplitudes of electric field, transmitted through and reflected from the structure, are written in terms of the tensor operator transmission $\tilde{A}_{\alpha\beta}(\vec{k}_{\perp}, \vec{k'}_{\perp})$ and reflection $\tilde{B}_{\alpha\beta}(\vec{k}_{\perp}, \vec{k'}_{\perp})$ coefficients of plane wave.

The 2×2 block S-matrix of the structure is defined in terms of the above tensor coefficients of wave transmission through and reflection from structure as follows

$$S = \left(\begin{array}{cc} A & \tilde{B} \\ B & \tilde{A} \end{array}\right) \tag{1}$$

Physically the S-matrix transforms the angular spectrum amplitudes of incident forward and backward going waves, with respect to positive direction of the z axis,into the angular spectrum amplitudes of scattered forward and backward going waves.

Section 3.

Split virtually the dielectric structure under consideration into a stack of n slices with splits between them, as in Fig. 1 of [2]. According to this reference, the composition rule [1] for the scattering operator (*T*-matrix) together with condition of non-overlapping the slices lead to a mixed system of exact equations-transfer relations for blocks of the *S*-matrices of subsystems of slices and amplitudes of local waves inside splits. As doing so the tensor coefficients of the local fields waves in splits can be eliminated from the transfer relations and expressed in terms of blocks of the *S*-matrices, $S_{1,m}$ and $S_{m+1,n}$. After this elimination, the transfer relations give the separate system of recurrent equations that describes the incremental change of the *S*-matrix of subsystem of slices with numbers $1, \ldots, m$ upon attachment of additional subsystem of slices with numbers $m+1, \ldots, n$. This system of recurrent equations has been got in [2] for the case of 2D dielectric structure and TE polarization, with m = n - 1, and for general case has a form

$$A_{1,n} = A_{m+1,n} (I - \tilde{B}_{1,m} B_{m+1,n})^{-1} A_{1,m},$$

$$B_{1,n} = B_{1,m} + \tilde{A}_{1,m} B_{m+1,n} (I - \tilde{B}_{1,m} B_{m+1,n})^{-1} A_{1,m}$$
(2)

and

$$\tilde{A}_{1,n} = \tilde{A}_{1,m} (\tilde{I} - B_{m+1,n} \tilde{B}_{1,m})^{-1} \tilde{A}_{m+1,n},
\tilde{B}_{1,n} = \tilde{B}_{m+1,n} + A_{m+1,n} \tilde{B}_{1,m} (\tilde{I} - B_{m+1,n} \tilde{B}_{1,m})^{-1} \tilde{A}_{m+1,n}$$
(3)

The symbols I and \tilde{I} denote some identity tensor operators, $I_{\alpha\beta}(\vec{k}_{\perp}, \vec{k'}_{\perp}) = P_{\alpha\beta}^{tr}(\hat{k}^+)\delta_{\vec{k}_{\perp},\vec{k'}_{\perp}}$ and $\tilde{I}_{\alpha\beta}(\vec{k}_{\perp}, \vec{k'}_{\perp}) = P_{\alpha\beta}^{tr}(\hat{k}^-)\delta_{\vec{k}_{\perp},\vec{k'}_{\perp}}$, acting in the subspaces of transverse inhomogeneous plane waves going forward and backward with the wave vectors $\vec{k}^{\pm} = \vec{k}_{\perp} \pm \gamma_k \hat{z}$, respectively, where \hat{z} is the unit vector along the z axis. Besides, the units vectors along these wave vectors are defined by, $\hat{k}^{\pm} = \hat{k}^{\pm}/k_{\circ}$, and a tensor, $P_{\alpha\beta}^{tr}(\hat{k})$, means the orthogonal projector in direction perpendicular to the unit vector \hat{k} . One should note here that in the scalar case a system of recurrent equations similar to Eqs. (2,3) has been got by Redheffer [4] as the functional relations (semigroup property) associated with the Riccati system of equations for the reflection and transmission coefficients of waves propagating in transmission lines. In this case, Regheffer has introduced an useful notion *star product*, (*), of the scattering matrices, which enables us to rewrite the above system of recurrent Eqs. (2,3) shortly as $S_{1,n} = S_{1,m} * S_{m+1,n}$.

Section 4.

Turn to the composition rule for S-matrix in Eqs. (2,3) and consider the case of thin attached nth slice, m = n-1. We introduce a useful renormalized version **S** of the scattering matrix (1) putting $\mathbf{S} = \text{diag}(\gamma^{1/2}, \gamma^{1/2})S$ $\text{diag}(\gamma^{-1/2}, \gamma^{-1/2})$ and suppose the S-matrix of the nth slice to be small deviated from an identity matrix, $\mathbf{I} = \text{diag}(I, \tilde{I})$, and subject to a condition, $\mathbf{S}_{n,n} = \mathbf{I} + \delta \mathbf{S} \Delta \mathbf{z}$. Here a thickness Δz of the nth slice tends to zero and an infinitesimal scattering matrix δS is obtained by a solution to the Lippman-Schwinger equation for T-matrix in the form $(\mathbf{II}^{++}, \mathbf{II}^{+-})$

$$\delta \mathbf{S} = \begin{pmatrix} \mathbf{U}^{++} & \mathbf{U}^{+-} \\ \mathbf{U}^{-+} & \mathbf{U}^{--} \end{pmatrix}$$
(4)

The blocks of this infinitesimal scattering matrix are given by

$$\mathbf{U}_{\alpha\beta}^{\xi\eta}(\vec{k}_{\perp},\vec{k'}_{\perp};z) = \frac{1}{2i} \exp\left[-i(\xi\gamma_k - \eta\gamma_{k'})z\right] \frac{1}{\sqrt{\gamma_k}} U_{\alpha\beta}^{\xi\eta}(\vec{k}_{\perp},\vec{k'}_{\perp};z) \frac{1}{\sqrt{\gamma_{k'}}}$$
(5)

with

$$U^{\xi\eta}_{\alpha\beta}(\vec{k}_{\perp},\vec{k'}_{\perp};z) = P^{tr}_{\alpha\mu}(\hat{k}^{\xi})U_{\mu\nu}(\vec{k}_{\perp}-\vec{k'}_{\perp},z)P^{tr}_{\nu\beta}(\hat{k}^{\eta'})$$
$$U_{\alpha\beta}(\vec{k}_{\perp},z) = V(\vec{k}_{\perp},z)(\hat{x}_{\alpha}\hat{x}_{\beta}+\hat{y}_{\alpha}\hat{y}_{\beta})+v(\vec{k}_{\perp},z)\hat{z}_{\alpha}\hat{z}_{\beta}$$

where $\xi, \eta = \pm, V(\vec{k}_{\perp}, z)$ and $v(\vec{k}_{\perp}, z)$ are the spatial Fourier transforms of the scalar potential $V(\vec{r}) = -k_{\circ}^{2}[\epsilon(\vec{r}) - \epsilon_{\circ}]/\epsilon_{\circ}$ and a function $v(\vec{r}) = -k_{\circ}^{2}[\epsilon(\vec{r}) - \epsilon_{\circ}]/\epsilon(\vec{r})$, respectively, with respect to transverse to the z axis component

of the position vector, \hat{x} and \hat{y} are unit vectors along the x and y axes, respectively. Substituting the obtained asymptotics for the S-matrix of thin nth slice into composition rule in Eqs. (2), (3) gives the following systems of differential equations for blocks of the **S**-matrix

$$\frac{d\mathbf{B}}{dz} = \mathbf{U}^{+-} + \mathbf{U}^{++}\tilde{\mathbf{B}} + \tilde{\mathbf{B}}\mathbf{U}^{--} + \tilde{\mathbf{B}}\mathbf{U}^{-+}\tilde{\mathbf{B}}, \qquad \tilde{\mathbf{B}}(z=\mathbf{0}) = \mathbf{0}$$
(6)

$$\frac{d\mathbf{A}}{dz} = \tilde{\mathbf{A}}(\mathbf{U}^{--} + \mathbf{U}^{-+}\tilde{\mathbf{B}}), \qquad \tilde{\mathbf{A}}(z=0) = \tilde{I}$$
(7)

$$\frac{d\mathbf{A}}{dz} = (\tilde{\mathbf{B}}\mathbf{U}^{-+} + \mathbf{U}^{++})\mathbf{A}, \qquad \mathbf{A}(z=\mathbf{0}) = I$$
(8)

$$\frac{d\mathbf{B}}{dz} = \tilde{\mathbf{A}}\mathbf{U}^{-+}\mathbf{A}, \qquad \mathbf{B}(z=\mathbf{0}) = \mathbf{0}$$
(9)

Klyatskin [5] derived a matrix Riccati equation similar to Eq. (6) in scalar case.

Section 5.

By straightforward calculation, one can verify that the infinitesimal scattering matrix (4) satisfies the following extended unitarity condition

$$\mathbf{H}^{pr} + i\mathbf{H}^{ev}\Sigma_x)\delta\mathbf{S} + [(\mathbf{H}^{pr} + i\mathbf{H}^{ev}\Sigma_x)\delta\mathbf{S}]^{\dagger} = 0$$
(10)

where \mathbf{H}^{pr} and \mathbf{H}^{ev} denote projectors on propagating and evanescent waves, respectively, and $\Sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is a block Pauli matrix (see [3]). On the other hand, one can also prove that the *star product* of two S-matrices does satisfies the extended unitarity from [3] in the form

$$(\mathbf{H}^{pr}\mathbf{S})^{\dagger}(\mathbf{H}^{pr}\mathbf{S}) = \mathbf{H}^{pr}\mathbf{I}\mathbf{H}^{pr} - i[\mathbf{H}^{ev}\Sigma_{x}\mathbf{S} - (\mathbf{H}^{ev}\Sigma_{x}\mathbf{S})^{\dagger}]$$
(11)

if the both S-matrices satisfy (11) separately. Bearing in mind that the star product is associative [4], we conclude that a solution to the derived Riccati system of equations satisfies the extended unitarity (11).

Section 6.

The transfer matrix \mathbf{M} transforms, in different from the \mathbf{S} -matrix, the angular spectrum amplitudes of forward and backward going waves on the left side of the structure into ones on the right side of the structure. This definition gives the known relation between matrices under consideration (see, e. g., [2]) and leads from the derived Riccati-system of equations to the following linear differential equation for the transfer matrix

$$\frac{d\mathbf{M}}{dz} = \Sigma_z \delta \mathbf{S} \mathbf{M}, \qquad \mathbf{M}(z=0) = \mathbf{I}$$
(12)

were $\Sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is a block Pauli matrix. Starting with the extended unitarity (10) for the infinitesimal scattering matrix one can verify by direct differentiation that a solution to the obtained linear equation has an energy invariat in a form of the following extended pseudu-nitarity for the transfer matrix

$$\mathbf{M}^{\dagger}\Sigma_{z}(\mathbf{H}^{pr} - i\Sigma_{x}\mathbf{H}^{ev})\mathbf{M} = \Sigma_{z}(\mathbf{H}^{pr} - i\Sigma_{x}\mathbf{H}^{ev})$$
(13)

This extended pseudu-unitarity for the transfer matrix generalizes the known pseudu-unitarity constraint [6] on the case when evanescent waves may be present.

7. Conclusion

Summarizing, the presented complete system of differential equations for blocks of the S-matrix and differential equation for the transfer matrix together with their energy invariants can be considered as an analytical basis to incorporate the modern theory of electromagnetic wave multiple scattering by dielectric structures with near field effects.

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Local Dielectric Measurement by Waveguide-type Microscopic Aperture Probe

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Abstract—For dielectric constant measurement of areas smaller than the wavelength, this paper proposes a method of employing waveguide-type microscopic aperture probe. The probe is made of WR-15 waveguide with one end shielded with metal plate of 0.3 mm, on which a 0.5 mm-dia or a 0.1 mm-dia aperture is made. The dielectric constant is derived from the slope of phase difference swept over 50-70 GHz between the cases of free-space transmission with and without the dielectrics. In order to evaluate the system, the dielectric constant of Teflon has been measured by three cases of using the probes of 0.5 mm-dia and 0.1 mm-dia, and two V-band corrugated horns. The results show good agreement.

1. Introduction

One of the well-established dielectric measurement methods in millimeter and submillimeter wave bands is the free-space transmission method using two horns[1]. That is sufficient for large objects compared to the wavelength. For the measurement of microscopic regional dielectric distribution of heterogeneous dielectric materials and cellular tissues, the aperture must be downsized so as the spatial resolution to be smaller than the wavelength [2].

As the embodiment of small aperture, waveguide-type probes are employed in this research. The probe is made of WR-15 waveguide with one end shielded with metal plate of 0.3 mm, on which a 0.5 mm-dia or a 0.1 mm-dia aperture is made. Figure 1 shows the outline. The probe replaces one horn at the transmission side in the free-space transmission measurement.

A microscopic aperture illuminates the region comparable to the aperture size, so that it realizes high spatial resolution of scanning microscopy for surface topography. Furthermore, employing the millimeter and submillimeter wavebands enables spectroscopic analysis, for example, oxygen content analysis by 60 GHz band as envisioned. On the other hand, it must be experimentally investigated to evaluate the decrease of the signal-to-noise ratio.

In order to evaluate the system, the dielectric constant of Teflon has been measured both by the proposed system and the free-space transmission method using two V-band corrugated horns with the aperture diameter 31 mm.



Figure 1: Waveguide-type probe.

2. Measurement

The dielectric constants is obtained from the slope of phase difference between the case of free-space transmission with and without the dielectrics. Relative dielectric constant ε_r is derived by

$$\varepsilon_r = \left(\frac{300\Delta\phi}{360d} + 1\right)^{\frac{1}{2}} \tag{1}$$

where $\Delta \phi$ (degree/GHz) is the slope of the phase difference, and d (mm) is the sample thickness. As a dielectric sample, a Teflon plate 100 mm × 100 mm × 4.1 mm (thickness) is used. The Teflon plate is contacted with the transmission side horn or the probe.

The phase difference is measured by using the vector network analyzer MVNA 8-350 (AB Millimeter, France). The lower frequency limit of the probe is determined by the cutoff frequency 40 GHz for the TE10 mode of WR-15. The frequency is swept over 50-70 GHz at 0.1 GHz step.

3. Result

The measurement of phase difference is made three times to obtain the average ε_r . One result by each system is shown in Figure 2. The solid line shows the measured phase difference, and the dotted line is derived by the least square method. As the aperture is smaller, the phase variance is increasing. The two-horn system gives $\varepsilon_r = 1.99$ and the proposed system gives $\varepsilon_r = 1.89$ with 0.5 mm aperture and $\varepsilon_r = 1.93$ with 0.1 mm aperture. They show good agreement, although the proposed systems have larger variance of phase difference.



Figure 2: The phase difference and the slope measured by the two-horn free-space transmission method (a) and the proposed probes (b) and (c).

4. Conclusion

The dielectric constant has been measured by the waveguide-type microscopic aperture probes with 0.5 mm and 0.1 mm-dia, and the standard two-horn free-space transmission method as a reference. There is a good agreement between three results, while they show slightly small values compared to the nominal value of Teflon 2.1. The next step is the measurement by scanning with improved accuracy.

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Power Absorption of Near Field of Elementary Radiators in Proximity of a Composite Layer

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Abstract—Near-field behavior of elementary electric and magnetic dipoles close to a plane layer (or layers) of engineered composite materials is analyzed using the rigorous analytical approach. Some results of computations are represented for composite media containing conductive inclusions. These composites provide shielding mainly due to absorption of electromagnetic energy. The effect of conductivity of inclusions and their geometry (through their aspect ratio) on the absorption and radiation efficiency of a radiator near composite layers is analyzed.

1. Introduction

The problems of studying electromagnetic interaction of different radiators with composite layered structures both in far- and near-field zones arise at the development of shielding enclosures for different electronic devices. In [1], the approach to engineering composites with the desired frequency response based on Maxwell Garnett (MG) formulation and a genetic algorithm is presented. An engineered infinitely large composite layer of finite thickness in [1] is considered for both normal and oblique incident plane waves. However, concepts of reflection and transmission coefficients, as well as of angles of incidence and polarization, are applicable only to the far-field region. In the near-field zone, it is better to consider field intensity attenuation due to such effects as excitation of evanescent waves, scattering, and different mechanisms of ohmic loss and energy transformation. In [2], the notions of absorption and radiation efficiencies in terms of power are introduced, and the corresponding power fluxes are calculated rigorously and explicitly via the spectra of the fields using the known solutions of boundary problems for parallel-plane, cylindrical, and spherical cases.

This paper considers the near-field behavior of elementary electric and magnetic dipoles close to a plane layer (or layers) of engineered composites, and the effect of conductivity of inclusions and their geometry (through the aspect ratio) on the absorption and radiation efficiency of a radiator near composite layers is studied.

2. Mathematical Model

2.1. Maxwell Garnett Formalism for Composites Containing Conductive Inclusions

The MG formulation is well-suited for modeling of linear electrodynamically isotropic multiphase mixtures of metallic or dielectric particles in a homogeneous dielectric base, where the parameters of the mixture do not change in time according to some law as a result of some external force—electrical, mechanical, etc.; inclusions are at the distances greater than their characteristic size; and the characteristic size of inclusions is small compared to the wavelength in the effective medium. The generalized MG mixing formula for multiphase mixtures with randomly oriented ellipsoidal inclusions is [1,3],

$$\varepsilon_{eff} = \varepsilon_b + \frac{\frac{1}{3} \sum_{i=1}^n f_i(\varepsilon_i - \varepsilon_b) \sum_{k=1}^3 \frac{\varepsilon_b}{\varepsilon_b + N_{ik}(\varepsilon_i - \varepsilon_b)}}{1 - \frac{1}{3} \sum_{i=1}^n f_i(\varepsilon_i - \varepsilon_b) \sum_{k=1}^3 \frac{N_{ik}}{\varepsilon_b + N_{ik}(\varepsilon_i - \varepsilon_b)}}$$
(1)

where $\varepsilon_b(j\omega) = \varepsilon_{\infty b} + \chi_b(j\omega)$ and $\varepsilon_i(j\omega) = \varepsilon_{\infty i} + \chi_i(j\omega)$ are the relative permittivity of the base and of the *i*-th type of inclusions, respectively. In (1), f_i is the volume fraction occupied by the inclusions of the *i*-th type; N_{ik} are the depolarization factors [4] of the *i*-th type of inclusions, where indices k = 1, 2, 3 corresponds to x, y, and z coordinates. If the inclusions are thin cylinders, their two depolarization factors are close to 1/2, and the third can be calculated as in [5], $N \approx (a)^{-2} \ln(a)$, where a = l/d is a cylinder's aspect ratio (length/diameter). Since the MG formula is linear, the resultant effective permittivity of the mixture can be also represented through effective high-frequency permittivity and susceptibility function,

$$\varepsilon_{eff}(j\omega) = \varepsilon_{\infty eff} + \chi_{eff}(j\omega). \tag{2}$$

If inclusions are conducting (metallic), their frequency characteristic in terms of relative permittivity is

$$\varepsilon_i(j\omega) = \varepsilon' - j\varepsilon'' = \varepsilon' - j\sigma/\omega\varepsilon_0. \tag{3}$$

The MG rule is applicable when the concentration of the conducting particles is below the percolation threshold, $p_c \cong C/a \ll 1$, where a is an aspect (axis) ratio for the inclusions in the form of highly prolate spheroids [6], and C is the experimental coefficient depending on the composite morphology (typically, C = 1-10). Otherwise, the different approximations from the general effective medium theories should be used, for example, McLachlan [7] or Ghosh-Fuchs approximations [8].

The base material might be quite transparent over the frequency range where high shielding effectiveness is desirable. However, if there are conducting inclusions, the shielding effectiveness will be provided by absorption of electromagnetic energy due to conductivity loss and to the dimensional resonance in the particles. Presence of conductive particles will also increase reflection from the composite layer. In this paper, non-conductive composite materials (with dilute phase of conducting inclusions) are modeled. Non-conductive composites mainly absorb (rather than reflect) the energy of unwanted radiation. The effect of conductivity of inclusions and their geometry on the absorption and radiation efficiency of a radiator near composite layers is studied using the method described below.

2.2. Power Fluxes and Radiation and Absorption Efficiency in a Parallel-plane Structure

The near-field behavior of elementary radiators in proximity of a composite planar layer is studied using the unified rigorous analytical approach developed in [2, 9]. Herein, this approach is specified for the parallel-plane geometry. Power radiation efficiency and absorption efficiency are calculated, using formulas similar to those introduced in [2],

$$\eta_{rad} = 10 \log_{10}[(P_{rad} - P_{loss})/P_{rad}] \text{ and } \eta_{abs} = 10 \log_{10}[P_{loss}/P_{rad}].$$
(4)

The radiated power P_{rad} and the power loss P_{los} are defined for a parallel-plane dielectric layer (see Figure 1):

$$P_{rad} = P_{Z1} + P_{Z2}; \quad P_{loss} = P_{Z1} - P_{Z3}.$$
(5)

The z-component of the Poynting vector in the parallel-plane geometry is

$$p_z = 0.5Re(E_x H_y^* - E_y H_x^*), \tag{6}$$



Figure 1: Parallel-plate geometry with a dielectric layer.

where $E_{x,y}$ and $H_{x,y}$ are the corresponding phasors for the tangential components of electric and magnetic field, and the asterisk stands for complex conjugating. The power through any cross-section S in the plane z is $P_z = \iint_{\sigma} p_z dS$.

As is done in [10], the spectral densities $U^{e,m}$ and $I^{e,m}$ of scalar electric (e) and magnetic (m) potentials are introduced, and the expansion in terms of the complete system of eigenfunctions (Fourier representation) is applied. The scalar potentials $U^{e,m}$ and $I^{e,m}$ play part of the generalized voltages and currents, respectively, and they are used instead of the unknown field components. The potentials are obtained from the rigorous solution of the boundary problem, taking into account physical effects of diffraction, absorption, refraction, and numerous reflections. The tangential components of the electromagnetic field contain spatial spectra of the scalar potentials,

$$\vec{E}_{\tau} = \iint_{\chi_1\chi_2} (U^e \vec{t} + U^m \vec{f}) d\chi_1 d\chi_2; \quad \vec{H}_{\tau} = \iint_{\chi_1\chi_2} (I^e \vec{t} + I^m \vec{f}) d\chi_1 d\chi_2.$$
(7)

The complete system of vector eigenfunctions is

$$\vec{t} = (-j\chi_1 \vec{x}_0 - j\chi_2 \vec{y}_0)e^{-j\chi_1 x - j\chi_2 y}; \quad \vec{f} = (-j\chi_2 \vec{x}_0 + j\chi_1 \vec{y}_0)e^{-j\chi_1 x - j\chi_2 y}$$
(8)

Vectors \vec{x}_0 and \vec{y}_0 are the Cartesian unit vectors. Then, the power flux is



Figure 2: Complex permittivity of the composite: base material is Teflon ($\varepsilon' = 2.2$); aspect ratio for inclusions a = 1500; volumetric fraction of inclusions is 0.15%; conductivity of inclusions σ is a parameter.



Figure 3: Complex permittivity of the composite: base material is Teflon ($\varepsilon' = 2.2$); volumetric fraction of carbon inclusions is $0.7/a < p_c$; conductivity is $\sigma = 40000 \text{ S/m}^2$; aspect ratio *a* is a parameter.

$$P_{z} = 2\pi^{2} Re \iint_{\chi_{1}\chi_{2}} \chi^{2} (U^{e} I^{e*} + U^{m} I^{m*}) d\chi_{1} d\chi_{2},$$
(9)

where $\chi^2 = \chi_1^2 + \chi_2^2$, and $\chi_{1,2}$ are the spatial frequencies along x- and y-coordinates in Fourier representation for the field components. Substitution of the Fourier representation for the field components (7) into Maxwell's equations yields the 2-nd order differential equations for $U^{e,m}$ and $I^{e,m}$. In the cross-sections z_1 and z_2 , where the reflected waves exist, and inside the dielectric layers, the solutions for $U^{e,m}$ and $I^{e,m}$ are

$$U^{e,m} = U^{e,m}_{inc} \cdot e^{-\gamma z} + U^{e,m}_{refl} \cdot e^{+\gamma z}, \quad I^{e,m} = (U^{e,m}_{inc} \cdot e^{-\gamma z} - U^{e,m}_{refl} \cdot e^{+\gamma z})/Z^{e,m}, \tag{10}$$

where $\gamma^2 = \chi^2 - k_0^2$ is the square of the propagation constant, and $k_0 = \omega \sqrt{\mu_0 \varepsilon_0}$ is the wave number in free space. The characteristic impedance of the medium is $Z^{e,m}$. The scalar potentials $U_{in}^{e,m}$ and $U_r^{e,m}$ correspond to the incident and reflected waves, respectively. They are obtained as the coefficients of two linearly independent solutions for the boundary problem formulated for the one-dimensional Helmholtz equation (in z-direction). In the cross-section z_3 , there are no reflected waves, and the values $U_{refl}^{e,m}$ and $U_{refl}^{e,m}$ are zero. To calculate the power flux through the cross-section z_1 in a lossless medium, two cases should be considered: $|\chi| < k_0$, and $|\chi| > k_0$.



Figure 4: Shielding effectiveness (SE) in terms of plane wave formulation for an infinite layer of a composite material: (a) corresponding to Figure 2; (b) corresponding to Figure 3.



Figure 5: FDTD modeled power decrease through the composite layer. Field is radiated by the electric dipole placed at h = 5 mm below the layer (see Figure 1).

Case 1. When $|\chi| < k_0$, the propagation constant $\gamma = j\beta$ is imaginary in a lossless case, and the impedances $Z^e = \gamma/(j\omega\varepsilon_0)$ and $Z^m = j\omega\mu_0/\gamma$ are real, so the power flux for propagating waves is

$$P_{\rm z\,prop} = 2\pi^2 \iint_{\chi_1\chi_2} \chi^2 \Big[\Big(|U_{inc}^e|^2 - |U_{refl}^e|^2 \Big) / Z^e + \Big(|U_{inc}^m|^2 - |U_{refl}^m|^2 \Big) / Z^m \Big] d\chi_1\chi_2. \tag{11}$$

Case 2. When $|\chi| > k_0$, the propagation constant $\gamma = \beta$ is real, and the characteristic impedance $Z^{e,m} = jX^{e,m}$ is imaginary. The power flux for evanescent waves in this case is

$$P_{\rm z\,evan} = 4\pi^2 \iint_{\chi_1\chi_2} \chi^2 \Big[{\rm Im}(U^e_{inc}U^{e*}_{refl}) / X^e + {\rm Im}(U^m_{inc}U^{m*}_{refl}) / X^m \Big] d\chi_1 d\chi_2.$$
(12)

The exact expressions for the coefficients $U_{inc}^{e,m}$ and $U_{refl}^{e,m}$ are found from the solution of a boundary problem with the known volume densities for the source. Obviously, the power flux through the surface that crosses a medium without loss is independent of the z-coordinate, because the coefficients $U_{inc}^{e,m}$ and $U_{refl}^{e,m}$ are independent of the propagation z-coordinate. The total power flux (11), (12) is comprised of two terms: one is determined by the propagating waves ves with $\gamma = j\beta$, while the second is determined by evanescent waves with $\gamma = \beta$. Only for the regions where there are no reflected fields, $(U_{refl}^{e,m} \text{ and } I_{refl}^{e,m}$ are zero) the power flux is determined only by propagating waves. In general case, the propagation constant is complex. For multilayered structures, the cascading of transfer matrices can be used even for near fields, as is done in [9].

3. Computations

The frequency dependences for permittivity of the Teflon-based composites containing conductive fibers modeled using (1) are shown in Figures 2 and 3. The corresponding frequency dependences of shielding effectiveness (SE = $-20 \log_{10}(E_{tr}/E_{inc})$) defined in a plane-wave formulation for infinite plane panels made of these composites are presented in Figure 4. S.E. increases with the increase of conductivity and aspect ratio of inclusions. Figure 5 shows the rate of power decrease through the absorbing layer $\eta_{trans} = -10 \log_{10}(P_z/P_{ref})$. The results in Figure 5 are modeled using FDTD codes. The source is an elementary electric dipole parallel to the layer. The 20-mm thick layer is a Teflon-based ($\varepsilon_b = 2.2$) composite with conducting inclusions (a = 100; $\sigma = 40000 \text{ S/m}$; concentration is 0.7/a, below the percolation threshold). The reference plane for calculating P_{ref} is z = -1 mm.

Figures 6 and 7 show the dependences of the absorption coefficient (4) versus distance of the electric dipole from the composite layer for different frequencies, conductivities of inclusions, and their aspect ratio. The electric dipole is parallel to the layer surface. When the point of observation is in the far-field region, the absorption in composites increases with the increase of conductivity and aspect ratio of inclusions. In contrast to the far-field region, in the near-field zone the higher conductivity and higher aspect ratio do not necessarily lead to greater absorption. Absorption depends on the source type, distance between the source and the layer, and the effective constitutive parameters of the composite [2]. Trends of the curves in Figures 6 and 7 at varying a and σ are different for different frequencies. This can be explained by variations in frequency dependences of the effective parameters of composites.



Figure 6: Absorption coefficient versus distance h between the electric dipole and the composite layer (d = 1 mm); frequency is 0.1 GHz, 0.5 GHz, 3 GHz, and 9 GHz. Conductivity σ of inclusions is a parameter.



Figure 7: Absorption coefficient versus distance h between the elementary electric dipole and the composite layer (thickness d = 1 mm); frequency is 0.1 GHz, 1 GHz, 3 GHz, and 9 GHz. Aspect ratio a of inclusions is a parameter.

4. Conclusions

In this paper, the analytical formulas for absorption and radiation coefficients for radiators near a composite dielectric layer are obtained by rigorous boundary problem solution. The complex frequency-dependent permittivity of a composite dielectric containing conductive inclusions is modeled using Maxwell Garnett effective medium formulation. The results of computations for near-field of an elementary electric dipole close to a plane composite layer show that the behavior of absorption of near fields in the composite layer with respect to the conductivity and aspect ratio of inclusions is different from the far-field behavior. Near-field absorption in a layer depends on the distance of the radiator from the composite layer and the particular effective permittivity of the composite layer at the particular frequency.

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The Imbedding Method in the Theory of Horn Array Antennas—Hypershort Impulses and the Near Fields

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Abstract—The problem of hyper short impulses distortion with horn array antennas radiation considers from spectrum analysis point of view. As main reason for misphasing of Fourier-components of field the collective effect resonances of horns overirradiaton were considered. The imbedding equations for transparent coefficients (field directional diagram) and reflection coefficients of linear HAA as functions of radiated field frequency have been build. Some results of numerical experiment are given and a part of near fields (inhomogeneous modes) was discussed.

1. Introduction

The usage of nanotechnologies in radiolocation has met some problems with distortion of hyper short (HS) impulses being radiated by horn array antennas (HAA). Qualitative explanation of this effect is connected with arising of reactive fields formed near antenna's system. But quantitative description based on the traditional methods meets serious difficulties. For correct description of radiation of ultra wide band (UWB) impulse process it's necessary to examine the internal problem of electrodynamics of HAA. Let's take into consideration that models usually used to describe narrow band signals radiation can't be considered adequate for UWB impulses.

Using the spectral method distortion of UWB signal during the radiation can be explained by misphasing and changing of its Fourier-component's amplitudes, arising in horn band. The latter can be considered as a transitional layer, matching waveguides with free space. If field in feeding waveguides $-E_{in}$ and in free space $-E_{out}$ is written in the mixed representation

$$E(\vec{q}, z; \omega) = \int d\vec{\rho} \cdot E(\vec{\rho}, z; \omega) \exp\left\{-i\vec{q}\vec{\rho}\right\}, \qquad \vec{\rho} = (x, y), \qquad (1)$$

than the main characteristic of HAA—the transparence coefficient $T(\vec{q}, \vec{q}'; \omega)$ can be determent as a kernel of integral equation

$$E_{out}(\vec{q}, z; \omega)|_{z=H} = \int d\vec{q}' T(\vec{q}, \vec{q}'; \omega) \cdot E_{in}(\vec{q}', z; \omega)\Big|_{z=0}$$
(2)

Here H is thickness of the transition layer or horns height.

It's clear that when describing UWB impulse radiation in terms of spectral theory the demand to the measurement accuracy $T(\vec{q}, \vec{q}'; \omega)$ is much bigger than in the case of narrow band signal. In particular, the wide spectrum of the signal forces to take into consideration the group effects—i. e., overirradiation of horns in grating. This is usually neglected in narrowband field. Periodic property of grating space structure in combination with wide space signal spectrum leads to the fact that the definite group of frequencies inevitably lays in the field of Wood anomalies, where the important role is played by near fields—inhomogeneous modes of space spectrum of the radiation field.

Thus, the basic problem at the spectral approach to the solution of a problem on radiation of UWB-impulses by HAA consists in a choose of method allowing to solve the internal problem of HAA electrodynamics maximum correctly and to describe amplitude, phase and spatial vector of radiation of a monochromatic signal as function of its frequency. As such an approach it is proposed to use the imbedding method.

2. Imbedding Equations for Linear HAA

The imbedding method is used as base for getting the equation for transparent coefficient of HAA. The kernel of this method is in the following. A great number of solutions of similar problems is examined, these problems differ only with the value of one parameter—the imbedding parameter. In the considered case such a parameter is the height of the horn h—transparent layer thickness. The "utmost" solutions are: the field radiated by the system of the feeding waveguides (h = 0) and the field of researched HAA (h = H). Farther the solutions evolution equation is built in this functional space. Thus there can be established the connection between the solutions of the problems with corresponding different values of the parameter. The solution with

one value of the imbedding parameter is relatively simple and is taken as known (h = 0). Than the solution of the researched problem (h = H) can be received as the solution of the Cauchy problem for imbedding equation (first order differential equation) with the initial condition as a solution of the problem at h = 0. Let's take into consideration that problems of waveguides radiation (h = 0) are rather simpler than problem of HAA radiation (h = H).

Thus, the transition from electrodynamics characteristics of waveguides' cut (h = 0) to the corresponding characteristics of horns can be seen in describing intermediate systems—the elements of the truncated horns family received one from another by increasing the height of the walls as it's shown in Fig. 1



Figure 1: The evolution of horn layer under increasing the imbedding parameter.

To make the problem simpler let's use the method of periodical prolongation of the structure, i.e., let's add the researched HAA, consisting of N horns, with identical systems to the left and to the right to make it periodical structure. Under such a representation of horn grating its space spectrum of radiation becomes discrete. From the mathematical point of view it means that we change integral equations to matrix equations.

Farther it is necessary to express the transparence coefficient of "increased" HAA $T(h + \Delta h)$ in terms of T(h), the reflection coefficient $r(h, \Delta h)$ and transparence coefficient $t(h, \Delta h)$ of the elementary layer.

According to the ideology of the works [1-2] the field being necessary for calculating $T(h + \Delta h)$ is considered in endless thin (virtual) clearance that divides the truncated horn of height h from the increased elementary layer. The clearance borders can be considered semitransparent mirrors with transparent coefficients $r(h, \Delta h)$ and R(h). Here R(h) it is a reflection coefficient and of truncated HAA of height h. Taking into consideration multiple reflections of field from the layer's borders the next equation [3] takes place

$$T(h+\Delta h) = [R(h+\Delta h) - r(h+\Delta h, \Delta h)] \cdot t^{-1}(h+\Delta h, \Delta h) \cdot R^{-1}(h) \cdot T(h),$$
(3)

written in finite difference.

Imbedding equation (3) is not closed, there is an unknown function R(h) in it. The equation for reflection coefficient for truncated HAA can be received by variation of co-relations of integral equations method also known as MMM [4]. This method gave good results in the description of reflection from ideally conducting surfaces.

The distinctive part of the problem for HAA is the presence of waveguides—special insertions in ideally conducting surfaces. On these parts of the surface the Dirichlet condition doesn't take place that leads to essential complication of the method equation. Generalizing of method equations can be received knowing that the field in the spaces where Dirichlet condition doesn't take place can be represented as the superposition of waveguide's modes.

The equation for R(h), evident view of which has being shown in [5] is a following matrix Riccati equation

$$\frac{1}{2i}\frac{d\hat{R}}{dz} = \hat{R}(\hat{I} - \hat{\tilde{D}})\hat{V}^{-1} - \hat{R}\left[\hat{\tilde{D}}(\hat{I} - \hat{H}\hat{V}) + (\hat{I} - \hat{\tilde{D}})\hat{H}\hat{V}\right]\hat{V}^{-1}\hat{R} + (\hat{I} - \hat{H}\hat{V})\hat{V}^{-1}\hat{R} \tag{4}$$

Here $\hat{\tilde{D}} = \hat{W}^{-1}\hat{\tilde{C}}^{-1}\hat{F}, \hat{H} = \hat{F}\hat{K}^{-1}\hat{W}^{-1}, \hat{K} = \hat{V}\hat{C} - \frac{1}{\Lambda}\hat{\mu}$. Matrix $\hat{\tilde{C}}$ has the following components $\tilde{C}_{kl} = \hat{\tilde{C}}_{kl}$

 $\frac{1}{\Lambda} \int_{0}^{\Lambda} e^{-i\frac{2\pi}{\Lambda}(k-l)x+iv_{l}h(x)} dx, \text{ and matrix } \widetilde{C} - C_{kl} = \frac{1}{\Lambda} \int_{0}^{\Lambda} e^{-i\frac{2\pi}{\Lambda}(k-l)x-iv_{l}h(x)} dx, h(x) \text{ is the form of a horn's profile,}$

$$A_{kn} = \int_{0}^{n} dx \int_{-\infty}^{\infty} dx' H_{0}^{(1)}(x, x', h(x), h(x')) \cdot e^{-iq_{k}x + iq_{n}x'}, \ \mu_{kn} = \frac{4}{b} \cdot \sum_{p=1}^{\infty} \widetilde{\chi}_{kp} \frac{1}{\widetilde{\nu}_{p}} \chi_{kp}, \ \widetilde{\nu}_{p} = \sqrt{k_{0}^{2} - \widetilde{q}_{p}^{2}}, \ \widetilde{q}_{p} = \frac{\pi}{b} p, \ b \text{ is } \sum_{p=1}^{n} \frac{1}{b} \sum_{p=1}^{\infty} \widetilde{\chi}_{kp} \frac{1}{\widetilde{\nu}_{p}} \chi_{kp}, \ \widetilde{\nu}_{p} = \sqrt{k_{0}^{2} - \widetilde{q}_{p}^{2}}, \ \widetilde{q}_{p} = \frac{\pi}{b} p, \ b \text{ is } \sum_{p=1}^{n} \frac{1}{b} \sum_{p=1}^{\infty} \widetilde{\chi}_{kp} \frac{1}{\widetilde{\nu}_{p}} \chi_{kp}, \ \widetilde{\nu}_{p} = \sqrt{k_{0}^{2} - \widetilde{q}_{p}^{2}}, \ \widetilde{q}_{p} = \frac{\pi}{b} p, \ b \text{ is } \sum_{p=1}^{n} \frac{1}{b} \sum_{p=1}^{\infty} \widetilde{\chi}_{kp} \frac{1}{\widetilde{\nu}_{p}} \chi_{kp}, \ \widetilde{\nu}_{p} = \sqrt{k_{0}^{2} - \widetilde{q}_{p}^{2}}, \ \widetilde{q}_{p} = \frac{\pi}{b} p, \ b \text{ is } \sum_{p=1}^{n} \frac{1}{b} \sum_{p=1}^{\infty} \widetilde{\chi}_{kp} \frac{1}{\widetilde{\nu}_{p}} \chi_{kp}, \ \widetilde{\nu}_{p} = \sqrt{k_{0}^{2} - \widetilde{q}_{p}^{2}}, \ \widetilde{q}_{p} = \frac{\pi}{b} p, \ b \text{ is } \sum_{p=1}^{n} \frac{1}{b} \sum_{p=1}^{n} \widetilde{\chi}_{kp} \frac{1}{\widetilde{\nu}_{p}} \chi_{kp}, \ \widetilde{\nu}_{p} = \sqrt{k_{0}^{2} - \widetilde{q}_{p}^{2}}, \ \widetilde{q}_{p} = \frac{\pi}{b} p, \ b \text{ is } \sum_{p=1}^{n} \frac{1}{b} \sum_{p=1}^{$$

waveguide width, $\chi_{pn} = \int_{\frac{\Lambda-b}{2}}^{\frac{\Lambda+b}{2}} \varphi_p(x - \frac{\Lambda-b}{2}) \cdot e^{iq_n x} dx, \varphi_p(\cdot)$ is *p*-th waveguide's mode, $\hat{\tilde{\chi}} = \hat{\chi}^{*T}, W_{mn} = e^{iv_n z_1} \cdot \delta_{nm},$

 δ_{nm} is Kronecker's symbol, \hat{I} is identity matrix. $F_{kn} = \frac{2}{\Lambda} x \cdot \sin c \left[\frac{2}{\Lambda} (k-n)x \right]$, matrix \hat{V} is diagonal with elements $v_{pk} = \frac{2}{v_k} \cdot \delta_{kp}$.

As the initial condition for it serves the reflection coefficient of system of feeding waveguides, which could be found by using the mode-matching method.

3. A Physical Picture of Distortions of a UWB Signal at Radiation by HAA. Wood Resonance and Near Fields

Periodic expansion of HAA used in the stated approach allows not only to simplify a problem in mathematical aspect, but also to make more clear interpretation of destruction mechanism of the signal's form. It is known, that at interaction of a field with periodic structure there only components of a discrete spectrum are interconnected. In case of linear HAA it is possible to present a set of the wave vectors forming this spectrum, as $\vec{k}_n = (\nu_n, q_0 + nk)$, here $k = 2\pi/\Lambda$ —is a vector of the inverse lattice, Λ is distance between the nearest radiators, $n \in Z$, $\nu_n = \sqrt{k^2 - (q_0 + nk)^2}$ and q_0 is a corresponding projection of allocated components of field angular spectrum. In case of the scattering problem, usually it is a projection of an external field's wave vector.

If frequency of a field $\omega = ck$ is such that one of its space components gets in area of Wood resonance $\nu_n \cong 0$, then anomalies are observed in distribution of a field on modes.

At radiation of the narrowband signal, carrying (central) frequency is chosen so that the condition $\lambda_0 > \Lambda (k_0 < \frac{2\pi}{\Lambda})$ is satisfied. In this case in a space spectrum of radiation only one mode is homogeneous (lateral petals in the directional diagram are absent). Thus all field modes, both homogeneous, and inhomogeneous, are far from Wood's resonance (Fig. 2 (a)). Therefore the problem of distortion of the form of the narrowband signal usually does not arise.

For a UWB signal the range of wave numbers change is great. It grasps a lot of resonant points (Fig. 2 (b)).



Figure 2: The range of wave numbers change for narrowband -(a) and UWB -(b) signal.

As follows from the formula (1), the transparency coefficient (the directional diagram) HAA is substantially determined by the feature of matrix reflection coefficient R(h). Let's present its elements as

$$R_{n,m}(\omega) = |R_{n,m}(\omega)| \cdot \exp\{i\Phi_{n,m}(\omega)\}$$

The magnitude $\tau_{n,m}(\omega) = -\frac{d}{d\omega} \Phi_{n,m}(\omega)$ defines a group delay for *n*-th mode of a scattering field. The index *m* defines an external field wave vector $\nu_m = \sqrt{k^2 - (q_0 + k \cdot m)^2}$. If $\tau_{n,m}(\omega)$ varies with change of frequency then the output form of a signal most likely is distort. In other words, any deviation of frequency dependence $\tau_{n,m}(\omega)$ from the linear law must be analyzed.

On Fig. 3 diagrams of dependences $|R_{n,m}(k')|$ and $\Phi(k')$ are presented. They are calculated with the help of imbedding method represented for a case of normal falling $(q_0 + k \cdot m = 0)$ of an external field on the periodic surface modeling linear HAA.

Here wave parameter k' is a dimensionless wave vector $k' = k\Lambda/2\pi = \omega'$. Let's notice, that deviations from linear dependence near the values of parameter k' = n, $n \in \mathbb{Z}$ corresponding to points of Wood's resonance, are observed.



Figure 3: The diagrams of dependences $|R_{n,m}(k')|$ and $\Phi(k')$.

Let's note also, that far from resonant points, the kind of dependence $\Phi = \Phi(\omega')$ can be counted linear, but in the different areas of a frequency spectrum separated by resonant values of parameter, the corner of an inclination of curves essentially differs. As the spectrum of a UWB signal spans the big number of such areas even without taking into account Wood's anomalies dependence $\Theta = \Theta(\omega')$ can be approximated only by wiselinear, but not linear dependence. It also is necessary to take into account at the analysis of the reasons of the distortion of the form of radiated signal.

Complete results of the carried out numerical experiment will be submitted in the report.

4. Conclusion

The problem of ultra short impulses radiated by HAA is observed. From the spectrum analysis point of view impulse distortion depends on its Fourier components misphasing. To describe this effect the matrix transparence coefficient $\hat{T}(\omega)$ of horns layer is introduced as transitional layer that matches waveguides with free space. To calculate $\hat{T}(\omega)$ the imbedding equations were built. They allow considering horns overirradiation effects and borders effects that bound with its finite dimensions. Group delay variation that leads to signal disintegration can be represented as resonant interactions (Wood anomalies).

Reactive fields formed near antenna's system can be represented as superposition of inhomogeneous modes. The importance of near fields (inhomogeneous modes) grows sharply near the points of Wood resonant.

This quality summary were confirmed by diagrams of $R_{nm}(\omega)$ dependence that were calculated using imbedding equations describing external field interactions with periodical surface that models linear HAA.

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Near-field Response in Lossy Media with Exponential Conductivity Inhomogeneity

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Abstract—This paper examines the near-field response to source currents in lossy media with exponential conductivity inhomogeneity. The motivation for this work is to understand the modification of the polar ionosphere D region (50–90 km altitude) by powerful high frequency transmitters. The transmitted waves heat the D region plasma, causing a localized conductivity perturbation. In the presence of the DC electric field of the polar electrojet, the conductivity perturbation produces a current perturbation referred to as "antenna current" that can drive extremely/very low frequency radiation. Here we seek to understand the production of antenna current in a strongly inhomogeneous plasma. In the lower D region, the static approximation is valid, and we solve using a scalar potential description. In the upper D region, we use the magnetoquasistatic approximation and solve using a vector potential approach.

1. Introduction

We begin the formulation by defining standard scalar and vector potentials for the electric and magnetic field perturbations introduced by the conductivity perturbation. In time-harmonic form, we have

$$\mathbf{E} = i\omega \mathbf{A} - \nabla \Phi \qquad \mathbf{B} = \nabla \times \mathbf{A},\tag{1}$$

where *i* is the imaginary unit and ω is frequency. Let us suppose that the charge relaxation time and electromagnetic transit time are both small compared to the time scale of interest. This assumption allows us to ignore the effect of displacement current, so that current consists of only the imposed antenna current \mathbf{J}_s due to the conductivity perturbation, and a self-consistent conduction current $\sigma \mathbf{E}$, where σ is the conductivity of the medium. Adopting a Coulomb gauge, the wave equation is given by

$$\nabla^2 \mathbf{A} + i\omega\mu_0 \sigma \mathbf{A} = -\mu_0 \mathbf{J}_s + \mu_0 \sigma \nabla \Phi, \tag{2}$$

where μ_0 is the permeability of the medium, assumed the same as free space. The two terms on the right side can be viewed as source terms for the vector potential. We will proceed as follows. In the lower ionosphere D region, the conductivity is small such that the magnetic relaxation time is fast compared to the time scale of interest, and thus we ignore effects of vector potential. In the upper D region, the conductivity is large such that the magnetic relaxation time is slower than the time scale of interest. In this case, magnetic diffusion dominates the behaviour of the system, and we ignore the effects of space charge and its associated scalar potential. We will analyze each of the two limits.

The above statements assume a simple scalar conductivity. In practice, the plasma conductivity is anisotropic and requires a matrix representation. In the northern polar region the direction z (altitude) is antiparallel the earth's magnetic field. The appropriate conductivity tensor is given by

$$\boldsymbol{\sigma} = e^{hz} \begin{bmatrix} \sigma_P & \sigma_H & 0\\ -\sigma_H & \sigma_P & 0\\ 0 & 0 & \sigma_0 \end{bmatrix},$$
(3)

where 1/h is the scale height of the conductivity. Here, the exponential factor models the variability in the plasma conductivity due to the plasma density inhomogeneity, and the matrix entries are constants pertaining to the anisotropic plasma conductivity tensor. The quantity σ_P is the Pedersen conductivity, σ_H is the Hall conductivity, and σ_0 is the specific conductivity. We are assuming that all conductivities vary in altitude at the same rate. Strictly speaking this is not the case as the specific conductivity increases with altitude somewhat more rapidly than the Pedersen or Hall conductivities. However, for the purposes of a simple treatment, we ignore the fine details of the altitude dependence of the individual conductivity elements.

2. Static Solution

We now turn to the problem of determining the scalar potential Φ in the static limit. If we incorporate the tensor definition for σ into Equation (2), ignore the vector potential, and take the divergence of both sides, we find that

$$\nabla^2 \Phi + \left(\frac{\sigma_0}{\sigma_P} - 1\right) \frac{\partial^2 \Phi}{\partial z^2} + \frac{h\sigma_0}{\sigma_P} \frac{\partial \Phi}{\partial z} = \frac{e^{-hz}}{\sigma_P} \nabla \cdot \mathbf{J}_s \equiv S(\mathbf{r}),\tag{4}$$

where $S(\mathbf{r})$ is the source distribution. Let us expand the right and left sides of Equation (4):

$$S(\mathbf{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mathbf{r}_0 S(\mathbf{r}_0) \delta(\mathbf{r} - \mathbf{r}_0)$$
(5)

$$\Phi(\mathbf{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mathbf{r}_0 S(\mathbf{r}_0) G_{\Phi}(\mathbf{r}, \mathbf{r}_0).$$
(6)

Inserting these expansions into Equation (4) yields an expression for the Green's function $G_{\Phi}(\mathbf{r}, \mathbf{r}_0)$:

$$\nabla^2 G_{\Phi}(\mathbf{r}, \mathbf{r}_0) + \left(\frac{\sigma_0}{\sigma_P} - 1\right) \frac{\partial^2 G_{\Phi}(\mathbf{r}, \mathbf{r}_0)}{\partial z^2} + \frac{h\sigma_0}{\sigma_P} \frac{\partial G_{\Phi}(\mathbf{r}, \mathbf{r}_0)}{\partial z} = \delta(\mathbf{r} - \mathbf{r}_0).$$
(7)

This is a constant coefficient equation, and therefore $G_{\Phi}(\mathbf{r},\mathbf{r}_0)$ is the same as $G_{\Phi}(\mathbf{r}-\mathbf{r}_0)$. We can write

$$\nabla^2 G_{\Phi}(\mathbf{r}) + \left(\frac{\sigma_0}{\sigma_P} - 1\right) \frac{\partial^2 G_{\Phi}(\mathbf{r})}{\partial z^2} + \frac{h\sigma_0}{\sigma_P} \frac{\partial G_{\Phi}(\mathbf{r})}{\partial z} = \delta(\mathbf{r}). \tag{8}$$

This equation solves easily using the method of Fourier transforms. Taking the Fourier transform of Equation (8), solving for $G_{\Phi}(\mathbf{k})$, and then inverse transforming, results in the following solution for $G_{\Phi}(\mathbf{r})$:

$$G_{\Phi}(\mathbf{r}) = -\frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mathbf{k} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{k_x^2 + k_y^2 + (\sigma_0/\sigma_P)k_z^2 - ih(\sigma_0/\sigma_P)k_z}.$$
(9)

We can now convert Equation (9) to cylindrical co-ordinates (ρ, ϕ, z) and (k_{ρ}, α, k_z) and perform the integrals:

$$G_{\Phi}(\mathbf{r}) = -\frac{1}{8\pi^3} \int_0^\infty dk_{\rho} k_{\rho} \int_{-\infty}^\infty dk_z \frac{e^{ik_z z}}{k_{\rho}^2 + (\sigma_0/\sigma_P)k_z^2 - ih(\sigma_0/\sigma_P)k_z} \int_0^{2\pi} d\alpha e^{ik_{\rho}\rho\cos(\phi-\alpha)}$$
(10)

$$= -\frac{1}{4\pi^2} \int_0^\infty dk_\rho k_\rho J_0(k_\rho \rho) \int_{-\infty}^\infty dk_z \frac{e^{ik_z z}}{k_\rho^2 + (\sigma_0/\sigma_P)k_z^2 - ih(\sigma_0/\sigma_P)k_z}$$
(11)

$$= -\frac{e^{-hz/2}}{2\pi\sigma_0/\sigma_P} \int_0^\infty dk_\rho \frac{k_\rho J_0(k_\rho\rho) e^{-\sqrt{(h/2)^2 + (\sigma_P/\sigma_0)k_\rho^2}|z|}}{\sqrt{(h/2)^2 + (\sigma_P/\sigma_0)k_\rho^2}}$$
(12)

$$= -\frac{e^{-hz/2 - h\sqrt{(\sigma_0/\sigma_P)\rho^2 + z^2/2}}}{4\pi\sqrt{(\sigma_0/\sigma_P)\rho^2 + z^2}}.$$
(13)

The integral over k_z above is facilitated by the residue theorem, and the integral over k_ρ uses the following identity

$$\int_{1}^{\infty} du e^{-\alpha u} J_0(\beta \sqrt{u^2 - 1}) = \frac{e^{-\sqrt{\alpha^2 + \beta^2}}}{\sqrt{\alpha^2 + \beta^2}},$$
(14)

which can be found in standard tables. The scalar potential for a given source distribution can then be found by integrating this Green's function over the source distribution. The basic form of the scalar potential is similar to that of sources in homogeneous isotropic media, except there is exponential decay in the upward direction, and the potential is squeezed in the ρ direction compared to the z direction by a factor corresponding to the degree of anisotropy σ_0/σ_P . We also note that the Hall conductivity σ_H does not play a factor in the static scalar potential.

3. Static Solution Example

In this section, we provide an example of the static solution. Let us consider a current source \mathbf{J}_s that consists of a horizontal cylinder-like structure modelled by

$$\mathbf{J}_s = \hat{\mathbf{x}} I \delta(y) \delta(z) [\mu(x + L/2) - \mu(x - L/2)], \tag{15}$$

where I is the current and L is the cylinder length. The source distribution is given by

$$S(\mathbf{r}_0) = (e^{-hz}/\sigma_P)\nabla \cdot \mathbf{J}_s \tag{16}$$

$$= (I/\sigma_P)[\delta(\mathbf{r}_0 + \hat{\mathbf{x}}L/2) - \delta(\mathbf{r}_0 - \hat{\mathbf{x}}L/2)].$$
(17)

The potential is given by

$$\Phi(\mathbf{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mathbf{r}_0 S(\mathbf{r}_0) G_{\Phi}(\mathbf{r} - \mathbf{r}_0)$$
(18)

$$= \frac{Ie^{-hz/2}}{4\pi\sigma_P} \left\{ \frac{e^{-h\sqrt{(\sigma_0/\sigma_P)[(x-L/2)^2+y^2]+z^2/2}}}{\sqrt{(\sigma_0/\sigma_P)[(x-L/2)^2+y^2]+z^2}} - \frac{e^{-h\sqrt{(\sigma_0/\sigma_P)[(x+L/2)^2+y^2]+z^2/2}}}{\sqrt{(\sigma_0/\sigma_P)[(x+L/2)^2+y^2]+z^2}} \right\}.$$
 (19)

The total current $\mathbf{J} = \mathbf{J}_s - \boldsymbol{\sigma} \cdot \nabla \Phi$, near the z axis, is given by:

$$\mathbf{J}_{(x,y)\approx 0} = (\mathbf{J}_s - \boldsymbol{\sigma} \cdot \nabla \Phi)_{(x,y)\approx 0} \tag{20}$$

$$= \hat{\mathbf{x}}I\delta(y)\delta(z) - (\hat{\mathbf{x}}\sigma_P - \hat{\mathbf{y}}\sigma_H)IL\sigma_0(1 + hw/2)\exp[h(z-w)/2]/(4\pi\sigma_P^2w^3),$$
(21)

where $w = \sqrt{(\sigma_0/\sigma_P)(L/2)^2 + z^2}$. The conduction current $-\boldsymbol{\sigma} \cdot \nabla \Phi$ flows largely above the origin, opposite the source current, effectively forming a vertical current loop. The conduction current distributions are shown for L = 15 km and the cases of homogeneous isotropic, inhomogeneous isotropic, and inhomogeneous anisotropic media.



Figure 1: Static conduction current distributions. Solid line: homogeneous isotropic media. Dashed line: inhomogeneous isotropic media (1/h = 2.5 km). Dotted line: inhomogeneous anisotropic media (1/h = 2.5 km). $\sigma_0/\sigma_P = 2$).

4. Magnetoquasistatic Solution

Let us now consider the problem of determining the vector potential relevant to the magnetoquasistatic limit. Returning to Equation (2), we ignore the scalar potential so that we have

$$\nabla^2 \mathbf{A} + i\omega\mu_0 \boldsymbol{\sigma} \cdot \mathbf{A} = -\mu_0 \mathbf{J}_s. \tag{22}$$

By Equation (4), the z component is decoupled from the x and y components. Since the current perturbation \mathbf{J}_s is generally horizontally directed in practical situations, A_z is not driven, and we assume it is zero. The x and y components are decoupled by transforming to a basis of eigenvectors of the conductivity tensor:

$$\begin{bmatrix} \hat{A}_x \\ \hat{A}_y \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \begin{bmatrix} A_x \\ A_y \end{bmatrix}.$$
(23)

After the transformation the equations for the vector potential components \hat{A}_x and \hat{A}_y can be written as

$$\nabla^2 \begin{bmatrix} \hat{A}_x \\ \hat{A}_y \end{bmatrix} + i\omega\mu_0 e^{hz} \begin{bmatrix} \sigma_P + i\sigma_H & 0 \\ 0 & \sigma_P - i\sigma_H \end{bmatrix} \begin{bmatrix} \hat{A}_x \\ \hat{A}_y \end{bmatrix} = -\frac{\mu_0}{\sqrt{2}} \begin{bmatrix} J_{sx} - iJ_{sy} \\ J_{sx} + iJ_{sy} \end{bmatrix} \equiv -\mu_0 \hat{\mathbf{J}}_s.$$
(24)

The Green's function for a component of $\hat{\mathbf{A}}$ is given by

$$\left[\nabla^2 + i\omega\mu_0 e^{hz} (\sigma_P \pm i\sigma_H)\right] G_{\hat{A}}(\mathbf{r}, \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0).$$
⁽²⁵⁾

In view of the e^{hz} factor, $G_{\hat{A}}(\mathbf{r}, \mathbf{r}_0) = G_{\hat{A}}(x - x_0, y - y_0, z, z_0) \neq G_{\hat{A}}(\mathbf{r} - \mathbf{r}_0)$. Thus we write

$$\left[\nabla^2 + i\omega\mu_0 e^{h(z+z_0)}(\sigma_P \pm i\sigma_H)\right]G_{\hat{A}}(x,y,z+z_0,z_0) = \delta(\mathbf{r}).$$
(26)

A solution by the method of Fourier transforms is confounded by the e^{hz} factor. Thus we transform in the x and y directions only, which converts the partial differential Equation (25) into an ordinary differential equation:

$$\left[\frac{\partial^2}{\partial z^2} - k_{\rho}^2 + i\omega\mu_0 e^{h(z+z_0)}(\sigma_P \pm i\sigma_H)\right] G_{\hat{A}}(k_x, k_y, z+z_0, z_0) = \delta(z).$$
(27)

The solutions are the Bessel functions $J_{\nu}[\lambda e^{h(z+z_0)/2}]$ and $Y_{\nu}[\lambda e^{h(z+z_0)/2}]$, with $\lambda = 2\sqrt{i\omega\mu_0(\sigma_P \pm i\sigma_H)}/h$ and $\nu = 2k_{\rho}/h$. In the $z \to \infty$ limit, the only bounded linear combination of solutions for $0 < \arg(\lambda) < \pi$ is a Hankel function of the form $C_1 H_{\nu}^{(1)}[\lambda e^{h(z+z_0)/2}]$. Similarly, in the $z \to -\infty$ limit, the only bounded solution for all complex λ is a Bessel function of the form $C_2 J_{\nu}[\lambda e^{h(z+z_0)/2}]$. To determine the constants C_1 and C_2 we impose that the solutions in the regions z > 0 and z < 0 are continuous at z = 0:

$$C_1 H_{\nu}^{(1)} \left(\lambda e^{hz_0/2}\right) - C_2 J_{\nu} \left(\lambda e^{hz_0/2}\right) = 0, \tag{28}$$

and that inhomogeneous Equation (27) is satisfied, which is done by integrating over a small interval at z = 0:

$$C_1 H_{\nu}^{(1)'} \left(\lambda e^{hz_0/2}\right) - C_2 J_{\nu}' \left(\lambda e^{hz_0/2}\right) = 2/\left(h\lambda e^{hz_0/2}\right).$$
⁽²⁹⁾

Recalling the Wronskian relationship $W_z[J_\nu(z), H_\nu^{(1)}(z)] = 2i/(\pi z)$, the solution for C_1 and C_2 is

$$C_1 = -i\pi J_{\nu} \left(\lambda e^{hz_0/2} \right) / h \qquad C_2 = -i\pi H_{\nu}^{(1)} \left(\lambda e^{hz_0/2} \right) / h. \tag{30}$$

 $G_{\hat{A}}(x, y, z + z_0, z_0)$ is found by performing the inverse Fourier transforms, which in cylindrical coordinates are

$$G_{\hat{A}}(x,y,z+z_0,z_0) = \frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} dk_\rho d\alpha k_\rho e^{ik_\rho\rho\cos(\phi-\alpha)} G_{\hat{A}}(k_\rho,\alpha,z+z_0,z_0)$$
(31)
= $-\frac{i}{4\pi^2} \int_0^\infty dk_\rho k_{\nu} J_0(k_\rho) J_{\nu} [\lambda e^{h(z+z_0)/2} \mu(-z)]$

$$2h \int_{0}^{2h} u_{\kappa\rho} \kappa_{\rho} S_{0}(\kappa_{\rho} \rho) S_{\nu} \left[\lambda e^{-\mu(-z)} + \lambda e^{hz_{0}/2} \mu(z) \right] H_{\nu}^{(1)} \left[\lambda e^{h(z+z_{0})/2} \mu(z) + \lambda e^{hz_{0}/2} \mu(-z) \right],$$
(32)

where $\mu(z)$ is the Heaviside step function. Therefore $G_{\hat{A}}(\mathbf{r}, z_0)$ is given by

$$G_{\hat{A}}(\mathbf{r}, z_{0}) = -\frac{i}{2h} \int_{0}^{\infty} dk_{\rho} k_{\rho} J_{0}(k_{\rho}\rho) J_{\nu} \Big[\lambda e^{hz/2} \mu(z_{0} - z) + \lambda e^{hz_{0}/2} \mu(z - z_{0}) \Big] H_{\nu}^{(1)} \Big[\lambda e^{hz/2} \mu(z - z_{0}) + \lambda e^{hz_{0}/2} \mu(z_{0} - z) \Big].$$
(33)

We find **A** by integrating $G_{\hat{A}}(\mathbf{r}, z_0)$ over the source $-\mu_0 \hat{\mathbf{J}}_s$ and transforming $\hat{\mathbf{A}}$ to **A** using Equation (23).

5. Magnetoquasistatic Solution Example

We consider, as an analytically tractable example, the response to a current sheet

$$\mathbf{J}_s = \hat{\mathbf{x}} K \delta(z), \tag{34}$$

where K is a surface current density. The response for a component of $\mathbf{\hat{A}}$ is found as follows

$$\hat{\mathbf{A}} = -\mu_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mathbf{r}_0 K \delta(z_0) G_{A'}(\mathbf{r}, z_0)$$
(35)

$$= \frac{iK\mu_0}{2h} \int_0^\infty \int_0^{2\pi} d\rho \, d\phi \rho \int_0^\infty dk_\rho k_\rho J_0(k_\rho \rho) J_\nu \Big[\lambda e^{hz/2} \mu(-z) + \lambda \mu(z) \Big] H_\nu^{(1)} \Big[\lambda e^{hz/2} \mu(z) + \lambda \mu(-z) \Big]$$
(36)

$$= \frac{i\pi K\mu_0}{h} \int_0^\infty dk_\rho \delta(k_\rho) J_\nu \left[\lambda e^{hz/2} \mu(-z) + \lambda \mu(z) \right] H_\nu^{(1)} \left[\lambda e^{hz/2} \mu(z) + \lambda \mu(-z) \right]$$
(37)

$$= \frac{i\pi K\mu_0}{h} J_0 \Big[\lambda e^{hz/2} \mu(-z) + \lambda \mu(z) \Big] H_0^{(1)} \Big[\lambda e^{hz/2} \mu(z) + \lambda \mu(-z) \Big].$$
(38)

The x component of the conduction current $i\omega\sigma\cdot\mathbf{A}$ is shown in Fig. 2. The upper cutoff of the conduction current distribution results from the exponential increase in magnetic diffusion time with altitude, and the lower cutoff arises from the exponential decrease in conductivity.



Figure 2: Magnetoquasistatic conduction current distributions. Solid line: $1/h = 2.5 \text{ km}, 1/\sqrt{\omega\mu_0\sigma_p} = 100 \text{ km}, \sigma_P = \sigma_H$. Dashed line: $1/h = 5.0 \text{ km}, 1/\sqrt{\omega\mu_0\sigma_p} = 100 \text{ km}, \sigma_P = \sigma_H$.

6. Conclusion

This work has determined the response of inhomogeneous, anisotropic media to conductivity perturbations in the static and magnetoquasistatic limits. The responses have been characterized as Green's functions, which can provide the response current distribution if the source currents are known a priori. Some simple source currents have been considered here. More discussion of ionospheric source currents can be found in [1].

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An Analysis of Coaxial Line Slot antenna for Hyperthermia Treatment by Spectral Domain Approach

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Abstract—An extended spectral domain approach (ESDA) is applied to evaluate the scattering parameter of laterally slotted coaxial antenna for hyperthermia treatment. The results calculated by ESDA are in good agreement with that by Finite Element Method (FEM) simulation. Computational labor of the present method is far lighter than that of FEM, and the method is suitable for the iterative computation that is required for the optimization of antenna design. The present method can afford to consider the effect of the metallization thickness in the outer conductor.

1. Introduction

In the fields of medical application, microwave is utilized for various purposes in the examining and treatment equipment [1]. The characteristics of coaxial line slot antenna for microwave hyperthermia applicators have been investigated [2]. This treatment thrusts a coaxial line applicator into the affected cancer part, heats up selectively the affected area and fixes the cancer cells. The currently used coaxial line applicator is not optimized in the point of view of impedance matching between applicator and human tissue, so that the radiation efficiency into the affected area is not good. In this paper, we analyze the radiation characteristics of applicator using efficient simulation technique and proposed the optimized design that presents high radiation efficiency.

The formulation procedure utilized in this paper is based on the extended spectral domain approach (ESDA). This procedure can afford to consider the effect of thickness of outer conductor of coaxial cable. The results calculated by ESDA are compared with that by FEM simulation and excellent agreement have been obtained between both results.



Figure 1: Schematic structure of coaxial line slot antenna.



Figure 2: Aperture electric fields.

2. Theory

Figure 1 shows the schematic structure of coaxial line slot antenna whose outer conductor has finite thickness t and has a ring slot of W in width cut laterally near the termination. A perfect electric conductor (PEC) sheet is introduced for convenience of analysis in the position apart g from the tip of coaxial line. It is assumed that the relative complex permittivity of material in each region is ϵ_{r1} , ϵ_{r2} , ϵ_{r3} , ϵ_{r4} , respectively. The radiation characteristics of coaxial line slot antenna are analyzed based on the extended spectral domain approach (ESDA) [3]. In the procedure, first the aperture fields are introduced in the aperture of outer conductor designated as e_z^{a1} (at $\rho = b$), and e_z^{a2} (at $\rho = b + t$) (Fig. 2), respectively. Whole analytic space is divided into four regions, that is, region I ($a \le \rho \le b, -\infty \le z \le 0$), region II ($b \le \rho \le b + t, -c - w/2 \le z \le -c + w/2$), region III ($b + t \le \rho \le r, -\infty \le z \le d + g$), and region IV ($0 \le \rho \le b + t, d \le z \le d + g$), as shown in Fig. 1. These regions can be treated independently resorting to equivalence theorem, and the electromagnetic fields in each region are Fourier transformed with respect to the z-direction. When the dominant TEM mode

$$E_{\rho} = \frac{E_0}{\rho} \exp(-jk_1 z), \quad H_{\phi} = \sqrt{\frac{\epsilon_{r1}\epsilon_0}{\mu_0}} \frac{E_0}{\rho} \exp(-jk_1 z) \quad (a \le \rho \le b)$$
(1)

enters the coaxial line slot antenna, there exist the incident and the scattered waves in region (I) and total





Figure 3: Edge singularities in aperture fields.



electromagnetic fields are expressed by Fourier integrals as,

$$E_{\rho}^{(I)}(\rho,z) = -j\frac{2E_{0}}{\rho}\sin(k_{1}z) + \sqrt{\frac{2}{\pi}}\int_{-\infty}^{0}\tilde{E}_{\rho}^{(I)}(\rho)\sin(\alpha_{1}z)d\alpha_{1},$$

$$E_{z}^{(I)}(\rho,z) = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{0}\tilde{E}_{z}^{(I)}(\rho)\cos(\alpha_{1}z)d\alpha_{1},$$

$$E_{\phi}^{(I)}(\rho,z) = \sqrt{\frac{\epsilon_{r1}\epsilon_{0}}{\mu_{0}}}\frac{2E_{0}}{\rho}\cos(k_{1}z) + \sqrt{\frac{2}{\pi}}\int_{-\infty}^{0}\tilde{H}_{\phi}^{(I)}(\rho)\cos(\alpha_{1}z)d\alpha_{1}$$
(2)

Similar expressions are available in region (III). The electromagnetic fields in regions (II) and (IV) are expressed by Fourier series instead of Fourier integrals to satisfy the boundary conditions on the side walls. Fields in regions (II) are expressed as

$$E_{\rho}^{(II)}(\rho, z) = \sum_{n=1}^{\infty} \tilde{E}_{\rho}^{(II)}(\rho) \sin \alpha_2 (z+c+\frac{W}{2}), \quad E_z^{(II)}(\rho, z) = \sum_{n=0}^{\infty} \tilde{E}_z^{(II)}(\rho) \cos \alpha_2 (z+c+\frac{W}{2}),$$

$$H_{\phi}^{(II)}(\rho, z) = \sum_{n=0}^{\infty} \tilde{H}_{\phi}^{(II)}(\rho) \cos \alpha_2 (z+c+\frac{W}{2}), \quad \alpha_2 = \frac{n\pi}{W}.$$
(3)

And similar expressions are available in region (IV).

These expressions of electromagnetic fields are substituted into Maxwells field equations. The general solutions of the transformed field equations can be expressed in terms of Bessel functions and Neumann functions in regions (I), (II) and (IV), and in terms of second kind of Hankel functions in region (III) as

$$\tilde{H}_{\phi}^{(i)}(\alpha_i;\rho) = A^{(i)}J_1(\xi_i\rho) + B^{(i)}N_1(\xi_i\rho) \quad \text{in regions(I), (II) and (IV)}$$

$$\tilde{H}_{\phi}^{(III)}(\alpha_i;\rho) = C^{(III)}H^{(2)}(\xi_i,\rho) \quad \text{in regions(III)}$$
(5)

 $\tilde{H}_{\phi}^{(III)}(\alpha_{3};\rho) = C^{(III)}H_{1}^{(2)}(\xi_{3}\rho) \quad \text{in regions(III)}$ (5) where $A_{(i)}$, $B_{(i)}$ and $C^{(III)}$ are unknown constants and $\xi_{i} = \sqrt{\omega^{2}\epsilon_{ri}\epsilon_{0}\mu_{0} - \alpha_{i}^{2}}$. These unknown constants can be related to the aperture fields e_{z}^{a1} , e_{z}^{a2} and e_{z}^{b} by applying the continuities of electric fields at interfaces. Then the electromagnetic fields in each region are expressed in terms of the aperture fields, for example,

$$H_{\phi}^{(I)}(\rho, z) = \sqrt{\frac{\epsilon_{r1}\epsilon_0}{\mu_0}} \frac{2E_0}{\rho} \cos(k_1 z) + \int_{z'=-c-W/2}^{-c+W/2} Y^{(I)}(\rho, z|\rho'=b, z') e_z^{a1}(z') dz' \quad \text{in region(I)}$$
(6)

where $Y^{(I)}$ is the Green's function and it can be derived easily in the transformed domain. Similar expressions are derived in other regions, which relate the fields to the involved aperture fields. The remaining boundary conditions, i. e., the continuity of the magnetic field at the interfaces between adjacent regions, are applied to obtain a set of the integral equations on the aperture fields. The aperture fields can be determined by applying the Galerkin's procedure to these coupled integral equations, and the scattering parameter (complex reflection constant) S_{11} are obtained by taking the inner product between the aperture field e_z^{a1} and the eigen function of coaxial line.



Figure 5: Convergence of reflection coefficients with respect to the distance of g. $a = 0.24 \text{ mm}, b = 0.8 \text{ mm}, c = 2.5 \text{ mm}, d = 5.0 \text{ mm}, w = 2.0 \text{ mm}, t = 0.1 \text{ mm}, \epsilon_{r1} = 2.1 - j0.0005, \epsilon_{r2} = \epsilon_{r3} = \epsilon_{r4} = 43 - j12.38, f = 2.45 \text{ GHz}.$



Figure 7: Phase variation of the reflection versus the frequency. $a = 3.10 \text{ mm}, b = 7.14 \text{ mm}, d = 0.57 \text{ mm}, \epsilon_{r1} = \epsilon_{r2} = 2.1.$



Figure 6: Gap discontinuity in coaxial line.



Figure 8: Frequency dependency of reflection coefficients of coaxial line slot antenna. a = 0.24 mm, b = 0.8 mm, c = 2.5 mm, d = 5.0 mm, w = 2.0 mm, t = 0.1 mm, $\epsilon_{r1} = 2.1 - j0.0005$, $\epsilon_{r2} = \epsilon_{r3} = \epsilon_{r4} = 43 - j12.38$.

3. Numerical Procedure and Result

The numerical procedure is based on Galerkin's procedure, and the unknown electric aperture fields e_z^{a1} , e_z^{a2} and e_z^b are expanded in terms of the appropriate basis functions,

$$e_{z}^{i}(z) = \sum_{k=1}^{N} a_{k}^{i} f_{k}^{i}(z)$$
(7)

The basis functions $f_k^i(z)$ are chosen taking the edge singularities near conductor edge into consideration (Fig. 3),

$$f_k^{a1}(z) = f_k^{a2} K(z) = \frac{T_{k-1}\{\frac{2}{W}(z+c)\}}{\sqrt{1-\{\frac{2}{W}(z+c)\}^2}}, \quad f_k^b(z) = \frac{T_{2(k-1)}\{\frac{1}{g}(z-d-g)\}}{\sqrt{1-\{\frac{1}{g}(z-d-g)\}^2}}$$
(8)

where $T_k(x)$ is Chebyshev polynomials of the first kind.

Preliminary computations are carried out to investigate the convergence of the reflection coefficients with respect to the number of basis functions. This method was settled by a little number of basis functions as shown in Fig. 4, and N = 8 is used in the following computations. Fig. 5 examines the effect of the fictious perfect electric conductor sheet placed at distance g ahead the tip of coaxial line slot antenna (Fig. 1). The influence of the conductor sheet decreases rapidly with g, and the sufficient spacing g = 40 mm is chosen in the following simulations.

To author's knowledge there is no published theoretical result to permit direct comparison with the present method for the reflection characteristics of coaxial line slot antenna. We apply the present method to analyze the gap discontinuity in the inner conductor of shorted coaxial line (Fig. 6) to show the validity of the method.



Figure 9: SAR distribution of coaxial line slot antenna.



Figure 10: Variation of the reflection versus the slot position. $a = 0.24 \text{ mm}, b = 0.8 \text{ mm}, w = 2.0 \text{ mm}, t = 0.1 \text{ mm}, \epsilon_{r1} = 2.1 - j0.0005, \epsilon_{r2} = \epsilon_{r3} = \epsilon_{r4} = 43 - j12.38, f = 2.45 \text{ GHz}.$

The formulation procedures are similar to those explained above and also the similar basis functions (8) are used in the numerical computation. Fig. 7 shows the phase variation of the reflection coefficient versus the frequency, comparing the results by mode-matching method [4] and Marcuvitzs analytical results [5]. Our results are in good agreement with [5] for wide frequencies.

Figure 8 shows the frequency dependency of reflection coefficients of the coaxial line slot antenna (applicator) thrust into the liver. The figure includes the values by FEM for comparison, and excellent agreement is observed between both methods over wide frequencies. Fig. 9 shows the SAR distribution calculated by both methods at f = 2.45 GHz.

The present method is numerically efficient and is suitable for the optimization of the coaxial line applicator, which requires the iterative computations. Fig. 10 shows the optimization of coaxial line by changing a slot position when the operation frequency is 2.45 GHz. The optimal value at this condition takes the reflective coefficient 0.32 at c - w/2 = 1.5 mm and d = 5 mm. This figure also includes the values by FEM and again good agreement is confirmed, although FEM calculations are time consuming and are presented only at discrete frequencies.

4. Conclusion

In this paper, we proposed the novel analyzing technique for the coaxial line slot antenna by ESDA, and carried out extraction of scattering parameters. This method can take the thickness effect of outer conductor into consideration. This method also secures the high accuracy by considering the singularities of fields near the conductor edge properly. The computational labor of the new method is far lighter than that of FEM, so that novel method is suitable for the time consuming iterative computation such as optimization procedure of antenna design.

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Critical Study of DCIM, and Development of Efficient Simulation Tool for 3D Printed Structures in Multilayer Media

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Abstract—Since the discrete complex image method (DCIM) has been widely used in conjunction with the Method of Moments (MoM) to efficiently analyze printed structures, some lingering issues related to the implementation of DCIM and their brief clarifications are first reviewed. Then, an efficient and rigorous electromagnetic simulation algorithm, based on the combination of MoM and DCIM, is proposed and developed for the solution of mixed-potential integral equation (MPIE) for printed structures with multiple vertical strips in multilayer media. The algorithm is possibly the most efficient approach to handle multiple vertical conductors, even spanning more than one layer, in printed circuits.

1. Introduction

Spatial-domain method of moments (MoM) is a widely used technique for the solution of mixed-potential integral equation (MPIE) for printed geometries in multilayer planar media [1], thanks to the introduction of an efficient closed-form approximation method [2] and its improved versions of the spatial-domain Green's functions [3, 4]. This approach, known as discrete complex image method (DCIM), basically approximates the spectral-domain Green's functions in terms of complex exponentials, and then casts the integral representations of the spatial-domain Green's functions into closed-form expressions via Sommerfeld identity [5]. Although DCIM is quite robust and works well to get the closed-form Green's functions, it has some limitations in the form of a limited range of validity depending upon the implementation of the method.

Some issues originating from the implementation of DCIM are discussed and possible clarifications are provided in Section 2. In Section 3, application of the closed-form Green's functions in conjunction with the spatial-domain MoM is reviewed, with the emphasis given to efficient handling of multiple vertical conductors. Finally, conclusions are provided in Section 4.

2. Discussions on Closed-form Green's Functions

It is well known that spectral-domain Green's functions can be written analytically in planar multilayer media, and their spatial-domain counterparts can be obtained from the inverse Hankel transform of the spectral-domain Green's functions [4, 6], as

$$G = \frac{1}{4\pi} \int_{SIP} dk_{\rho} k_{\rho} H_0^{(2)}(k_{\rho}\rho) \widetilde{G}(k_{\rho})$$
(1)

where $k_{\rho}^2 = k_x^2 + k_y^2$, ρ is the variable in cylindrical coordinate system, G and \tilde{G} are Green's functions in the spatial and spectral domain, respectively, $H_0^{(2)}$ is the Hankel function of the second kind and SIP is the Sommerfeld integration path. Since the integrand usually exhibits oscillatory nature and slow convergence, rendering the transformation computationally very expensive, spectral-domain Green's functions can be approximated by complex exponentials, via the generalized pencil-of-function (GPOF) method [6], to obtain closed-form expressions from the inverse Hankel transform. Since the crucial step in this approach is the approximation of the spectral-domain Green's functions, which is detailed in [3, 4], discussions on the accuracy of the method for large distances have concentrated mainly on the approximation procedure, because the resulting closed-form Green's functions are, in general, accurate enough for distances as far as $k_0\rho = 20 - 30$ ($\rho/\lambda = 3 - 4$), beyond which they may deteriorate significantly.

In the literature, there were basically three attributable sources of problems in the implementation of DCIM: (i) not extracting the quasi-static terms, (ii) introducing a wrong branch point in the process of approximation, and (iii) not extracting the surface wave poles (SWP). In the original implementation of DCIM, as introduced in [2], there were only one level of approximation, and it was necessary to extract the quasi-static terms to make the remaining portion of the spectral-domain Green's functions converge to zero for large k_{ρ} values. However, with the introduction of two-level and multi-level approximation algorithms [3, 7], the necessity of finding the quasi-static terms and their extraction before the approximation has been eliminated. The issue of introducing wrong branch point originates from the following observations: spectral-domain Green's functions, when the source is in a bounded layer, have no branch point at $k_{\rho} = k_s$, although they have k_{zs} term in the denominator, where k_s is the wave number of the source layer; and the approximating exponentials with k_{zs} factor in the denominator seem to have branch point at $k_{\rho} = k_s$. However, one should note that the exponential approximation is always performed over the deformed path of *SIP* and the function to be approximated over this path is single valued with the right choice of the branch. Therefore, the resulting exponentials divided by k_{zs} is a singlevalued function with this right choice of the branch. The last problem concerning the SWPs is inherent to the approach unless the SWP contributions are totally extracted from the functions to be approximated. The detailed discussions on these issues and some clarifications can be found in [4].

3. MoM-DCIM Application for Multiple Vertical Strips

In the analysis of printed geometries with multiple vertical strips, a method based on MoMDCIM is employed, as proposed in [7], and it is extended to efficiently handle multiple vertical strips. The algorithm and its efficient handling of multiple vertical strips can be described by examining one of the inner-product terms in the MoM matrix entries, as follows:

$$\left\langle \frac{\partial}{\partial x} T_x(x,y), \, G_z^q * \frac{\partial}{\partial z} B_z(y,z) \right\rangle = \iint dx dy \frac{\partial}{\partial x} T_x(x,y) \cdot \int dy' B_z(y') \int dz' \frac{\partial}{\partial z'} B_z(z') G_z^q(x-x',y-y',z,z') \tag{2}$$

where $T_x(x, y)$ and $B_z(x, y)$ are the testing and basis functions used in the evaluation of corresponding MoM matrix entry. Writing the spatial-domain Green's function G_z^q in terms of its spectral-domain representation \tilde{G}_z^q , followed by the change of the order of integrations, (2) can be cast into the following form

$$\left\langle \frac{\partial}{\partial x} T_x(x,y), \, G_z^q * \frac{\partial}{\partial z} B_z(y,z) \right\rangle = \iint du dv F_z^q(u,v,z=cons) \int dy B_z(y-v) \frac{\partial}{\partial x} T_x(x'+u,y) \tag{3}$$

where x - x' = u, y - y' = v and

$$F_z^q \cong \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)} \left(k_\rho \left| \boldsymbol{\rho} - \boldsymbol{\rho}' \right| \right) \cdot GPOF \left\{ \int dz' \frac{\partial}{\partial z'} B_z(z') \widetilde{G}_z^q(k_\rho, z = cons, z') \right\}$$
(4)

Note that the auxiliary function $F_z(u, v)$ is obtained analytically in terms of complex exponentials and it is an explicit function of u = x - x' and v = y - y', and the inner integral of (3) can easily be obtained analytically for most basis and testing functions. Therefore, the same inner-product terms corresponding to other vertical strips can be obtained simply by evaluating $F_z(u, v)$ for different values of u and v, as long as the basis functions used to represent the current densities along them have identical z'-dependencies. Consequently, having more than one vertical conductors in a printed circuit would not require significant amount of additional computation.

The formulation described above is applied to a microstrip line lying along x-direction with four vertical yspanning strips to assess and demonstrate the computational efficiency of the method. Here are the parameters of the microstrip line: the dielectric constant of the medium is 4.0; the length and width of the line is 18.0 cm and 0.1 cm, respectively; the thickness of the substrate is 0.4 cm; the frequency of operation is 2 GHz; and 71 horizontal basis functions along x-direction are employed. As the thickness of the substrate is uniform, which is usually the case for most of antenna and microwave applications, two basis functions are used over every vertical strip, and naturally they have the same z and z' dependencies, satisfying the only criterion for the efficiency of the method for multiple vertical strips. To validate the method, the current distribution along the microstrip line is first obtained, and compared to that from a commercially available EM simulation software, em by Sonnet, as shown in Fig. 1. An excellent agreement is observed; slight differences in the amplitude can be attributed to the inherent models of the approaches: em by Sonnet solves the problem in shielded environment while the method proposed here solves it in open environment, which inevitable causes some slight differences on the resonant frequencies of the structure.

Once the validation is complete, the computational efficiency of the proposed method is assessed in terms of the CPU time obtained from a 1.5 GHz Centrino CPU. The microstrip line is first analyzed with one vertical strip (at x = 7.0 cm), and then the number of vertical strips is increased to four by one-by-one. As the ultimate measure for the efficient handling of multiple vertical metallization, in addition to the first one, matrix fill time for additional vertical strips are listed in Table 1. For the matrix fill times in case I, the necessary auxiliary functions are calculated only once and used repeatedly, but for case II, the auxiliary functions are re-calculated for every entry corresponding to each basis and testing functions introduced with the addition of new vertical strips. It is observed that efficient use of auxiliary functions has significantly reduced the computational complexity of
the whole method. This can be stated with adding new vertical strips to the microstrip line with one vertical strip costs about 4.0 seconds whereas it requires 70.0 seconds in case of not using auxiliary functions repeatedly. Note that CPU times are obtained by using only the symmetry of the MoM matrix and it has not been used any acceleration technique for the evaluation of MoM matrix entries.



Figure 1: Magnitude of the current along the microstrip line with 4 vertical strips.

Table 1: MoM matrix fill times for each additional vertical strip.

Number of vertical	MoM matrix fill-time (sec)	
\mathbf{strips}	CASE I	CASE II
1	11.8	69.8
2	4.0	68.6
3	4.1	72.2
4	4.2	75.8

4. Conclusions

Issues related to the implementation of DCIM have been first clarified, as it is used in conjunction with the MoM in the algorithm proposed in this paper. The algorithm, based on the DCIM-MoM technique, is assessed in terms of its accuracy and the efficiency in the analysis of printed geometries with multiple vertical conductors. It has been shown mathematically and numerically that, as long as the vertical dependencies of the basis or testing functions are chosen to be the same, the inclusion of additional vertical conductors is extremely efficient. Therefore, this approach seems to be a good candidate to use in conjunction with an optimization algorithm in a CAD tool.

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Analysis of Cylindrical Microstrip Line with Finite Thickness of Conductor

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Abstract—Novel analytical method based on extended spectral domain approach (ESDA) is presented for cylindrical microstrip line. The method utilizes the aperture fields as the source quantities, as opposed to the conventional methods, which have used the current on the strip as the source. The whole region can be divided into sub-regions by the introduction of aperture fields, and each sub-region can be treated independently. This method makes possible the analysis both of zero and finite thickness of the strip conductor. The numerical procedure incorporates the effects of the edge singularities properly and can afford the efficient and accurate calculations for the phase constants and characteristic impedances of a microstrip line with zero- and finite-thickness on the characteristics of a cylindrical microstrip line.

1. Introduction

Recently, curved surface substrates have attracted an attention as materials of antennas and front ends for portable terminals. A lot of analyses of the propagation characteristic of the stripline and the coplanar waveguide composed on a cylinder substrate are reported [1–6], including the moment method, the FDTD method [3], and the finite element method [5]. However, their works assumed the conductor thickness to be zero, and the report concerning the effect of the conductor thickness on the propagation characteristic has not be found. Recently, authors reported on the effect of the finite thickness of a conductor on electric characteristics of cylindrical coplanar waveguides (CCPWs) by using the extended spectral domain approach (ESDA).

In this paper, we report on the analytical method of the cylindrical microstrip line based on ESDA, and the effect of conductor thickness by numerical calculation. The present method utilizes the electric fields at the interface of the aperture as the source quantities, as opposed to the conventional methods [1,2], which have used the current on the strip as the source. The accurate and efficient numerical procedure, which makes consideration for the field singularities near the conductor edge of zero- and finite-thickness, reveals the effect of the curvature and the finite thickness of a conductor on the characteristic impedances and the phase constants of the cylindrical microstrip line.

2. Theory

Cross section of a microstrip line on a cylindrical dielectric substrate is shown in Fig. 1(a). Curvature R of the cylindrical substrate is defined as the ratio of inner and outer diameter of substrate,



Figure 1: Schematic structure of cylindrical microstrip line.

$$R = \frac{a}{b} = 1 - \frac{h}{b} \tag{1}$$

where h = b - a is substrate thickness. A signal conductor of W in width is put on the substrate, which is backed by the ground conductor. Both conductors are assumed to be perfect electric conductor (PEC), but the signal conductor has the finite thickness t, as opposed to the previous reports. A single-layered substrate is assumed in the following explanation for the simplicity, although the method is applicable to multilayered and/or overlaid structure problem. The theoretical scheme is based on the ESDA [8–10]. The method has been successfully worked out to analyze the effect of the conductor thickness of the various types of planar transmission lines. Here, in this study, the method is extended further to the analysis of the effect of conductor thickness in cylindrical microstrip line. In the ESDA, first the aperture electric fields are introduced at the circumferential surfaces of dielectric substrate at $\rho = b$, $e^b(\phi)$, and the upper surface of signal conductor at $\rho = b + t$, $e^c(\phi)$ shown in Fig. 1(b). By introducing these aperture fields and utilizing the equivalence theorem, the whole region is divided into subregions, i. e., (I) the outer ($\rho > b + t$), (II) the aperture ($b < \rho < b + t$) and (III) the substrate ($\rho < b$) subregions. After dividing the region, each subregion can be treated separately, and then the longitudinal components of electromagnetic fields in each subregion are expressed in terms of the appropriate eigenfunctions $\Phi_n^{(i)}(\phi)$, $\Psi_n^{(i)}(\phi)$, which satisfy the boundary conditions in the ϕ -direction.

$$E_{z}^{(i)}(\rho,\phi)e^{-j\beta z} = \sum_{n=0}^{\infty} \tilde{E}_{z}^{(i)}(\rho)\Phi_{n}^{(i)}(\phi)e^{-j\beta z}$$
(2)

$$H_{z}^{(i)}(\rho,\phi)e^{-j\beta z} = \sum_{n=0}^{\infty} \tilde{H}_{z}^{(i)}(\rho)\Psi_{n}^{(i)}(\phi)e^{-j\beta z}$$
(3)
$$i = I, II, III$$

where β is the unknown phase constant and $\tilde{E}_z^{(i)}$ is the transform of $E_z^{(i)}$. The transversal (ρ, ϕ) field components can be related to the longitudinal components $E_z^{(i)}$ and $H_z^{(i)}$ by utilizing the field equations. The general solution of the transform $\tilde{E}_z^{(i)}$ in region (i) can be expressed as

$$\tilde{E}_{z}^{(i)}(\rho) = A^{(i)}J_{n}(\beta_{c}\rho) + B^{(i)}Y_{n}(\beta_{c}\rho) \qquad (4)$$
$$\beta_{c} = \sqrt{\omega^{2}\varepsilon\mu - \beta^{2}}$$

where $A^{(i)}$, $B^{(i)}$ is unknown constants, and they are determined by the boundary conditions at the interfaces. The continuities of electric fields are expressed as

$$E_{\phi}^{(III)}(\rho = a + 0, \phi) = 0, \qquad E_{z}^{(III)}(\rho = a + 0, \phi) = 0 \qquad \text{at} \quad \rho = a$$
(5)

$$E_{\phi}^{(II)}(\rho = b + 0, \phi) = E_{\phi}^{(III)}(\rho = b - 0, \phi) = e_{\phi}^{b} \qquad \text{at} \quad \rho = b \quad (\frac{\phi W}{2} < \phi < \pi) \tag{6}$$

$$E_z^{(II)}(\rho = b + 0, \phi) = E_z^{(III)}(\rho = b - 0, \phi) = e_z^b$$
(7)

$$E_{\phi}^{(I)}(\rho = b + t + 0, \phi) = E_{\phi}^{(II)}(\rho = b + t - 0, \phi) = e_{\phi}^{c} \qquad \text{at} \quad \rho = b + t \quad (\frac{\phi W}{2} < \phi < \pi) \tag{8}$$

$$E_z^{(I)}(\rho = b + t + 0, \phi) = E_z^{(II)}(\rho = b + t - 0, \phi) = e_z^c.$$
(9)

These continuity conditions are transformed into spectral domain and they are used to relate the unknowns $A^{(i)}$, $B^{(i)}$ to the aperture fields. The fields are then related to the aperture fields as follows

$$E^{(II)}(\rho,\phi) = \int_{\phi'} \{\overline{\overline{T}}^{(II)}(b,\phi|b+t,\phi') \cdot \mathbf{e}^{\mathbf{c}}(\phi') + \overline{\overline{T}}^{(II)}(b,\phi|b,\phi') \cdot \mathbf{e}^{\mathbf{b}}(\phi')\} d\phi'$$
(10)

$$H^{(II)}(\rho,\phi) = \int_{\phi'} \{\overline{\overline{Y}}^{(II)}(b,\phi|b+t,\phi') \cdot \mathbf{e}^{\mathbf{c}}(\phi') + \overline{\overline{Y}}^{(II)}(b,\phi|b,\phi') \cdot \mathbf{e}^{\mathbf{b}}(\phi')\} d\phi'$$
(11)

where, $\overline{\overline{T}}'_s$, $\overline{\overline{Y}}'_s$ are the dyadic Green's functions. Then, the integral equations on the aperture fields are obtained by using the continuities of magnetic fields at the aperture surfaces,

$$H_{\phi}^{(II)}(\rho = b + 0, \phi) = H_{\phi}^{(III)}(\rho = b - 0, \phi) \qquad (\frac{\phi W}{2} < \phi < \pi)$$
(12)

$$H_z^{(II)}(\rho = b + 0, \phi) = H_z^{(III)}(\rho = b - 0, \phi)$$
(13)

$$H_{\phi}^{(I)}(\rho = b + t + 0, \phi) = H_{\phi}^{(II)}(\rho = b + t - 0, \phi) \qquad (\frac{\phi W}{2} < \phi < \pi)$$
(14)

$$H_z^{(I)}(\rho = b + t + 0, \phi) = H_z^{(II)}(\rho = b + t - 0, \phi).$$
(15)

Applying the Galerkins procedure to these integral equations, we get the determinant equation for the phase constant β . In the Galerkins, the unknown aperture fields are expressed in terms of the appropriate basis functions $\xi_{\phi i}(\phi)$ and $\xi_{zi}(\phi)$ as,

$$e_{\phi}^{b}(\phi) = \sum_{i=1}^{N} b_{\phi i} \xi_{\phi i}(\phi), \qquad e_{z}^{b}(\phi) = \sum_{i=1}^{N} b_{z i} \xi_{z i}(\phi)$$
(16)

$$e_{\phi}^{c}(\phi) = \sum_{i=1}^{N} c_{\phi i} \xi_{\phi i}(\phi), \qquad e_{z}^{c}(\phi) = \sum_{i=1}^{N} c_{z i} \xi_{z i}(\phi)$$
(17)

where $b_{\phi i}$, b_{zi} , $c_{\phi i}$, and c_{zi} are the unknown expansion coefficients. The basis functions $\xi_{\phi i}(\phi)$, $\xi_{zi}(\phi)$, which incorporate the singularities of fields properly near the conductor edge [8–10], are used in the following computations. For the case with the conductors of zero thickness, the aperture region (II) will be eliminated in the procedure and the aperture field e^b equals to e^c .

The definition of the characteristic impedance is somewhat ambiguous for the hybrid mode propagation along microstrip line. We adopt the voltage-current definitions

$$Z_{VI} = \frac{V_o}{I_o} \tag{18}$$

where V_o is the voltage between the center strip and the ground conductor, and I_o is the total current flowing in the z-direction on the strip conductor. The voltage V_o is evaluated by integrating the radial component of electric field $E_{\rho}^{(III)}$ between the ground ($\rho = a$) and the signal ($\rho = b$) conductors,

$$V(\phi) = \int_{a}^{b} E_{\rho}(\rho, \phi) d\rho \tag{19}$$

where ϕ may be any in $0 < \phi < \phi_W/2$. Therefore $V(\phi)$ is integrated with ϕ over $0 < \phi < \phi_W/2$ to get

$$V_{o} = \frac{2}{\phi_{W}} \int_{0}^{\frac{\phi_{W}}{2}} V(\phi) d\phi.$$
 (20)

The current I_o can be evaluated by the line integral C of the magnetic field around the strip conductor [7]

$$I_o = \oint_c \mathbf{H} \cdot dl. \tag{21}$$

3. Numerical Procedure and Results

The conventional methods have treated the propagation characteristics of a microstrip line on a cylindrical substrate assuming the conductor thickness to be zero [2]. The present method, when the aperture field is adopted as the source quantity in the formulation, can afford to present the characteristics of the case with finite as well as zero thickness. Also, the present formulation procedure could employ the current on the strip instead of the aperture field as the source quantity, although this procedure could be applied only to the case with zero thickness. Fig. 2 shows the frequency dependency of the effective dielectric constant ε_{eff} and the characteristic impedance Z_{VI} of a microstrip line on a cylindrical substrate with larger R [2]. The effective dielectric constant ε_{eff} is obtained in terms of the phase constant β as

$$\varepsilon_{eff} = \{\beta/\omega\sqrt{\varepsilon_0\mu_0}\}^2 \tag{22}$$

The results of zero thickness conductors are calculated by both the aperture field and the current bases, and both results are in excellent agreement and they agree well with the conventional ones [2] over the frequencies. The figure includes the results of the case with finite thickness of the strip conductor (50 μ m) showing the effects of the conductor thickness on ε_{eff} and Z_{VI} .



Figure 2: Frequency dependency of propagation characteristics. $\varepsilon_r = 9.6, h = 1 mm, W = 1 mm, R = 0.9.$



Figure 3: Curvature dependency of propagation characteristics. $\varepsilon_r = 9.6, h = 1 mm, W = 1 mm, f = 10 GHz.$



Figure 4: Thickness effect on propagation characteristics. $\varepsilon_r = 9.6, h = 1 mm, W = 1 mm, R = 0.9.$

The present methods is equally applicable to the a cylindrical microstrip line with larger and smaller curvature rate R. Fig. 3 shows the curvature dependency of ε_{eff} and Z_{VI} . The value of ε_{eff} increases rapidly when curvature rate R is 0.5 or less. That is, the concentration of the electromagnetic field in the dielectric substrate becomes stronger as the curvature ratio becomes smaller. Therefore, the effect of the thickness of the conductor becomes smaller for the smaller R. Fig. 4 shows the conductor thickness effect where the relative changes of ε_{eff} and Z_{VI} are presented with the thickness variation of conductor. Both ε_{eff} and Z_{VI} are decrease monotonously up to 100 μ m thickness conductor. It should be noted that the effect of the conductor thickness becomes smaller for higher frequency (f = 18 GHz), as opposed to a cylindrical coplanar waveguides (CCPWs), where the effect becomes larger for higher frequency. This is why the electromagnetic field concentrates more in the dielectric substrate between the strip and the ground conductors and the effect of conductor thickness becomes smaller for higher frequency.

4. Conclusion

Novel analytical method based on extended spectral domain approach (ESDA) is presented for a cylindrical microstrip line. The method is able to treat the effect of the finite thickness of a strip conductor by utilizing the aperture electric fields as source quantities. The numerical procedure incorporates the effects of the edge singularities properly and can afford the efficient and accurate calculation method for the characteristic impedances in addition to the phase constants of a cylindrical microstrip line. The calculated results for zero-thickness conductor by both procedures, based on current or aperture field, are in good agreement and also they agree well with the published data. The results obtained by the present method show the curvature dependency of the propagation characteristics and reveal the effect of conductor thickness, which is different from that of a cylindrical coplanar waveguides (CCPWs).

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Detection of Small Tumors in Microwave Medical Imaging Using Level Sets and Music

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Abstract—We focus on the application of microwaves for the early detection of breast cancer. We investigate the potential of a novel strategy using shapes for modeling the tumor in the breast. An inversion using a shape-based model offers several advantages like well-defined boundaries and the incorporation of an intrinsic regularization that reduces the dimensionality of the inverse problem whereby at the same time stabilizing the reconstruction. We explore novel level-set techniques as a means to detect the tumor without any initialization of its position and size. We present some numerical resonstructions and we compare them with the conventional MUSIC algorithm, in particular with respect to the frequency which is used for the investigation. We show that for different frequencies these two methods show a different qualitative behaviour in the reconstructions.

1. Introduction

Microwave imaging shows significant promise as a new technique for the early detection of breast cancer (see [5] and references therein). This is so because of the high contrast between the dielectric properties of the healthy breast tissue and the malignant tumors at microwave frequencies. As a consequence, microwave imaging may be used as a clinical complement to the conventional mammography which is based on the attenuation of X-rays that go through the breast. We note that mammographies offer high resolution images but with low contrast.

Several image reconstruction algorithms have been investigated during the last years for the detection and location of breast tumors using active microwave imaging. In this application, one is typically not so much interested on the detailed reconstruction of the spatial distribution of the dielectric properties (which would require by far more data than there are usually available), but mainly to answer in a fast, harmless and inexpensive way the following three questions: (i) whether or not there is a malignant tumor, (ii) its (approximate) location, and (iii) its (approximate) size. Once these questions have been answered reliably, more details can be investigated if necessary by alternative (but then typically more expensive) imaging techniques.

In this paper we investigate the use of the level set technique (see [4, 7-11] and references given there for details) as a means to detect the presence, location and size of small tumors if their properties are assumed to be known. The main difficulty in this work is the extremely limited view to the domain of interest due to a very specific source-receiver geometry: all sources and receivers are located at the same side of the domain. Our observation from earlier work [4] has been that in these situations the level set iteration, when initiated with an arbitrary starting guess for the shape, tends to suffer from local minima, which makes it difficult to reliably detect the correct location of the tumor. Therefore, we have investigated an adaptation of our level set approach to this new situation which is able to start without any pre-specified starting guess for the shape. Our algorithm is able to create shapes in any location of the domain. It does so during the early iterations taking into account the data and the sensitivity mapping of the inverse problem. Once a good first approximation for the shape is found, it continues in a completely automatic way with optimizing this shape until the data least squares cost functional is sufficiently reduced. We compare the results of numerical experiments for this new reconstruction algorithm with those of a straightforward (and non-optimized) implementation of the MUSIC algorithm (for a detailed theoretical and numerical investigation of this imaging scheme see for example [1-3, 6] and the references given there). Some conclusions of this comparison are given at the end of this paper.

2. Level Set Formulation of the Problem

For modelling TM-waves in Microwave imaging we use a scalar Helmholtz equation for $u(\mathbf{x})$ describing one component of the electric field. It is

$$\Delta u + \kappa(\mathbf{x})u = q(\mathbf{x}) \quad \text{in } \Omega = \mathbb{R}^2 \tag{1}$$

with $\kappa(\mathbf{x}) = \omega^2 \mu_0 \epsilon_0 \left[\epsilon(\mathbf{x}) + i \frac{\sigma(\mathbf{x})}{\omega \epsilon_0} \right]$. The field *u* is required to satisfy the Sommerfeld radiation condition, and it is assumed to be continuous together with its normal derivatives across interfaces. In the shape inverse problem



Figure 1: Left: The MUSIC estimate (10), normalized to one, of the location of one tumor in a homogeneous medium. The exact location and the size of the $2 \times 2 \text{ mm}^2$ tumor is denoted by the square. The tumor is located less deeper in the top row than in the bottom one. The frequencies used in these images are 3 GHz (left column) and 4 GHz (right column). Right: Evolution of a shape by a level set formulation.

we assume that the parameter distribution is described by

$$\kappa(\mathbf{x}) = \begin{cases} \kappa_i & \text{in} & S\\ \kappa_e & \text{in} & \Omega \backslash S \end{cases}$$
(2)

where S defines the shape of the tumor. For the formal derivation of our reconstruction approach we introduce the one-dimensional Heaviside function $H(\psi)$ which is defined as $H(\psi) = \begin{cases} 1 & , \ \psi > 0 \\ 0 & , \ \psi \le 0 \end{cases}$ We call ψ a *level* set representation of the shape S if

$$\kappa(\psi) = \kappa_e H(\psi) + \kappa_i (1 - H(\psi)). \tag{3}$$

Using the level set representation, $\psi(\mathbf{x})$ the shape S is characterized by all those points $\mathbf{x} \in \Omega$ where $\psi(\mathbf{x}) \leq 0$, and the region $\Omega \setminus S$ is characterized by those points $\mathbf{x} \in \Omega$ where $\psi(\mathbf{x}) > 0$ (see Fig. 1 on the right). The boundary of the shape S is then modeled by the zero level set $\partial S = \{\mathbf{x} \in \Omega : \psi(\mathbf{x}) = 0\}$. It is clear that the level set representation of a given shape S is not unique. However, every continuous function ψ uniquely specifies a corresponding shape (which we denote $S[\psi]$) by the above definitions. We now define the least squares data misfit cost functional $\mathcal{J}(\psi) = \frac{1}{2} \|\mathcal{R}(\kappa(\psi))\|^2$, where $\mathcal{R}(\kappa(\psi))$ denotes the difference between measured data and those calculated by a forward solver using the parameter distribution κ (modeled by the level set function ψ). The goal during the shape reconstruction problem will be to find an evolution of the level set functions ψ in artificial evolution time t which reduces and eventually minimizes this cost functional. We consider the general evolution law

$$\frac{d\psi}{dt} = f(\mathbf{x}, t, \psi, \mathcal{R}, \ldots)$$
(4)

for the level set function ψ describing the shape S during the artificial evolution. Then the unknown which we are looking for is the forcing term $f(\mathbf{x}, t, \psi, \mathcal{R}, ...)$, which might depend on a variety of parameters as indicated. Formally differentiating (3) with respect to ψ yields $\frac{d\kappa}{d\psi} = (\kappa_e - \kappa_i)\delta(\psi)$ where $\delta(\psi) = H'(\psi)$ is the onedimensional Dirac delta distribution. Formally differentiating the least squares cost functional $\mathcal{J}(\kappa(\psi(t)))$ with respect to the artifical time variable t and applying the chain rule yields

$$\frac{d\mathcal{J}}{dt} = \operatorname{Re} \int_{\Omega} \mathcal{R}'_{l}(\kappa)^{*} \mathcal{R}(\kappa) \left(\kappa_{e} - \kappa_{i}\right) \delta(\psi) f(\mathbf{x}, t, \psi, \mathcal{R}, \ldots) d\mathbf{x}, \qquad (5)$$

where Re indicates the real part of the corresponding quantity. In (5), $\mathcal{R}'_l(\kappa)^*$ denotes the formal adjoint of the linearized Residual operator $\mathcal{R}'_l(\kappa)$ and the expression $\mathcal{R}'_l(\kappa)^*\mathcal{R}(\kappa)$ coincides with the pixel-based Frechét



Figure 2: Reconstruction of a small tumour with the level set formulation. Left: using frequency 1 GHz. Right: using frequency 4 GHz. More details can be found in the text.

derivative of the parameter-to-data mapping of the corresponding parameter reconstruction problem [10]. Using the fact that formally $\delta(\psi) > 0$ in (5), we can define the search or descent direction as

$$f_d(\mathbf{x}) = -\operatorname{Re}\left(\left(\kappa_e - \kappa_i\right) \mathcal{R}'_l(\kappa)^* \mathcal{R}(\kappa)\right) \quad \text{for all} \quad \mathbf{x} \in \Omega.$$
(6)

In contrast to more tradional level set approaches which typically use a Hamilton-Jacobi-type formulation, our search direction $f_d(\mathbf{x})$ has the property that it can be applied even if there is no initial shape available when starting the algorithm. Therefore, it allows for the creation of objects at any point in the domain, by lowering a positive level set function until its values arrive at zero. This property is useful for avoiding certain types of local minima which often occur in level set formulations which are solely based on the propagation of an already existing shape. Numerically discretizing (4) by a straightforward finite difference time-discretization with time-step $\tau > 0$ and interpreting $\psi^{(n+1)} = \psi(t + \tau)$ and $\psi^{(n)} = \psi(t)$ yields the iteration rule

$$\psi^{(n+1)} = \psi^{(n)} + \tau f_d(\mathbf{x}), \quad \psi^{(0)} = \psi_0.$$
(7)

3. MUSIC formulation of the problem

We consider an array of N electromagnetic transducers located at positions \mathbf{x}_n , n = 1, 2, ..., N. Two adjacent transducers are separated by a distance $\lambda_0/2$, where λ_0 denotes the wavelength of the signal emmitted by the array. With this arrangement the transducers do not behave like separate entities but like an array having an aperture $a = (N - 1)\lambda_0/2$ that interrogates the medium. Within the medium there are M targets (tumors) located at positions \mathbf{y}_m , m = 1, ..., M. The scattered echos by the tumors are recorded at the array. We call the resulting data set the multistatic response matrix (MSR matrix) $K = (K_{ij})$, whose entries are defined by the scattered field detected at the *i*th transducer (in receive mode) when the *j*th transducer (in active mode) emits an electromagnetic signal. The goal is to estimate the location \mathbf{y}_m of the tumors from the knowlegde of the MSR matrix. The singular value decomposition of the MSR is given by

$$K = U\Sigma V^H av{8}$$

where the superscript H denotes the hermitian matrix. In (8), Σ is a diagonal matrix whose diagonal entries σ^2 are the eigenvalues of the time reversal matrix (TR matrix) $T = K^H K$. If there are less targets than array elements (M < N) there are at most M non zero eigenvalues indexed from 1 to M, and N - M zero eigenvalues indexed from M + 1 to N. The column vectors of the matrix U in (8), denoted by \mathbf{U}_k $(k = 1, \ldots, N)$, are the eigenvectors of $T = K^H K$ normalized to one. The column vectors of V, denoted by \mathbf{V}_k $(k = 1, \ldots, N)$, are the complex conjugates of \mathbf{U}_k . It can be shown that the N dimensional space of signal vectors applied to the N element antenna array can be expresses as the direct sum $S \oplus \mathcal{N}$ [6]. The signal subspace S can be spanned by the significant eigenvectors of the TR matrix T, i. e., by \mathbf{U}_k with $k = 1, \ldots, N$. The MUSIC



Figure 3: Reconstruction of a small tumour with the level set formulation. Left: using frequency 1 GHz. Right: using frequency 4 GHz. More details can be found in the text.

algorithm exploits the fact that the MSR matrix is a projection operator onto the signal subspace S which is also spanned by the complex conjugates of the vectors

$$g(\mathbf{y}_m) = (G(\mathbf{x}_1, \mathbf{y}_m), G(\mathbf{x}_2, \mathbf{y}_m), \dots, G(\mathbf{x}_N, \mathbf{y}_m))^t , \qquad (9)$$

where m = 1, ..., M, the superscript t denotes the transpose, and $G(\mathbf{r}, \mathbf{r'})$ is the deterministic two-point Greens function of the background medium. Therefore, we have that $\langle \mathbf{U}_k, g^*(\mathbf{y}_m) \rangle$ for k = M + 1, ..., N. Then, we can display the objective functional

$$\mathcal{F}(\mathbf{y}^{s}) = \frac{1}{\sum_{k=M+1}^{N} |\langle \mathbf{U}_{k}, g^{*}(\mathbf{y}^{s}) \rangle|^{2}}$$
(10)

for the search points \mathbf{y}^s in the domain. Since $g^*(\mathbf{y}^s)$ is orthogonal to \mathbf{U}_k , with $k = M + 1, \ldots, N$, whenever the search point \mathbf{y}^s equals a tumor location, (10) will exhibit a peak at those positions. We will normalize (10) to one. We note that

$$\sum_{k=M+1}^{N} |\langle \mathbf{U}_{k}, g^{*}(\mathbf{y}^{s}) \rangle|^{2} = |g^{*}(\mathbf{y}^{s}) - \sum_{k=1}^{M} (\mathbf{U}_{k}^{t} g^{*}(\mathbf{y}^{s}))\mathbf{U}_{k}^{*}|^{2} .$$
(11)

Since in our application we will consider only one tumor (M = 1) it is more efficient to compute $\mathcal{F}(\mathbf{y}^s) = (|g^*(\mathbf{y}^s) - (\mathbf{U}_1^t g^*(\mathbf{y}^s))\mathbf{U}_1^*|^2)^{-1}$, normalized to one, instead of (10).

4. Numerical Experiments

In the numerical experiments shown here, the domain of investigation consists of (simulated) tissue of the size $10 \times 8 \text{ cm}^2$ in which a tumour of size $2 \times 2 \text{ mm}^2$ is imbedded at different positions as shown in Figs. 2 and 3. The relative electric permittivity values are 9 in the background medium and 49 inside the tumor. For simplicity, the conductivity value is assumed here to be a small constant of value 0.001 S/m everywhere in the medium. 8 transducers are equidistantly positioned at the top side of the medium. They illuminate the medium with microwaves of different frequencies (we use here 1, 3 and 4 GHz). We solve (1) with a second order finite differences scheme and a perfectly matching layer (PML). The received numerical data have been perturbed by 5 % white Gaussian noise. Fig. 1 shows on the left the MUSIC estimate for the two target locations. In the top row the target is located at a less deep position than in the bottom row. We have used frequencies of 3 GHz (left column) and 4 GHz (right column). Figs. 2 and 3 show the estimates of the level set based algorithm for the two different locations of the hidden tumor. Each of these two figures is divided into two panels of 4 subfigures (the left panel shows results for frequency 1 GHz and the right one for 4 GHz). Each panel of figures is structured in the following way. Top left: true permittivity distribution; top right: reconstructed permittivity distribution; bottom left: final level set function viewed from the side; bottom right: horizontal cross section of the final level set function through the location of the recovered tumor.

5. Observations and Conclusions

We observe in our numerical experiments that the MUSIC algorithm provides a good estimate of the crossrange for high frequencies (4 GHz) with a resolution that decreases with depth (compare the top right and bottom right images of Fig. 1). However, range information is lost (in particular at the deep location), and therefore it should be obtained separetely. See for example Ref. [2]. For a lower frequency (3 GHz) both, range and cross-range resolution decrease. For frequencies lower than 3 GHz we were not able to get useful estimates of the tumor locations with the MUSIC algorithm.

On the other hand, the level set based reconstruction scheme shows a somehow reversed behavior compared to the MUSIC results. For lower frequencies (here 1 GHz) it shows a quite stable estimate of the approximated tumor location, whereas for higher frequencies (here 4 GHz) the corresponding oscillations in the electromagnetic fields tend to introduce artifacts in the reconstructions. As a consequence, the estimate using the level set algorithm gives rise to a ghost location of the tumor in addition to its correct location. We note that the resolution provided for the level set algorithm is much better than that given by the MUSIC algorithm. We also want to mention here that the level set reconstruction method has also the potential of iteratively finding the contrast values of the tumors from the given data if they are not a-priori known. Although this has not been implemented so far in our algorithm, some related approaches can be seen in [7, 10–11]. The MUSIC algorithm is not easily extendable to incorporate this feature.

We conclude that level set based algorithms can provide a useful and flexible strategy for the early detection of small tumors in tissue with microwaves. In the future we plan to extend our method to the more complex situation in which the dielectric properties of the healthy tissue and the tumor are unknown and need to be reconstructed from the given data.

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3D Shape Reconstruction in Optical Tomography Using Spherical Harmonics and BEM

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Abstract—We consider the recovery of smooth 3D region boundaries with piecewise constant coefficients in Optical Tomography (OT). The method is based on a parametrisation of the closed boundaries of the regions by spherical harmonic coefficients, and a Newton type optimisation process. A boundary integral formulation is used for the forward modelling. An advantage of the proposed method is the implicit regularisation effect arising from the reduced dimensionality of the inverse problem. Results of a numerical experiment are shown which demonstrate the performance of the new method in a realistic situation.

1. Introduction

In this paper, we explore a technique for the retrieval of the internal boundaries of 3D regions in frequency domain Diffusive Optical Tomography (DOT), [3]. The optical parameter of interest in this application are μ_a being the absorption coefficient, μ'_s being the (reduced) scattering coefficient, and their combination $D = \frac{1}{3(\mu_a + \mu'_s)}$ being the diffusion coefficient. They are assumed to take piecewise constant values in the three dimensional bounded domain Ω with jumps at the interior interfaces. There are several physiologically interesting observations which can be derived from the knowledge of the absorption and diffusion of light in tissue. This includes tissue oxygenation, blood volume and blood oxygenation [1, 2]. Primary applications are the detection and classification of tumourous tissue in the breast, monitoring of the oxygenation level in infant brain tissue, and functional brain activation studies.

Our model for light propagation in biological tissue is the diffusion equation [3]

$$-\nabla \cdot D(\mathbf{r})\nabla \Phi(\mathbf{r}) + \mu_{a}(\mathbf{r})\Phi(\mathbf{r}) + \frac{\mathbf{i}\omega}{\mathbf{c}}\Phi(\mathbf{r}) = \mathbf{q}(\mathbf{r})$$
(1)

where $\Phi(\mathbf{r})$ is photon density, c is the speed of light in the medium, and $q(\mathbf{r})$ describes the source term. It represents the number of photons per unit volume at the source position \mathbf{r} . ω is the modulation frequency. The appropriate boundary condition is of the Robin type (given by (4)). However, if we assume that the distribution of the optical parameters inside the body Ω is arranged into L disjoint regions Ω_j , so that $\Omega = \bigcup_{j=1}^{L} \Omega_j$, which are separated by smooth closed interfaces Γ_j , and have piecewise constant optical properties $\{D_j, \mu_{\mathbf{a},j}\}$, we may describe the propagation of light by a set of coupled Helmholtz equations

$$-\Delta \Phi_j + k_j^2 \Phi_j = q_j \quad \text{in} \, \Omega_j \,, \tag{2}$$

with boundary conditions

$$\Phi_{j+1} = \Phi_j, \quad D_{j+1} \frac{\partial \Phi_{j+1}}{\partial \nu} = D_j \frac{\partial \Phi_j}{\partial \nu} \qquad \text{on } \Gamma_j, \qquad (3)$$

$$\Phi_1 + 2AD_1 \frac{\partial \Phi_1}{\partial \nu} = 0 \qquad \text{on } \partial\Omega \,. \tag{4}$$

Here, A models the refractive index difference at the boundary $\partial\Omega$. The respective (complex) 'wavenumbers' are $k_j^2(\omega) = \frac{\mu_{a,j} + \frac{i\omega}{c}}{D_j}$.

The described inverse problem of 3D DOT is severely ill-posed due to the diffusive behavior of the fields in the tissue and the relatively small number of available noisy data. This typically leads to quite unstable reconstructions, unless strong regularization is applied. One possible way of regularizing the problem is to take advantage of prior information about the general structure of the expected parameter distribution, which often



Figure 1: Segmented MRI data of a baby's scalp and as voxel volume (left), (Thanks to Richard Bayford, Middlesex University). The mapping of the surface on the sphere (middle). The parametric representation with 11 degrees of spherical harmonics (right).

is available in medical applications from alternative imaging modalities or from general anatomical knowledge. This will be our approach in this paper, assuming that the domain of interest can be divided into basically two different zones: a background distribution and an embedded object whose shape can be approximately described by a given (small) number of spherical harmonics parameters.

2. Parametric Representation of Surfaces

Our main interest lies in the use of geometric prior information in order to create a sufficiently realistic model of the different subregions of an anatomical structure. Having in mind applications in head and brain imaging, we decided to use the head's geometry as a test bed. We can use good resolution MRI or CT-scan images as prototypes. Both imaging modalities use voxel maps to create an image. The voxel faces comprising the boundary surface are mapped to the surface of a sphere by a method described in [9].

Since our application is not limited to star-shaped objects, a harmonic distribution of the extracted surface's net onto the sphere's surface was chosen instead of a direct radial function. Having defined coefficients $\{C_l^m\}$, we can use them to create the parametric description of the surface by weighted averaging with the relevant spherical harmonics

$$\upsilon = \begin{cases} \upsilon_x(\vartheta,\varphi) &= \sum_{l=0}^{\varpi} \sum_{m=-l}^{l} C_{l,x}^m Y_l^m(\vartheta,\varphi), \\ \upsilon_y(\vartheta,\varphi) &= \sum_{l=0}^{\varpi} \sum_{m=-l}^{l} C_{l,y}^m Y_l^m(\vartheta,\varphi), \\ \upsilon_z(\vartheta,\varphi) &= \sum_{l=0}^{\varpi} \sum_{m=-l}^{l} C_{l,y}^m Y_l^m(\vartheta,\varphi). \end{cases}$$
(5)

Here, ϖ is the maximum degree of spherical harmonics that we used for the particular representation. In practice, to ensure that only real surfaces are represented, we define a real basis as

$$\tilde{Y}_l^m(\vartheta,\varphi) := \begin{cases} Re[Y_l^m](\vartheta,\varphi), & \text{when } m \le 0, \\ Im[Y_l^m](\vartheta,\varphi), & \text{when } m > 0, \end{cases}$$
(6)

for which the orthogonal condition $\langle \tilde{Y}_l^m, \tilde{Y}_{l'}^{m'} \rangle = \delta_{mm'} \delta_{ll'}$ still holds. For simplicity we introduce the notation $\gamma_j = \{C_l^m\}_j$, with $l = 1, \dots, \varpi$ and $m = -l, \dots, l$ which describes the finite set of spherical harmonics coefficients for the surface Γ_j up to degree ϖ .

3. The Forward Problem

As in conventional pixel based reconstruction we assume multiple sources p_s , $s = 1, \ldots, S$ and detectors m_d , $d = 1, \ldots, M$, located at the surface $\partial \Omega$. During the experiment, light is emitted from one source at a time and the photons leaving the domain are collected at all the detectors. We denote by $g_{s,d}$ the measurements which corresponds to detector d and source s. The combined measurements for a source s are denoted by \mathbf{g}_s . A boundary integral formulation is used to simplify the discretisation of the volume of the domain Ω to that of the interfaces Γ_j of the disjoint regions that comprise Ω . The shapes and locations of the boundaries are described by finite sets of shape coefficients $\gamma = \{\gamma_j\}$. The forward problem uses a Boundary Element Method (BEM) to discretise the mapping from the shape coefficients $\{\gamma_j\}$ and the optical parameters values $\{D_j, \mu_{\mathbf{a},j}\}$ to the data $\mathbf{g} = \mathcal{M}(\gamma)\Phi$ on the surface $\partial\Omega$, where \mathcal{M} denotes the linear measurement operator which typically takes point evaluations of the fields Φ at few discrete points of the surface $\partial\Omega$. The inverse problem in this setup amounts to finding the representation $\{\gamma_j\}$ and the values $\{D_j, \mu_{\mathbf{a},j}\}$ from observed or simulated measurements **g**.

Discretising our forward problem (2)-(4) by the so-called 'collocation Boundary Element Method' [4,8] we construct a linear matrix equation of the form

$$\mathsf{T}(\gamma)\mathbf{f} = \mathbf{q} \tag{7}$$

The matrix $\mathsf{T}(\gamma)$ (which depends in a nonlinear way on the shape parameters γ) takes the form of a dense non-symmetric block matrix. The corresponding system is solved using a preconditioned GMRES solver. To relate the BEM approach to the forward model, we introduce the linear measurement operator \mathcal{M} . Then we have

$$\mathbf{g}_s = \mathcal{K}_s(\gamma, D, \mu_a) = \mathcal{M}\mathsf{T}^{-1}(\gamma)\mathbf{q}_s,\tag{8}$$

where \mathbf{g}_s are the measured data at the discrete points m_d , $d = 1, \ldots, M$ corresponding to the source \mathbf{q}_s , and $\mathcal{K}_s(\gamma, D, \mu_a)$ denotes the nonlinear forward operator mapping unknown shape parameters to the corresponding measurements [3]. In the following we will omit the subindex s in the notation for simplicity.

4. The Shape Inverse Problem

Starting from a geometric configuration defined by the set of shape coefficients (γ^0) , we will try to search for the set (γ^*) that minimises the distance between computed data $\mathcal{K}(\gamma, D, \mu_a)$ and given data **g**. Our approach will be a cost minimisation procedure:

find
$$\gamma^*$$
 so that $\Xi(\gamma^*) = \min_{\alpha} \|\mathbf{g} - \mathcal{K}(\gamma, D, \mu_a)\|^2$ (9)

A typical way to minimise such a cost function is a Newton-type method, [7], where we search for a minimum for $\Xi(\gamma)$ by iterations of local linearisation and Taylor expansion around the current estimate γ^k as

$$\gamma^{k+1} = \gamma^k + (\mathbf{J}_k^T \mathbf{J}_k + \mathbf{\Lambda})^{-1} \mathbf{J}_k^T (\mathbf{g} - \mathcal{K}(\gamma^k, D, \mu_a)).$$
(10)

 Λ is a Levenberg-Marquandt control term [5]. In our implementation, we take Λ to be the identity.

The modified Newton method (10) for the minimisation of the residual (9) produces the descent direction in the parameter space by providing a step $\delta \gamma^k = \gamma^{k+1} - \gamma^k$. In practice, moving $\Xi(\gamma)$ to the full step length $\delta \gamma^k$ could lead the residual far from the actual minimum. A quadratic fit line search method is introduced in order to avoid detours in the downhill direction and speed up the optimisation.

5. Construction of the Jacobian

One of the key elements in the implementation of the optimisation scheme (10) is the calculation of the Jacobian $J = \frac{\partial \mathcal{K}(\gamma, D, \mu_a)}{\partial \gamma}$ of the forward operator \mathcal{K} with respect to the shape coefficients γ . Since the measurement operator \mathcal{M} is linear, this amounts essentially with (8) to calculating $\frac{\partial T(\gamma)^{-1}}{\partial \gamma_i}$ in an efficient way. In our numerical calculations we have implemented a semi-adjoint scheme for calculating these expressions. Assume that the matrix T is invertible and differentiable with derivative $\frac{\partial T(\gamma)}{\partial \gamma_i}$. Differentiation of the identity $T(\gamma)^{-1}T(\gamma) = I$ yields by the product rule

$$\frac{\partial \mathsf{T}(\gamma)^{-1}}{\partial \gamma_i} = -\mathsf{T}(\gamma)^{-1} \frac{\partial \mathsf{T}(\gamma)}{\partial \gamma_i} \mathsf{T}(\gamma)^{-1}$$
(11)

Denote $\mathbf{f}_s = \mathsf{T}(\gamma)^{-1} \mathbf{q}_s(\gamma)$ the solution vector for the s^{th} source vector by $\mathbf{q}_s(\gamma)$, and let $\mathbf{e}_d = \begin{bmatrix} 0 \ 0 \ \cdots \ 0 \ 1 \ 0 \ \cdots \ 0 \ 0 \end{bmatrix}$ the standard d^{th} unit vector where the value 1 is at d^{th} position. Then, the measurement at the d^{th} detector corresponding to source s can be written as

$$g_{sd} = \mathbf{e}_d^{\mathrm{T}} \cdot \mathbf{f}_s = \mathbf{e}_d^{\mathrm{T}} \cdot \mathsf{T}(\gamma)^{-1} \cdot \mathbf{q}_s(\gamma)$$
(12)

By differentiation with respect to γ_i and using the identity (11) we get

$$\frac{\partial g_{sd}}{\partial \gamma_i} = \mathbf{e}_d^{\mathrm{T}} \cdot \mathsf{T}^{-1}(\gamma) \cdot \frac{\partial \mathsf{T}(\gamma)}{\partial \gamma_i} \cdot \mathsf{T}^{-1}(\gamma) \cdot \mathbf{q}_{\mathbf{s}}(\gamma) + \mathbf{e}_d^{\mathrm{T}} \cdot \mathsf{T}^{-1}(\gamma) \cdot \frac{\partial \mathbf{q}_{\mathbf{s}}(\gamma)}{\partial \gamma_i}$$
(13)



Figure 2: Recovery of inhomogeneity shape from OT measurements on the surface with known a-priori optical parameters. (left) the target; (right) red: the initial guess, green: the reconstructed shape.



Figure 3: Relative data error, $\|\mathbf{g}^{-1}(\mathbf{g} - \mathcal{K}(\gamma_k, D, \mu_a))\|$ on the left, and parameter space error $\sum_n \|(\gamma_{target,n} - \gamma_{k,n})\|^2$, on the right.

Denoting furthermore

$$\mathbf{f}_{d}^{+} = \mathbf{e}_{d}^{\mathrm{T}} \cdot \mathbf{T}^{-1}(\gamma), \qquad \mathbf{Q}_{s} = \mathbf{T}^{-1}(\gamma) \cdot \frac{\partial \mathbf{q}_{s}(\gamma)}{\partial \gamma_{i}}$$
(14)

we finally arrive at

$$\frac{\partial g_{sd}}{\partial \gamma_i} = \mathbf{f}_d^{+,\mathrm{T}} \cdot \frac{\partial \mathsf{T}(\gamma)}{\partial \gamma_i} \cdot \mathbf{f}_s + \mathbf{e}_d^{\mathrm{T}} \cdot \mathbf{Q_s}.$$
(15)

We notice that $\frac{\partial g_{sd}}{\partial \gamma_i}$ are the actual entries of the Jacobian J. The derivative of the BEM system matrix T with respect to the geometrical parameter γ_i is now done using a finite difference method

$$\frac{\partial \mathsf{T}(\gamma)}{\partial \gamma_i} = \frac{\mathsf{T}(\gamma_1, \cdots, \gamma_i + \varepsilon_i, \cdots, \gamma_n) - \mathsf{T}(\gamma_1, \cdots, \gamma_i, \cdots, \gamma_n)}{\varepsilon_i}$$
(16)

The practical choice of ε_i requires a trade-off between the mathematical accuracy of the derivative approximation and the computer roundoff error consideration [7]. In our case it is chosen empirically as $10^{-4}\gamma_i$.

6. Results from 3D Simulations

In our experimental setup, a geometric model for an infant's head (Figure 1) is created and treated as a homogeneous domain with an embedded randomly shaped inhomogeneity, which we try to recover. The optical parameters chosen for the homogeneous background are $\mu_a = 0.01 \text{ cm}^{-1}$ and $\mu_s = 1 \text{ cm}^{-1}$, and for the internal region Ω_2 we have $\mu_a = 0.05 \text{ cm}^{-1}$ and $\mu_s = 2 \text{ cm}^{-1}$. The inhomogeneity's surface is described by 16 spherical harmonic coefficients γ_0 for each cartesian coordinate x, y, z. This defines a parametric surface using up to the 3^{rd} degree spherical harmonics. A regular mesh with 48 elements and 98 nodes is mapped onto that surface to create the discrete approximation necessary for the BEM calculation, see Figure 2. Using this geometric setup, we assign 20 sources and 20 detector positions at the surface of the head. The modulation frequency on the sources in set to 100 MHz. Synthetic data are then collected at the 20 detectors using the forward model $\mathcal{K}(\gamma_0)$ with one source illuminated at a time. We split this data into real and imaginary parts of its logarithm to get a vector $\mathbf{g} \in \mathbb{R}^{800}$. Gaussian random noise with a standard deviation of 1% of the measured signal is added to these data. As the initial guess for the reconstruction we select a closed surface centred at a random position. In this case, we use 9 parameters for each direction in a 2nd degree spherical harmonics description. This choice leads to a search space of dimension $3 \times 9 = 27$. The solution follows the residual minimisation technique described above. The reconstructed boundary is displayed in Figure 2. Figure 3 shows the relative data error $\|\mathbf{g}^{-1}(\mathbf{g} - \mathcal{K}(\gamma_k, D, \mu_a))\|$ versus iteration index k on the left hand side. On the right hand side of this figure, a measure for the quality of the approximation of the shape is displayed. Due to the larger number of coefficients γ_0 used for the construction of the target than for the definition of the evolution shape γ_k , we define γ_{target} to be the set of spherical harmonics coefficients that define the target truncated up to the degree used for the evolution. So the residual of Figure 3 is chosen to be $\sum_n \|(\gamma_{target,n} - \gamma_{k,n})\|^2$, with n summing up to the degree of spherical harmonics used for the evolution shape.

As can be seen, the location and the approximate shape of the simple 3D homogeneous region can be recovered with good accuracy from noisy data. The minimisation of the least squares functional has completed successfully with the distance norm becoming 33 times smaller that the initial value after only 5 iterations. On the other hand, the distance between the shape coefficients shows good convergence if we take into account that a different degree of spherical harmonics was used for the creation of the simulated data than for the evolving shape during the reconstruction.

7. Conclusion

In the paper we have proposed a novel reconstruction scheme for a shape based three dimensional inverse problem in DOT. In our method, the search space for the solution of the inverse problem is defined in terms of a spherical harmonic expansion of the unknown region surfaces which are not restricted being star-shaped. Doing so we incorporate in our scheme an implicit regularisation, where the regularisation parameter is the degree of spherical harmonics used for representing the surfaces. A semi-adjoint formulation of the parameteror shape-sensitivities has been derived. In our numerical experiments, using the semi-adjoint form, we have demonstrated that our scheme is able to reconstruct in a stable and efficient way low-parametric approximations of more complicated shapes from few given data.

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On the Intermittency of the Light Propagation in **Disordered Optical Materials**

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Abstract—We consider propagation of light through an ensemble of $N \gg 1$ statistically independent optical fibers of length L whose refraction coefficient is a random function of length. We introduce the generalized transmission coefficient $|t(k,L)|^p$ for energy k^2 and study its quenched and annealed Lyapunov exponents. For small disorder we calculate the Lyapunov exponents in asymptotic form.

1. Introduction

The idea of intermittency was originally proposed in the study of turbulent flow [1] and has become widespread in statistical particle physics. Intermittency means random deviations from smooth and regular behavior. To illustrate it, we consider a bundle of $N, N \gg 1$, statistically equidistributed independent optical fibers of a fixed length L whose refractive index changes randomly along the length of the fiber. If one face of the bundle is illuminated then, due to reflection of the light and its localization in the fibers, one might expect that the outlet of the bundle will be uniformly dark. However, because of strong statistical fluctuations of the transparency (that is a typical manifestation of the intermittency), the exit of the bundle will look like a dark sky with sparse bright stars. This model was proposed by I. M. Lifshits [2] to explain high irregularity of the light distribution after propagation through a thick layer of a disordered optical material. Propagation of light in each fiber is described by the equation

$$-\psi'' + \sigma V_j(x)\psi = k^2\psi, \quad j = 1, 2, \dots, N,$$
 (1)

where $V_i(x)$ are homogeneous random potentials equal zero outside the fibers and constant σ characterizes strength of the disorder.

Equation 1 has scattering solutions

$$\psi_{k,j}(x) = \begin{cases} e^{ikx} + r_j(k) e^{-ikx}, & x < 0, \\ t_j(k) e^{ikx}, & x > L, \end{cases}$$
(2)

where $t_i(k)$ and $r_i(k)$ are random complex transmission and reflection coefficients, respectively, such that $|t_j(k)|^2 + |r_j(k)|^2 = 1$. We also introduce the empirical mean $\frac{1}{N} \sum_{j=1}^{N} |t_j(k)|^2$ for the transmitted energy provided

the energy density of the incident wave equals one for each waveguide, and for fixed L and $N \to \infty$

$$\frac{1}{N} \sum_{j=1}^{N} |t_j(k)|^2 \xrightarrow{a.s.} \langle |t(k,L)|^2 \rangle, \tag{3}$$

where a.s. means almost surely (with probability one). Expressions $|t(k,L)|^p$ and $\langle |t(k,L)|^p \rangle$ are decreasing exponentially as $L \to \infty$ whose logarithmic rate of decay we call the quenched and annealed (moment) transmission Lyapunov exponents, respectively,

$$\gamma_q^T(k,p) = \lim_{L \to \infty} \frac{\ln |t(k,L)|^p}{L} = p \lim_{L \to \infty} \frac{\ln |t(k,L)|}{L} = p\gamma^T(k), \tag{4}$$

$$\mu_a^T(k,p) = \lim_{L \to \infty} \frac{\ln\langle |t(k,L)|^p \rangle}{L}.$$
(5)

Using this notation we can quantitatively characterize intermittency: after propagation through the fiber bundle light exhibits intermittency if $|\mu_a^T(k,2)| < |\gamma_q^T(k,2)|.$ (6)

The stronger inequality (6) is, the more intermittent is the distribution of energy on the exit of the fiber bundle.

2. Analytical Tools

The study of equation 1 with representative potential V(x) is based on the phase-amplitude formalism. Let $\psi_k^{(i)}(x), i = 1, 2$, be the fundamental set of solutions of (1) with initial values $\psi_k^{(1)}(0) = 1$, $\frac{d}{dx}\psi_k^{(1)}(0) = 0$, $\psi_k^{(2)}(0) = 0$, $\frac{d}{dx}\psi_k^{(2)}(0) = 1$. The matrix

$$M_k([0,L]) = \begin{pmatrix} \psi_k^{(1)}(L) & k\psi_k^{(2)}(L) \\ \frac{1}{k}\frac{d}{dx}\psi_k^{(1)}(L) & \frac{d}{dx}\psi_k^{(2)}(L) \end{pmatrix}$$
(7)

is the propagator of (1) whose determinant equals one.

For the general solution of (1) we put

$$\psi_k(x) = r_k(x)\sin\theta_k(x), \quad \frac{d\psi_k(x)}{dx} = kr_k(x)\cos\theta_k(x). \tag{8}$$

Then for θ_k and $\ln r_k$ we obtain the following system [2], [3]

$$\frac{d\theta_k(x)}{dx} = k - \frac{\sigma V(x) \sin^2 \theta_k}{k},\tag{9}$$

$$\frac{d\ln r_k(x)}{dx} = \frac{1}{2k}\sin 2\theta_k(x)V(x).$$
(10)

In most cases of interest [2], [3], the phase $\theta_k(x) \in [0, \pi)$ represents either a Markov process with generator \mathcal{L} (white noise potential) or a component of a multidimensional Markov process (the Kronig-Penny model). To illustrate intermittent behavior of light distribution, we use the simplest case when the potential $V(x) = \dot{b}(x)$ is the white noise (the derivative of the Brownian motion b(x)).

Equations 9-10 are understood as Itô's stochastic differential equations with Stratonovich corrections. In our case, the generator of the diffusion process (9) has the form [4]

$$(\mathcal{L}f)(\theta) = \frac{B^2(\theta)}{2} \frac{d^2 f}{d\theta^2} + \left(A(\theta) + \frac{(BB')(\theta)}{2}\right) \frac{df}{d\theta},\tag{11}$$

where $A(\theta) = k$, $B(\theta) = -\frac{\sigma \sin^2 \theta}{k}$. Similarly,

$$d(\ln r(x)) = \left(\alpha(\theta(x)) + \frac{1}{2}\beta B(\theta(x))\right)dx + \beta(\theta(x)) \cdot db(x)$$
(12)

with $\alpha = 0$ and $\beta(\theta) = \frac{\sigma \sin 2\theta}{2k}$. Hence,

$$r^{p}(x) = e^{\int_{0}^{x} D(\theta) \cdot db(z) + \int_{0}^{x} C(\theta) dz},$$
(13)

where $D(\theta) = p\beta(\theta(z))$ and $C(\theta) = p(\alpha + \frac{1}{2}\beta B)(\theta(z)) dz$. If $u_p(x,\theta) = \langle r^p(x)|\theta(0) = \theta \rangle$ is the expectation of $r^p(x)$, then $u_p(x,\theta)$ satisfies the Feynman-Kac formula which for the white noise potential has the form

$$\frac{\partial u_p}{\partial x} = \frac{\sigma^2 \sin^4 \theta}{2k^2} \frac{\partial^2 u_p}{\partial \theta^2} + \left(k + \frac{\sigma^2 (1-p) \sin^2 \theta \sin 2\theta}{2k^2}\right) \frac{\partial u_p}{\partial \theta} + \frac{\sigma^2 p \sin^2 \theta \cos \theta (p \cos \theta - \sin \theta)}{2k^2} u_p = \widetilde{\mathcal{L}}_p u_p.$$
(14)

Formula (14) allows to calculate the Lyapunov exponent for the amplitude r(L). In the quenched case we have

$$\frac{\operatorname{n} r(L)}{L} = \frac{1}{L} \int_0^L \frac{1}{2} \beta B(\theta(x)) \, dx + \beta(\theta(x)) \cdot db(x) \xrightarrow{a.s.} \langle \frac{1}{2} \beta B \rangle_\eta$$
$$= -\frac{\sigma^2}{4k^2} \int_0^\pi \eta(\theta) \sin 2\theta \sin^2 \theta \, d\theta = \gamma_q(k). \tag{15}$$

Here $\eta(\theta)d\theta$ is the invariant measure for the phase $\theta(x)$ which satisfies the equation

$$\mathcal{L}^* \eta = \frac{d^2}{d\theta^2} \left(\frac{\sigma^2 \sin^4 \theta}{2k^2} \eta \right) - \frac{d}{d\theta} \left[\left(k + \frac{\sigma^2 \sin^2 \theta \sin 2\theta}{2k^2} \right) \eta \right] = 0$$
(16)

that can be solved exactly.

Consider now the moment Lyapunov exponent

$$\mu_a(p) = \lim_{L \to \infty} \frac{\ln \langle r^p(L) \rangle}{L}.$$
(17)

According to Perron–Frobenius theorem about positive semigroups, $\mu_a(p)$ equals maximum eigenvalue of the nonsymmetric operator $\widetilde{\mathcal{L}}_p$ (14)

$$\widetilde{\mathcal{L}}_p \psi = \mu_a(p)\psi \tag{18}$$

and the corresponding eigenfunction $\psi(x)$ is strictly positive.

The Lyapunov exponent $\gamma(k)$ of the amplitude r(L) and $\mu(p)$ have the following properties:

- (a) $\gamma(k) > 0$. This property leads to the localization theorem for the Hamiltonian $H\psi = -\psi'' + \sigma V(x)\psi = \lambda\psi$ on the whole real axis [2], [3].
- (b) For fixed k the annealed Lyapunov exponent is analytic in p and convex.
- (c) $\mu(p)$ is symmetric with respect to p = -1: $\mu(p) = \mu(-p-2)$ and $\frac{d\mu}{dp}(0) = \gamma(k)$. In particular, $\mu(0) = \mu(-2) = 0$ (Fig. 1).
- (d) For small disorder constant σ and fixed $k \gamma(k) = \frac{\pi \sigma^2 \widehat{B}(2k)}{4k^2} (1 + o(\sigma))$, where $\widehat{B}(2k)$ is the spectral density of the potential V. For the white noise $\gamma(k) = \frac{\sigma^2}{8k^2} (1 + o(\sigma))$ and $\mu_a(p) \approx \frac{1}{2} p(p+2)\sigma^2 \gamma(k)$ as $\sigma \to 0$.



Figure 1: Graphs of the annealed moment Lyapunov exponent $\mu_a(p)$ (solid line) and transmission Lyapunov exponent $\mu_a^T(p)$ (crossed line) for fixed k and small σ .

The energy transmission coefficient can be calculated through the matrix $M_k([0, L])$ (7) as follows

$$|t(k,L)|^{2} = \frac{4}{2 + ||M_{k}([0,L])||^{2}},$$
(19)

where the norm is understood as the sum of the squares of matrix's entries. Then $||M_k([0,L])||^2 = [r_k^{(1)}(L)]^2 + [r_k^{(2)}(L)]^2$. From asymptotic behavior of the amplitudes $\ln r_k^{(i)}(L) \approx \gamma(k)L$, i = 1, 2, with probability one as $L \to \infty$ we conclude that $\ln ||M_k([0,L])|| \approx \gamma(k)L$. Therefore,

$$\frac{\ln|t(k,L)|}{L} = \frac{1}{L} \ln \sqrt{\frac{4}{2 + \|M_k([0,L])\|^2}} \to -\gamma(k).$$
(20)

Thus, the quenched transmission Lyapunov exponent is

$$\mu_q^T(k,p) = \lim_{L \to \infty} \frac{\ln |t(k,L)|^p}{L} = -p\gamma(k) < 0.$$
(21)

Calculation of the annealed Lyapunov exponent is more difficult. Typically $r_k \sim e^{L\gamma(k)}$. However, with exponentially small probability $r_k(L)$ can be of the order $e^{-\delta L}$, $\delta > 0$. Then $\langle r_k^p(L) \rangle = e^{-p\delta L}P\{\ln r_k(L) < -\delta L\}$, and for very negative p the product tends to $+\infty$ (Fig. 1). We use large deviation theory [5] to calculate $\mu_a^T(k,p)$. Let us take $0 \leq \beta < \gamma$ and estimate $P\{r_k(L) < e^{\beta L}\}$. Using exponential Chebyshev inequality with optimization over parameter $p \leq 0$ we obtain

$$P\{r_k(L) < e^{\beta L}\} = P\{r_k^p(L) > e^{p\beta L}\} \le \min_{p \le 0} \frac{\langle r_k^p(L) \rangle}{e^{p\beta L}} \sim \min_{p \le 0} e^{(\mu_a(k,p) - p\beta)L} = e^{\mu^*(k,p)L},$$
(22)

where $\mu^*(k,\beta) = \max_p(-p\beta + \mu_a(k,p))$ is the Legendre transform [6] of $\mu(k,p)$ for fixed k with respect to parameter p. It is well-known that in the Markov case it is not only estimation from above but the logarithmic equivalence: $P\{r_k < e^{\beta L}\} \stackrel{\log}{\sim} e^{-\mu^*(k,p)L}$. Now for p > 0

$$\langle |t(k,L)|^p \rangle \stackrel{\log}{\sim} \int \frac{1}{e^{-p\beta L} + e^{p\beta L}} \, dP\{r_k < e^{\beta L}\} = \max_{0 \le \beta \le \gamma} e^{-p\beta L - \mu^*(k,\beta)} = \begin{cases} e^{\mu(k,-p)L}, & 0 < p \le 1, \\ e^{\mu(k,-1)L}, & p > 1. \end{cases}$$
(23)

For small σ we can use parabolic approximation for $\mu_a^T(k, p)$ that gives

$$\gamma_q^T(k,p) = -p \frac{\pi \widehat{B}(2k)}{4k^2} \sigma^2 (1+o(1))$$
(24)

and

$$\mu_a^T(k,p) = \begin{cases} p(p+2)\frac{\pi \widehat{B}(2k)}{8k^2} \sigma^2 (1+o(1)), & p \le 1, \\ -\frac{\pi \widehat{B}(2k)}{4k^2} \sigma^2 (1+o(1)), & p > 1, \end{cases}$$
(25)

where B(x) = Cov(V(y)V(y+x)) is the covariance of random potential V(x), and $\widehat{B}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikx} B(x) dx$ is the corresponding energy spectrum of V(x) (Fig. 1). In particular, for p = 2

$$\mu_a^T(k,2) \approx \frac{1}{4} \gamma_q^T(k,2) < 0.$$
(26)

This relation is the manifestation of the strong intermittency (cf. [1]). It shows that the main contribution to the transmitted energy comes not from "typical" fibers where the logarithmic rate of energy decay is $\gamma_q^T(k,2)$, but rather from few rare fibers (the probability of their occurrence is $e^{\frac{1}{4}\gamma_q^T(k,2)L}$) through which significant part of the energy of order O(1) is transmitted. Thus, we have the I. M. Lifshits picture described in the introduction.

3. Conclusion

We have considered propagation of light through a bundle of independent optical fibers whose refractive index is a random function of length. It is found that distribution of energy at the exit of the bundle has intermittent behavior. For quantitative estimation of irregularity we introduced the generalized energy transmission coefficient and studied its Lyapunov exponent. Essential difference in the quenched and annealed energy transmission Lyapunov exponents is suggested as a manifestation of intermittency. In the case of small randomness of the fiber refractive index it is found that the energy transmission Lyapunov exponent of a typical single fiber is four times bigger than the average one of the bundle. Unlike the moment Lyapunov exponent $\mu_a(p)$ for the amplitude which has quadratic dependence on the moment p, the transmission moment Lyapunov exponent is constant for $p \geq 1$.

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A Novel Multiband and Broadband Fractal Patch Antenna

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Abstract—a novel multiband and broadband fractal patch antenna is presented in this paper. The proposed antenna is compact, simple to design and fabricate. The impedance bandwidth of the proposed antenna could reach 18%, which has rarely been reported for single layer and single patch antennas. Multiband characteristics are also observed and analyzed. All results are proved by simulation and experiment.

1. Introduction

Microstrip patch antenna (MPA) has attracted wide interest due to its important characteristics, such as light weight, low profile and low cost, mechanically robust, simple to manufacture, easy to be integrated with RF devices, allow multi-frequency operation to be achieved, etc. However, its further use in specific systems is limited because of its relatively narrow bandwidth. The impedance bandwidth of a typical patch antenna may be just 1-2%. Much intensive researches have been done in recent years to develop bandwidth-enhancement techniques: using a thick air or foam substrate results in a maximum bandwidth of less than 10%; using stacked or co-planar parasitic patches [1] obtains a bandwidth of 10%-20%; using a gap-coupled probe feed [2] achieves a bandwidth of 16%; more recently, the addition of a U-shaped slot [3] and the use of an L-shaped probe have both been shown to provide bandwidth in excess of 30%. However, these techniques increase antenna volume, complicate the design and fabrication of the antennas.

Fractal geometries have recently been introduced in the design of antennas. It has been shown that fractal shaped antennas exhibit characteristics that are associated with the geometric properties of fractals. One property associated with fractal geometry that is used in the design of antennas is self-similarity. A fractal antenna can be designed to receive and transmit over a wide range of frequencies using the self- similarity properties associated with fractal geometry structures.

In this paper, a novel multi-band and broadband fractal patch antenna is designed, measured and analyzed. The impedance Antenna Structure bandwidth of the proposed antenna could reach 18%, which has rarely been reported for single-layer and single-patch antennas. As the scale Ra increases, multi-band characteristics are observed. All results are validated by simulations and experiments.

2. Antenna Structure



Figure 1: Geometry of the broadband and multi-band fractal microstrip antenna with a tuning stub.

Figure 1 shows the geometry of the fabricated antenna. The exact dimensions for the proposed antenna are also given in Figure 1. The patch is printed on a microwave substrate FR4 of the thickness H = 1 mm and the relative permittivity $\varepsilon_r = 4.4$. The antenna is fed through a 50 ohm microstrip line of the width W = 2 mm. As the FR4 substrate is not suitable to be used at frequencies above 4 GHz, R0 should not be too small. For the present design R0 = 40 mm. The radiation elements are composed of ten similar orthogonal bars. Both the length and width of the orthogonal bar are magnified by the factor of Ra * Ra. Four antennas with different Ra (1.01, 1.02, 1.03 and 1.05) are simulated, fabricated and measured. A tuning stub with the width T and the length L is added to the feed-line of the antenna of Ra = 1.01 to get a wider bandwidth of 18%.

3. Simulated and Measured Results

The proposed antenna is simulated using Ansoft HFSS 9.2 and measured with a HP8510C network analyzer. Figure 2 compares the simulated and measured return loss of the antenna of Ra=1.01. The measured impedance bandwidth of the un-tuned antenna is approximately 11% (2.996 GHz–3.321 GHz) while that of the tuned antenna is approximately 18% (3.03 GHz–3.65 GHz). In addition, the antenna has a resonant frequency around 0.9 GHz, which proves the electrically small characteristics of the antenna. Good agreement can be seen between the simulated and measured results.



(a) Return loss of the un-tuned antenna (Ra = 1.01).

(b) Measured return loss of the proposed antenna (Ra = 1.01, S = 3 mm, T = 2.5 mm, L = 4 mm).

Figure 2: Return loss of tuned and untuned antenna.



Figure 3: Surface current distribution at f1, f2 and f3.

From Figure 2(a), one can see that three frequencies f1, f2 and f3 which are close to each other result in a wide bandwidth. Figure 3 shows the simulated surface current density of the three frequencies. When working frequency increases, the area surface current mainly distributed on moves from outer bars to inner bars. Because the dimension of the bars vary slowly by the factor of Ra * Ra = 1.02, the antenna has several resonant frequencies close enough to each other to form a wide bandwidth. One can see that f1, f2 and f3 increase by the factor of 1.03, which agrees with the geometry of the structure.

Four antennas of Ra = 1.01, 1.02, 1.03 and 1.05 are fabricated and measured to study the influence of the parameter Ra on the antenna's characteristics. Figure 4 shows each return loss of the antennas. Figure 4 also shows that when Ra increases, the antenna change from a broadband antenna (Ra = 1.01 and 1.02) into a multi-band antenna (Ra = 1.03 and 1.05). As Ra becomes larger, the dimensions of the bars and the working region vary faster, the resonant frequencies is not close enough to each other to form a wide bandwidth.

The radiation characteristics are also studied. Typical results of the tuned antenna at 3.30 GHz are shown in Figure 5.



Figure 5: Radiation pattern at 3.3 GHz of the tuned antenna.

4. Conclusion

A novel fractal microstrip antenna with multi-band and broadband characteristics has been successfully demonstrated. The obtained impedance bandwidth of the antenna (Ra = 1.01) can be 18% around 3.3 GHz. As Ra increases, Multi-band characteristics of the proposed antennas are also observed. The antenna is compact, simple to design and easy to fabricate.

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Improvement of Reflectarray Performances at Millimeter Waves by Reduction of the Cell Size

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Abstract—In this paper we discuss the advantages and limitations of reducing the cell size of reflectarrays elements. Reflectarrays have demonstrated their utility at mm-Wave because of their compactness, flexibility and quasi-optical feed that reduces losses. Several applications have been covered such as the automotive cruise control including beam scanning. Most of them use $\lambda/2$ cell sizes. We have investigated and compared performances of reflectarrays with 15 mm and 50 mm diameters using $\lambda/2$ and $\lambda/4$ cell sizes at 94 GHz. Measurements on reduced cell size reflectarrays have demonstrated a loss of 1 dB over 60° beam scanning whereas it is of 3 dB for the $\lambda/2$ structure in the case of the smaller reflector. Nevertheless, this effect is not demonstrated on the largest one because of the phase compensation range that is limited by the variation in the patch dimensions. The maximum corrected phase values are of 320° and 240° for $\lambda/2$ and $\lambda/4$ cells respectively. Furthermore a program based on the ray tracing theory has been developed in order to evaluate the influence of the cell size on the array performances.

1. Introduction

Reflectarrays consist of printed elements, typically patches or dipoles representing the elementary cell of the array. They are designed to scatter the incident field, coming from a feeding antenna, with the proper phase required to form a planar phase surface. They have been developed in the millimeter-wave domain since over 10 years [1] in regards to their low profile and low cost. Most of the classical array characteristics have been studied in order to obtain beam scanning [2], or high aperture efficiency both for linear or circular polarization [3]. Efforts have been made for finding appropriated patch shapes for increasing the variation of the reflected phase, the bandwidth or the fabrication simplicity [4, 5].

In this paper, we investigate the influence of the cell size on the gain and beam scanning performances. The elementary cell consists in a rectangular patch printed on a thin substrate. As the patch shape is not the aim of this work, it has been chosen in order to be simple to design regarding to the cell size variation. The first section describes the measured performances of two small reflectarrays with cell sizes of $\lambda/2$ and $\lambda/4$ respectively, at the operating frequency of 94 GHz. In the second section, a program based on the ray tracing theory is presented and tested on different structures in order to demonstrate the effects of the cell size reduction. Finally limitations are discussed.

2. Influence of the Cell Reduction on a 15 mm Diameter Reflectarray

Two reflectarrays of 15 mm diameter were designed and measured at 94 GHz. They are chosen to be small (about 5λ) in order to avoid the effect of the phase compensation limitation. Indeed, when frequency increases, it becomes difficult to cover 360° of reflection phase with a square patch since its dimensions become too small to be fabricated by classical printed circuit techniques. One solution could be to use sophisticated lithography, like the one based on glass mask. It drastically increases the fabrication cost, thus decreases the competitivity of reflectarrays toward other high gain antenna systems such as dielectric lenses or parabolic reflectors.

The primary source is a standard WR-10 open waveguide that radiates a power pattern that can be approximated by $\cos^5(\theta)$. Considering the spillover and taper efficiencies relations given in [1], a diameter to focal length ratio of 2 provides spillover and taper efficiencies of 87 and 89% respectively. Thus focal length is chosen to be of 7.5 mm. Figure 1(a) and (b) show the upper side of the two reflectarrays. Patch size is optimised by numerical simulations provided by the commercial three-dimensional finite element method solver (HFSS) using the periodic structure module. The substrate is Duroid of dielectric constant 2.2 and 0.381 mm thickness. Phase range compensations are of 320° for the $\lambda/2$ cell and 240° for the $\lambda/4$ one. Nevertheless, due to the small size of the reflector, the number of rings with missing phase values is only of one. Thus their effect is decreased.

Reflectarrays are measured at 94 GHz for a scan angle up to 60° as described in Figure 2. Results are reported in Figure 3(a) and 3(b). A reflectarray with $\lambda/2$ cells performs a loss of gain of 3 dB while scan angle

moves. The $\lambda/4$ cells exhibit only 1 dB loss. These results are expected due to the increase of phase accuracy. Additionally, an increase of 2.3 dB is observed on the gain when the cell size is $\lambda/4$.



Figure 3: (a) $D = 1(\lambda/2)$ cell size, (b) $(\lambda/4)$ cell size.

3. Analysis Program

0

-5

-10

-15

-20

Relative amplitude (dB)

A program based on ray tracing theory was developed in order to investigate the influence on the cell size reduction. It was implemented using Scilab [6]. The surface of the reflectarray is divided into square cells of $\lambda/2$

 $\alpha = +60$

or $\lambda/4$ size depending on the structure under study. Values of the desired compensation phases are calculated taking into account the directions of incident spherical and reflected plane wave, including offset feeds and scan angles as described in [1]. A complex amplitude coefficient is affected to each cell. Its module is the value of the power pattern described before. The phase is the difference between the formerly calculated one and the compensated one computed by simulations. If the 360° phase could be covered by the square patch, this difference should be of zero in the desired maximum radiation direction. A matrix of complex coefficient is generated. If we denote θ the angle with respect to the z axis described in Figure 2, power density along θ is calculated by making the sum of all the coefficients of the complex matrix for each angle θ . The advantage of using a software as scilab is the possibility to create 3D matrix whose two first dimensions represent the physically 2D reflector and the third one represents the scan angle θ . As a consequence, the time of calculation is reduced. Radiated power is calculated over the power density integration assuming that the radiation pattern is the same over the φ angle. This does not take into account the real primary source radiation pattern such as the square shape of the reflectarray. Radiation pattern is finally plotted after the normalization of the power density by the radiated power. Figure 4 shows the results for the 15 mm reflector. It is obvious that the effect on the beam scanning improvement is the same as the measured one. Gain values are much higher in the simulation. It can be explained by several factors: the simulation does not take into account the primary source blockage, neither the coupling between primary the source and the reflectarray which are both critical in regard to the very short focal length.



Figure 4: Simulated radiation pattern of the 15 mm reflectarray with different cell sizes.



Figure 5: Simulated radiation pattern of Fresnel reflectors with D = 100 mm.

The same program is tested on reflector using Fresnel zones phase compensation, whose formula is reminded bellow:

$$R_n = \sqrt{\left(2n\frac{\lambda}{P}\right) + \left(n\frac{\lambda}{P}\right)^2}$$

where R_n is the radius of the Fresnel zone referred to the reflector center, λ the free space wavelength, f the focal length and P the Fresnel correction factor (for example P = 2 for a half-wavelength Fresnel reflector). In this case, the effect of the cell size reduction can be seen with different approaches.

First, we consider the effect on size reduction with the same Fresnel correction factor. The improvement of maximal gain is of about 1 dB for a 100 mm diameter half-wavelength reflector.

Second, the space dedicated to each Fresnel zone, defined by $(R_{n+1} - R_n)$ decreases when n increases. As a consequence, high values of P cannot be obtained because the cell size becomes larger than the space for the zone. It can be overcome by using a reduced cell size. The same reflector as described above can be simulated with P = 4 if $\lambda/4$ cell size is used whereas cell size of $\lambda/2$ limits P to 2. Results are plotted in Figure 5. The gain increases of 4.4 dB which corresponds to a 50% improvement due to the passage from half- to quarter-wavelength Fresnel reflector enhanced by the cell reduction.

4. Limitations

Larger reflectors of 50 mm diameter have been made and tested without performing the formerly described ameliorations. Considering the limit values of the corrected angles, which are of 320° for the $\lambda/2$ cell and 240°

for the $\lambda/4$ one, the program has been modified including this limitation. The number of uncorrected rings increases in comparison to the smaller reflector as shown in Figure 6. Results are plotted in Figure 7. It is obvious that the formerly improvements disappear when the phase compensation range of 360° is not covered.



Figure 6: 50 mm diameter reflectors with uncorrected zones.



Figure 7: Simulated radiation pattern showing the limitation of cell sizes reduction.

5. Conclusion

We have shown that the cell reduction of reflectarrays provide a gain enhancement and better beam scanning for classical reflectarrays. In case of Fresnel reflectors, the increase of the gain is more important since the cell size reduction enables to increase the Fresnel correction factor P. Nevertheless, making reflectors with cell of $(\lambda/4)$ encounters the difficulty to obtain a phase reflection compensation of 360°, specially at mm-Wave. In this case, performances are strongly decreased. New patch shapes have to be investigated in order to overcome this problem.

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Array Patterns Synthesizing Using Genetic Algorithm

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Abstract—A planar array antenna with arbitrary geometry synthesis technique based on genetic algorithm is discussed. This approach avoids coding/decoding and directly works with complex numbers to simplify computing programming and to speed up computation. This approach uses two crossover operators that can overcome premature convergence and the dependence of convergence on initial population. Simulation results show that this method is capable of synthesizing arrays whose elements are located on irregular grids, and generates quite complex shapes and can realize good sidelobe suppression at the same time.

1. Introduction

In the interests of efficiency, the shape of a footprint pattern radiated by a satellite-borne array antenna should conform precisely to the shape of the region on Earth for which coverage is required. Alternatively, in order to achieve "isoflux" illumination [1] on the earth surface through multi-beams, the size and the shape of the footprint should be precisely controlled.

Previous works on the synthesis of arrays for arbitrary footprint shapes include Chebyshev method, modified Woodward–Lawson method and a series of methods based on sampling a circular Taylor distribution [2–4]. But these methods have the drawback that they require the arrays to lie on a rectangular lattice or circular lattice. Some methods require the aperture of the array to be regular shape.

Genetic algorithm (GA) has a high ability in global optimization. It is an increasingly popular optimization method being applied to many fields of endeavor, including electromagnetic optimization problems.

Use genetic algorithm to synthesize array pattern has no limitation on lattice shapes and aperture shapes. It can synthesize planar array with arbitrary geometry and generating arbitrary patterns. Conventional GAs [5,6] with binary coding and binary genetic operation are inefficient and inconvenient for array pattern synthesis problems to optimize complex numbers. Unlike conventional GAs, this approach avoids coding/decoding and directly deals with complex excitation vectors.

Compared with other numerical methods [7,8], this approach has unique features to treat complicated problems (complicated arrays and complicated pattern shapes).

2. Problem Formulation

The far-field radiation pattern $F(\theta, \varphi)$ at a far-field angle (θ, φ) from array broadside is given by

$$F(\theta,\varphi) = EF(\theta) \cdot AF(\theta,\varphi) \tag{1}$$

where

$$EF(\theta) = \cos^{1.5}(\theta) \tag{2}$$

is the element radiation pattern; $AF(\theta, \varphi)$ is the array factor. For an arbitrary array, the Array Factor (AF) can be expressed by the general function:

$$AF(\theta,\varphi) = \mathbf{Is}(\theta,\varphi) \tag{3}$$

$$\begin{split} \mathbf{I} &= [I_1, I_2, ..., I_N], I_n \in C^n, \\ \mathbf{s} &= [e^{jkr_1 \hat{a}(\theta, \varphi)}, e^{jkr_2 \hat{a}(\theta, \varphi)}, ..., e^{jkr_N \hat{a}(\theta, \varphi)}]^T, \\ \mathbf{r}_n &= \hat{\mathbf{a}}_{\mathbf{x}} x_n + \hat{\mathbf{a}}_{\mathbf{y}} y_n + \hat{\mathbf{a}}_{\mathbf{z}} z_n, \\ \hat{\mathbf{a}}(\theta, \varphi) &= \hat{\mathbf{a}}_{\mathbf{x}} \sin \theta \cos \varphi + \hat{\mathbf{a}}_{\mathbf{y}} \sin \theta \sin \varphi + \hat{\mathbf{a}}_{\mathbf{z}} \cos \theta \end{split}$$

where, **I** is the excitation vector, **s** is the steering vector, C^n is the set or subset of all complex numbers, \mathbf{r}_n is the element location vectors, $\hat{\mathbf{a}}(\theta, \varphi)$ is unit vector of distance ray of spheric coordinate, and θ and φ are the elevation and azimuth angles respectively. GA is applied to find proper excitation coefficient vector **I** to achieve desired pattern shape, sidelobe suppression and steering.

3. The Genetic Algorithm

The GA process could be simplified as following: 1) Initialize a random pool of Individuals. 2) Evaluate each Individual. 3) Choose couples (Mating). 4) Breed them together (Crossover). 5) Evaluate each Individual. 6) Selection. 7) Mutation. 8) If the pool has converged, or a number of pre-determined cycles have been completed, finish the cycle. If not, return to step #3.

A. Construction of Chromosomes

In this approach, chromosomes are represented directly by complex excitation vectors \mathbf{I} . N elements complex excitation coefficients are genes of the chromosome.

B. Fitness Function

Evaluation plays a very important role in the GA process. Fitness function maps all the properties of an individual to a floating-point number, essentially, giving it a rank and a place amongst the other individuals in the pool. Creating the fitness function is one of the most difficult works in the creation of a GA solution.

In this approach, we desired that the magnitude $|F(\theta, \varphi)|$ of the far-field pattern remain bounded between some specified limits as

$$|F_{\min limit}(\theta,\varphi)| \le |F(\theta,\varphi)| \le |F_{\max limit}(\theta,\varphi)|$$
(4)

A cost measure to be minimized is the sum of the squares of the excess far field magnitude outside the specified bounds. This can be written as

$$f_{1} = c_{1} \sum_{j=1}^{J} \sum_{k=1}^{K} \max(|F(\theta_{j}, \varphi_{k})| - |F_{\max}|_{\text{limit}}(\theta_{j}, \varphi_{k})|, 0)^{2} + c_{2} \sum_{j=1}^{J} \sum_{k=1}^{K} \max(|F_{\min}|_{\text{limit}}(\theta_{j}, \varphi_{k})| - |F(\theta_{j}, \varphi_{k})|, 0)^{2}$$
(5)

where, $|F(\theta, \varphi)|$ are the far field pattern values from (1) evaluated at $J \times K$ far-field angles (θ_j, φ_k) . θ_j and φ_k are spaced in some rules. This will be decided by the beam-pointing angle. c_1, c_2 are weights.

If the dynamic range ratio $|I_{\text{max}}/I_{\text{min}}|$ is too large, the excitation will not easy to realize. We can limit it using the following fitness function:

$$f = f_1 + c_3(|I_{\max}/I_{\min}|)$$
(6)

where, c_3 is a weight parameter. Lower values of f indicate better fitness.

C. Mating Scheme

Yeo [9] discussed three mating schemes and thought if one or more near-solutions were added to an initial population of random individuals, EMS scheme usually yields the best chromosome among these three methods. However, this scheme always results in prematurity. In this approach, we make use of a stochastic mating scheme. All individuals have chance to mate and no one can mate two times.

D. Crossover

The crossover operator is the most important operator and it is the operator that combines two individuals to create (a) new individual(s), which will, it is hoped, become better than his/their parents. This might and can work because the selection operator chooses the better individuals.

Real coded GAs usually use interpolate cross operator to breed offspring. Its operating process can be described as follows,

 ${f I}^1$ and ${f I}^2$ are parents. The chromosomes of them have N genes. The offspring of them can be written as

$$I_i^{1'} = cI_i^1 + (1-c)I_i^2 \tag{7}$$

$$I_i^{2'} = cI_i^2 + (1-c)I_i^1 \tag{8}$$

where, $c \in [0, 1]$, i=1, 2, ..., N.

Extrapolate cross operator is another real number cross operator. The offspring genes of I^1 and I^2 can be written as

$$I_i^{1'} = I_i^1 - (I_i^2 - I_i^1)c \tag{9}$$

$$I_i^{2'} = I_i^2 + (I_i^2 - I_i^1)c \tag{10}$$

The two real number operator can work with complex number as well. Interpolate cross operator has advantages of fast convergence. Extrapolate cross operator expand the search space. Combining two operators can overcome premature convergence and the dependence of convergence on initial population.

The range of genes is $|I_i| \leq 1$. If genes generated by crossover are out of the bound, then

$$I_i = I_i / |I_i| \, (i = 1, \, 2, \, \dots, \, N) \tag{11}$$

E. Selection

The selection operator distinguishes the better individuals from the worse individuals using their fitness. In this approach, both the child and parent populations are ranked together in ascending order. Then, based on the principle of survival of the fittest, those producing superior output survive, while those producing inferior output die off. Please note that the competitors for survival selection include both parents and their children so that the members of next generation may include members of the previous generation. This guarantees that the new generation performs no worse than older ones. In other words, the cost f versus generation curve decreases monotonically. This selection scheme includes optimum maintaining strategy.

F. Mutation

The mutation operator plays a secondary role with respect to crossover operators. It can maintain the diversity of the population. Mutation is a minor change to the genes of an individual, in a hope to find an even better solution, or rather, to expand the search space to a point where normal breeding might not reach. Mutation effectively slows down convergence, but might yield better and closer-to-best individuals. If an individual is "pushed" to a different peak area, a higher one, it might "pull" other individuals with the crossover process to the new peak, thus climbing a better and higher peak, and achieving a better solution. Both the probability and the range of mutation can affect convergence [10].

Nevertheless, mutation is required to prevent an irrecoverable loss of potentially useful information that occasionally reproduction and crossover can cause.

The fitness of mutated individual usually has low value. If put mutation in front of selection. The mutated individuals would die off because of the child and parent competition. In this approach mutation is put after selection.

Assuming P_m is the mutation probability, the mutation is as follows: P real numbers are generated in [0, 1] randomly. Each number corresponds to an individual of the population. Set the number corresponding to current optimum to "1" to avoid being mutated. An individual whose corresponding number less than P_m will be muted use (12).

$$\mathbf{I}' = \mathbf{I} + \mathbf{d} \tag{12}$$

where, **d** is a vector dimension same as **I**.

G. Convergence Observation

For fast convergence, the initial population can include approximate excitations by other techniques (such as Fourier expansion method [11], etc.) We care about not only the shape of main beam, but also the sidelobe level. In order to obtain the required sidelobe level rapidly, we put the optimization result of adjacent beams into initial population which can reduce optimization time largely.

In this approach, two crossover operators are used to generate offspring. Adjusting proportions act on population of two operators, i.e., 40% population use interpolate cross operator to generate offspring and 60% population use extrapolate cross operator to generate offspring, can make the algorithm has a good performance.

4. Simulation Results

This subsection presents a shaping example based on a Low Earth Orbit (LEO) satellite-born antenna array. As shown in Figure 1, a 61 elements antenna array with hexagonal (or equilateral triangular) grid is used.

In order to achieve "isoflux" illumination on the earth surface, a circularly symmetric cell layout was decided as shown in Figure 2 after calculation. Wedge shaped cells are arranged in rings about nadir. There are 23 beams requiring shape, and 30 dB sidelobe suppression is needed.

Figure 3 is the pattern of beam 1. Gain in main beam edge is higher than that in beam center. This can compensate the path loss due to the slant range differences from satellite to earth. Figures 4, 5 and 6 are the patterns of beam 2, 3 and 8. It is observed that the main lobe satisfy the requirement, and side lobe level suppress reach 30 dB which is outstanding the results of [7,8]. If the dynamic range ratio $|I_{\text{max}}/I_{\text{min}}|$ is too



Figure 1: 61 elements antenna array.



Figure 3: Pattern of beam 1, $u = \sin\theta \cos\varphi$, $v = \sin\theta \sin\varphi$.



Figure 2: Cells arranged in rings, $U = \theta \cos\varphi$, $V = \theta \sin\varphi$.



Figure 4: Pattern of beam 2.

large, the excitation will not easy to realize. For beam 1 shown as Figure 2, $|I_{\text{max}}/I_{\text{min}}| = 20$. For beams in ring 2 and a sidelobe level of -30dB, an $|I_{\text{max}}/I_{\text{min}}| < 40$ can be reached. For beams in ring 3 and a side lobe level of -30 dB, an $|I_{\text{max}}/I_{\text{min}}| < 50$ can be reached. These excitations are easily to realize.



If Woodward-Lawson method was used, it requires the elements to lie on a rectangular lattice and require the aperture of the array to be rectangle. And the $|I_{\text{max}}/I_{\text{min}}|$ of the solution will reach 800 or even higher [3].

The $-3 \, dB$ contour of each beam after being shaped is shown in Figure 7. Notice the shape of beams, for example beams in ring 2, for perfection beamforming the footprints shape should wedge-shaped. Of course, it is achievable only by an infinitely large array. Due to the limitation of aperture size and restriction of elements number, the footprint shapes are kidney-shaped. However, it can satisfy the requirements of isoflux illumination.

Figure 8 shows the gain of beams along $U = 0^{\circ}$ of the satellite coverage. Path loss due to the slant range differences from satellite to earth is considered. From the slice figure we can see that gain is higher than 13 dBi, and ripples lower than 3 dB.



Figure 7: $-3 \, dB$ contour of each beam.



5. Conclusion

A complex coded GA based method is discussed for the synthesis of planar arrays with arbitrary geometry that generate footprints of arbitrary shape. This approach is capable of synthesizing quite complex shapes of 3D patterns for main lobe and can realize good sidelobe suppression at the same time. The method has been proved to be useful for the synthesis of large array antennas whose elements are located on irregular grids.

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Space-filling Patch Antennas with CPW Feed

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Abstract—In this paper, the performances of some space-filling monopole antennas with coplanar waveguide have been investigated. It may be contended that the bends and corners of these geometries would add to the radiation efficiency of the antenna, thereby improving its gain. Advantage of these configurations is that they lead to multiband conformal antennas. A new version of Gosper curve patch antenna is introduced and its performance has been compared to conventional Gosper curve patch antenna.

1. Introduction

Fractal shaped antennas exhibit some interesting features that stem from their inherent geometrical properties. The self-similarity of certain fractal structures results in a multiband behavior of corresponding fractal antennas and frequency-selective surfaces (FSS) [1, 2].

On the other hand, the high convoluted shape and space-filling properties of certain fractals allow reducing the volume occupied by a resonant element. Theses properties are useful in designing multiband antennas and FSS, and reducing the size of certain antennas.





Figure 1: 4th order of Hilbert patch antenna with Figure 2: Lozenge shape patch antenna with CPW monopole feed.



Figure 3: CPW-fed Gosper curve patch antenna.



Figure 4: Another version of



Figure 5: Circular disc CPW-fed monopole antenna.

Microstrip patch Antennas are very popular in many fields as they are low-profile, low weight, robust and cheap. In last years new techniques employing fractal geometries are studied and developed. One of them is the fractalizing of antenna's boundary where new qualitative effect as the higher localized modes appear, that result in directive radiation patterns [3]. Another technique that has been studied in this paper is using space-filling curves as patch radiator.

Gosper curve patch antenna.

Space-filling curves map the multi-dimensional space into the one-dimensional space. A space-filling curve acts like a thread that passes through every cell element (or pixel) in the n-dimensional space so that every cell is visited only once. Therefore, the space-filling curve does not self-intersect. Thus, a space-filling curve imposes a linear order of the cells in the n-dimensional space. These geometries have the following properties: Self-Avoidance (as the line segments do not intersect each other), Simplicity (since the curve can be drawn with a single stroke of a pen) and self-similarity [4].



Figure 6: Return loss vs. frequency of antenna configuration in Fig. 1.



Figure 8: Return loss vs. frequency of antenna configuration in Fig. 3.





Figure 9: Return loss vs. frequency of antenna configuration in Fig. 4.



Figure 10: Return loss vs. frequency of antenna configuration in Fig. 5.

There are many types of space-filling curves (SFCs), e.g., the Peano, Hilbert, and Gosper curves, to name a few. They differ from each other in the way they visit and cover the points in space.



Figure 11: Maximum total field gain and total gain normal to antenna plane vs. frequency of Hilbert curve and lozenge shape antennas (left to right).



Figure 12: Maximum total field gain and total gain normal to antenna plane vs. frequency of antenna configuration in Fig. 3 and Fig. 4 (left to right).

In other hand, coplanar waveguide feed is a well-known technique for increasing the bandwidth of patch antennas [5].



Figure 13: Maximum total field gain and total gain normal to antenna plane vs. frequency of circular monopole antenna (Fig. 5).

In this paper, this technique has been imposed on some types of fractal space-filling monopole antennas such as Hilbert curve antenna and Gosper curve antenna.



Figure 14: Elevation pattern gain display of Hilbert curve CPW-fed monopole antenna and lozenge shape CPW-fed monopole antenna in 9 GHZ.



Figure 15: Elevation pattern gain display of two versions of Gosper curve CPW-fed monopole antenna (Fig. 3 and Fig. 4) in 9 GHZ (left to right).

2. Proposed Antenna Configurations

Schematic of studied structures are shown in Figs. 1–5. These configurations are in a single layer metallic structure. Hilbert curve and Gosper curve radiators are fed through coplanar wave guide monopole feed. For comparison the Euclidean counterparts of these structures have been studied.

Each section of Ground plane has the dimension of $6 \text{ cm} \times 4 \text{ cm}$, the width of microstrip feed in every configuration is 1.45 mm while the gap between the strip and coplanar ground plane is 0.1 mm.

The overall height of each space filling curve is assumed to be about 10 cm.

3. Simulation Results

Simulation of the above structures has been done using IE3D MOM-based code. In Figs. 6–10, Return losses versus frequency of these antennas are shown.

Simulation results show that space-filling patch antennas are conformal multiband antennas.

Making a direct relationship between antenna characteristics and geometrical properties of inscribed geometries is not easy. However we can say the results of return loss versus frequency of theses structures show that in same overall dimensions, the space-filling CPW-fed monopole antennas have better performance in input


Figure 16: Elevation pattern gain display of circular disc CPW-fed monopole antenna (Fig. 5) in 9 GHz.

matching characteristics, number of resonant frequencies and bandwidth than their Euclidian counterparts. For instance compare the results of return loss versus frequency of Hilbert curve CPW-fed monopole antenna and lozenge shape CPW-fed monopole antenna (Fig. 6 and Fig. 7) and the results of two versions of Gosper curve CPW-fed monopole antenna and circular disc CPW-fed monopole antenna (Fig. 8 and Fig. 9 with Fig. 10).

Main resonant frequencies of Hilbert curve CPW-fed monopole antenna and lozenge shape CPW-fed monopole antenna are very close together. This can be seen about two version of Gosper curve CPW-fed monopole antenna and circular disc CPW-fed monopole antenna.

The results of maximum total field gain vs. frequency and total gain normal to antenna plane (Z-direction) vs. frequency of these structures are shown in Figs. 11–13. From these results we can see that the space-filling CPW-fed monopole antennas have better gains in the direction perpendicular to antenna plane.

In Figs. 14–16, elevation pattern gain displays of these structures in 9 GHZ are shown. In this frequency all configurations have relatively good input matching characteristics. According to the fact that there is no ground plane except CPW ground plane, elevation pattern display in each structure is bilateral.

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The Power Line Transmission Characteristics for an OFDM Signal

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Abstract—In this paper, we measured what influence the sinusoidal transmission characteristics of the electric power line with various forms gave to the transmission characteristic of OFDM (Orthogonal Frequency Division Multiplexing) signal through PLC (power line communication system) modem. The electric power line transmission line with various forms in a real environment is classified into two basic elements, which are an outlet type branch and a switch type branch. Next, PHY rate (Physical rate) is measured for each basic element connected with the PLC modem. At this time, the transmission characteristics of the electric power line are simulated from measured data. OFDM sending and receiving systems are composed on the computer, and the PHY rate is simulated. By comparing with measured and calculated values, it is revealed that PHY rate of PLC modem is most affected in the case of the power line transmission characteristics having broad band and high level attenuation and is not affected in the case of that having narrow band group delay variation.

1. Introduction

Recently, Internet users are increasing by the rapid spread of the Internet and the Internet user needs the broadband that are high-rates and inexpensive communication service. While the access networks such as FTTH (Fiber To The Home) and xDSL (Digital Subscriber Line) spread, PLC attracts attention as the Internet connectivity from each room, and a home network, which controls household-electric-appliances. In PLC, since a communication network can be realized by using the existing power line, it is not necessary to install a transmission cable in addition, and a convenience outlet serves as a connection port of network at a general home, and office and factory. Moreover, since network connection and an electric power supply can be made together, we can use PLC like Wireless LAN [1–3].

However, power lines differ from telephone lines in that they are bus-type wiring and a great variety of device are connected to them. Thus, the impedance, transmission line loss and noise level of power line fluctuate greatly according to how the devices are connected and their operating conditions. To realize stable highspeed communication even under such circumstances requires the use of technology that is employed in wireless communication, such as OFDM [4,5]. In OFDM method, it is excellent in the efficiency of the frequency use, because a lot of sub-careers are orthogonal in the frequency domain. Therefore, a lot of sub-careers can be used, and it is possible to follow to the transmission line characteristic flexibly. Moreover, in the OFDM method, when the electromagnetic wave from PLC influences other existing systems, it is possible not to use the career frequency of this band or it can be set to lower the sending level. From such a feature, the adoption of the OFDM method is a mainstream in the PLC modem [4–6].

In such a background, a real environmental test with PLC modem using the OFDM method is progressed. And, there is a report concerning the electromagnetic compatibility technology [6–9]. There are various examinations for the electromagnetic radiation characteristic and quantification method [9–12]. On the contrary, as one of the concerns for which the user uses PLC, the communication should be high-quality and be seamless in wiring in the home. If maximum 200 Mbps is achieved in the PLC modem under development without trouble, it is possible to adjust to a large data transfer of the personal computer peripherals in recent years. When paying attention to such a viewpoint, there are a lot of uncertain parts what influence the characteristic of a complex electric power line gives to the OFDM signal. In the past, the influence of the transmission characteristic of the electric power line has been verified by real environmental experiment. Therefore, the example of the quantitative examination is few.

In this paper, in such a background, we first measured the transmission characteristic of the electric power line with branch [11]. The electric power line transmission line with various forms in a real environment is classified into two basic elements, which are an outlet type branch and a switch type branch. Next, PHY rate (Physical

rate) is measured for each basic element connected with the PLC modem. OFDM sending and receiving systems are composed on the computer, and the PHY rate is simulated. By comparing with measured and calculated values, we try to make clear what influence the sinusoidal transmission characteristics of the electric power line with branch gave to the transmission characteristic of OFDM (Orthogonal Frequency Division Multiplexing) signal through PLC modem.

2. Measurement of Electric Power Line Transmission Characteristic to Sinusoidal Wave Signal

2.1. Basic Elements of Power Line Transmission Model

When measuring the transmission characteristic of the power line with PLC modem, it is necessary to use shield room. However, since we have only G-TEM (Giga-hertz Transverse Electro-Magnetic) Cell, we composed the concise electric power line transmission line model as shown in Figure 1. The VVF (Vinyl insulated and Vinyl sheathed Flat type) cable with a conductor diameter of 1.6 mm is used for the power line. The electric power line transmission line with various forms in a real environment is classified into basic elements as shown in Figure 1, which are important elements influencing the power line transmission characteristics.



Figure 1: Power line transmission model.

- (a) No branch: The total length is 110 cm.
- (b) An outlet type branch: The main cable length is 110 cm. The branch cable with length of 10 cm or 160 cm is branched at the middle point of the main cable (55 cm). The terminal form of the branch cable is open or short.
- (c) A switch type branch that is used in a power line for lamp: Cable configuration is almost the same as the outlet type branch, but the branch cable is connected only one line of main cable. Terminal condition of branch cable is on or off.

2.2. Measurement of Transmission Loss Characteristics

Figure 2 shows the measurement system of the transmission loss characteristic for the basic element as shown in Figure 1 by using a network analyzer. The electric power line composing the basic element is transmission line with balance type, but a coaxial cable from the network analyzer is transmission line with unbalance type. Therefore, we used a balun at the connected point of the electric power line and the coaxial cable. The



Figure 2: Measurement system of transmission loss characteristic.



measurement frequency is from 1 MHz to 100 MHz corresponding to the assurance frequency of balun. Figures 3 and 4 show the measurement results of transmission loss characteristics for the basic element. The

Figure 3: Transmission loss characteristics of power line (Outlet type branch).



Figure 4: Transmission loss characteristics of power line (Switch type branch).

insertion loss of balun is subtracted from the measurement value of the transmission loss by using normalizing function of the network analyzer. First, the characteristic for "outlet type branch-open" in Figure 3(a) has approached the characteristic for "no branch" by decreasing in branch length. On the other hand, sharp attenuation like the resonance appeared on the low frequency band by increasing in branch length. Especially, the transmission loss reaches up to 20 dB around the frequency of 24 MHz when the branch length is 160 cm. This band is used with the PLC modem. Next, the transmission loss for "outlet type branch-short" in Figure 3(b) becomes very large on the low frequency band according to decreasing in branch length. The transmission loss at the branch length of 10 cm became 25 dB at the frequency of 5 MHz, but the transmission loss is improved as the frequency become higher. In addition, the transmission loss for "switch type branch-off" in Figure 4(a) is very large in all frequency band, the maximum transmission loss reaches 20 dB around the frequency of 50 MHz when the branch length is 10 cm, because one of two lines is disconnected. But, transmission loss is improved according to increasing branch length. The transmission loss is improved according to increasing branch length. The transmission loss for "switch type branch-off" in Figure 4(a) is very large in all frequency band, the maximum transmission loss reaches 20 dB around the frequency of 50 MHz when the branch length is 10 cm, because one of two lines is disconnected. But, transmission loss is improved according to increasing branch length. The transmission loss characteristics for "switch type branch-off" in Figure 4(b) are similar to that for "outlet type branch-open" in Figure 3(a) in the all frequency band.

Though this model as shown in Figures 1 is small-scale, the basic characteristics of an electric power line can be measured. Therefore, it is thought that this model is applicable as an electric power line model in the transmission measurement using the PLC modem as shown in paragraph 3.

2.3. Measurement of Group Delay Characteristic

The group delay can be calculated by the following expressions.

$$\Delta t[s] = -\frac{d\phi}{df}$$

The group delay reaches a constant value if a transmission media has a linear characteristic. Oppositely, the group delay increases if the transmission media has the nonlinearity. As an influence of the group delay to the transmission characteristic, it is considered that the guard interval length on the transmission system using OFDM is affected by the group delay. Therefore, it is very important to understand the amount of the change of the group delay, and evaluate the transmission characteristic from such a viewpoint.

Figures 5 and 6 show the measurement result of the group delay characteristics. In the case of "outlet type



Figure 5: Group delay characteristics of power line (Outlet type branch).



Figure 6: Group delay characteristics of power line (Switch type branch).

branch 160 cm open" as shown in Figure 5(a), the group delay changes sharply at the frequencies of 24 MHz and 75 MHz, which correspond the frequency occurring sharp attenuations like resonance in transmission loss characteristic for the same branch condition as shown in Figure 3(a). On the other hand, in the case of "outlet type branch 10 cm short" as shown in Figure 5(b), the group delay around low frequency band does not change in spite of large transmission loss around the same frequency band as shown in Figure 3(b). And, there is a similar tendency in the case of "switch type branch 10 cm off" as shown in Figure 4(a). In this case, the transmission loss is large at all frequency band, but the group delay does not change as shown in Figure 6(a). It is clear from these results that the group delay characteristic does not relate the amount of the transmission loss, but the change of the transmission loss.

3. Transmission Characteristics Measurement System of OFDM Signal Using PLC Modem

3.1. Measuring Method of Transmission Characteristic with Modem

We measured the transmission characteristic for the OFDM signal using PLC modem made of Sumitomo Electric Industries, LTD. Figure 7 shows the measurement system. Sending and receiving PCs is connected through the PLC modem and each model of the transmission line as shown in Figure 1. The bandwidth of the PLC modem is from 4 to 34 MHz. Next, the communication link between sending and receiving PCs through

the modem is established and measured PHY rate (Physical rate). The measurement system set in a G-TEM (Giga-Hertz Transverse Electromagnetic) cell in order to suppress power supply coupling between sending and receiving modems. Normally, PLC modem is connected to AC power line, but in this case, the measurement system connected to DC power line as shown in Figure 7 is provided to suppress the power supply coupling. We considered on a grand side, and each electric power line in the GTEM Cell is set above 10 cm high from a metal floor of the GTEM cell, in order to suppress the influence of the metal floor.



Figure 7: Transmission characteristics measurement system of OFDM signal using PLC modem.

3.2. Method of Simulating Transmission Characteristics Using PLC Modem

First, we modeled the PLC modem with the computer software and composed the OFDM sending and receiving system and simulated the transmission characteristics of the OFDM signal, that is, PHY rate for the electric power line as shown in Figure 1. Figure 8(a) shows the simulation block chart of the OFDM sending and receiving system and, Figure 8(b) shows that of the mock electric power transmission line. It is assumed that the sending and receiving systems are composed by the OFDM system based on a general FFT (Fast



Figure 8: Simulation model of transmission characteristics for OFDM signal using PLC modem.

Fourier Transform) [13, 14]. First, the input random binary signal is converted into the frame data in the S/P (Serial/Parallel) block, and converts into a multilevel symbol by QAM (Quadrature Amplitude Modulation) in the Mapping block for each sub-career. Next, Orthogonal transform is processed in the IFFT (Inverse Fast Fourier Transform) block. As a result, base-band OFDM signal is generated, and converted into the pass-band signal by the Up-Conversion block. Here, the Channel block consists of Figure 8(b), and the composition is as follows. Wideband and constant signal attenuation are imitated in the ATT (attenuator) block. In the Digital Filter block, each electric power line is composed by using the measurement data of the transmission loss and the group delay characteristics. Moreover, the thermal noise of the equipment is imitated by the AWGN (Additive White Gaussian Noise) block. On the other hand, the receiving system is reversely converted about the sending system. The receiving signal is equalized by the Channel Estimator block. The equalization method is division of the complex number that uses the pilot-careers. Finally, the receiving binary data is compared with the sending binary data and BER (Bit Err Rate) is calculated. PHY rate is calculated from the receiving bits and the sample rate. Receiving bits are the subtraction of the error bits from all sending bits. For the simulation parameter, we calculated by using the simplified model compared with an actual modem because each parameter used for the actual modem is not obtained as public information. Therefore, the calculation of PHY rate is adjusted to measurement data when using the parameter as shown in Table 1.

number of sub-careers	1400
first modulation	1024QAM $=10$ bits
use band	$4\mathrm{M}{\sim}34\mathrm{MHz}$
sub-career interval	$43\mathrm{kHz}$
maximum PHY rate	$186\mathrm{Mbps}$
symbol length	$23\mu{ m s}$
guard interval length	$360\mathrm{ns}$
AWGN	S/N=50dB

Table 1: Simulation parameter for OFDM signal using PLC modem.

3.3. Measurement and Calculation Results of Transmission Characteristic Using PLC Modem

Table 2 shows the results of PHY rate corresponding to each basic element as well as their termination conditions, and the calculated values agree well with the measured values. It became clear that basic element (b) with termination condition of "10 cm short" influences most for the PHY rate and basic element (c) with termination condition of "160 cm on" influences next. It is thought that the transmission loss was the largest in the use band of the modem. Therefore, it is clear that the transmission loss is main factor for decreasing PHY rate. On the other hand, when comparing "outlet type branch 160 cm open" with "switch type branch 160 cm on", PHY rate for "switch type branch 160 cm on" was lower. Oppositely, variable quantities of the group delay characteristic for "outlet type branch 160 cm" were larger. Therefore, it is considered that PHY rate is hardly influenced if the amount of the group delay is below guard interval length. In fact, PHY rate of PLC modem is most affected in the case of the power line transmission characteristics having broad band and high level attenuation.

Table 2: Measured and calculated results of PHY rate for PLC modem.

	PHY rate[Mbps]		
basic element and termination condition	meas.	calc.	
(a) no branch	183	186	
(b) outlet type branch 10 cm open	181	186	
(b) outlet type branch 10 cm short	166	170	
(b) outlet type branch 160 cm open	178	180	
(b) outlet type branch 160 cm short	181	186	
(c) switch type branch 10 cm off	180	185	
(c) switch type branch $10 \mathrm{cm}$ on	181	186	
(c) switch type branch $160\mathrm{cm}$ off	183	186	
(c) switch type branch 160cm on	168	171	

4. Conclusion

In this paper, we measured and calculated what influence the transmission characteristics of the electric power line with basic element gave to the transmission characteristic of OFDM signal through PLC modem. The following items are clear by comparing with the measurement and the calculation values:

(a) The electric power line transmission line with various forms in a real environment is classified into two basic elements, which are an outlet type branch and a switch type branch. Therefore, even if a small-scale

electric power line model is used, it is able to measure the characteristic of large-scale and complicated electric power line.

(b) PHY rate of PLC modem is most affected in the case of the power line transmission characteristics having broad band and high level attenuation and is not affected in the case of that having narrow band group delay variation.

In future, it is necessary to examine transmission characteristic for OFDM signal by using a complex transmission line model.

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Relation Between Balance-unbalance Conversion Factor and Leaked Electric Field in Power Line with Branch for PLC

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Abstract—In this paper, we calculated balance-unbalance conversion factor and leaked electric field in the power line with branch in high frequency. We paid attention to two typical branch such as outlet branch and lamp switch branch about the branch of the electricity distribution lines, and calculated the model which is combination of those divergences. To verify the validity of the calculation, we measured the model similar to the calculation, and compared with calculation results. As a result, the measurement and calculation values were approximated well both balance-unbalance conversion factor and leaked electric field, It was shown that the calculation model in the method of moments was effective to the analysis of the power line communication. And from the comparison of the calculation results, it was shown to have good correlation between balance-unbalance conversion factor and leaked electric field.

1. Introduction

The number of Internet users has recently seen explosive growth and demand for home LAN has increased, too. The means that the network can be easily constructed domestically is demanded. As one of the some solutions, there is a high-speed power line communication (PLC). PLC transmits data by using the conventional power line laid to supply commercial electric power both inside and outside the home in stead of telecommunication line. The practical application of PLC as realization approach of Home LAN is strongly hoped with the transmission speed improving in recent years. But there is a possibility that the leaked electromagnetic wave influences a radio service of high frequency band (3–30 MHz) by using PLC, because PLC uses this high frequency band. However, there is movement to develop the leaked electric field decreasing technology and to aim for realization of PLC, because the convenience of PLC is very large. From such a background, the Ministry of Internal Affairs and Communications (MIC) inaugurated the "Study Group for Power Line Communication Facilities" in April, 2002 [1]. And, High Speed Power Line Communication Promoters' Alliance of Japan (PLC-J) was established by the Japanese electric power company and the manufacturer, and they are doing various examinations of the leaked electric field decreasing technology. Now, various experiments are conducted [2], but it is difficult to verify all cases by experimental examination. Therefore, it is necessary to study a computer calculation method that can imitate the experiment.

In an experimental examination, the leaked electric field emitted from power lines has been analyzed quantitatively by using the degree of unbalance to ground, such as Longitudinal Conversion Loss (LCL) and common mode current, as an index [3]. Thus, the simulation method that can calculate LCL, common mode current, and leaked electric field is needed. There is a method of using four port network theory [4] about the calculation method, but we focused attention on method of moments (MoM) that was one of the electromagnetic field analysis method. We thought that the calculation was easily possible in a large-scale system, so that the MoM has the feature with comparatively short calculation time.

Based on above, in this paper, we report on the result of the calculation of LCL, the common mode current distribution, and the leaked electric field in the electric power line using the MoM. And, to verify the effectiveness of the calculation, we measured the characteristics of power line system with branch under PLC-J cooperation [5]. By comparing the measurement results with the calculation one, we will clarify the effectiveness of the calculation method as well as the relation between the balance-unbalance conversion factor and the leaked electric field.

2. Power Line Model

A general domestic power line has diverged variously, but it is possible to classify it into an outlet branch and a lamp switch branch by dividing a complex divergence of each element. In the outlet branch, both two lines of a pair line composing power line is diverged to make the outlet branch as shown in Figure 1(a), but in the lamp switch branch, only one line of the pair line is lengthened to make an ON / OFF switch as shown in Figure 1(b). In this paper, we paid attention to element of branch such as the outlet branch and the lamp switch branch, and "1 branch model" connected only the outlet branch line and "2 branch model" connected both the outlet branch line and the lamp switch branch line are measured and calculated respectively.

Figure 2 shows configuration of the 2 branch model, and size of the 2 branch model is shown in Figure 3. The outlet branch line of 5.6 m in total length and the lamp switch branch line of 4.0 m in total length are connected to the backbone line like a gate form. On the other hand, in the 1 branch model, the lamp switch branch line as shown in Figure 2 is not connected. The VVF (Vinyl insulated and Vinyl sheathed Flat type) cable with a conductor diameter of 1.6 mm is used for power line. And the size of this model reduces twice from the size of actual power line system, because an actual power line system is large-scale and then it is difficult to construct the power line model in a measurement site for radiated emission. To imitate the lamp, a terminal resistor of 100 Ω is connected in the lamp switch branch line. We connected the line of 1 m in length as a switch line, and connected the switch as the terminal. There is a possibility that ON / OFF of this switch influences the characteristic of the power line. Thus, we examined the changing characteristics to switching condition of ON/OFF. Each balun terminal has lengthened line to 40 cm in the height of 20 cm as shown in Figure 4, based on regulations of the line terminal in CISPR, and this size is also reduced to half.

In order to imitate the influence of grounding one wire of the pair line in pole transformer applied to Japanese power line, we grounded one wire of the pair line in balun on opposite side of the signal impression, that is, far end side from a domestic outlet. It is a factor that one of the pair line grounding lowers the balance-unbalance conversion factor of the power line. But, the structure to extend one wire of the pair line for the switch line is also cause of unbalance in the power line. We studied the model on the side where grounded line and switch line were the same in measurement and calculation, because the combination of the same sides acts additively to the characteristic of power line.



Figure 2: Configuration of the 2 branch model.

3. Measurement Method

3.1. LCL Measurement Method

In this paper, we paid attention to the value of balance-unbalance conversion factor, common mode current, and leaked electric field as EMC characteristics of the PLC, and measured these values and compared it with the calculation value. First, we measured LCL that was the ratio of the differential mode voltage and the common mode voltage in the input side as balance-unbalance conversion factor. LCL is defined by the following formula.

$$LCL = 20\log_{10}\frac{V_{Cin}}{Din} \quad [dB] \tag{1}$$



Figure 3: Size of the 2 branch model.



Figure 4: Configuration of power line terminal.

Here, V_{Cin} is a common mode voltage in the input side, and V_{Din} is differential mode voltage in the input side. LCL was obtained by measuring the voltage appeared in the differential mode port when impressing the signal to common mode port of the balun, by using a network analyzer as shown in Figure 5.



Figure 5: LCL measurement method.



Figure 6: Common mode current measurement method.

3.2. Common Mode Current Measurement Method

We impressed the no modulated sine wave with power of 11 dBm from the signal generator to differential mode port of the balun, and measured the common mode current by using the current probe and spectrum analyzer synchronized the signal generator. Figure 6 shows the common mode current measurement system. The current distribution was obtained by measuring the current at intervals of 50 cm from signal input side balun.

3.3. Leaked Electric Field Measurement Method

In order to measure the leaked electric field from the power line, the power line model as shown in Figure 2 is set up on a large-scale turntable with radius of 5.0 m, and the no modulated sine wave signal of 11 dBm is impressed from the signal generator to differential mode port of balun. And we measured an electric field strength by a loop antenna, which is set at the point of 12 m from the center of turntable that arranged the power line model. We measured the electric field strength to horizontal, vertical and radial polarizations of loop antenna, and added all of three polarizations. The height of the antenna is 1.0 m. In the leaked electric field strength measurement, we rotated the turntable at intervals of 30 degrees. In frequency characteristics of the leaked electric field strength, the maximum value among many measured values obtained by rotating the turntable was adopted as the measured values at a frequency. Figure 7 shows the relation among arrangement of power line model and the receiving antenna position and the rotation angle of turntable.

4. Calculation Method

4.1. Calculation Model

The measurement system consisted of the power line as shown in Figure 2 was converted to an equivalent circuit of a differential transmission line to construct the calculation model by MoM as shown in Figure 8, and calculated each characteristics with MoM by using NEC2 as a calculation software [6]. Shape and size of the calculation model are completely equal to the measurement model. And we defined the model of the far end grounding by connecting the ground line of the impedance 0Ω at far end side, because it is necessary to consider one wire grounding of pair line in the calculation model [7].

4.2. LCL Calculation Method

The electric field and magnetic field can be calculated by MoM as well as the current distribution along conductive wire, but the balance-unbalance conversion factor (for example LCL) cannot be calculated directly by



Figure 7: Configuration of power line model and receiving antenna on leaked electric field measurement systems.



Figure 8: Calculation model by MoM.

Figure 9: LCL calculation method by MoM.

MoM. However, as discussed previously, the method of simulation that can calculate LCL is demanded, because LCL is an important index of the characteristic of the power line. Thus, we devise a method of calculating LCL by the MoM. The method is shown here. First, to imitate the common mode input, we impressed the in-phase voltage at the feeding point of two wires as shown in Figure 8, because LCL is a conversion ratio from the common mode signal to the differential mode signal at the input side. When grounding one wire of pair line at the far end, the current phase in an in-phase signal is greatly influenced by the grounding. Therefore, the difference of complex current appears in both lines on the segment of input side, and the subtracted value does not become 0, when one wire of the pair line is grounded on far end. This current can be considered as a differential mode current, and differential mode voltage (V_{Din}) is obtained by multiplying this differential mode current and differential mode impedance. LCL can be calculated from the ratio of V_{Din} and voltage (V_{Cin}) developed by one-quarter impedance of differential mode impedance.

4.3. Common Mode Current Calculation Method

We calculated the common mode current distribution by the method similar to the calculation of LCL, but, feeding power to each wire is impressed in reversed phase. A complex current is added to each segment at the same position of both lines, in order to calculate the common mode current distribution.

4.4. Leaked Electric Field Calculation Method

We calculated the leaked electric field at the point of 12 m from the center of power line model and 1m in height. The maximum value obtained by sweeping the angle at intervals of 30 degrees like measurement is adopted as a calculated value in calculation frequency range from 1 MHz to 30 MHz, according to the frequency range measurable by the loop antenna. And we calculated the magnetic field and converted it into the electric field by multiplying magnetic field and free space impedance.

5. Measurement and Calculation Results

5.1. LCL Calculation Result

Figure 10 shows LCL measurement and calculation results for the 1 branch model, and Figure 11 shows LCL for the 2 branch model. In Figure 10, the point is measurement value, and the solid line is calculation



Figure 10: LCL for the 1 branch model.

Figure 11: LCL for the 2 branch model.

value. In Figure 11, Δ is measurement value for turning on switch (abbreviated as SW-ON), O is the one for turning off switch (abbreviated as SW-OFF). And the solid line is calculation value for SW-ON, the dotted line is the one for SW-OFF. From these figures, it is understood that average LCL for the 1 branch model is about 28 dB and the one for the 2 branch model is about 30 dB. In addition, it is also clear that LCL is not so affected by switching condition, but the frequency appearing hump changes a little. From the comparison of these values, measurement and calculation values are almost corresponding though the hump position is different. The calculation value shifts the hump position to the high frequency side, and this cause is considered that dielectric constant of shielding material in each wire of the power line cannot be defined in the method of moment. Though the difference has extended in the 2 branch model, this cause is thought that the equivalent circuits for the switch and the lamp are not imitated accurately. However, it can be considered that the LCL calculation by MoM is effective, because measurement and calculation values are a similar tendency.

5.2. Common Mode Current Calculation Result

Figure 12 shows measurement and calculation values for the common mode current distribution along the backbone line in 1 branch model at frequencies of 1 MHz, 20 MHz, 40 MHz and 60 MHz. Figure 13 shows the current distribution along the outlet branch line in 1 branch model, and Figure 14 shows that along the backbone line in 2 branch model (SW-ON). The result in other cases was omitted, because the results were almost similar to that mentioned above.



Figure 12: Common mode current for 1 branch model (backbone line).

Figure 13: Common mode current for 1 branch model (outlet branch line).

From the current distribution along the backbone line shown in Figs. 12 and 13, in both the measurement and the calculation, the current has changed suddenly in 5.0 m. This position is the connecting point to each branch line and backbone line. Thus, it is understood that the common mode current flowing along the backbone line flows greatly to the line where common mode impedance is lower in the position of branch. From comparing the results, calculation values at the position of antinodes and nodes on the standing wave agree with measurement values, and the level of the common mode current is also almost equal to each other. Therefore, it was confirmed that the calculation of common mode current distribution by MoM was effective.



Figure 14: Common mode current for 2 branch model (backbone line).

Figure 15: Leaked electric field for 1 branch model.

5.3. Leaked Electric Field Calculation Result

Figure 15 shows the value of leaked electric field for the 1 branch model. The leaked electric field for the 2 branch model (SW-ON) is shown in Figure 16, and that for the 2 branch model (SW-OFF) in Figure 17. A thin solid line in figures is ambient noise such as the broadcasting waves and noise level of the interference wave for the measuring instrument on an open area site. From those Figs. 15, 16, and 17, tendencies of the calculated frequency characteristics for the electric field strength are almost equal to measured ones. It is considered that the influence of dielectric material for power line appears as mentioned already for being a difference in the hump position, but it was revealed that the calculation for the leaked electric field by MoM was effective, because measurement and calculation values are a similar tendency.

5.4. Relation between Balance-unbalance Conversion Factor and Leaked Electric Field

Figure 18 shows a relation between balance-unbalance conversion factor and leaked electric field for the 1 branch model, and Figure 19 shows the relation for the 2 branch model. From comparing the relation between LCL and leaked electric field in these figures, if LCL reaches a low value, the leaked electric field becomes a high value, regardless of the switching condition. Conversely, if LCL reaches a high value, the leaked electric field becomes a low value. Thus, it is confirmed that there is relation of inverse proportion between LCL and leaked electric field. In addition, it is thought that such relation maintain regardless of the difference of the branch form and the switching condition.



Figure 16: Leaked electric field for 2 branch model (SW-ON).

Figure 17: Leaked electric field for 2 branch model (SW-OFF).



Figure 18: Relation (LCL-E-field) for 1 branch model.



Figure 19: Relation (LCL-E-field) for 2 branch model.

6. Conclusion

In this paper, we focused attention on a power line communication using in high frequency band, and calculated balance-unbalance conversion factor and leaked electric field in the power line with branch by MoM. In order to confirm the validity of the calculation, we measured balance-unbalance conversion factor and leaked electric field for the same model as the calculation model, and compared the measurement value with the calculation value. As a result, the calculation value agreed well with measurement value, and then it was revealed that the calculation model of the PLC using the MoM was effective. In addition, it is confirmed that the influence of grounding one wire of the pair line in pole transformer applied to Japanese power line is small to LCL of the power line in the home, because LCL of the power line including each branch model is about 30 dB. Moreover, it was clear that there was a good correlation between LCL and the leaked electric field. The future task is expanding this branch model greatly, and applying it to power line model in the home.

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On the Application of the Radiative Transfer Approach to Scattering from a Random Medium Layer with Rough Boundaries

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Abstract—For studying the problem of scattering from a random medium layer with rough boundaries the radiative transfer (RT) approach is widely used. In order to better understand this procedure we compared it with the statistical wave approach. Two such wave approaches are presented in this paper: the surface scattering operator (SSO) approach, and the unified approach. In both wave approaches two conditions are essential for arriving at RT system: the ladder approximation to the intensity operator, and the quasi-stationary approximation of fields. With these approximations one arrives at the integro-differential equations of the RT system. However, to arrive the at the RT boundary conditions, one has to impose further approximations. In the SSO approach weak surface correlation must be imposed. In the unified approach, one has to ignore the terms involving volumetric spectral densities, and consider only single scattering from the rough boundary when deriving the boundary conditions.

1. Introduction

The analysis of scattering from a random medium layer with rough boundaries is a difficult problem. This is the kind of problem one often encounters in remote sensing applications. People have used the phenomenological radiative transfer approach to study this problem (Ulaby et al., 1986; Lam and Ishimaru, 1993; Shin and Kong, 1989). This approach is conceptually simple and yet very effective for studying multiple scattering processes. Here one uses the transport equations corresponding to the random medium of the layer and then one imposes the relevant boundary conditions. Although this procedure appears to be heuristically sound it is not clear what approximations are involved, and under what conditions such a procedure may be used for the problem at hand. One way to better understand this radiative transfer approach is to compare and relate it to the statistical wave approach. For the case of unbounded random media it has been demonstrated how the ladder approximated Bethe-Salpeter equation reduces to the radiative transport equation (Barabanenkov et al., 1971). We found that this procedure can be applied to the problem of random medium layer with planar boundaries and arrive at the radiative transport system as given in Ulaby et al., (1986). However, if the boundaries are statistically rough, the problem is considerably more complicated and we need special procedures to deal with them. We have employed two different statistical wave approaches for such problems. In the first approach we assume that we know the solution of the problem without the volumetric fluctuations. The second approach is based on the solution of the problem where all the fluctuations vanish. We shall compare the results of these two approaches with those of the radiative transfer (RT) approach. This will enable us to understand and meaning and import of the radiative transfer approach as applied to our problem. To keep discussions in a simple setting we will consider the scalar problem and keep the lower boundary alone as rough.

2. Geometry of the Problem

The geometry of the problem consists of a random medium layer with a rough bottom boundary. The permittivity of the layer medium consists of a deterministic part ϵ_2 and randomly fluctuating part $\tilde{\epsilon}\epsilon_2$. z = 0 and $z = -d + \zeta(r_{\perp})$ describe the upper and lower boundary of the layer. We assume that $\tilde{\epsilon}$ and ζ are small and smooth zero-mean stationary processes independent of each other. The medium above the layer is homogeneous, and we impose the Neumann boundary condition on the lower boundary. This layer is excited by a wave incident from above and we are interested in the scattered waves.

3. Radiative Transfer Approach

The classical equation of radiative transfer is given as

$$\hat{s} \cdot \nabla I(r, \hat{s}) + \eta I(r, \hat{s}) = \int d\Omega' P(\hat{s}, \hat{s}') I(r, \hat{s}')$$
(1)



Figure 1: Geometry of the problem.

where $P(\hat{s}, \hat{s}')$ is the phase function and η is the extinction coefficient. This equation was originally intended for unbounded scattering medium. However it can be applied to bounded medium with arbitrary geometry by imposing appropriate boundary conditions. For layer geometry we have the following set of coupled integrodifferential equations.

$$\cos\theta \frac{d}{dz}I_u(z,\Omega) + \eta I_u(z,\Omega) = I_u^c + \frac{|k|^4}{4\pi} \int d\Omega' \left\{ \Phi(\theta,\theta';\phi-\phi')I_u(z,\Omega') + \Phi(\theta,-\theta';\phi-\phi')I_d(z,\Omega') \right\}$$
(2)

$$\cos\theta \frac{d}{dz}I_d(z,\Omega) - \eta I_d(z,\Omega) = -I_d^c + \frac{|k|^4}{4\pi} \int d\Omega' \left\{ \Phi(-\theta,\theta';\phi-\phi')I_u(z,\Omega') + \Phi(-\theta,-\theta';\phi-\phi')I_d(z,\Omega') \right\} (3)$$

Eqs. (2) and (3) follow from (1) noting that the problem is translationally invariant in azimuth. I_u and I_d represent the incoherent part of radiant intensities corresponding to upward and downward travelling waves inside the layer. I_u^c and I_d^c represent the corresponding contributions due to coherent intensities. Φ represents the spectral density of the volumetric fluctuations. Eqs. (2) and (3) are solved using the following boundary conditions.

$$I_d(0,\Omega) = |R_{12}(\Omega)|^2 I_u(0,\Omega)$$
(4)

$$I_u(-d,\Omega) = \int d\Omega' \langle |R_{32}(\Omega,\Omega')|^2 \rangle I_d(-d,\Omega)$$
(5)

The extinction coefficient η is readily derived from the differential scattering cross section of the random medium. R_{12} is the reflection coefficient at the upper boundary for waves incident from below. R_{32} is the reflection coefficient at the lower boundary for waves incident from above. Thus we see that the formulation in the radiative transfer approach is simple and straight forward, and can be applied to a variety of different geometries. The fundamental quantity in this approach is the radiant intensity and hence is not suitable to represent wave phenomena such as diffraction, interference, etc. A more general approach to this problem is the statistical wave approach. In this paper we will describe two such approaches and compare them with that of radiative transfer.

4. Surface Scattering Operator Approach

We start with the following equations governing the Green's functions of the problem.

$$\Delta G_{12} + k_1^2 G_{12} = 0 \Delta G_{11} + k_1^2 G_{11} = -I \Delta G_{21} + k_2^2 G_{21} = -q G_{21} \Delta G_{22} + k_2^2 G_{22} = -I - q G_{22}$$

where $q = \omega^2 \mu \tilde{\epsilon} \epsilon_2$ represents the volumetric fluctuations. We write the above system as

$$LG = -I - QG \tag{7}$$

where $G \equiv \{G_{ij}\}, L = \text{diag}\{L_1, L_2\}, L_j = \triangle + k_j^2, Q = q \text{diag}\{0, 1\}$. For multiple scattering analysis it is convenient to convert (7) into the following integral equation.

$$G = \ddot{G} + \ddot{G}QG \tag{8}$$

(6)

where \tilde{G} is the Green's function of the problem without volumetric fluctuations. In principle, one can construct such Green's functions using surface scattering operators (Voronovich, 1994; Soubret et. al., 2002). First average (8) w.r.t. volumetric fluctuations.

$$\langle G_v \rangle \simeq \check{G} + \check{G} \langle Q \langle G \rangle_v Q \rangle \langle G \rangle_v \tag{9}$$

On operating this by L we obtain

$$L\langle G \rangle = -I - \langle Q \langle G \rangle_v Q \rangle \langle G \rangle_v \tag{10}$$

From this we find that

$$L_1 \langle G_{11} \rangle_v = -I \tag{11a}$$

$$L_2 \langle G_{22} \rangle_v = -I - \langle G_{22} \rangle_v \langle qq \rangle \langle G_{22} \rangle_v \tag{11b}$$

Next average (11) w.r.t. surface fluctuations

$$L_1 \langle G_{11} \rangle_{vs} = -I \tag{12a}$$

$$L_2 \langle G_{22} \rangle_{vs} = -I - \langle \langle G_{22} \rangle_v \langle qq \rangle \langle G_{22} \rangle_v \rangle_s \tag{12b}$$

We infer from (12a) that the mean propagation constant in Region 1 is unaffected by the fluctuations of the problem. To interpret (12b) we approximate $\langle \langle G_{22} \rangle_v \langle qq \rangle \langle G_{22} \rangle_v \rangle_s$ as $\langle G_{22} \rangle_{vs} \langle qq \rangle \langle G_{22} \rangle_{vs}$. As we shall see, this kind of approximation is essential for arriving at the RT system as given in the previous section. Thus

$$(\triangle + k_2^2) \langle G_{22} \rangle_{vs} = -I - \langle G_{22} \rangle_{vs} \langle qq \rangle \langle G_{22} \rangle_{vs}$$
(13)

This implies that

$$k_{2m}^2 = k_2^2 + \langle G_{22} \rangle_{vs} \langle qq \rangle \tag{14}$$

This is the operational definition for the mean propagation constant in the layer region. With this we can proceed to construct the mean Green's functions.

We next turn our attention to the second moments of the fields. Taking the tensor product of (8) with its complex conjugate and performing volumetric averaging leads to

$$\langle G \otimes G^* \rangle_v = \langle G \rangle_v \otimes \langle G \rangle_v^* \{ I + K \langle G \otimes G^* \rangle_v \}$$
(15)

where K is the intensity operator corresponding to the volumetric fluctuations. Hence the equation for field correlation is

$$\langle \psi \otimes \psi^* \rangle_v = \langle \psi \rangle_v \otimes \langle \psi \rangle_v^* + \langle G \rangle_v \otimes \langle G \rangle_v^* K \langle \psi \otimes \psi^* \rangle_v \tag{16}$$

Averaging this over surface fluctuations we have

$$\langle \psi \otimes \psi^* \rangle_{vs} = \langle \langle \psi \rangle_v \otimes \langle \psi \rangle_v^* \rangle_s + \langle \langle G \rangle_v \otimes \langle G \rangle_v^* K \langle \psi \otimes \psi^* \rangle_v \rangle_s \tag{17}$$

Now we employ the following two approximations essential for arriving at the radiative transfer system.

$$\langle\langle G\rangle \otimes \langle G\rangle_{v}^{*}K\langle\psi\otimes\psi^{*}\rangle_{v}\rangle_{s} \simeq \langle\langle G\rangle_{v} \otimes \langle G\rangle_{v}^{*}\rangle_{s}K\langle\psi\otimes\psi^{*}\rangle_{vs}$$
(18a)

$$K \simeq \langle Q \otimes Q^* \rangle \tag{18b}$$

The first is the weak surface correlation approximation. The second is called the ladder approximation. Thus we arrive at the following equation for the second moment of the fields inside the layer

$$\langle \psi_2 \otimes \psi_2^* \rangle_{vs} = \langle \langle \psi_2 \rangle_v \otimes \langle \psi_2 \rangle_v^* \rangle_s + \langle \langle G_{22} \rangle_v \otimes \langle G_{22} \rangle_v^* \rangle_s K \langle \psi_2 \otimes \psi_2^* \rangle_{vs}$$
(19)

Observe that $\psi_2 = \langle \psi_2 \rangle_{vs} + \tilde{\psi}$ and $\langle \psi_2 \rangle_v = \langle \psi_2 \rangle_{vs} + \langle \widetilde{\psi_2} \rangle_v$ where tilde is used to denote the fluctuating part. Using these relations in (19) we obtain

$$\langle \tilde{\psi}_2 \otimes \tilde{\psi}_2^* \rangle_{vs} = \langle \langle \tilde{\psi}_2 \rangle_v \otimes \langle \tilde{\psi}_2 \rangle_v^* \rangle_s + \langle \langle G_{22} \rangle \otimes \langle G_{22} \rangle_v^* \rangle_s \langle q \otimes q^* \rangle \langle \psi_2 \otimes \psi_2^* \rangle_{vs}$$
(20)

We next introduce Wigner transforms of the wave functions and the Green's functions in (20) and obtain

$$\tilde{\mathcal{E}}(z,k) = \tilde{\mathcal{E}}^s(z,k) + \frac{|k_2|^4}{(2\pi)^6} \int dz_1 \int d\alpha \int d\beta \mathcal{G}(z,k;z_1,\alpha) \Phi_v(\alpha-\beta) \mathcal{E}(z_1,\beta)$$
(21)

where $\tilde{\mathcal{E}}$, $\tilde{\mathcal{E}}^s$, \mathcal{E} and \mathcal{G} are the Wigner transforms of $\langle \tilde{\psi}_2 \otimes \tilde{\psi}_2^* \rangle_{vs}$, $\langle \langle \tilde{\psi}_2 \rangle_v \otimes \langle \tilde{\psi}_2 \rangle_v \rangle_s$, $\langle \psi_2 \otimes \psi_2^* \rangle_{vs}$ and $\langle \langle G_{22} \rangle_v \otimes \langle G_{22} \rangle_$

$$\mathcal{G} = \mathcal{G}^o + \mathcal{G}_{uu} + \mathcal{G}_{ud} + \mathcal{G}_{du} + \mathcal{G}_{dd} \tag{22}$$

 \mathcal{G}^{o} is the Wigner transform of $G^{o} \otimes G^{o*}$ where G^{o} is the singular part of $\langle G_{22} \rangle$. \mathcal{G}_{uu} is the Wigner transform corresponding to that part of $\langle G_{22} \rangle$ involving the surface scattering operator $\langle S_{uu} \rangle$ and so on. Using this decomposition we split (21) as follows.

$$\tilde{\mathcal{E}}^{u}(z,k) = \tilde{\mathcal{E}}^{su}(z,k) + \frac{|k_{2}|^{4}}{(2\pi)^{6}} \int_{-d}^{z} dz_{1} \int d\alpha \int d\beta \mathcal{G}^{>}(z,k;z_{1},\alpha) \Phi(\alpha-\beta) \mathcal{E}(z_{1},\beta) + \frac{|k_{2}|^{4}}{(2\pi)^{6}} \int_{-d}^{0} dz_{1} \int d\alpha \int d\beta \{\mathcal{G}^{uu} + \mathcal{G}^{ud}\}(z,k;z_{1},\alpha) \Phi(\alpha-\beta) \mathcal{E}(z_{1},\beta)$$
(23a)

$$\tilde{\mathcal{E}}^{d}(z,k) = \tilde{\mathcal{E}}^{sd}(z,k) + \frac{|k_{2}|^{4}}{(2\pi)^{6}} \int_{z}^{0} dz_{1} \int d\alpha \int d\beta \mathcal{G}^{<}(z,k;z_{1},\alpha) \Phi(\alpha-\beta) \mathcal{E}(z_{1},\beta) + \frac{|k_{2}|^{4}}{(2\pi)^{6}} \int_{-d}^{0} dz_{1} \int d\alpha \int d\beta \{\mathcal{G}^{du} + \mathcal{G}^{dd}\}(z,k;z_{1},\alpha) \Phi(\alpha-\beta) \mathcal{E}(z_{1},\beta)$$
(23b)

On using the expressions for \mathcal{G} 's the above pair of equations can be represented as the following integrodifferential transport equation system

$$[dz+2\eta'']\tilde{\mathcal{E}}_u(z,k_{\perp}) = \mathcal{E}_u^c + \frac{|k_2|^4}{16\pi^2|\eta|^2} \int d\alpha_{\perp} \{\Phi_v(k_{\perp}-\alpha_{\perp};\eta'-\eta'_{\alpha})\tilde{\mathcal{E}}_u(z,\alpha_{\perp}) + \Phi_v(k_{\perp}-\alpha_{\perp};\eta'+\eta'_{\alpha})\tilde{\mathcal{E}}_d(z,\alpha_{\perp})\}$$
(24a)

$$[dz - 2\eta'']\tilde{\mathcal{E}}_d(z,k_\perp) = -\mathcal{E}_d^c + \frac{|k_2|^4}{16\pi^2|\eta|^2} \int d\alpha_\perp \{\Phi_v(k_\perp - \alpha_\perp; -\eta' - \eta'_\alpha)\tilde{\mathcal{E}}_u(z,\alpha_\perp) + \Phi_v(k_\perp - \alpha_\perp; -\eta' + \eta'_\alpha)\tilde{\mathcal{E}}_d(z,\alpha_\perp)\} (24b)$$

Here \mathcal{E}_{u}^{c} and \mathcal{E}_{d}^{c} are the contributions due to coherent intensities. The associated boundary conditions are obtained as

$$\tilde{\mathcal{E}}_d(0,k_\perp) = |R_{12}(k_\perp)|^2 \tilde{\mathcal{E}}_u(0,k_\perp)$$
(25a)

$$\tilde{\mathcal{E}}_u(-d,k_\perp) = \langle |R_{32}(k_\perp,k'_\perp)|^2 \rangle \tilde{\mathcal{E}}_d(-d,k_\perp)$$
(25b)

where R_{12} and R_{32} are the reflection coefficients at the lower and upper boundaries for waves in the layer. In the process of obtaining (25) we had to impose the following approximation

$$\langle [R_{32} \otimes R_{32}^*] [(I + S_{dd}) \otimes (I + S_{dd})^*] \rangle \simeq \langle R_{32} \otimes R_{32}^* \rangle \langle (I + S_{dd}) \otimes (I + S_{dd})^* \rangle$$
(26)

This is similar to the weak surface correlation approximation in the sense that we assume that the influence of the boundary fluctuations result in local relations. On observing that $I(z,\Omega) = \frac{\epsilon c k_2^2}{(2\pi)^2} \mathcal{E}(z,k_{\perp}) \cos \theta$ we find that the system of integro-differential Eqs. (24) and (25) is identical to the radiative transfer equation system (2)–(5). The conditions under which this has been possible are:

1. ladder approximation to the intensity operator

2. quasi-stationary approximation for fields

3. weak surface correlation

For unbounded random media and random medium layer with planar boundaries we find that the first two conditions are sufficient. But for random media with rough boundaries we need in addition the third approximation.

5. Unified Approach

The system of equations that we start here is the same as that in the surface scattering operator (SSO) approach, viz., (6) and (7). However, the integral equation representation is different. In the SSO approach we did not directly deal with the boundary conditions. The role of the boundaries are represented entirely by the

SSO. Indeed the boundary conditions are essential to determine the SSO. However, in the unified approach we will directly make use of the boundary conditions. At the top surface the boundary conditions are given as $G_{12}(r_{\perp}, 0; r') = G_{22}(r_{\perp}, 0; r')$

$$\epsilon_2 \partial_z G_{12}(r_\perp, 0; r') = \epsilon_1 \partial_z G_{22}(r_\perp, 0; r') \tag{27}$$

There is a similar pair of relations at the top surface involving G_{11} and G_{21} . At the bottom surface we have

$$\partial_n G_{21}(r_\perp, \zeta; r') = \partial_n G_{22}(r_\perp, \zeta; r') = 0 \tag{28}$$

These boundary conditions are translated on the plane z = -d by using the following approximation which applies when the surface fluctuations are small and smooth.

$$\partial_z G_{21}(r_{\perp}, -d; r') = \mathcal{H}G_{21}(r_{\perp}, -d; r')
\partial_z G_{22}(r_{\perp}, -d; r') = \mathcal{H}G_{22}(r_{\perp}, -d; r')$$
(29)

where $\mathcal{H} = \nabla_{\perp} \zeta \cdot \nabla_{\perp} - \zeta \partial_z^2$. Using (29) we can convert the differential equation system of our problem into the following integral equation system.

$$G = G^o + G^o Q G \tag{30}$$

where

$$Q = Q_v + Q_s \tag{31a}$$

$$Q_v = qN$$
 $Q_s = -\mathcal{H}\delta(z+d)N$ (31b)

 Q_v and Q_s represent the volumetric fluctuation and the surface fluctuation, respectively. G^o is the Green's function for the unperturbed problem, viz., the problem when all the fluctuations vanish. Notice that, in this approach, volumetric and surface fluctuations are treated on equal footing. Thus statistical averaging over volumetric and surface fluctuations are carried out at the same step. Therefore we do not have any subscripts for the averaging operations. This is in contrast with the SSO approach where these operations are carried out separately at different stages of the analysis and we had to use subscripts to indicate whether it is w.r.t. volumetric fluctuations or surface fluctuations.

First average (30) to get

$$\langle G \rangle = G^o + G^o \langle Q \langle G \rangle Q \rangle \langle G \rangle \tag{32}$$

This is the mean Green's function that we will use in our analysis of the second moments of the fields. Details of the analysis of (32) are given in Mudaliar (2005). We proceed to the calculation of the field correlation described by the following equation

$$\langle \psi \otimes \psi^* \rangle = \langle \psi \rangle \otimes \langle \psi \rangle^* + \langle G \rangle \otimes \langle G \rangle^* K \langle \psi \otimes \psi^* \rangle$$
(33)

where

$$K \simeq \langle Q \otimes Q^* \rangle = \langle Q_v \otimes Q_v^* \rangle + \langle Q_s \otimes Q_s^* \rangle \tag{34}$$

We employ the Wigner transforms in (33) as before and obtain

$$\mathcal{E}(r,k) = \tilde{\mathcal{E}}^m(r,k) + \frac{1}{(2\pi)^6} \int dr_1 \int d\alpha \int d\beta \mathcal{G}(r,k;r_1,\alpha) \{T_v + T_s\} \mathcal{E}(r_1,\beta)$$
(35)

where T_v and T_s are spectral representations of $\langle Q_v \otimes Q_v^* \rangle$ and $\langle Q_s \otimes Q_s^* \rangle$, respectively. As before we employ the quasi-stationary field approximation, use (22), and hence arrive at a system of integro-differential equations. The system thus obtained in identical to that in SSO approach. However, the boundary conditions are quite complicated and we have

$$\tilde{\mathcal{E}}(0,k_{\perp}) = \tilde{\mathcal{E}}^{o}(0,k_{\perp}) + \int_{-d}^{0} dz_{1} \int d\alpha_{\perp} W(0,k_{\perp};z_{1},\alpha_{\perp}) \tilde{\mathcal{E}}(z_{1},\alpha_{\perp})$$
(36a)

$$\tilde{\mathcal{E}}(-d,k_{\perp}) = \tilde{\mathcal{E}}^{o}(-d,k_{\perp}) + \int_{-d}^{0} dz_{1} \int d\alpha_{\perp} W(-d,k_{\perp};z_{1},\alpha_{\perp}) \tilde{\mathcal{E}}(z_{1},\alpha_{\perp})$$
(36b)

where $\tilde{\mathcal{E}}^o$ is the single scattering solution, and W is a 2 × 2 matrix given in the appendix. Observe that the boundary conditions are not localized. Furthermore, W involves both surface scattering and volumetric scattering. Thus our system incorporates volumetric and surface scattering interactions. However, if we let $\Phi_v \to 0$ and consider only single scattering from the rough boundary, then we obtain the boundary conditions used in the radiative transfer approach, viz., (25).

6. Conclusion

Radiative transfer approach is very efficient and at the same time simple for describing multiple scattering phenomena. Quite rightly this approach is very popular and is used in a wide variety of applications. Consequently, there are several different interpretations of the meaning and domain of applicability of this approach. One good way to understand this approach is to compare and relate it to the statistical wave approach. Its relation to the wave approach has been well established for the case of unbounded random media. The primary conditions for establishing this equivalence are: ladder approximation to the intensity operator and quasi-stationary approximation of the fields. We find these two conditions are the only requirements even for random media with plane parallel boundaries. However, if the boundaries are statistically rough we need to impose additional restrictions. To illustrate this point we considered two statistical wave approaches: the surface scattering operator approach and the unified approach. In both approaches the integro-differential equations for intensities are the identical to those used in the RT approach. However the boundary conditions are different from those in the RT approach. In the case of SSO approach we need to impose the weak surface correlation approximation to arrive at the boundary conditions of the RT approach. In the case of unified approach we had to let $\Phi_v \to 0$ and consider only single scattering from the rough surface while deriving the boundary conditions. With these additional conditions all the three approaches result in the same system of equations. This study has thus helped us to better understand the three approaches and in particular the relation between the radiative transfer approach and the statistical wave approach when applied to the problem of scattering from a random medium layer with rough boundaries.

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$$\begin{aligned} \mathbf{Appendix} \\ W_{uu} &= \frac{1}{(2\pi)^2} e^{-2q''z} \{ |k_2|^4 T_{uu}^v + T_{uu}^s \} \\ &\quad T_{uu}^v = \operatorname{rect} \{z, -d\} |S^{>}|^2 e^{2\eta''z_1} \Phi_v(k_{\perp} - \alpha_{\perp}; \eta' - \eta'_{\alpha}) + |S^{uu}|^2 e^{2\eta''z_1} \Phi_v(k_{\perp} - \alpha_{\perp}; \eta' - \eta'_{\alpha}) \\ &\quad + |S^{ud}|^2 e^{-2\eta''z_1} \Phi_v(k_{\perp} - \alpha_{\perp}; -\eta' - \eta'_{\alpha}) \\ &\quad T_{uu}^s &= \{ |S^{uu}|^2 e^{-2\eta''d} + |S^{ud}|^2 e^{2\eta''d} \} \Phi_s(k_{\perp} - \alpha_{\perp}) \{ (k_{\perp} - \alpha_{\perp}) \cdot \alpha_{\perp} - \eta_{\alpha}^{'2} \}^2 \\ W_{ud} &= W_{uu} \{\eta'_{\alpha} \to -\eta'_{\alpha} \} \\ W_{du} &= \frac{1}{(2\pi)^2} e^{2q''z} \{ |k_2|^4 T_{du}^v + T_{du}^s \} \\ &\quad T_{du}^v = \operatorname{rect} \{0, z\} |S^{<}|^2 e^{-2\eta''z_1} \Phi_v(k_{\perp} - \alpha_{\perp}; -\eta' - \eta'_{\alpha}) + |S^{du}|^2 e^{2\eta''z_1} \Phi_v(k_{\perp} - \alpha_{\perp}; \eta' - \eta'_{\alpha}) \\ &\quad + |S^{dd}|^2 e^{-2\eta''z_1} \Phi_v(k_{\perp} - \alpha_{\perp}; -\eta' - \eta'_{\alpha}) \\ &\quad T_{du}^s &= \{ |S^{du}|^2 e^{-2\eta''d} + |S^{dd}|^2 e^{2\eta''d} \} \Phi_s(k_{\perp} - \alpha_{\perp}) \{ (k_{\perp} - \alpha_{\perp}) \cdot \alpha_{\perp} - \eta_{\alpha}^{'2} \}^2 \\ W_{dd} &= W_{du} \{\eta'_{\alpha} \to -\eta'_{\alpha} \} \end{aligned}$$

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Angular Normalization of ENVISAT ASAR Data over Sahelian-grassland Using a Coherent Scattering Model

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Abstract—ENVISAT-ASAR acquires data at different incidence angles. For temporal soil and vegetation parameters retrieval it is necessary to normalize the radar data for the angular behavior of the radar backscatter. A simple method for characterizing the angular behavior is by plotting the measurements of the backscattering coefficient during the dry season (stationary target) as a function of the incidence angle and considering that this angular dependence is conserved throughout the rainy season as well. However, this method is not sufficiently precise for data gathered in the rainy season since the scattering mechanisms of the vegetation and soil are different. Instead, we propose the use of a coherent scattering model for vegetation in order to carry out a more precise angular normalization. With this method, a higher precision is obtained for the normalization of the backscattering coefficient at VV polarization during the rainy season, in which the main contribution to the backscatter is attributed to vegetation.

1. Introduction

The ENVISAT satellite was launched by ESA (European Space Agency) in March 2002 on a sun-synchronous orbit at a mean altitude of 800 km and an inclination angle of 98.5°. In this study, we analyzed the data gathered by the multimode ASAR (Advanced Synthetic Aperture Radar) sensor [1], which operates at C-band at VV, HH, HV and VH polarizations. Its incidence-angle range is from 15° to 45°.

Due to the fact that data acquisition for a given spot is provided at different polarizations and incidence angles, it is necessary to perform an angular normalization before exploit the data. When only measured data are available, a simple procedure to normalize them is by considering that the angular dependence is conserved during dry and humid seasons. For the gathered 2004 ENVISAT data, it is possible to employ the HH-polarization response in the dry season to obtain the adjusted angular-dependence curve.

2. Background

The study site is located in northern Mali, in the Gourma region. Its geographical coordinates are 15.35° N and 1.48° W.

The site Agoufou is steered by a semi-arid tropical climate defined by the water resources, the day duration and the temperature amplitude. The rainy season, during the African monsoon, generally starts at the end of June and finishes in September. The vegetation dynamic is mainly determined by rainfall during the monsoon [2]. Vegetation development starts after the first rain (not prior to June) and unless the annual plants wilted before maturity due to lack of rain, the senescence follows the fructification that matches with the end of the rainy season. This vegetation is composed by shrubs (1% cover), trees (3% cover) and annual herbaceous layer (5–40% cover). Soil cover range is between 60% and 95% depending on the season. Trees can be classified into four species: Acacia senegal, Acacia raddiana, Balanites aegyptica and Leptadenia pyrotechnica which is the main species. During the dry season (from October to June), there is no green vegetation apart from exception of shrubs and trees. The soil is essentially composed of sand (91.2%) and of clay (4.5%) [2]. Figure 1 shows physical characteristics of soil and vegetation. The vegetation input data were derived from terrain measurements performed during the 2004 rainy season. The characteristics of both trees and soil roughness are assumed to be constant throughout all the year.

The herb description is 10 cm length, 0.3 cm width, 0.03 cm thickness and with an erectophile orientation $(0 < \alpha < 360^{\circ}, 0^{\circ} < \beta < 35^{\circ}, 40^{\circ} < \gamma < 50^{\circ})$. We assume that branch and trunk moistures are constant throughout the year and are set at 50% and 60%, respectively and its geometrical characteristics are shown in Tab. 1.



Figure 1: Vegetation parameters and soil moisture content (data taken in 2004).

Table 1: Tree description.

Layer	Layer	Vegetal	Height Diameter		Density	orientation	
	height [m]	scatterer	[m]	[cm]	[#/m3]	$\beta[^{\circ}]$	$\gamma[^\circ]$
1 (bottom)	0.15	Trunk	0.15	5	0.04	0-30	0-30
2	0.65	Branch	0.65	4	0.036	0-30	0-30
3	0.60	Branch	0.60	2.5	0.098	0-35	0-50
4 (top)	0.60	Branch	0.60	1.76	0.05	0-35	0-50

3. Angular Normalization Algorithm

Currently, angular normalization is based on the hypothesis of the angular dependence preservation throughout all the year. For ENVISAT 2004 data this process is applied because most of them were gathered at HH polarization; at this polarization, the main contribution can be assigned to scattering from the soil.

The use of only HH-polarization data does not give enough information about the Sahelian grassland, hence it is also necessary the VV polarization to understand the savanna response. This establishes the requirement of a general normalization procedure considering as well the grass contribution when it exits.

To normalize angularly the ENVISAT data, we propose a hybrid algorithm using both measurements and a coherent scattering model. This algorithm is composed of three main steps:

- 1. Determination of the soil contribution from the backscattering coefficient data at HH polarization during the dry season. For this extraction, we consider that soil properties do not suffer any change during this season.
- 2. Simulation of the backscattering coefficient as a function of both time (during the growing season) and incidence angle $\sigma(\mathbf{t}, \theta_i)$ employing the roughness parameters obtained from Step 1 and ancillary data (vegetation type, density, etc). This allows for a first-order correction to gentle angular variations of backscattering coefficient.

3. Determination of the angular normalization factor plot of normalized backscattering coefficient σ_{θ_0} (normalized to mean incidence angle) as a function of time to monitor long term temporal variations.

4. Coherent Model



Figure 2: Coherent model description.

This paper is based on a coherent model for forest scattering [3], which has been adjusted to the grassland case. In this model, the vegetation is described as a discrete medium placed on a dielectric soil and composed of superposed horizontal layers. Each layer is defined by its height and the characteristics of the individual scatterers. Each scatterer type is described by its dimensions, its spatial density, its moisture content and its spatial description (orientation and position). The vegetation is represented as a cluster of scatterers composed of smooth dielectric circular cylinders and thin dielectric ellipsoids [4, 5]. The vegetation generation is based on the description of each layer. The scatterers are positioned, taking into consideration the no-superposition condition between elements enforcing a minimal distance between them and using a probabilistic function to have a realistic distribution.

The coherent model inputs are the scatterer description, the number and the height of layers, the roughsurface description and radar parameters. The coherent model outputs are the backscattering coefficient (calculated by adding coherently the scattered fields of the observed region), the different scatterer contributions and the scattering-mechanism contributions. A schematic representation of the model is shown in Figure 2. The scattering matrix describing each element of the vegetation is built considering four main scattering mechanisms: direct scattering from vegetation scatterers, soil-vegetation interaction, vegetation-soil interaction and soil-vegetation-soil interaction. In this analysis, multiple scattering between scatterers is neglected. In order to take into account superior layer influence, all of these contributions are attenuated and shifted in phase using Foldy's approximation [6]. By adding coherently these two results, the vegetation backscattering coefficient is obtained.

Soil contribution is then reduced taking into account the attenuation due to vegetation layers. Finally, the total backscattering coefficient is the addition of the vegetation contribution and the soil contribution. An average backscattering-coefficient value is obtained by performing the backscattering coefficient over several realizations to warrant the convergence of the response (100 realizations for the grassland case).

For the case of the Sahelian grassland, trees and herbs were separately generated due to our model generator. Herbs were modeled enclosed in only one layer and trees were divided in 4 layers. Herbs are modeled as thin ellipsoids (stems contribution is neglected) and trees as smooth dielectric cylinders (trunks and branches). Total backscattering coefficient is then built by the incoherent addition of herb and tree contributions.

5. Simulation Results

Figure 3 shows the ENVISAT measured data for the site 17 (Agoufou) at Global Monitoring Mode without any normalization, and the angular dependence curve for measured soil contribution. The start of the dry season is supposed to be on October 3^{rd} (day 277).



Figure 3: HH polarization ENVISAT-ASAR measured data without any normalization. b) Angular dependence curve for the HH polarization acquired during the dry season.



Figure 4: a) HH polarization factor plot surface for the angular normalization. b) Comparison between simple method and hybrid method for 23° incidence angle (vertical bars represent temporal rainfall).

The surface representing the backscattering coefficient at HH polarization as a function of time and incidence angle is presented in Figure 4 (a). It is shown that as the incidence angle increases, the grass contribution is more significant to the total backscattering coefficient in the growing period, changing in this way, the incidence angular dependence depicted during the dry season.

Figure 4 (b) presents the comparison between the simple procedure and the hybrid method for the HH polarization for 23° incidence angle. The simple method depicts a higher dynamic (5.4 dB), in contrast the hybrid method presents a lower dynamic (4.4 dB).

Both two methods describe a similar behavior but with different maximum levels (about 1.6 dB of difference) on 29 August (day 242), for which soil moisture content is maximal. The main differences occur during the period of maximal grass biomass (from 28 July up to 17 September). This confirms the contribution of grass to the total backscattering coefficient shape and the importance of taking into account grass influence in the normalization process.

6. Conclusion

This paper proposes a hybrid algorithm to normalize angularly the ENVISAT-ASAR data for the Sahelian grassland and a comparison with a simple normalization. This algorithm allows a correction to get a more accurate angular normalization than that obtained by the simple method.

The most important contribution of the hybrid algorithm is the grass-effect calculus on the backscatteringcoefficient shape causing a difference of about 1.5 dB; however it has two important limits: the need of the ground data and the computing time to carry the simulations out.

This study shows differences when grass contribution is considered within the normalization procedure. In this case, only HH polarization was analyzed. Following this algorithm, differences with the simple method could be amplified at VV polarization for which the vegetation contribution is more significant.

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A Review of the Mechanisms of Interaction Between the Extremely Low Frequency Electromagnetic Fields and Human Biology

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Abstract—Studies of the biological effects and any health related consequences of extremely low frequency (ELF) electromagnetic fields (EMF) have been going on for over half a century however with contradictory outcomes. Hence, it is now necessary to stress on standardizing the EMF-health research experiment procedures in order to enable such experiments become replicable and results comparable. In 1998, a review of several of the ELF EMF human biological interaction mechanisms regarding field intensities and frequencies was presented to the Australian Radiation Laboratory of Commonwealth Department of Community Services and Health in 1988 by Andrew W. Wood. Wood's 1988 assertion of the importance of understanding the interaction mechanisms did not alter even after a decade when the NIEHS RAPID (http://www.niehs.nih.gov/) gathering of world experts produced their statement, in which quoted, there have been experiments on possible mechanism/s in support or refutation of the various proposals however none were replicated. Valberg (Valberg et al., 1997) also summed up some but failed to include all the claimed proposed mechanisms at the time. This paper is to present a complete list of the allegedly possible interaction mechanisms to date.

This paper will also report on an academic research on computer modeling of biological effects of ELF EMF using one of the proposed mechanisms. The research reported here has generally aimed at modeling the proposals using computer. The initial phase of this effort has concentrated on Ca effect as the number of publications referencing that was considerable. Calcium is a key element in the biological performance of every organ in the human body. Thus it deemed imperative to study the effect of EMF on Ca channels of a living cell.

Furthermore, considerations for setting standards in EMF experimental research protocols are recommended. Developing a standard protocol allows results of future experiments to be comparable; and, the chance of replicability in EMF-health improve, which this aspect has indisputably been absent in EMF research projects thus far. Replication is desirable mainly because it eliminates bias, artifact and systematic errors. Replication is almost impossible in the case of epidemiological studies however in experimentation is possible if the details are specified in full. To authenticate any effect of MF, it is not satisfactory to present experimental results without reporting the experimental settings in their entirety.

1. Introduction

Allegations of the biological effects and health hazards of extremely low frequency (ELF) electromagnetic fields (EMF) have been debated for over fifty years. The epidemiological and experimental studies and clarification of conclusions of both research methods have been contradictory. Hence, stipulations have arisen for standardizing EMF-health research experiment procedures so that results of various experiments can be replicated and compared. This paper is to present a list of interaction mechanisms suggested thus far followed by a discussion on setting standard protocols in EMF experimental research. Using a standard protocol allows outcomes of experiments become comparable and replicable. The replicability attribute has undeniably been missing in EMF research projects to date [35].

2. Introducing the Proposed Interaction Mechanisms

Non-ionising radiations are those EMF with frequencies less than 2×10^{16} Hz. They can be grouped into: (i) frequencies over 1 GHz e.g., microwave, infrared and visible light; (ii) frequencies over 3 kHz but below 1 GHz e.g., those in communication systems; and, (iii) frequencies less than 3 kHz known as extremely low frequency or ELF.

The higher frequency exposure can cause dielectric heating by enforcing intra-molecular friction via vibrating momentum increase in water molecules as happens in a kitchen microwave cooking oven. Radiation in this domain can primarily affect the human superficially e.g., skin, cranium, eyes etc and the heat generated can subsequently move deeper onto the body and effectively heat all the internal human organs. Radiation in the GHz range, e.g., mobile phone handsets, antenna and towers' exposure can cause a heating effect penetrating more inside the human body. In the ELF range (< 3 kHz) for instance. when one is exposed to power-line frequencies and/or home appliances, the effects are not yet well clarified. In other words, the jury is still out on what the interaction mechanism is. Unlike the higher frequency radiations stated above, the electric and magnetic fields in the ELF range can be considered de-coupled. The electric component may barely diffuse in the human body. A widespread observation is via skin hair and only for high flux EMF. But, magnetic component may well penetrate the body nearly un-attenuated.

A review [41] of some of the ELF EMF human biological interaction mechanisms with respect to field intensities and frequencies was presented to the Australian Radiation Laboratory of Commonwealth Department of Community Services and Health in 1988. Wood's 1988 affirmation of the importance of understanding the interaction mechanisms did not alter even after a decade years when the NIEHS RAPID [27] gathering of world experts released their report, in which cited, there have been experiments on possible mechanism/s in support or refutation of the various proposals³ however none were replicatedThe interaction mechanisms proposed to date are:

- (1) Magnetite;
- (2) Free radical;
- (3) Cell membrane;
- (4) Cell nuclei;
- (5) Heat shock proteins;
- (6) **Resonance**;
- (7) Blood-brain barrier;
- (8) **Spatial summation**;
- (9) Field induction;
- (10) Energy; and,
- (11) **Corona**.

Describing all the above require a book to be written. However, we will endeavour to introduce these in layman terms briefly in the presentation. The suggested references in support and/or refutation and for better understanding each of the proposed mechanisms are as listed below [35].

Magnetite: Kirschvink et al. [2001], Phillips [1996] and NIEHS [1997].

Free radical: Valberg et al. [1997], Adair [1994] and NIEHS [1997].

Cell membrane: Miles [1969], Cotman and McGaugh [1980], McLeod [1995], Kavaliers et al. [1996], Adey [1981], Blackman et al. [1988], Wood [1988], Ueno [1996], Manni et al. [2002] and Szabo et al. [2001].

Cell nuclei: Lai and Singh [1997], Goodman and Blank [2002], Adair [1998], Ruiz-Gmez et al. [2002], Yomori et al. [2002] and Blank and Goodman [1998].

Heat shock proteins: Zryd et al. [2000].

Resonance: Blackman et al. [1985], Liboff et al. [1987], Lednev [1991], NIEHS [1997], Prato et al. [1996], Prato et al. [1997], Hendee et al. [1996] and Prato et al. [2000].

Blood-brain barrier: Andreassi [1995], Salford et al. [1994] and Lai [1992].

Spatial summation: Valberg et al., [1997] and Astumian et al. [995].

Field induction: Gailey et al. [997], Dimbylow [1998], Baraton and Hutzler [1996], Sagan [1996] and Kaune et al. [2002].

Energy: Valberg et al. [1997].

Corona: Fews et al. [1999], Hopwood [1992], Wood [1993].

Any experimental design to authenticate our theoretical model needs to be replicable. Replication is desirable mainly because it eliminates bias, artifact and systematic errors⁴. Replication is almost impossible in the case of epidemiological studies however in experimentation is possible if the details are specified in full. To substantiate any effect of MF, it is not adequate to present experimental results without reporting the experimental settings in their entirety.

A research has begun by our team using computer simulation of the proposed mechanisms starting with modeling the effect of ELF EMF on the calcium channels. The research project team preparing this paper has also aimed at modeling the proposals using computer. The initial phase of this study concentrated on Ca effect. Calcium is a crucial element in the biological functioning of every organ in human body. Thus it is important to study the effect of EMF on Ca channels of a living cell.

3. Replication in EMF Research

Replication in EMF research is advantageous since it eradicates bias, artefact and systematic errors [35]. Replication is almost impossible in the case of epidemiological studies but in laboratory experimentation is achievable if the details are specified completely. To validate any biological effect of EMF, it is inadequate to present experimental results without reporting the settings fully. This would make certain the effects are replicable. A proper EMF replication necessitates applying excellent quality assurance measures to ensure matching exposure parameters [34]. These include the human biological endpoint of interest, field characteristics, exposure timing, physical dimensions of the exposure (local or whole body), field strength, DC or AC (sinusoidal or pulsating) frequency, harmonics, field alignment, field direction (linear vs. polarised), instrumentation, laboratory temperature, air-conditioning, light quantity, quality and intensity, background and environmental EMF, time of day, subject's history of exposure, subject's prior to experiment exposure, subject's adaptability to environmental factors, food intake and many others. Obviously, one has limited control over the subject's individual biological condition prior to the experiment [35].

Range of some of the parameters listed above may be controlled using correctly planned, designed and executed protocols. In planning a laboratory research project on the human health effects of EMF, biological measures chosen for the study need to be relevant.

4. Results and Discussion

The ELF bioelectromagnetics biological effects research has entrapped the scientists and the public in a maze since 1960's; no one has yet rescued the concerned community by provision of replicable proof [35]. Besides, a synthesis of the above-listed mechanisms may have to be considered if reasonable in an endeavor to formulate an indisputable interaction mechanism theory verifiable by experimental work.

This area of science is widely accepted as an area of controversial results. The proposed interaction mechanisms were: Magnetite; Free radical; Cell membrane; Cell nuclei; Heat shock proteins; Resonance; Blood-brain barrier; *Spatial summation; Field induction; Energy; and, Corona.* Acceptance or rejection of the proposals is impossible due to lack of independently replicable experiment. The parameters to be considered in a replication include the biological endpoint, field characteristics, strength, signal waveform, frequency, harmonics, alignment, direction, exposure timing, physical dimensions, instrumentation, laboratory temperature, air-conditioning, light quantity, quality and intensity, background and environmental EMF, time of day, subject's history of exposure, subject's prior to experiment exposure, subject's adaptability to environmental factors, food intake and many others. Obviously, one has limited control over the subject's individual biological condition prior to the experiment.

5. Conclusions

None of the alleged interaction mechanisms were proven with replication. Hence, it was concluded that, there was vividly a need for future experimental research in this field using a standard set of experimental research protocols.

Finally, experimental design efforts for testing the interaction mechanism/s theories in our research group are currently tending to focus on protein folding and Ca channels which are slow biological processes. Any experimental design to verify our theoretical model must be replicable. A replication necessitates applying excellent quality assurance measures to match exposure parameters and conditions.

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Calculating SAR in Two Models of the Human Head Exposed to Mobile Phones Radiations at 900 and 1800 MHz

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Abstract—Since the 1990's, use of mobile phones has augmented worldwide generating a public concern as to whether frequent utilization of such devices is unsafe. This provoked EMF researchers to find suitable techniques of assessing radiation blueprint and exposure hazards if any. Most research groups focused on two techniques: experimental measurements and finite-difference time-domain (FDTD) computations. Computation of the specific absorption rate (SAR) generated by cellular phones inside two models of the human head is presented in this paper. Two models of mobile phones were considered working at 900 and 1800 MHz bands according to the Global System for Mobile Communication. Radiated energy distributions and averaged SAR values in 1 g and 10 g of tissue were computed inside the models of head using FDTD. Computations were compared with a realistic head model constructed with the MRI scans. The distribution of the local SAR in the head was similar to that of the simplified head models. The maximum local SAR calculated was 53.43 W/kg and the maximum SAR(10 g) was 2.96 W/kg, both for 1 W output power from the antenna. The results indicated the area of the maximum local SAR was situated in outer layer of skull, where muscle and skin were. The important parameters in absorbed energy in the head were the type of antenna, current distribution and the distance between head and antenna. The head models used for simulation proved as insignificant parameter in the calculations.

1. Introduction

Within only the last ten years, mobile phone usage has been rapidly spread globally. In chorus with the expanding usage, a question has been raised repeatedly as to whether frequent usage of such a device which radiates GHz electromagnetic field onto the human head is unsafe. This rapid expansion has thus pushed the research toward the necessity of finding a reliable means of analyzing mobile phone for radiation pattern performance to address the safety concerns. It is broadly accepted that mobile phones cause heating of the human organ exposed to their radiation and specifically the human head. The current exposure limits are based on Specific Absorption Rate (SAR) of the exposure heat. A SAR limit of 2 W/kg averaged over any contiguous 10 g head tissue was recommended by the Council of European Union [1] for the general public. This recommendation in a way acknowledged that a simple cubical geometry used may yield calculated dosimetric quantities of conservative values corresponding to the exposure guidelines.

It has been a while since, most research groups studying biological effects of mobile phones have focused on two methods: *experimental measurements* and *finite-difference time-domain* (FTDT) computations [2–5]. While experimental measurements make use of the actual mobile phone being tested [6], there remains a question of appropriateness of representing the human head with simplified phantoms that for compliance testing include, at most, two or three tissue type materials [4,5]. The FDTD method, on the other hand, can be questioned on its lack of a realistic anatomically heterogeneity representation of the radiation exposure of mobile phone through the human head model [7].

In the research work reported here, the authors focused on the absorption of energy in the human head from near-field radiation of wireless phones. Two models for mobile phone (half-wavelength dipole and a quarter-wavelength monopole) and two simple models for head (homogenous and multi-layer spherical) were considered. A modern method for calculating maximum SAR (10 g) was introduced and the results were compared with a realistic MRI model of head [6,7].

2. Numerical Method and Modeling

Two models used for the human head were spheres of 20 cm diameter. The first model was a sphere consisting entirely of material with the electric properties of brain tissue. The second model comprised three layers as illustrated in Fig. 1. The spherical model had a uniform content at its core (representing the human brain) and the core was surrounded by two spherical shells representing the skull (bone) and the muscle and skin (skin) with their respective electromagnetic properties.





Figure 1: The model of the layered sphere.

Figure 2: The model of the monopole antenna mounted on top of the metal box.

The handset has been modeled in two different ways, namely a half-wavelength dipole and a quarterwavelength monopole both mounted on top of a metal box. The thickness of the antennas in both cases was one FDTD cell. The antennas were centered on the top surface of a conducting box with dimensions $120 \text{ mm} \times 55 \text{ mm} \times 20 \text{ mm}$ (see Fig. 2). The face of the metal box next to the user was covered with a dielectric material of 5 mm thickness. The feeding gap of the dipole antenna was placed at a distance of 0.5 mm from the sphere. However, the feeding gap of the monopole antenna located 2 cm away from the sphere, since the minimum distance between the sphere and metal box was again 0.5 cm [4, 5, 8].

The simulation was performed at the two common telecommunication carrier frequencies of 900 MHz and 1800 MHz. The handset antenna model length was adjusted according to the wavelength in free space obviously corresponding to the frequency in use. Combinations of the cases were also investigated as detailed in Table 1. The properties of tissue material considered in the computer simulation modeling at both frequencies [3] were as tabulated in Table 2.

Case	Homogeneous Sphere	Layered Sphere	900 MHz	$1800\mathrm{MHz}$	Dipole	Monopole
1	*		*		*	
2	*		*			*
3	*			*	*	
4	*			*		*
5		*	*		*	
6		*	*			*
7		*		*	*	
8		*		*		*

Table 1: Description of the cases examined.

A software sourced from the Utah University of Technology (<u>http://www.fdtd.org</u>) was used for the simulation. The lattice for all cases was formed by a uniform rectilinear grid with a space step of 2.5 mm in all three directions. The simulation time was twenty periods of the source signal. A hard source model was positioned at the feeding gap, which had the size of one cell [3, 9-12]. The source had a sinusoidal time behavior and was switched on at the beginning of the computer run. In all simulations the output power of the antenna was 1 W. Mur's second-order absorption boundary conditions [13, 14] were used to truncate the computational domain.

The distribution of the local SAR values can be calculated directly from the electric field distribution, which results from the computer run. This was achieved using Eq. (1) as the sinusoidal source leads to a steady state electric field, numerically analogous with the same sinusoidal variation [3–6].

$$SAR = \frac{\sigma E_{\max}^2}{2p} \tag{1}$$

	$900\mathrm{MHz}$	900 MHz	$1800\mathrm{MHz}$	$1800\mathrm{MHz}$	Mass Density
Material	ε_r	σ	ε_r	σ	(Kg/m^3)
Skin	39.5	0.7	38.2	0.9	1080
Bone (cortical)	12.5	0.17	12.0	0.29	1180
Brain (Grey matter)	56.8	1.1	51.8	1.5	1050
Dielectric Phone Cover	2.7	0.0016	2.7	0.003	-

Table 2: The properties of the materials used in the simulations.

To test the hypothesis that steady state was reached after twenty periods of the numerical source signal, the time evolution of the electric field at several points in the lattice was monitored during each computer run [3, 7, 15]. It was found that for all the cases examined, the simulation time was more than enough to arrive at steady state.

The derivation of the average SAR values needs some post-processing of the simulation results. The SAR values were averaged over 1 g and 10 g of tissue. The way averaging volume was selected was crucial for the derived average SAR distributions [3, 7, 15]; hence the averaging procedure

average SAR distributions [5, 1, 15], hence the averaging procedure adopted had to be clarified. The edge of each cell in the lattice was 2.5 mm which allowed for a 1 cm cube to be considered per four cells. This cube provided 1 g of tissue for a mass density of 1 g/cm^3 . Noting that the area of SAR (10g) was situated outside of the skull area (in the muscle) and the size of the cube was sufficiently small. Therefore, the computation output results allowed direct calculation of the maximum SAR. Conversely, it was impossible to have a cubic volume of tissue with a mass of 10g; and, as in the previous method a cube of 2.25 cm sides (nine cells) was considered (method 1). In this case, due to the long length of the cube and its location with different SAR specially in between skull/brain and skull/muscle, the calculations resulted in a noticeable difference with previously published studies. The process details were reviewed and checked again and found to be precise. Furthermore, two cubes were considered, 1.75 cm^3 (seven cells)



Figure 3: The cube volumes used for calculating the SAR (10 g) values.

and 2.25 cm³ (nine cells) co-centered as illustrated in Fig. 3. The SAR (10 g) value was subsequently calculated considering the contribution of the smaller cube and the contribution of the cubical shell around it each with a predefined weighting coefficient using Eq. (2). $\sum (CAB) = \sum (CAB)$

$$SAR(10\,g) = \frac{\sum_{v_1} (SAR)_i m_i + \sum_{v_2 - v_1} (SAR)_j m_j}{\sum_{v_1} m_i + \sum_{v_2 - v_1} m_j}$$
(2)

where $m_i = \rho_i \Delta V$ and $m_j = P_j \Delta V \frac{10-V_1}{V_2-V_1}$ [3]. Index *i* refers to the lattice cells inside the inner cube and index *j* to those around it (method 2).

Table 3: Total absorbed power, maximum SAR(1g), maximum SAR(10g) and local SAR in the head model (antenna output power 1 W).

Case	P_{abs} (W)	$SAR(1g)_{ m max}$ (W/kg)	$SAR(10g)_{\max}$ (W/kg) (Method 1)	$SAR(10g)_{ m max}$ (W/kg) (Method 2)	Local SAR _{max} (W/kg)
1	0.77	2.45	2.96	1.89	26.19
2	0.41	0.74	0.82	0.53	4.23
3	0.75	4.67	6.15	3.89	70.16
4	0.24	0.52	0.63	0.41	3.92
5	0.76	1.64	1.91	1.22	18.86
6	0.48	0.5	0.58	0.37	3.72
7	0.83	4.13	5.01	3.18	54.14
8	0.30	0.59	0.69	0.44	4.87

3. Results

Table 3 presents a summary of the total absorbed power, the maximum SAR (1g), the SAR (10g) and local SAR values. These results indicated that the model of the handset device played a more important role in dosimetry than the model of human head. This was evident when looked at the difference in calculated values; for instance, considering case 1 (homogeneous sphere, 900 MHz, $\lambda/2$ dipole) with case 5 (layered sphere, 900 MHz, $\lambda/2$ dipole) yielded smaller figures than when the case 1 was considered with case 2 (layered sphere, 900 MHz, $\lambda/4$ monopole). As reported previously [4, 5, 8], this research confirmed that in most cases the homogeneous sphere resulted in a larger SAR values than the layered sphere. It was also apparent from the results in Table 3 that modeling the handset as a dipole yielded higher SAR values than modeling it as a monopole.

However a direct comparison between the respective pairs of cases was not possible, because it could be argued that the smaller distance of the dipole feeding gap to the head may account for the larger SAR values. So, the distance of the $\lambda/2$ dipole was varied for case 3 (homogeneous sphere, 1800 MHz, $\lambda/2$ dipole) to study its significance. As shown in Fig. 4, at 2 cm distance the values obtained with the $\lambda/2$ dipole are still larger than case 4 (homogeneous sphere, 1800 MHz, $\lambda/4$ monopole). It can be noted from Fig. 4 that the maximum local SAR doesn't fall off inversely proportional to the square of distance, as it would in the far field.



Figure 4: Variation of maximum local SAR values.

The effect of different operating frequencies was as illustrated in Fig. 5 and Fig. 6. An observation was that the SAR decreased faster in the higher frequency range as expected due to the smaller penetration depth.



Figure 5: The profile of local SAR across the homogeneous spherical head model. The distance was measured from the point of the source closest to the head model. The SAR values were normalized to the maximum to show the effect of frequency.



Figure 6: The profile of local SAR across the layered spherical head model. The distance was measured from the point of the source closest to the head model. The SAR values were normalized to the maximum to show the effect of frequency.

4. Comparison between Case Study and MRI Human Head Model

The MRI model used in this work was the Bradford University Telecommunication Research Center Tissueclassified high-resolution voxel image of a human head [6,7]. The original resolution of the model was 0.909 mm in the x and y direction on the axial plane and 1.480 mm in the z (vertical) direction. The phantom was resampled to have cubic voxels with each side of 0.25 mm length. A $\lambda/4$ monopole on top of a metal box was used for the device model, with the source operating at 900 MHz. The distribution of the local SAR in the head (Fig. 7) was similar to that of the simplified head models. The maximum local SAR calculated was 53.43 W/kg and the maximum SAR(10 g) was 2.96 W/kg, both for 1 W output power from the antenna.



Figure 7: Distribution of the local SAR on a) the middle coronal plane and b) the middle axial plane of the MRI head model for the monopole on top of the Box operating at 900 MHz. The values are normalized to the maximum local SAR.

5. Discussion and Conclusions

The maximum SAR(10 g) value was higher than the basic limit of 2 W/kg SAR(10 g) over 6 minute period according to the widely adopted exposure guidelines [1]; however, the following points must be noted:

1)In the simulations, it was assumed that the device was operating continuously. Nevertheless, timeaveraged power of a GSM device under real operating conditions was 1/8 of its nominal power. Therefore, for a nominal operating power of 1W the actual time-averaged

output power was 0.125W [see also 3-5, 7, 8, 15].

2) The introduction of a dielectric scatterer like the human head in the vicinity of the radiating antenna alters the input impedance of the latter. In the calculations it was assumed that all the power generated and transmitted by the electronic devices to the antenna was fully radiated out; i.e., the antenna and the transmission line were completely matched. This assumption represented the worse case since only a portion of the power was radiated from the antenna.

3)It was shown that the use of a metal box model for the phone instead of a CAD model gives more conservative results, i.e., higher SAR values [6].

Although the total absorbed power in most cases was lower for 1800 MHz than for 900 MHz, the maximum SAR values are higher for the higher frequency. The distribution of SAR in the spherical head models shown as in Fig. 7, indicated that the large SAR values were restricted within a volume close to



Figure 8: Percentage of volume in the spherical head models for which the local SAR has a value smaller than 10% of the maximum local SAR.

the surface of the model. In fact, in more than 94% of the head model volume the SAR was smaller than 10% of the local maximum SAR for each case (see Fig. 8).

Finally, the results demonstrated that the area of the maximum local SAR was situated in outer layer of skull, where muscle and skin were located. Since the maximum difference in dielectric properties between muscle and skull was more than other tissues, the maximum reflection happened around the boundary of the
two tissues. Thus maximum absorption power was seen in this border; and, because of the skin effect, with increasing frequency the situation of the maximum local SAR would get closer to the outer layer of the head. Also the results showed that the maximum SAR (10 g) was about 20% to 30% lower than maximum SAR (1 g) under similar conditions.

In conclusion, the important parameters affecting the absorbed energy in the human head exposed to mobile phone radiation were the type of antenna, current distribution and the distance between head and antenna; and, head models used for simulation did not play any significant role in the calculations.

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Qualitative Analysis of Human Semen Using Microwaves

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Abstract—Microwave engineering now a days plays a vital tool in diagnostic and therapeutic medicine. A quality evaluation of human semen at microwave frequencies using the measurements made at different intervals of time by cavity perturbation technique in the S-band of microwave spectrum is presented in this paper. Semen samples were also examined in the microscopic as well as macroscopic level in clinical laboratory. It is observed that conductivity of semen depends upon the motility of sperm and it increases as time elapses, which finds applications in forensic medicine.

1. Introduction

Accurate information about the dielectric properties of tissues and biological liquids is important for studies on the biological effects at radio and microwaves frequencies. In macroscopic level, these electrical properties determine the energy deposition patterns in tissue upon irradiation by an electromagnetic field. In microscopic level, they reflect the molecular mechanisms, which underlie the absorption of electromagnetic energy by the tissue or liquids. Knowledge of the microwave dielectric properties of human tissues is essential for the understanding and development of medical microwave techniques. Microwave thermography, microwave hyperthermia and microwave tomography all rely on processes fundamentally determined by the high frequency electromagnetic properties of human tissues. Tissue temperature pattern retrieval in the microwave thermography is achieved using models of the underlying tissue structure, which depend particularly on the dielectric properties of the tissue [1]. A recent review of published data on animal and human dielectric parameters shows that for most tissue types animal measurements are good substitute for human tissues [2].

Gabriel et al., Cook and Land et al., reported the dielectric parameters of various human tissues at different RF frequencies. [3–6]. Microwave study of human blood using coaxial line and wave-guide methods was carried out by Cook [7]. Tissue samples of human brain at microwave frequencies were analysed using sample cell terminated transmission line methods [8]. Open-ended coaxial line method allows measurements of tissue samples over a wide range of frequencies [9].

Microwave medical tomography is emerging as a novel non-hazardous method of imaging for the detection of fracture, swelling and diagnosis of tumors. Active and passive microwave imaging for disease detection and treatment monitoring require proper knowledge of body tissue dielectric properties at the lower microwave frequencies [10–12]. Studies on the variation of dielectric properties of body fluids and urinary calcifications at microwave frequencies have revealed that diagnosis is possible through cavity perturbation technique [13–15]. The present paper reports dielectric properties of semen at microwave frequencies as well as the quantitative analysis in the clinical laboratory. It is observed that conductivity of semen depends upon the motility of sperm as well as the time elapses after ejaculation.

2. Materials and Methods

The experimental set-up consists of a transmission type S-band rectangular cavity resonator, HP 8714 ET network analyser. The cavity resonator is a transmission line with one or both ends closed. The resonant frequencies are determined by the length of the resonator. The resonator in this set-up is excited in the $TE_{10\rho}$ mode. The sample holder which is made of glass in the form of a capillary tube flared to a disk shaped bulb at the bottom is placed into the cavity through the non-radiating cavity slot, at broader side of the cavity which can facilitate the easy movement of the holder. The resonant frequency f_o and the corresponding quality factor Q_o of the cavity at each resonant peak with the empty sample holder placed at the maximum electric field are noted. The same holder filled with known amount of sample under study is again introduced into the cavity resonator through the non-radiating slot. The resonant frequencies of the sample loaded cavity is selected and the position of the sample is adjusted for maximum perturbation (i.e., maximum shift of resonant frequency with minimum amplitude for the peak). The new resonant frequency f_s and the quality factor Q_s are noted. The same procedure is repeated for other resonant frequencies.

3. Theory of Cavity Perturbation

When a material is introduced into a resonant cavity, the cavity field distribution and resonant frequency are changed which depend on shape, electromagnetic properties and its position in the fields of the cavity. Dielectric material interacts only with electric field in the cavity.

According to the theory of cavity perturbation, the complex frequency shift is related as [16]

$$-\frac{d\Omega}{\Omega} \approx \frac{(\bar{e_r} - 1) \int E \cdot E_0^* dV}{2 \int \limits_{V_c} |E_0|^2 dV}$$
(1)

But

$$\frac{d\Omega}{\Omega} \approx \frac{d\omega}{\omega} + \frac{j}{2} \left[\frac{1}{Q_s} - \frac{1}{Q_0} \right] \tag{2}$$

Equating (1) and (2) and separating real and imaginary parts results

$$\varepsilon_r' - 1 = \frac{f_o - f_s}{2f_s} \left(\frac{V_c}{V_s}\right) \tag{3}$$

$$\varepsilon_r'' = \frac{V_c}{4V_s} \left(\frac{Q_o - Q_s}{Q_o Q_s}\right) \tag{4}$$

Here, $\bar{\varepsilon_r} = \varepsilon'_r - j\varepsilon''_r$, $\bar{\varepsilon_r}$ is the relative complex permittivity of the sample, ε'_r is the real part of the relative complex permittivity, which is known as dielectric constant. ε''_r is the imaginary part of the relative complex permittivity associated with the dielectric loss of the material. V_s and V_c are corresponding volumes of the sample and the cavity resonator. The conductivity can be related to the imaginary part of the complex dielectric constant as

$$\sigma_{\varepsilon} = \omega \varepsilon'' = 2\pi f \varepsilon_0 \varepsilon_r'' \tag{5}$$

4. Results and Discussion

Table 1: Variation of dielectric constant with frequency at different time intervals.

Frequency (MHz)	T = 5 minutes				
	M23	M24	M29	M33	
2439.019	11.93	12.90	11.17	14.11	
2683.882	11.91	12.44	10.93	12.52	
2969.983	12.12	13.92	11.96	13.21	
	T = 15 minutes				
2439.019	12.73	12.97	11.21	13.12	
2683.882	11.87	12.76	12.53	11.77	
2969.983	11.77	13.01	10.11	12.23	
	T = 30 minutes				
2439.019	11.01	12.60	12.86	14.01	
2683.882	12.37	12.56	12.38	13.81	
2969.983	13.53	12.90	12.71	13.92	
	T = 45 minutes				
2439.019	13.31	12.61	11.74	13.22	
2683.882	13.80	12.11	11.82	13.71	
2969.983	13.06	12.84	11.95	13.51	

SEMEN ANALYSIS								
	M23	M24	M29	M33				
Time of collection	12.00 PM	12.15 PM	12.30 PM	12.35 PM				
Time of liquefaction	12.30 PM	12.45 PM	1.00 PM	1.05 PM				
MACROSCOPIC EXAMINATION								
Volume	$1 \mathrm{ml}$	$1.5 \ {\rm ml}$	$0.5 \ \mathrm{ml}$	$1 \mathrm{ml}$				
Colour	Opaque grey	Opaque grey	Opaque grey	Opaque grey				
Viscosity	Normal	Normal	Normal	Normal				
pH	8.0	8.0	8.0	8.0				
Liquefaction	Within 30	Within 30	Within 30	Within 30				
	minutes	minutes	minutes	minutes				
	MICROSCOPIC EXAMINATION							
Motility								
Quick	55~%	60 %	70~%	45~%				
Sluggish	$15 \ \%$	$15 \ \%$	$10 \ \%$	$15 \ \%$				
Dead	30~%	25~%	20~%	40~%				
Sperm count	85million/ml	95 million/ml	100 million/ml	75million/ml				
Pus cells	2-3/hpf	2-4/hpf	1-2/hpf	1-2/hpf				
Morphology								
Normal	85 %	90 %	88 %	89 %				
Giant Head	3 %	2 %	3~%	3 %				
Round head	4 %	4 %	2~%	2~%				
Pin head	5 %	2 %	4 %	5 %				
Double head	3 %	2 %	3~%	1 %				

Table 2: Quantitative analysis of semen in the clinical laboratory.

The microwave studies on the samples are done by cavity perturbation technique and the results are shown in Table 1 and in Figures 1 and 2. The clinical evaluation of the semen samples are done and the results are tabulated in Table 2. Table 1 indicates the variation of dielectric constant of different semen samples at different time intervals after ejaculation. It is observed that the dielectric constant is consistent at all frequencies at different intervals of time after ejaculation. From Figure 1, it is observed that the conductivity of the semen samples increases as frequency increases. This indicates that semen is lossier at higher frequencies due to the presence of the high motile quick sperms and its absorption of electromagnetic energy. The conductivity of the sample is more for high motile quick sperms and low conductivity for the dead sperms. From Figure 2, it is observed that the conductivity of semen increase as time elapses. This is due to the clotting enzyme of the prostatic fluid, which forms a coagulum in early stages after ejaculation, which makes the sperm remain relatively immobile, because of the viscosity of the coagulum [17]. The conductivity is relatively low due to this effect in early stages. As coagulum dissolves during the next 5 to 15 minutes, sperms become highly motile, which causes an increase in the conductivity.

This has potential application in forensic medicine in that the elapsed time after ejaculation is directly related to the conductivity of semen.



Figure 1: Conductivity of semen.



Figure 2: Temporal behaviour of conductivity of M24.

5. Conclusion

The microwave study of the semen samples is done using cavity perturbation technique. This technique requires very small volume of sample and it is particularly applicable to biological samples like semen. The study shows that the dielectric constant of given semen sample does not show appreciable variation with time or with frequency. But it is observed that the conductivity of the semen sample increases as frequency increases, which shows that semen is lossier at higher frequencies. Samples with high conductivity indicates the presence of more high motile quick sperms and low conductivity indicate the more dead sperms. The conductivity of semen increases as time elapses and this finds application in forensic medicine to find the elspsed time after ejaculation.

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Effects from the Thin Metallic Substrate Sandwiched in Planar Multilayer Microstrip Lines

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Abstract—This paper studies the dispersion characteristics of open multilayer microstrip lines with a thin highly-conductive metallic substrate using the spectral domain approach. From the numerical results, it is found that, in both lossy (metal-insulator-silicon structure such as $Si-SiO_2$) and lossless configurations (thin metal between lossless dielectric microstrip line), at the low frequency range, this thin metallic substrate can excite slow waves. And accordingly the frequency dependent transmission-line characteristics of interconnects, such as propagation constant, attenuation and the characteristic impedance, can change remarkably with the existence of the thin metallic substrate.

1. Introduction

The lossy transmission line structures gain more attentions due to the fast development of the VLSI semiconductor circuits. In this case, multilevel interconnect networks are introduced into multilayer silicon structures to enable great efficiency of semiconductor integration. But this is also accompanied with many challenges to interconnect in circuit design. From the viewpoint of signal integrity, these complicated structures of interconnect network make the circuit much vulnerable to the substrate coupling of noise, power supply noise, ground bounce, crosstalk, ringing, antenna effects etc.

As one solution, very thin highly-conductive substrates are added into multilayer structures as ground to depress or shield away the harsh effects. Cregut et al., showed that, by certain metallic substrate configuration, the crosstalk can be reduced as well as the transient performance improved [1]. However, at some low RF and microwave frequencies, the skin depth may be much larger than the metal thickness of profile. The electromagnetic wave can penetrate the thin metallic substrate and reach further deeply into the layers underneath. In another words, the electromagnetic wave can "see through" and consequently interact with the multilayer configuration underneath. Considering this phenomenon, Song et al. found the frequency dependent characteristics or dispersion characteristics of interconnects, such as resistance, inductance, capacitance, and conductance (RLCG) per unit length, change remarkably with the existence of thin metallic substrate [2].

The previous work of planar multilayer microstrip line starts with the hypothesis of the perfect electric conductor (PEC) ground plane or impedance boundary condition (IBC), and considers the effects of the strip with finite conductivity or thickness. This paper focus on the effects of this thin but highly conductive substrate in the middle substrate. To adequately analyze this effect, the spectral domain approach (SDA) is used [3, 4]. The simplified model of an open microstrip line is proposed, and the corresponding 2-dimensional Green's functions of multilayer microstrip line are deduced. The method of moments is applied to solve the related eigenvalue problem numerically. Thus the dispersion performances of the propagation constant, attenuation and the impedance of open multilayer microstrip lines are simulated numerically. It also shows that the slow wave can be excited.

2. Modelling and Spectral Domain Approach

The open microstrip line with considered thin metallic substrate is shown in Fig. 1. The substrates are assumed to be uniform and infinite in both the x and z directions. The signal strip as well as the lowest ground plane is taken as infinitesimally thin and PEC. After taking a spatial Fourier transformation in the x direction, the coupled integral equations on the surface of the strip line become the algebraic ones as following

$$\begin{bmatrix} \tilde{E}_x(\alpha, y_s) \\ \tilde{E}_z(\alpha, y_s) \end{bmatrix} = \begin{bmatrix} \tilde{Z}_{xx}(\alpha, y_s) & \tilde{Z}_{xz}(\alpha, y_s) \\ \tilde{Z}_{zx}(\alpha, y_s) & \tilde{Z}_{zz}(\alpha, y_s) \end{bmatrix} \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix}$$
(1)

where \tilde{Z}_{xx} , \tilde{Z}_{xz} , \tilde{Z}_{zx} and \tilde{Z}_{zz} are the dyadic Green's functions for microstrip geometry. α denotes the spectral domain variable in x direction. y_s stands for the interface where the signal strip located on. The immittance approach [3] decouples the field into two independent configurations as transverse electric (TE^y) and transverse magnetic (TM^y) modes. Using the transmission line modelling (TLM), the Green's functions are derived as



Figure 1: Configuration of 3-layer open microstrip line with very thin metallic substrate with thickness t (grey).

parallel combination of the admittance seen above and below the interface y_s . Then the Galerkin's method is used to solve the equation (1) for the propagation constant. The current density \tilde{J}_x and \tilde{J}_z are separately expanded into a series of basis functions as

$$\tilde{J}_x = \sum_{m=1}^{M} a_m \tilde{J}_{xm}, \quad \tilde{J}_z = \sum_{n=1}^{N} b_n \tilde{J}_{zn},$$
(2)

where the a_m and b_n are the coefficients of current basis function. After substituting the current expansions (2) into (1) and doing the scalar product on both sides, based on the Parseval theorem, the integral equation is discretized into a homogeneous system with M + N linear equations. Consequently the value of propagation constant is directly correspondent to the eigenvalue of the system which makes the determinant of the following equation equal to zero.

$$\sum_{m=1}^{M} a_m K_{i,m}^{xx} + \sum_{n=1}^{N} b_n K_{i,n}^{xz} = 0 \quad (i = 1, \dots, M)$$
$$\sum_{m=1}^{M} a_m K_{j,m}^{zx} + \sum_{n=1}^{N} b_n K_{j,n}^{zz} = 0 \quad (j = 1, \dots, N)$$
(3)

The real part of the complex wave number is related directly with the propagation wavelength and phase velocity while the imaginary part is the attenuation per unit length along the z direction.

3. Dispersion Characteristics

In this section, the influences of this very thin metallic substrate on the performance of microstrip are simulated. One thin metallic substrate is inserted into a metal-insulator-semiconductor (MIS) structure and a lossless microstrip line, which results in two multilayer cases respectively: the metal-insulator-metal-semiconductor (MIMS) and metal-insulator-metal-insulator (MIMI) to be studied in the following.

3.1. Very Thin Metallic Substrate in Lossy System

Here one modified case of MIS structure is considered with a thin metallic substrate inserted between the dielectric SiO₂ and lossy Si as shown in Fig. 2(b). The ϵ_r of the silicon dioxide is 4 and its thickness is 1 μ m. The width of strip is 160 μ m. The thickness of the silicon layer is 250 μ m with $\epsilon_r = 12$ and conductivity $\delta = 5$ S/m. Fig. 2 shows the frequency dependence of the attenuation constant and the normalized guiding wavelength, which is equivalent to the phase velocity normalized by the speed of the light in free space. When the t equals to zero, this modified structure is degraded into the typical metal-insulator-silicon structure previously studied by the Hasegawa et al. [5]. Our results agree well with the one calculated by Cano, Medina and Horno shown as rectangular dots in Fig. 2. The further validation can be found in [7].

In Figure 2(a), as the frequency decreases, the normalized guiding wavelength converges to about 0.06. So the wave propagates on the microstrip lines much slower than in free space, which is known as the slow wave effect. In addition, several "limit" curves are marked. When the thickness of the middle metallic substrate grows to infinity, the effect of the silicon substrate and PEC ground underneath become negligible. Another curve is obtained by treating the middle metal as PEC to make the transmission system become lossless. This curve fits well with the result of Pramanick's and Bhartia's formula [8] shown as circle dots. With increasing the frequency, each curve with different thickness converges consequently to the critical curve representing the infinite metal thickness. This attributes to the fact that the skin depth decreases as the frequency increases. Physically it means the electromagnetic field experiences more attenuation when penetrating the same conductive metallic substrate. Thus at some points, the whole substrate can become opaque and block the field from reaching the



Figure 2: Dispersion performance of a 3-layer MIMS open microstrip under different thicknesses t. (lower silicon thickness: $250 \ \mu m$; ε_r : 12; σ : 5 S/m; thin metal σ : 5.8(10⁷) S/m; upper silicon-dioxide: 1 μm , ε_r : 4; w: 160 μm ; circle dots: Pramanick and Bhartia's results [8] for lossless microstrip lines; rectangular dots: Cano, Medina and Horno's results [6] for MIS structure).



Figure 3: Frequency behavior of the (a) real and (b) imaginary parts of the characteristic impedance for the MIMS structure shown in Fig. 2 (using $Z_0 = V/I$ definition, rectangular dots: Cano, Medina and Horno's results [6] for MIS structure).

lower substrate. At the high frequency, the thin metallic substrate will work as infinite thick metal ground, when the metal thickness equals to 2 to 3 times of the skin depth. For example, at 1 GHz, the skin depth of copper is about $2 \,\mu$ m. The curve with thickness of $2 \,\mu$ m converges at around 4 GHz, almost 2 times thicker than the skin depth at 4 GHz. Similarly for $1 \,\mu$ m curve, the convergence point is about 25 GHz with skin depth $0.4 \,\mu$ m. In most of the frequency range shown, the normalized guiding wavelength decreases as the frequency decreases. When the skin depth is about 10 times more than the metal thickness, the wave becomes a slow wave.

Figure 2(b) also shows the behavior of the attenuation. The attenuation constants under different thickness of thin ground substrate approach consequently to the critical curve of infinite thickness. At the high frequency range, the attenuation is proportional to square root of the frequency. This is because the electric current flows through a region proportional the skin depth that is proportional to the inverse square root of the frequency. On the contrary, at the low frequency region, it is observed that the slopes of the curves are proportional to the square of the frequency, which is due to the ohmic loss of the electric current flowing in the metallic ground and substrate. In addition it is observed that the curves for finite thickness converge to the infinite thick metallic substrate when the thickness is equal to the skin depth. For example, the curve of 2 μ m converges to the infinite thick at about 1 GHz. It shows that the concept of skin depth has more direct connection with the attenuation constant other than the phase velocity in Fig. 2(a).



Figure 4: Dispersion performance of a 3-layer MIMI open microstrip under different thicknesses t of thin metallic substrate. (lower dielectric substrate: $80 \,\mu\text{m}$; ε_r : 10.2; thin metal σ : $5.8(10^7) \,\text{S/m}$; upper dielectric substrate: $20 \,\mu\text{m}$, ε_r : 10.2; w: $200 \,\mu\text{mm}$; circle dots: Pramanick et al. results [8] for lossless microstrip lines).

Figure 3 shows the relation of characteristic impedance versus frequency computed by using the definition of voltage-current. The voltage is defined as the path integral of the electrical field E_y on the y axis from the strip center to the ground plane and the current is the longitudinal current flowing through cross section of the strip. When the thickness t becomes zero, our result is validated by the result from Cano et al., [6] again. The figures show that when the thickness of the thin metallic substrate becomes larger, the characteristic impedance decreases. This behavior is very like the one when directly increasing the conductivity of silicon layer in MIS structure. The thicker this metallic substrate is, the more current flows through its cross section. Or equivalently, the impedance of this thin metal becomes smaller. When t becomes PEC, the impedance become pure resistance with the smallest value as shown (circle dots) in Fig. 3(a).

3.2. Very Thin Metallic Substrate in Lossless System

As demonstrated before, the slow-wave is a comprehensive effect due to the influence coming from the lower lossy silicon substrate and this thin metal. To identify the influence of this thin metallic substrate only, a thin metal layer is inserted into a lossless microstrip line as shown in Fig. 4(b). The lossless dielectric material ($\varepsilon_r = 10.2$) is divided into two parts as 20 µm and 80 µm. The width of the metal strip line is 200 µm. The corresponding dispersion characteristics are calculated and shown in the Fig. 4(a) and 4(b). The similar patterns reoccur and accord with the previous figures and discussions. This also illustrates that the thin metal layer with finite conductivity in a lossless substrate can introduce the slow wave phenomenon at the low frequency range.



Figure 5: Frequency behavior of the (a) real and (b) imaginary parts of the characteristic impedance for the MIMI structure in Fig. 4 (using $Z_0 = V/I$ definition, circle and diamond dots: Pramanick and Bhartia's results [8] for lossless microstrip lines with 100 μ m and 20 μ m dielectric substrates repectively).

In addition the remarkable difference between the PEC and real metal shows that the slow wave exists even when the thickness is only small fraction of the skin depth. This is a phenomenon that the PEC cannot describe appropriately. It is concluded that this thin metallic substrate has great impact on the dispersion characteristics of microstrip lines.

In Figure 5, the real and imaginary parts of the impedance are also plotted. when t equals to zero, our result agrees with the Pramanick's. The imaginary parts of the impedance is relatively small compared with the real parts. At the same time, when the metallic substrate becomes thicker, the impedance converges to the PEC (diamond dots in the figure) faster at the higher frequency range. This is because the reduction of the skin depth with increasing frequencies makes the thin metallic substrate more likely act as the good conductor with infinity thickness.

4. Conclusion

The effects of a thin metal ground with finite conductivity on the dispersion characteristics of loss and lossless multilayer microstrip line have been examined using the rigorous spectral domain approach. The numerical results show that electromagnetic field can penetrate the metal layer and interact with layers underneath. It is found that the thin metallic substrate in both lossy and lossless cases has a great impact on the dispersion characteristics, such the propagation constant, attenuation constant and characteristic impedance of multilayer microstrip lines even the thickness is much less than the skin depth. If considering the signal phase constant or velocity, the thin metallic substrate with thickness greater than 2 to 3 times of the skin depth can be regarded as infinite thick. At the same time, if merely the attenuation considered, the thin metal with only one skin-depth thickness is enough to make the ground like infinite thick. The results show that, at the low frequency range, the thin metallic substrate can excite a slow wave in both lossy and lossless multilayer microstrip lines.

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Extraction of Chip Power Delivery Current and Activity Variations

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Abstract—It is extremely difficult to directly measure power delivery current on chip under operating conditions. We discuss an approach of extracting PDS current through measurement of impedance of power delivery system and voltage. Because impedance extraction, in turn, requires knowledge of power delivery current, we use a controlled process for which power delivery current can be predicted. The paper provides a method of current extraction for impedance measurement.

1. Introduction

Current on power delivery system (PDS) of a chip and activity variation are major indicators of chip and package performance. Knowledge of PDS current is also important to assess worst case power delivery noise [1]. Current on a functioning die is generated by chip operation. Therefore it cannot be measured directly. The only exception is the case when chip activity is unchanged (for example, when the only process on chip is clock operation). In this case PDS current averaged over clk period is DC current which can be measured directly in any accessible location of power delivery loop far from chip. Because in general on-chip current is AC we need to look for indirect methods of current extraction. Because measurement of PDS voltage is easy to perform on a functioning chip, PDS current can be extracted from measured PDS voltage if PDS impedance of a chip is known [2]. There are two major obstacles preventing from measurement of PDS impedance on a functioning chip. First, because chip and package are not separated, a longer interconnect is required to allow connection with the probe. On-die PDS impedance is very low, in milliohm or sub-milliohm range, which is thousands time less than impedance of connecting wires. The second obstacle is that to extract impedance one needs to know both PDS voltage Vcc and current Icc, so we return to the problem of current extractions. In conventional methods involving VNA [3], current is injected into the system and can be measured directly. Although an attempt to extract PDS current on operating chip using PDS impedance stumbles, looping back to the problem of measurement of PDS current, it has an advantage: Instead of extracting PDS current for an arbitrary unpredictable computer process we can now focus on a particular controlled process of our choice because impedance of PDS does not depend on variations of chip activity.

The paper discusses an approach of extracting PDS current through measurement of PDS impedance and voltage. For impedance measurement we consider a controlled process for which PDS current can be predicted. The paper provides a method of PDS current extraction for impedance measurement and considers a methodology of extraction of PDS current and switching activity variations for any computer process.

2. Choice of Controlled Computer Process for Impedance Measurement

The controlled computer process must enable one to measure a magnitude of current, and since only direct on-die current can be measured the computer process has to contain long intervals with unchanged chip's activity. The computer process should also be simple because to extract impedance and current one needs to use Fourier transform and solve de-convolution problem which is sensitive to noise. Therefore the computer activity profile must be as simple as possible to avoid impedance to be dumped by noise. In [4-7] we suggested to use a computer process in which chip activity changes step-wise. The step-wise computer process is a good candidate to meet the above requirements. This process has a wider bandwidth than any continuous process. Several methods of generation of step-wise computer activity have been presented. For EV7 microprocessor measurements we used a specially designed computer code [4]. Advantage of this method is that it does not require any special hardware arrangements. We also used a method in which step-wise activity was generated by toggling clock frequency between two levels [5, 6]. In this method the profile of chip activity is far less noisy than the computer code method. A particular case of clock toggling is switching clock on and off [7, 8].

3. Extraction of PDS Current for Step-wise Computer Process

The rise/fall time limits the bandwidth of PDS current and impedance measurement. For activity step generated by toggling clock frequency the transition time is about one clock period. For step-wise process we need to make an assumption about current behavior in the vicinity of the transition period. In [4–6] it was assumed that current is unchanged beyond transition and changes linearly within the transition interval (trapezoidal current). This assumption was adopted in other applications of this approach. [7,8]. However, more detailed consideration of the current behavior shows that the mean PDS current (averaged over the clock period) is not trapezoidal. This can be derived from the correlation between the mean PDS current *Icc* and mean voltage on PDS Vcc of a chip:

$$Icc = C * F_{cl} * Vcc, \tag{1}$$

where F_{cl} is the clock frequency and C is the effective "switched" capacitance, which depends on the number of switching gates at a given state. According to Eq. (1), PDS current is not trapezoidal because voltage profile is not trapezoidal. From measurements it is known that voltage response to step-wise changing activity (step response) has a rippled profile after transition, exhibiting one or more droops. This means that, PDS current may not contain long unchanged intervals far from transitions, which are needed to provide us with reference points for current measurement. Therefore even for a simple, step-wise, change of chip activity PDS current has a complicated profile and there is no sections on current profile for which current can be measured directly. To resolve the problem, note that according to Eq. (1) chip can be formally characterized by an equivalent conductance G:

$$G = C * F_{cl}.$$
(2)

Because effective capacitance and/or clock frequency change when computer activity changes, for a computer process where chip activity toggles between high and low levels, the equivalent conductance will also toggle. We prefer to use conductance instead of resistance, because the transition time for conductance, unlike for resistance, properly represents the bandwidth of the process. We will use a representation of a chip through variable equivalent conductance to extract PDS current for the step-wise process. Figure 1 shows a simplified equivalent circuit for the power delivery loop.



Figure 1: Simplified equivalent circuit of power delivery system.

The right hand side shows a chip represented by variable equivalent conductance. On the left hand side is a PDS block, which includes power delivery network on PCB, package and die. It also includes a DC voltage source. For the right-hand side, we can represent current as

$$Icc = G * Vcc \tag{3}$$

which is Ohm's law in time domain. This equation allows one to determine PDS current for a step wise process if variable conductance is known. Fourier component of PDS impedance Z can be determined from the convolution equation valid for PDS block in Figure 1. It is presented in the form of Ohm's law in frequency domain:

$$Z = -Vcc(f)/Icc(f), \tag{4}$$

where f is noise frequency. Eq. (4) is not defined at zero frequency. Note that we independently use Ohm's law twice to determine both PDS current and impedance. In this solution the problem of extraction PDS current is reduced to the problem of extraction variable equivalent conductance of the chip for a step-wise computer process. Because equivalent conductance changes step-wise, we need to know its high and low levels. We measure them separately in two independent computer runs, in which the only process running on-chip is a process with unchanging activity on either high or low level which will be used later on in activity toggling. In each run we simultaneously measure the mean voltage between power and ground on chip and the mean current. We cannot measure current on-chip, so we measure it far from chip, on voltage regulator, because current is unchanged. Each conductance level is determined as a ratio of measured current and voltage.



Figure 2: Typical measured raw Vcc (black) and Vcc after noise is filtered out (white).

It is important to accurately measure Vcc profile to avoid noise in impedance profile, because current for any computer process is determined using division by impedance. Most of the noise is a random noise which can be excluded by averaging over many Vcc waveforms. Regular noise can be identified by changing clock frequency or measurement setup [6]. Figure 2 shows a typical measured raw Vcc (black trace) and Vcc after noise is filtered out (white trace).

In the methodology we have just discussed we measure directly only PDS voltage on-die and current on voltage regulator. With these measurements no calibration and compensation of parasitics is required which is a nightmare for traditional methods of low impedance measurements using VNA or impedance analyzers. Note also that by solving the problem of *Icc* extraction, we actually eliminated the problem of very long interconnect in measurement of very low impedance.

4. Comparison of Different Methods of *Icc* Extraction for Step-wise Process

Consider differences between impedance determined through variable conductance representation of a chip, and assuming trapezoidal current. We do not expect difference in impedance resonance frequencies because in variable conductance case current ripples follow Vcc ripples, so resonance frequency is the same as for Vccresonances for both assumptions. However, resonance peaks may be different, because Icc varying in accord with voltage variations can make impedance resonances more pronounced than in the case of trapezoidal current. Figure 3(a) shows voltage and current waveforms in time domain for measurement simulation made for a Spice model of a future generation chip. One can see that Vcc has an overshoot, and it rapidly returns to its value before transition. This system has a high Q-factor. Figure 3(b) shows voltage and current in frequency domain. We can see that both voltage and current for "variable conductance" case have pronounced resonances. The trapezoidal current does not have resonances.



Figure 3: PDS current and voltage for measurement simulation in time (a) and frequency (b) domains.



Figure 4: PDS impedance for measurement simulation.

Figure 4 shows impedance of this chip for the two cases. One can see that assuming trapezoidal current on-chip (white trace) we get to a significant, 3-fold, underestimation of the resonance impedance.

Consider measurement results for EV7 microprocessor. This system has a 3-fold higher decoupling capacitance and an order of magnitude higher resistance in the decoupling loop then in the previous case. Figure 5(a) shows that Vcc after the first droop restores slowly so that the noise at maximum is still half of the noise at the first droop minimum. The Q-factor for EV7 chip is lower than in the previous example. Figure 6 shows PDS impedance of the EV7 chip, extracted using the two assumptions on Icc. There is no significant difference between the results obtained using both methods. Figure 5(b) provides an explanation. It shows Fourier components of measured Vcc and the two currents. The voltage resonance at 60 MHz is shallow, so that the respective current resonance for "variable conductance" case is hardly noticeable on the steeply declined current profile. Figure 7 shows the relative difference between values of impedance determined by the two methods plotted versus Q-factor. The difference goes up linearly with the increase in Q-factor.



Figure 5: PDS current and voltage for EV7 chip in time (a) and frequency (b) domains.

5. Extraction of PDS Current for any Computer Process

Once PDS impedance on chip is known one can extract PDS current for any computer process, not only step-wise process, by using a de-convolution procedure. Maximum ΔIcc can be obtained running extreme processes with maximum activity changes (power viruses). To extract the current signature we measure PDS voltage, and apply FFT to convert Vcc to frequency domain. Dividing voltage by known impedance we can obtain PDS current in frequency domain. Then we apply inverse Fourier transform in order to convert PDS current from frequency to time domain. Since PDS impedance is not defined at zero frequency, the de-convolved PDS current is determined up to an additive constant and a reference current is required to determine Icc. The way to obtain the reference current is to measure Icc at voltage regulator simultaneously with measurement of



Figure 6: PDS impedance for EV7 chip.

Figure 7: Relative difference in PDS impedance.

Vcc on-die. This current is averaged over time and is used as a reference for the averaged de-convolved PDS current. Figure 8 shows a fragment of measured Vcc on EV7 chip running the power virus SWIM, and Figure 9 shows the de-convolved PDS current on-chip.

6. Extraction of Activity Variations

We used a term "chip activity" to qualitatively characterize a computer process. Equivalent conductance can serve as a quantification of the chip's activity. It best represents the chip activity because supply current is consumed through switching of on-chip capacitive loads. Changes in activity result from changes in the magnitude of the capacitive loads or from changes in the frequency at which the capacitive loads are switched. Because equivalent conductance variation depends on the same changes, the variance in switched activity is equivalent to a variance in chip equivalent conductance. Actually, chip's activity rather than chip's PDS current can be considered as a stimulus producing PDS noise. The chip's current is less appropriate because it is affected by the *Vcc*. Therefore, for PDS noise characterization we need to know ΔG rather than ΔIcc . We can determine variations of the chip's activity or equivalent conductance dividing measured PDS current by voltage in time domain. Equivalent conductance variation for EV7 chip is shown in Figure 10. From Figure 10 one can extract maximum ΔG , knowledge of which is necessary to obtain the absolute maximum PDS noise for a chip.





Figure 8: *Vcc* on EV7 chip running power virus SWIM.

Figure 9: Icc on EV7 chip running code SWIM.



Figure 10: Equivalent conductance variation on EV7 chip running power virus SWIM.

7. Conclusion

There are no direct methods of measurement of PDS current on functioning die. Current can be extracted indirectly from measured Vcc if PDS impedance is known. Measurements of PDS impedance, in turn, requires knowledge of PDS current. The solution we suggested is to generate a step-wise computer process and predict PDS current for this process using equivalent conductance representation of a chip. Equivalent conductance can serve as a quantification of chip switching activity. PDS current and chip activity for any computer process can be determined by de-convolution involving measured Vcc and known PDS impedance. The measurement are easy to perform because only Vcc on-die and current on voltage regulator are measured directly so we can avoid calibration and compensation that are most challenging parts for conventional measurement techniques. The proposed methodology is the only available methodology for measurement PDS current and activity variations on a functioning chip

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Principles of Synthesis of Steerable Reflect-array Antennas

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Abstract—The synthesis of steerable reflect-array antennas, as a synthesis of any antenna, should be realized by solving two problems: external antenna problem and internal antenna problem. The first one includes the investigation of the antenna radiation pattern. The second one concerns a design of the structure providing the required amplitude-and-phase distribution along the antenna aperture. The first problem is based on the theory of antenna array. The distance between the small reflectors and geometry of the array are responsible for the directivity, the beam width, and the side lobe level of the antenna. The internal antenna problem consists of finding the phase shift required for each small reflector. Remarkable feature of a tunable reflector made as a microstrip vibrator or a patch in combination with a tunable device is that the almost 360° phase shift of the reflected wave can be provided with only one tunable device (varactor diodes or ferroelectric tunable capacitors). Commonly, simulation of the phase shift required is performed with a numerical technique. The design of reflect-array antenna can be sufficiently simplified, if the simulation by the numerical technique is amplified with a correct analytical model.

1. Introduction

Reflect-array antennas are being developed during many years [1–3]. Recently the reflect-arrays were suggested as structures with an electronically steered radiation pattern [4, 5]. Such a steerable reflect-array antenna can be used as a low cost version of a phased array antenna for a wide commercial application. The synthesis of steerable reflect-array antennas, as a synthesis of any antenna, should be realized by solving two problems: external antenna problem and internal antenna problem. The first one includes the investigation of the antenna radiation pattern. The second one concerns a design of the structure providing the required amplitude-and-phase distribution along the antenna aperture. Any reflect-array antenna consists of primary radiator or illuminator and a reflecting surface. The primary radiator provides the reasonable amplitude distribution with a minimum spillover loss. Sophisticated design technique was used to diminish a constructive space occupied by the radiator [6]. The reflected surface is covered by a large number of small reflectors in form of microstrip vibrators or patches. The vibrators or patches are connected with tunable devices (varactor diodes [4] or ferroelectric tunable capacitors [7]), which serve for controlling the phase of the wave reflected by each small reflector. The goal of this paper is to characterize the main stages of synthesis and design of a steerable reflect-array antenna.

2. External Antenna Problem

The scheme of a typical reflect-array antenna is shown in Fig. 1. The system of patches provides transformation of a spherical wave front of the primary radiator into the plane wave front in the antenna aperture. The main beam width of an antenna and the directivity of the antenna, which are required, determine the size of the antenna aperture.

In Fig. 2, radiation pattern of a circle aperture with homogeneous field distribution as a function of a generalized angle function u is shown [8]. $(u = kR \sin \theta, k = 2\pi/\lambda)$, where λ is wavelength is free space, R is radius of the circle aperture). In the case of homogeneous field distribution the main beam width in degree is $\Delta \theta^{\circ} = 59 \cdot \lambda/2R$ and level of the first side lobe is -18 dB. The antenna directivity is $D = 4\pi (\pi R/\lambda)^2$. If the field distribution decays to edge of the aperture, the main beam width is higher and the directivity is lower, the first side lobe level being decreased.

Forming the radiation pattern is drastically influenced by the inhomogeneity of the phase distribution over the radiating aperture. In the case of the reflect-array antennas the phase inhomogeneity can be provoked by inaccuracy of realization of size and position of the elementary radiators (patches). Let us consider a statistical estimation of the inaccuracy of the array realization [9].

The array radiation pattern can be presented as follows:

$$\Phi(\theta,\varphi) = \sum_{i=1}^{m} A_{0,i}\varphi_i(\theta,\varphi) \tag{1}$$



Figure 1: Scheme of a typical reflect-array antenna with: ground plane (1), dielectric layer (2), patch reflector (3), spherical wave front (4), plane wave front (5), primary radiator (6).



Figure 2: Radiation pattern of a circle aperture as a function of generalized angle function u.

where $A_{0,i}$ is the optimized current amplitude of *i*-th elementary radiator, $\varphi_i(\theta, \varphi)$ is the pattern of *i*-th elementary radiator taking into account the position of the center of its phase pattern, *m* is the number of elementary radiator in the array.

If the real current distribution differs from the optimized one

$$A_i = A_{i,0} + \Delta_i,\tag{2}$$

the mean-square-error of amplitude/phase distribution over whole array is

$$\beta_{mse} = \left| \sum_{i=1}^{m} |\Delta_i|^2 / \sum_{i=1}^{m} |A_i, 0|^2 \right|^{1/2} .$$
(3)

The value β_{mx} can be used to find decay of the array directivity (by the factor g) and increase of the side lobe by ξ_{sl} : 1 $3\beta_{mx}$

$$g = \frac{1}{1 + \beta_{mx}^2}, \qquad \xi_{sl} = \frac{3\beta_{mx}}{\sqrt{m}} \tag{4}$$

Let us suppose for example: m = 2000, $|A_{i,0}| = 1$ for all i, 30% of radiators are characterized by the phase error of 90°. Simple calculation gives $\beta_{mse} \approx 0.85$, g = 0.58 (decrease of the directivity in 2.4 dB), $\xi_{sl} = 0.06$ (increase of the side lobe level up to -12 dB).

3. Internal Antenna Problem

The primary radiator provides feed of the reflector patches with amplitude distribution required and with a minimum spillover loss. The efficiency of the primary radiator can be determined by the following equation:

$$\eta(\rho,\gamma) = \int_{0}^{\alpha(\rho)} [F(\theta,\gamma)]^2 \sin\theta d\theta / \int_{0}^{\pi/2} [F(\theta,\gamma)]^2 \sin\theta d\theta, \qquad F(\theta,\gamma) = [\cos(\theta)]^y \tag{5}$$

where $\rho = F/2R$, $\cot(\alpha(\rho)) = 2\rho$, F is the focal distance of the patch reflector, R is the radius of the aperture. $F(\theta, \gamma)$ is the radiation pattern of the primary radiator, the exponent γ determines the directivity of the primary radiator.

Table 1 illustrates the efficiency of the primary radiator expressed in dB. The data presented are followed by the conclusion that the preferable values of F and γ are $F \leq R, \gamma \geq 1$.

The phase of waves reflected by the patch mirror has to meet two principal demands: 1) Transformation of spherical wave front given by the primary radiator into a plane wave, 2) Providing phase gradient along the array, which corresponds to the beam deflection required.

The distance between the patches lies in the range $s = (0.63 - 0.67)\lambda$. If the linear size of the array (2R) is much higher than the wavelength (λ) , the total phase shift change along the array can be much higher than 360°. In this case the phase distribution is corrected by reset of the phase by n times 360° where n = 1, 2, 3, ... Such a phase correction is well known as a characteristic feature of Fresnel mirror.

ρ	0.2	0.3	0.4	0.5	0.6
$\gamma = 1.5$	-0.09	-0.31	-0.71	-1.21	-1.82
$\gamma = 1.0$	-0.23	-0.63	-1.20	-1.90	-2.60
$\gamma = 0.5$	-0.65	-1.22	-2.18	-3.00	-3.92

Table 1: The primary radiator efficiency $\eta(\rho, \gamma)$ in m dB.





Figure 3: The phase shift of wave reflected by a patch as a function of length and width of the patch in mm for dielectric constant of the substrate $\varepsilon_{sab} = 1.06$.

Figure 4: Distribution of the patch positions and sizes in array.

The phase shift of the wave reflected by a patch depends on the patch size [1-4, 6]. That is illustrated by Fig. 3 for the operational frequency f = 10 GHz. Commonly, simulations of the required phase shift are performed with a numerical technique. The result of the phase shift simulation is used for designing the reflect-array antenna. In photo (Fig. 4) one can see the distribution of the patch sizes over the mirror array.

4. The Steerable Patch Array

The phase shifts (Fig. 3) and an appropriate design of the patch array (Fig. 4) correspond to a nonsteerable array with a fixed position of the main beam. In order to control the position of the main beam of radiation pattern, the phase shift of the reflection coefficient of each patch in the array should be controlled. The tunable devices (semiconductor varactor diodes [4] or ferroelectric tunable capacitors [7]) should be included in each patch. The state of the tunable device serves for controlling the phase of wave reflected by each small reflector. Remarkable feature of a tunable reflector made as a microstrip vibrator or a patch in combination with a tunable device is that the 360° phase shift of the reflected wave can be provided with only one tunable device. It should be reminded that for a realization of a digital transmission-type phase shifter one needs at least 8 tunable devices [9]. The optimum phase shift of each tunable reflector must be found as a result of a correct simulation and can be realized by applying to each tunable device the appropriate biasing voltage. The result of the phase shift simulation must be included in the driving program of the biasing voltage controller.

The problems mentioned above can be sufficiently simplified, if the simulation by the numerical technique is replaced by using a correct analytical model. The problems mentioned above can be sufficiently simplified, if the simulation by the numerical technique is replaced by using a correct analytical model. A scheme of a tunable patch is shown in Fig. 5. The sketch drawn in Fig. 5(a) presents a single patch located in a virtual waveguide confined by electrical and magnetic walls. The patch is considered as a microstrip vibrator loaded by the tunable capacitor. The microstrip is formed on a dielectric substrate with a conductive ground plane. Microwave current in the vibrator is induced by the incident wave. The current distribution along the vibrator



Figure 5: Scheme of a tunable patch. a) Single patch located in a virtual waveguide confined by electrical an magnetic walls and b) Equivalent schematic.



Figure 6: Simulations of the reflected wave phase shift in the framework of the schematic analytical model.

is found on a basis of solution to telegraph equations using the method of induced electromotive forth [8]. The equivalent schematic is shown in Fig. 5(b). Two type of resonances can be observed in the circuit shown in Fig. 5(b). Firstly, fundamental resonance, which corresponds to infinite impedance of the vibrator, which is presented by the transmission line stub with length l and characteristic impedance Z_P . In this case one observe the reflection from the ground plane through the substrate; the phase of the reflection coefficient is near to $\pm 180^{\circ}$. Secondly, anti-resonance, which corresponds to parallel resonance of two transmission line stubs (l, Z_P) and (H, Z_0) . In this case one observes the zero phase of reflection from the plane, in which the vibrator is located. Thus, change of capacitance of the tunable capacitor makes possible to obtain the change of the reflection phase approximately in the range +180...0...-180 degrees. In Fig. 6, results of simulations in the framework of schematic analytical model are presented [10]. The following data were taken: the square virtual waveguide $20 \times 20 \text{ mm}^2$ with the substrate H = 1 mm, $\varepsilon_S = 3.0$; dimensions of the patch: w = 2 mm, 2l = 9 mm, operational frequency f = 9.5 - 10.5 GHz.

Simulations based on the schematic analytical model can be used for the design of a steerable reflect-array and for developing a driving program of the biasing voltage controller. Some fitting parameters of the analytical model can be found using a comparison with the full-wave analysis simulation.

5. Conclusion

The development of simple and correct theoretical models and elaboration of material components of reflectarray antenna is an urgent problem, which solution is important for realization of a cheap steerable antenna for mass production.

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Design of a Steerable Reflect-array Antenna with Semiconductor Tunable Varactor Diodes

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Abstract—The demonstrator of a steerable reflect-array antenna was designed as a system of dipoles loaded by varactor diodes. The microwave response of a dipole loaded by varactor has been simulated in closed form based on equivalent circuit approach. The circuit analytical model has been verified by the full-wave analysis. Change of the varactor capacitance in the range of 0.3–1.3 pF was provided by biasing voltage 0–20 V. The array consists of 20 dipoles structured as two parallel lines. The operational frequency is 11 GHz, the length of the dipole is 9.2 mm, spacing between dipoles is 18 mm. The double-side metallized PTFE with $\varepsilon = 2.8$ and thickness of 1 mm was used as a substrate. Dipole structure was manufactured by a photolithographic process and formed with surface mounted varactor diodes. The radiation pattern of the array is characterized by the width of the main beam $\approx 8^{\circ}$, the side lobe level - $(12 \div 20) \text{ dB}$, the steering range $\pm 15^{\circ}$. Control voltage was set manually with variable resistors separately for each varactor. The fine alignment of the control voltage for each varactors turned out to be very important. Inherited data are used for correction of the operational principle of a varactor steerable antenna controller.

1. Introduction

The reflect-array antennas are being studied many years and some theoretical and experimental results have been obtained [1–3]. The possibility to obtain an electron steering of the radiation pattern of such an antenna is currently under investigation [4,5]. The goal of this paper is to discussed an experimental realization of a steerable reflect-array antenna demonstrator designed as a system of dipoles loaded by varactor diodes. The GaAs varactors MA46H070 produced by MACOM Inc. were tried. Control voltage applied to each varactor in the array should provide the phase shift along the dipole structures, which is necessary for transformation of a spherical phase front of a prime radiator into the plane phase front with the required declination. One should simulate the distribution of the phase shift of reflection coefficient of each dipole and find the dependence of the phase shift on the biasing voltage applied to the varactor loading the dipole. A set of error sources complicates solution to the problem. The following errors should be taken into account. 1) Incorrectness in simulation of the phase shift of reflection coefficient as a function of the dipole sizes and value of the varactor capacitance. 2) Dispersion of the dependence of the varactor capacitance on the applied biasing voltage. 3) Fabrication errors in dimensions of the design components.

In order to overcome difficulties mentioned above, the experience of designing and examination of different version of steerable reflect-array antenna should be accumulated and used for developing the design procedure.

2. Design of the Demonstrator

In Fig. 1 structure of the array under investigation is shown. The array consists of 20 dipoles structured as two parallel lines. Spacing between lines and spacing between dipoles in the lines are 18 mm. Each dipole is loaded by a tunable varactor. The dipole length is 9.2 mm, The doubleside metallized PTFE with $\varepsilon = 2.8$ and thickness of 1 mm was used as a substrate. A *dc-rf* filter and *dc*-biasing strip lines are arranged for each dipole. Fig. 2 shows the equivalent diagram of the bench, which was used for formation of the control voltages. The control voltages were set manually with variable resistors separately for each varactor. The fine alignment of the varactor tuning was find to be very important. That can be explained by a sharp dependence of the phase of the dipole reflection coefficient on value the biasing voltage. Fig. 3 illustrates this dependence, which was simulated for the dipole considered. Fig. 4 shows the derivative of the reflection coefficient phase with respect to the biasing voltage as a function of the biasing voltage. One can see that at the section of curve (Fig. 3) with the highest slope, the setting the phase shift with accuracy $\pm 45^{\circ}$ requires the accuracy of setting the biasing voltage better than ± 0.25 V. That relates not only to the accuracy of the biasing voltage control, but to repitability of the varactor Volt-Farad characteristics.



Figure 1: Structure of the array under investigation. The array consists of 20 dipoles structured as two parallel lines. The total length L = 200 mm.



Figure 3: Dependence of the phase of the dipole reflection coefficient on biasing voltage.



Figure 2: Scheme for formation of the control voltages. The voltages applied to varactors are separately registered.



Figure 4: Derivative of the reflection coefficient phase with respect to the biasing voltage.

3. Some Experimental Results

The prime radiator was placed in a distance from the surface of the array of 100 mm. The open end of a rectangular waveguide with the transverse section $23 \times 5 \text{ mm}^2$ was used as the prime radiator.

The required phase distribution along the array was simulated in closed form based on equivalent circuit approach. The adequacy of the circuit analytical model has been verified by the full-wave analysis. The simulation of the required phase distribution was followed by the simulation of distribution of the biasing voltage applied to all varactors.

Performance of the radiation pattern of the array at the frequency 11 GHz is shown in Fig. 5. Three positions of the main beam are shown: -10° ; 0° ; $+10^{\circ}$. The amplitudes of the radiation pattern in all positions are practically the same. While measuring the radiation pattern, the biasing voltage distribution was slightly corrected to obtain maximum of the signal.

The gain of the antenna was not measured, because the efficiency of the prime radiator had not been optimized.

Fig. 6 gives the amplitude-frequency response of array in the center position of the main beam. The frequency band of the antenna is about 2% at the level of -3 dB. Such a narrow frequency band can be referred to a high quality factor of a resonant tank formed by the dipole and the tunable varactor. It may be assumed that the reactive parameters of the pair dipole-varactor can be optimized and the frequency band can be extended.



Figure 5: Radiation pattern of the steerable reflectarray (three positions).



Figure 6: Amplitude-frequency response of the steerable reflect-array.

4. Conclusion

A steerable reflect-array antenna demonstrator was experimentally realized. The results of the demonstrator investigation are in agreement with the theory of the steerable antenna arrays [6,7]. That gives possibility to make a confident conclusion that the steerable reflect-array antenna based on application of the tunable varactors can be designed and manufactured. Such an antenna can be offered as a cheap version of steerable antenna for a mass production.

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Modeling of Low-profile Reflect Array Antenna

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Abstract—The theoretical analysis of a planar reflect-array antenna consisting of a rectangular microstrip patch radiators is presented. Such an antenna is to be designed to convert spherical wave radiated by feed horn antenna into plane wave by phasing of reflected wave due to adjusting the patch sizes and arranging them by principle of Fresnel mirror. The modelling of array antenna is based on the modelling of elementary equivalent waveguide cell consisting patch radiator at the interface between superstrate and grounded substrate layers. Spectral Domain Approach (SDA) of Method of Moments is used to analyse the characteristics of elementary waveguide cell. Theoretical and experimental results are compared.

1. Introduction

Microstrip reflectarray antenna exploits operational principles of traditional parabolic reflector antenna and microstrip patch phased array [1–4]. Such a combination allows eliminating two disadvantages of both standard antennas. For microstrip patch antennas there is a common difficulty to overcome a 30 dB gain limit of phased array because of lossy feeding network. The conventional high-gain antennas are the parabolic reflectors. Being the very efficient radiators they are bulky and massive. A flat microstrip refletarray is being developed as a compact high-gain antenna [2]. Low loss is conditioned by the sizes of most radiators are far from ones of resonant half-wavelength radiators (used usually with transmission lines for phase adjustment) as well as by absence of feeding networks. Feeding of the printed radiators is realized quasi-optically.

Such an antenna is to be designed to convert spherical wave radiated by feed horn antenna into plane wave by phasing of reflected wave due to adjusting the patch sizes and arranging them by principle of Fresnel mirror.

The basic configuration of antenna includes a feed horn antenna and a printed reflectarray. Rectangular patches arranged in a planar aperture based on metal backed substrate will reradiate illuminated energy into space. Each radiator's phase is adjusted to make total reradiated field cophasal and concentrated in a specific direction. The phasing method is to use a variable size patches to form the front of reflected wave.

A folded version of such reflectarrays has been proposed and realized [1]. The dual polarization properties of rectangular patch array enable to remove the feed element from the focal point in front of antenna and to place it at the backside of antenna with polarizing grid for reradiation.

In present paper we describe a code developed for the design of multilayer printed reflectarrays, which adjusts the sizes of patches to achieve a progressive phase for dual linear polarisation according to both twisting and focusing requirements.

2. Theory

To build a procedure for calculation of a reflection coefficient of linear polarized wave normally incident on a patch radiator we assume that no coupling between adjacent patches takes place. Such a situation is valid for normal incidence on infinite periodical array and gives us a reasonable approximation for most part of finite nonperiodic array if the distance between adjacent radiators is big enough.

Figure 1 illustrates the normal incidence of the plane wave on a single microstrip patch. With two pair of opposite perfect electric and magnetic walls corresponding to the case of non-interacting radiators we can consider the whole structure as a stack of elementary TEM (in z direction) waveguides. Due to partial filling of such a waveguide with dielectric substrate, air superstrate and current carrying layers an existence of pure TEM mode represents another assumption and such a waveguide could be called as a quasi-TEM waveguide. To calculate the reflection coefficient of fundamental TEM mode we use a standard Spectral Domain Approach of Method of Moment.

General relation between electric fields in the plane of microstrip radiator is

$$\tilde{\bar{\mathbf{E}}}_{tot} = \bar{\bar{\mathbf{G}}} \cdot \tilde{\bar{\mathbf{J}}} + \tilde{\bar{\mathbf{E}}}_{inc} (1 + \bar{\bar{\mathbf{\Gamma}}})$$
(1)

where $\tilde{\mathbf{E}}_{tot}$ is vector of total tangential E-field (superposition of scattered and incident field), $\tilde{\mathbf{\bar{G}}}$ is Green's dyad for elementary waveguide, $\tilde{\mathbf{J}}$ is surface current density excited in microstrip dipole, $\tilde{\mathbf{E}}_{inc}$ is vector of incident



Figure 1: Equivalent elementary wavegude.

linear-polarized along the microstrip radiator side and $\overline{\overline{\Gamma}}$ is a tensor of reflection coefficients. Tilde denotes operating in Fourier domain. $\tilde{\overline{\overline{\Gamma}}}$ can be easily derived by analogy with determination of Green's dyad using standard immitance approach.

To approximate unknown current density one has to expand both components of $\bar{\mathbf{J}}$ by series of chosen basis functions.

Assuming that polarization of the incident wave is parallel to one side of rectangular patch, along y-direction in instance, we can suppose that y-polarized electric field excite only y-component of current density:

$$\bar{\mathbf{J}} = J_y \cdot \bar{\mathbf{e}}_y \tag{2}$$

Then equation (1) transforms to scalar one:

$$\tilde{E}_{y,tot} = \tilde{G}_{yy}\tilde{J}_y + \tilde{E}_{y,inc}(l + \tilde{G}_{yy})$$
(3)

where

$$J_y = \sum_n A_n \varphi_n(x, y) = \sum_n A_n \varphi_n^x(x) \varphi_n^y(y)$$
(4)

In order to take into account a priory known current density distribution with zero value at edges parallel to electric walls and singularity at the edges parallel to magnetic walls, we suggested a set of following separable expanding functions:

$$\varphi_n(x,y) = \begin{cases} \frac{\cos(\frac{4n\pi x}{w})\sin(\frac{(2n+1)\pi}{l}(\frac{l}{2}-y))}{w\sqrt{1-(\frac{2x}{w})^2}} & |x| \le \frac{w}{2}, |y| \le \frac{l}{2} \\ 0 & |x| \ge \frac{w}{2}, |y| \ge \frac{l}{2} \end{cases}$$
(5)

Here l is the length of rectangular patch, i.e., the length of a side corresponding to the direction of polarization, w is the width, and n is the number of basis function.

In order to find the vector of unknown coefficient $\{A_n\}$, Galerkin's procedure has to be implemented with weighting functions the same as the expansion functions. Then the phase of reflected field can easily be found.

Modifying (1) by addition of electric field $\tilde{\tilde{\mathbf{E}}}_{tr} = R_s \cdot \tilde{\mathbf{J}}$ we can take into account the losses due to finite conductivity of patches by using of equivalent surface impedance R_s .

3. Result and Discussion

Figure 2 demonstrates the results of calculation of phase angle depending on length of patch for two different thickness of single substrate. In order to adjust the sizes of patches to achieve a progressive phase for dual linear polarisation according to both twisting and focusing requirements the calculation of phase for polarizations along the both side of patches has to be done. If the dimensions of the patches are chosen in such a way that the absolute difference between the reflection angle of the two perpendicular polarization is 180° then the polarization of the wave incident on the patch tilted by 45° with respect to polarization of incident wave will be twisted by 90° .



Figure 2: Results of calculation of phase angle depending on patch dimensions (a) Phase angle versus length of patch (different thickness of the substrate H), (b) Surface plot of phase angle in dependence on both dimensions of patch. Frequency 10 GHz, thickness of substrate H = 2 mm, dielectric permittivity 1.06, L = 21 mm, range of patch sizes 5–19 mm.

The phases of the reflected wave have to be attainable in the range of $0-360^{\circ}$. However the range of real array is, usually, less then 360 degree [4]. There are two opposite factor which influence the performance of antenna: the less the thickness of substrate and the bigger ε the greater range of obtainable phase, but the greater slope of curve and the higher technological requirements for patch size precision. Besides, for twisting reflectarray there is a gap of unreliazable phases. In Figure 3 the results of calculation of phase for bilayered substrate ($H_{upper} = 0.12$ MM (dacron, $\varepsilon_r = 3.2$,), $H_{lower} = 1$ MM (foam, $\varepsilon_r = 1.06$)) at 25 GHz are presented. In order to cover these two gaps about 55° each, one needs to use near values of patch sizes what will result in general error of phase approximately of 28°.

The validity of presented approach is confirmed by comparison with results of simulation by MS CST and Ansoft HFSS (Figure 4). Small discrepancy between data may be explained by more accuracy of methods of simulation based on FE and FDTD. However for acquisition of large arrays of patch dimensions the using MoM based code is more efficient.

Finally the developed code has been used for design of reflectarray antenna with diameter 300 mm operating at 25 GHz. More than 400 radiators were used to form a designed gain of 36 dB. Measured gain is around 32 dB, half-power beam width 2.7°, sidelobes are around 16°. Difference between measured and calculated characteristics can be partly explained by mentioned deviation of patch sizes to cover the unreliazable phase gap. Another reason of discrepancy is due to loss of some part of energy at the edge region of antennas plate. Moreover, the quasi-optical method to build a Fresnel reflector requires considering the dependence of phase angle also on angle of oblique incidence of wave on the edge patches.



Figure 3: Dependence of phase angle on patch dimensions.



Figure 4: Comparison of simulation by presented model and by using microwave simulation packages. $H = 0.787 \text{ mm}, \epsilon_r = 2.22, L = 6 \text{ mm}, \text{ range of patch sizes from } 0.6 \text{ to } 5.6 \text{ mm}, f = 25 \text{ GHz}.$

4. Conclusion

The design code based on the Spectral Domain Method of Moments (MoM) for lossy mutilayer periodic structures and normal incidence of a plane wave has been developed. Specific entire-domain basis functions have been proposed to achieve a high convergence and accuracy of MoM. Code developed is very efficient because it combines high calculation speed and high accuracy of full-wave analysis and is very promising for acquisition of large arrays of patch dimensions.

To validate the design method, a series of reflectarray antennas operating at different frequencies with and without twisting effect have been designed and manufactured [6]. A good agreement was obtained between predicted and measured radiation patterns for both polarizations. The measured gains were not less then 32 dB.

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Abstract—In the context of the development of the emerging Ultra-Wide Band (UWB) technology, the modelling and the analysis of the transient waveforms radiated and received in a communication link are proposed. The modelling developped, based on analytical expressions, allows to consider combinations of thin-wire elements to represent conductive antennas with a 2D or 3D geometry. The propagation channel has been modelled by the presence of a dielectric material in air. Detailed parametric studies have been performed, and optimization of the antenna characteristics has been addressed. This work appears as a synthesis and an extension of previous studies.

1. Introduction

In recent years, the Ultra-Wide Band (UWB) technology appears as an attractive solution for technical improvements in conventional narrow band wireless communication systems. Such a technology relies initially on the transmission of a series of baseband modulated (in amplitude or phase) short pulses (less than 1 ns) with a low duty cycle. Such signals have an energy which is spread thinly across the entire broadband spectrum (greater than 500 MHz with a fractional bandwidth of more than 20%), thus allowing the UWB technology to coexist with other wireless systems without licence. This technology offers several advantages such as: transmission at a high bit rate, low cost, low probabilities of interception and detection, separation of multipath... Therefore, several applications are being implemented including communications, networking, radar (GPR, vehicular, surveillance) imaging (medical, through-wall, construction materials, security, GPR), and measurement devices (sensors, positioning). Currently, the FCC has regulated UWB power levels emitted by defining a spectral mask in the -10 dB bandwidth between 3.1 and 10.6 GHZ in order to limit interferences with existent licensed wireless systems (WLANs, GPS). Efforts are under way by the IEEE community for standardizing the use of UWB systems in indoor (home and office) multimedia transmissions.

Because of the very wide frequency band of the transmitted signal, novel studies concerning the modelling and the characterization of the propagation channel and of the antennas have to be achieved; it appears that in such a case the studies are best performed in the time domain Therefore, we have focused our studies on the modelling and on the parameter analysis of the physical phenomena involved in the transient radiation, propagation and reception of a pulse signal in a simplified multipath propagation channel including a dielectric sample in air. The modelling tool uses extended analytical expressions, in the transmission and reception configurations, to describe the responses of conductive antennas made of a combination of infinitely and loaded linear thin-wire elements, representing several 2 D or 3 D antenna geometries (V-dipole, bow-tie, butterfly, TEM horn...). Such an original tool which includes extended developments of previous work allows to highlight the pulse-shaping process in the spatio-temporal domain [1, 3, 5].

2. Transient Responses of Thin-wire Elements

The basic transmission link is composed of single transmitting and receiving antennas separated by a distance d in the far-field zone of each other over the operating frequency bandwidth; the major component of the radiating field $E_{\theta}(r, \theta, \phi)$ is oriented along the direction $\vec{\theta}$. The antennas do not necessary face each other and can be tilted. The transmitting antenna is positioned at the origin 0, and two angles β and ϕ define the position of each wire element to axis Oz and axis Ox respectively. The propagation channel is modelled by the presence of a dielectric material with large dimensions which has been placed in the air between both antennas; thus, diffraction does not occur at the edges. The link modelled is presented in Figure 1 with the parameters associated with the frequency domain to highlight the presence of the several frequency-dependent impedances involved in the input and output circuits.

The modelling approaches described in this section concern in transmission symmetric straight dipoles (-L, L), and in reception symmetric V-dipoles (-L, L). In transmission, as the expressions allow to distinguish the radiated components of the electric field issued from each monopole, the extension of the modelling to a V-dipole can be easily deduced by tilting each arm. In reception, it appears more natural to model the physical phenomena involved in each arm individually. From the analytical expressions developed, other dipole orientations can be considered [2].



Figure 1: Geometry of the link between two V-dipoles with their parameters in the frequency domain.

2.1. Transmission Configuration

The straight dipole (-L, L) aligned with axis Oz is supposed to be excited at the feed point z = 0 by an impulse current which has generally the shape of the Gaussian function or one of its successive derivatives. The *p*-th derivative g(t) of the Gaussian function is expressed by:

$$g^{(p)}(t) = \frac{d^p}{dt^p} (A_0 e^{-(\frac{t}{\tau})^2})$$
(1)

where τ represents the characteristic time of the pulse, and A_0 its amplitude. The duration w of each signal has been defined as the time interval centered on the pulse shape and containing 99% of the total energy of the pulse. For example, $w = 3.3\tau$ for p = 0, $w = 4.08\tau$ for p = 1, and $w = 4.5\tau$ for p = 2. In general, τ has been set in order to fit a number n not necessarily an integer of pulse durations in each arm length of the dipole. Thus, we defined the reference time $\tau_a = L/c = nw$, where c represents the velocity in the air.

In the case of an infinitely conductive dipole, the initial current wave $I_s(t - |z/c|)$ which propagates along both arms undergoes a total reflection (with a coefficient -1) as it reaches both terminations z = L and z = -L, and a partial reflection at the feed point (denoted ρ). The back and forth propagation process continues until the traveling current undergoes a complete attenuation [1,3]. Such a ringing effect produces narrow frequency radiated waveforms, and usually the objective is to reduce it as much as possible. In general, the dipole radiation is produced at the positions z = 0, z = L, and z = L. In the far-field zone, the main electric field component $E_{\theta}(r, \theta, t)$ in the spherical coordinates (r, θ) is expressed as follows [3]:

$$E_{\theta}(r,\theta,t) = \frac{\eta_0}{4\pi r} \left[\sum_{i=1}^{\infty} \Gamma_i \frac{\sin\theta}{(1-\cos\theta)} \{ I_s(t-r/c-b_i) - I_s(t-r/c-(L/c)(1-\cos\theta)-b_i) \} + \Gamma_i \frac{\sin\theta}{(1+\cos\theta)} \{ I_s(t-r/c-c_i) - I_s(t-r/c-(L/c)(1-\cos\theta)-c_i) \} \right]$$
(2)

$$b_i = \begin{cases} (L/c)(i-1+\cos\theta) & i \text{ even} \\ (L/c)(i-1) & i \text{ odd} \end{cases} \quad c_i = \begin{cases} (L/c)(i-1-\cos\theta) & i \text{ even} \\ (L/c)(i-1) & i \text{ odd} \end{cases}$$
(3)

$$\Gamma_{i} = \begin{cases} (-1)^{i/2} \rho^{(i/2-1)} & i \text{ even} \\ (-\rho)^{(i-1)/2} & i \text{ even} \end{cases}$$
(4)

where η_0 represents the wave impedance in the vacuum. In relation (2), we can distinguish the contribution of both arms (0, L) and (0, -L) of the dipole to the radiated field induced by upward and downward propagating current components respectively: as the factor $\sin \theta / (1 - \cos \theta)$ corresponds to the upper arm, the factor $\sin \theta / (1 + \cos \theta)$ is associated with the lower arm. We notice that the total electric field radiated is built from differentiated delayed current components that express the time derivative of the current in each arm if the two components are separated by a delay $\Delta t = \tau_a (1 - \cos \theta) \ge \omega$ and $\Delta t = \tau_a (1 + \cos \theta) \ge \omega$. Also, we remark that the radiation of the lower arm is the same as the radiation of the upper arm when changing the observation angle θ to its complementary value $\pi - \theta$. Such a modeling has been compared to a numerical one based on the FIT, and the results agree satisfactorily.

In the case of a Wu and King (WK) loaded dipole, a distributed complex impedance along each arm length of the dipole is expressed by [4]:

$$Z(z,\omega) = \frac{1}{Y(z,\omega)} = r(0,\omega) \left(\frac{L}{L-|z|}\right) \left(1-j\frac{k_0}{L}\right)$$
(5)

where k_0 is the wave number in the vacuum, and $r(0, \omega) = \frac{\eta_0 \psi}{2\pi}$ the impedance at the feed-point z = 0 of the dipole. ψ is a function of the frequency, and η_0 the impedance of the free space. For the sake of simplicity, the parameter ψ is replaced by its mean value over the given frequency bandwith considered. The expression of the transient radiated electric field given by Samaddar et al. extended to consider the presence of an impedance associated with the input circuit is given by [5, 6]:

$$E_{\theta}(r,\theta,t) = \frac{1}{r\psi\sin^{2}\theta} \frac{\tau_{a}}{\tau_{t}} \begin{bmatrix} \sin^{2}\theta(V_{G}(t^{*}) - \frac{1}{\tau_{t}}\int_{0}^{\infty}V_{G}(t^{*} - t')e^{-t^{0}/\tau_{t}}dt' \\ -\frac{(1+\cos^{2}\theta)}{\tau_{t}}\int_{0}^{\infty}V_{G}(t^{*} - t')e^{-t^{0}/\tau_{t}}dt' \\ +\frac{(1+\cos\theta)^{2}}{2\tau_{t}}\int_{0}^{\infty}V_{G}(t_{1} - t')e^{-t^{0}/\tau_{t}}dt' \\ +\frac{(1-\cos\theta)^{2}}{2\tau_{t}}\int_{0}^{\infty}V_{G}(t_{2} - t')e^{-t^{0}/\tau_{t}}dt' \end{bmatrix}$$
(6)

where: $t^* = t - r/c$; $t_1 = t^* - \tau_a(1 - \cos\theta)$; $t_1 = t^* - \tau_a(1 + \cos\theta)$. Also, $\tau_t = C_g \tau_a$ with $C_g = 1 + 2\pi Z_G/(\eta_0 \psi)$. Z_G is a constant representing the mean value of the impedance associated with the input circuit in the frequency bandwidth of the excitation signal. If $Z_G = 0$, we have $\tau_t = \tau_a$.

2.2. Reception Configuration

A dipole antenna (-L, L) such as presented in Figure 1 is supposed to be excited at the upper arm by a transient electric plane wavefront with an oblique incidence angle ψ relative to the direction of the antenna. Two cases can be distinguished depending if a straight or a V-dipole is considered: in the case of a straight dipole, as the upper arm is first excited at its top, the lower arm is first excited at the position z = 0. In the case of a V-dipole, both upper and lower arms are first excited at their top. The analytical developments presented in this paper concern a monopole(0, L) initially aligned with axis O_z . If the monopole is tilted with an angle β relative to axis O_z the expressions remain valid, and the case of a monopole (0, -L) tilted with an angle $-\beta$ relative to axis O_z can be deduced from the expression of the upper monopole by adding an additional delay $\tau'_D = L \sin \psi/c$ and changing the incidence angle ψ by its complementary value $\pi - \psi$. The excitation signal $E_{\theta}(\theta_r, t)$, which is supposed to be issued from a transmitting antenna of the same kind, interacts with a given monopole (0, L) and produces at each point of the element of abscissa z' an induced current dI'(z, t; z'). This current propagates from each excitation point z' in two opposite directions [2]. A time delay τ_D is assigned to each discrete source to represent the arrival time of the oblique plane wave front with incidence ψ on the antenna:

$$\tau_D = (L - z')\sin\psi/c \tag{7}$$

$$dI(z,t;z') = \begin{cases} dI'(z',t-(z-z')/c-\tau_D)\cos\psi \cdot U(z-z') \\ +dI'(z',t-(z+z')/c-\tau_D)\cos\psi \cdot U(z'-z) \end{cases} [U(z)-U(z-L)]$$
(8)

where U is the Heaviside unit-step function. Each local current component mentioned in relation (8) writes as follows: H'(A, b) = H'(A, b) =

$$dI'(z',t) = Y(z')E(z',t-\tau_D) = 1/Z(z')E(z',t-(L-z')\sin\psi/c)$$
(9)

where Z(z') represents the impedance which can vary along the antenna.

In the case of a uniform infinitely conductive antenna, we have extended previous developpement in order to consider the total current component distributed along the antenna arms for an oblique incidence. For the sake of simplicity the formulation concerning the upper monopole (0, L) does not include the ringing elect issued from total and partial reflections at the top end and the feed point respectively:

$$I_{upper}(z,t) = \frac{c\cos\psi}{1+\sin\psi} \{\xi(t-(L-z)\sin\psi/c) - \xi(t-z/c-L\sin\psi/c)\} + \frac{c\cos\psi}{-1+\sin\psi} \{\xi(t-(z-L)/c) - \xi(t+(z-L)\sin\psi/c)\}$$
(10)

And $\xi(t) = \int_0^t E(t') dt'$. At the position z = 0, the current detected becomes:

$$I_{upper}(0,t) = -\frac{c\cos\psi}{1+\sin\psi}\xi(t-L\sin\psi/c) - \frac{c\cos\psi}{1-\sin\psi}\xi(t-L/c) + \frac{2c}{\cos\psi}\xi(t-L\sin\psi/c)$$
(11)

In the case of a WK loaded dipole, we have expressed the current generated by each local source along the upper monopole (0, L) as follows:

$$dI'(z,t;z') = Y(z',t) \otimes E(z',t - (L-z')\sin\psi/c)\cos\psi$$
(12)

where \otimes denotes the time convolution product. Then, replacing the local current dI'(z,t;z') in relation (8),



Figure 2: (a) Geometry of the link simulated involving two V-dipoles in the far-field and a single-layer dielectric material; (b) Polar diagram of the radiated power of a uniform V-dipole for several aperture angles.

we obtain an additionnal component $I'_{upper}(z, t)$ which superimposes on the current component $I_{upper}(z, t)$ associated with the uniform monopole. It is defined as follows:

$$I'_{upper}(z,t) = I'_{top}(z,t) + I'_{bottom}(z,t)$$
(13)

with:
$$I'_{top}(z,t) = \frac{c\cos\psi}{1+\sin\psi} \left\{ \begin{array}{c} z\xi(t+(z-L)\sin\psi/c) \\ +\frac{1}{L}\frac{c}{1+\sin\psi}[\varsigma(t+(z-L)\sin\psi/c)-\varsigma(t-z/c-L\sin\psi/c)] \end{array} \right\}$$
(14)

$$I_{bottom}'(z,t) = -\frac{1}{L} \frac{c \cos \psi}{-1 + \sin \psi} \left\{ \begin{array}{l} L\xi(t + (z - L)/c) - z\xi(t + z(z - L)\sin \psi/c) \\ +\frac{1}{L} \frac{c}{-1 + \sin \psi} [\varsigma(t + (z - L)/c) - \varsigma(t + z/c - L\sin \psi/c)] \end{array} \right\}$$
(15)

And $\varsigma(t) = \int_0^t \xi(t') dt'$. Then the total current component without reflection at the feed-point is given by:

$$I_{WK,upper}(z,t) = \frac{1}{r(0)} (I_{upper}(z,t) + I'_{upper}(z,t)) \otimes e^{-t/\tau_a}$$
(16)

At the position z = 0, the current received by the detector is:

$$I_{WK,upper}(0,t) = \frac{1}{r(0)} (I_{upper}(0,t) + I'_{upper}(0,t)) \otimes e^{-t/\tau_a}$$
(17)

Relation (16) appears as a extended version of the expression given by Samaddar et al., [5], as it allows to study the current distribution along each arm of a dipole and not only at the position z = 0.

3. Simulation Results

As an illustration of the modelling presented above, we have considered in transmission and reception two identical symmetric V-dipoles formed of two thin-wire elements with length L = 10 cm and characterized by an aperture angle $2\alpha = 120^{\circ}$ ($\beta = 30^{\circ}$) as presented in Figure 2(a). The dipoles which face each other have been fixed in each other far-field at a distance d = 1 m. The equivalent impedances at the feed-point in transmission and reception have been estimated to 600Ω . The excitation voltage in transmission is assumed to have the



Figure 3: (a) Waveforms radiated by uniform and loaded V-dipoles; (b) Resulting waveforms radiated by a





Figure 4: (a) 3D current distribution along the V-dipole $(2\alpha = 120^{\circ}, L=10 \text{ cm}, \text{ incidence } \varphi = 0^{\circ} \text{ so } \Psi = 30^{\circ});$ (b) Current distribution induced in a loaded V-dipole in the reception configuration considering the incident wavefront (a); (c) Detected current at the position z=0.

shape of the first derivative of the Gaussian function; the reference characteristic time τ has been fixed to 41.25 ps (which corresponds to the duration $\omega = 0.17$ ns), so that 2 pulses (n = 2) fit in each arm length L. The propagation channel is represented here by a single-layer dispersive dielectric material with thickness e = 4 cm; in the modelling, any multi-layer dielectric sample with a plane surface can be considered. In transmission, we have studied the influence of the aperture angle 2α , and the pulse duration (by the means of n) on the focalization of the electromagnetic energy radiated in the direction $\theta = 90^{\circ}$. Some results associated with a uniform V-dipole are presented in Figure 2(b) for three angles $2\alpha = 60^{\circ}$, 80° , 100° ; they show that a focalization in the direction $\theta = 90^{\circ}$ is obtained for n = 2.9, 1.7 and 0.9 pulses in a arm length L = 10 cm (for $2\alpha = 120^{\circ}$, we have found that n = 0.8). The polar diagram highlights that a narrower focalization is obtained for the lowest aperture angle 2α . Moreover, the spatio-temporal waveforms radiated in the far-field zone by a uniform (with 3 reflections) and a loaded V-dipole of the same dimensions have been compared; the plots of Figure 3a which consider the direction $\theta = 90^{\circ}$ show that as the uniform V-dipole gives a distorted version of the excitation signal with a ringing phenomenon, the loaded V-dipole generates an attenuated and a slightly distorted version of the initial pulse. Then, we have studied the transmission of the signal radiated by the loaded dipole under the incidence angle $\theta_i = 0^\circ$ and through a single layer material considering the following dielectric properties: (1) $\epsilon' = 3$, $\sigma = 0.012 \,\text{S} \cdot \text{m}^{-1}$, (2) $\epsilon' = 3$, $\sigma = 0.12 \,\text{S} \cdot \text{m}^{-1}$, and (3) $\epsilon' = 5$, $\sigma = 0.012 \,\text{S} \cdot \text{m}^{-1}$. The resulted waveforms can be visualized in Figure 3(b). The corresponding ratios of the energy transmitted to the energy of the incident signal are respectively 77.7%, 30.3%, and 68.5%. So, these results highlight the strong attenuation elect introduced by the losses produced inside the material. Afterwards, considering the signal transmitted through the sample with the dielectric properties $\epsilon' = 3$, $\sigma = 0.012 \,\mathrm{S} \cdot \mathrm{m}^{-1}$ (see Figure 4(b)), we have computed the current distribution in a loaded V-dipole ($\beta = 30^{\circ}$). The 3D plot of Figure 4(a) shows the induced current before executing the convolution product with the term e^{-t/τ_n} and mentioned in relation (16). The current at the position z = 0 is visualized in Figure 4(c) before and after the convolution operation. We remark that such an operation smoothes the signal.

4. Conclusion

In this paper, the modelling of a communication link involving two transmitting and receiving antennas in the time domain has been revisited. The modelling, based on analytical expressions, allows to consider several conductive antennas which can be represented by a simplified model using a combination of non interacting thin-wire elements. Further developments have been made mainly in the reception configuration in order to consider the interaction of a uniform or a loaded Wu and King dipole which are supposed to be illuminated by an oblique plane wavefront. At present, the propagation channel has been represented by the presence of a multi-layer dielectric dispersive material which produces multipath induced by the multiple reflections occuring in its thickness. Statistical models of propagation channel, such as the Turin approach, associated in a given environment can be planned. Samples of the parametric studies made have been presented in the case of a communication link involving two transmitting and receiving V-dipoles. We have particularly analyzed the case of a WK V-dipole as its better represents an UWB band as the ringing effect of the current has been eliminated by a distributed absorption along each arm. The realization of a graphical interface will make easier more parametric studies to thoroughly analyzed the link budget in a given configuration. Moreover, optimization of the link will be addressed.

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Mathematical Modeling of Nonlinear Waves and Oscillations in Gyromagnetic Structures by Bifurcation Theory Methods

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Abstract—The vector field bifurcation approach and its numerical implementation for the rigorous mathematical simulation of nonlinear phenomena in microwave and mm-wave ferrite or composite semiconductor/ferrite devices are developed. The bifurcation points of nonlinear Maxwell's operator for the three-dimensional boundary problems, stated and solved rigorously (i.e., considering the full Maxwell's equations together with the nonlinear equations of motion for magnetization in ferrites and transport carriers in semiconductors) are analyzed using numerical methods. The electromagnetic field is represented as decomposed into a series of weakly nonlinear wave fields. The solutions of a linearized Maxwell's operator matrix equation are determined. The propagation constants of weakly nonlinear waves in waveguiding structures (WGS) or eigenfrequencies of weakly nonlinear oscillations in resonator structures (RS) are found. Using the bifurcation dynamics of Maxwell's equations the nonlinear wave interactions in the strongly nonlinear planar ferrite insert, loaded into strip-slot RS, are analyzed (from the harmonic frequency terms at the 'soft' non-linear stage into the region of 'hard' nonlinearity). The nonlinear propagation of electromagnetic waves in the strip-slot ferrite RS are modeled. The nonlinear wave phenomena, including the parametric excitation of oscillations and the wave instability process are investigated taking into account constrained geometry WGS and RS.

1. Introduction

The research of bifurcations in nonlinear dynamical systems with distributed parameters, described by nonlinear differential equations in partial derivatives, involves serious mathematical difficulties. As for distributed systems a characteristic determinant is an analogue of frequency characteristics, that's why it is possible to analyze distributed self-sustained oscillation systems using the linearization method combined with the characteristic determinant analysis (at first it was shown in [1] for the one-dimensional case). Hitherto the bifurcation analysis was used to investigate nonlinear dynamical systems with lumped parameters, described by nonlinear ordinary differential equations (ODEs). When the ordinary differential equation is of second order a qualitative analysis is possible on the two-dimensional phase surface [2]. The linearization method in combination with the frequency-domain analysis is used for the analysis of self-sustained oscillating systems and automatic control systems [2]. Determining the solutions of nonlinear differential equations in fixed points using numerical computation is a very complicated problem even for ODEs, because at the branching points qualitative modifications of solutions can happen due to variation of parameters.

The behavior caused by the instability of waves and oscillations in nonlinear or parametric systems, containing nonlinear magnetic or semiconductor media, is complex [3]. The physical theories of the instability of magnetostatic or spin waves were developed using the approximate analysis of the equation of motion of the magnetization vector in ferromagnet for one-dimensional structures only [4, 5]. The analysis of the transition region from the stable regime to the onset of labile oscillating mode caused the instability is the most complicated problem. This analysis can only be based on the solutions of full nonlinear Maxwell's equations, complemented by the nonlinear equations of motion of the magnetization vector in a ferromagnet [3]. The goal of this paper is to develop a new approach based on the bifurcation theory [6, 7] for accurate electromagnetic modeling of nonlinear wave phenomena in gyromagnetic or semiconductor waves in waveguiding structures (WGS) or resonator structures (RS) using a numerical approach for the analysis of the linearized matrix equation and bifurcation points of the nonlinear Maxwell's operator. It opens up new prospects of bifurcation analysis and rigorous mathematical modeling of strongly nonlinear electrodynamical systems using the bifurcation dynamics of Maxwell's equations.

2. The Numerical Method of Linearization of Nonlinear Maxwell's Operator in Combination with the Analysis of the Characteristic Determinant

The numerical method to determine the propagation constants of weakly nonlinear waves in WGS (or eigenfrequencies of weakly nonlinear oscillations in RS) loaded with strongly nonlinear gyromagnetic or semiconductor boundary media consists in the following.

The three-dimensional boundary problems, stated rigorously (i.e., considering the full Maxwell's equations with the nonlinear equation of motion for magnetization in ferrites or the equation of transport carriers in semiconductors, with boundary conditions following from conditions of non-asymptotic radiation) was reduced to the boundary problem for a system of nonlinear DEs together with the system of the nonlinear algebraic equations using the cross-sections method in [8, 9].

The system of nonlinear DEs together with the system of nonlinear algebraic equations [8,9] is represented in a symbolic form, as:

$$\frac{dy_i}{dz} = F_i(y_1, y_2, \dots, y_n), \qquad \Psi_j(y_1, y_2, \dots, y_n) = 0, \tag{1}$$

where $i = 1, 2, ..., m; j = m + 1, m + 2, ..., n; y_i = y_i(z)$ are unknown functions of the longitudinal coordinate z compiled on the functions $a_n^t(\omega_m), b_n^t(\omega_m), a_n^z(\omega_m), b_n^z(\omega_m)$, given in references [8,9].

Let $y_i = 0$ (i = 1, 2, ..., n) be the solution of the system (1), satisfying the boundary conditions as given in reference [8,9]. Then the functions F_i and Ψ_j (i = 1, 2, ..., m; j = m + 1, m + 2, ..., n) identically vanish, consequently, the solution $y_i = 0$ (i = 1, 2, ..., n) of the system (1) is fixed (stationary) relative to the coordinate variable z.

As the first approximation, reduce the system of nonlinear differential equations (1) to a system of linear differential equations. For this purpose it is necessary to represent functions F_i and Ψ_j by their generalized Taylor's series in the neighborhood of fixed (stationary) points $x_i = 0$, and to take into account the first order partial derivatives. This procedure results a system of linear differential equations:

$$\frac{dy_i}{dz} = \sum_{K=1}^n \frac{\partial F_i(0,0,\dots,0)}{\partial y_K} \cdot y_K, \qquad \sum_{K=1}^n \frac{\partial \Psi_j(0,0,\dots,0)}{\partial y_K} \cdot y_K = 0,$$
(2)

where i = 1, 2, ..., m; j = m + 1, m + 2, ..., n.

Let us represent the system of differential equations (2) in expanded form:

$$\begin{cases} a_{11}(z) \cdot y_1 + a_{12}(z) \cdot y_2 + \ldots + a_{1n}(z) \cdot y_n = y'_1, \\ a_{21}(z) \cdot y_1 + a_{22}(z) \cdot y_2 + \ldots + a_{2n}(z) \cdot y_n = y'_2, \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1}(z) \cdot y_1 + a_{m2}(z) \cdot y_2 + \ldots + a_{mn}(z) \cdot y_n = y'_m, \\ a_{m+1,1}(z) \cdot y_1 + a_{m+1,2}(z) \cdot y_2 + \ldots + a_{m+1,n}(z) \cdot y_n = 0, \\ a_{m+2,1}(z) \cdot y_1 + a_{m+2,2}(z) \cdot y_2 + \ldots + a_{m+2,n}(z) \cdot y_n = 0, \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n1}(z) \cdot y_1 + a_{n2}(z) \cdot y_2 + \ldots + a_{nn}(z) \cdot y_n = 0, \end{cases}$$
(3)

where the coefficients $a_{ij}(z)$ (i, j = 1, 2, ..., n) compiled on the partial derivatives from (2). The system of equations (3) can be represented in matrix form as:

$$A \cdot y = \frac{dy}{dz} \tag{4}$$

where y is the vector with components y_1, y_2, \ldots, y_m ; $\frac{dy}{dz}$ is the vector with components y'_1, y'_2, \ldots, y'_m ;
$$A = A_{11} - A_{12} \cdot A_{22}^{-1} \cdot A_{21},$$

$$A_{11} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{pmatrix}, \qquad A_{12} = \begin{pmatrix} a_{1m+1} & a_{1m+2} & \dots & a_{1n} \\ a_{2m+1} & a_{2m+2} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,m+1} & a_{m,m+2} & \dots & a_{mm} \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} a_{m+1,1} & a_{m+1,2} & \dots & a_{m+1,m} \\ a_{m+2,1} & a_{m+2,2} & \dots & a_{m+2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix}, \qquad A_{22} = \begin{pmatrix} a_{m+1,m+1} & a_{m+1,m+2} & \dots & a_{m+1,n} \\ a_{m+2,m+1} & a_{m+2,m+2} & \dots & a_{m+2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix},$$

We find the partial solutions of the system of equation (4) in the form:

$$y = \alpha \cdot e^{\lambda \cdot z} \,, \tag{5}$$

where α is the vector with components $\alpha_1, \alpha_2, \ldots, \alpha_m$. Substituting (5) into (4), we obtain the following eigenvalue matrix equation:

$$A \cdot \alpha = \lambda \cdot \alpha \,, \tag{6}$$

where λ and α are correspondingly the eigenvalues and eigenvectors of matrix A. Using a numerical method (for example, the QR-algorithm) to solve the matrix equation (6) the eigenvalues λ_m and eigenvectors α of A can be determined.

The solutions (5) of the linearized Maxwell's operator (6) are treated as weakly nonlinear waves. The electromagnetic field in WGS is decomposed into a series of weakly nonlinear wave fields. The eigenvalues λ_m of matrix A are the propagation constants of the weakly nonlinear waves in WGS (or the eigenfrequencies of weakly nonlinear oscillations in RS). The components of the eigenvectors α of matrix A are the transverse and longitudinal components of weakly nonlinear waves .

The computational algorithm, using the linearization of nonlinear Maxwell's operator and the decomposition into a series of weakly nonlinear wave fields, is more complex than those for the propagation constants and fields of eigenwaves of WGS, filled with a linear medium. But the convergence of this algorithm and its stability for rounding errors is better. It permits to solve the three dimensional diffraction boundary problems for WGS or RS loaded with strongly nonlinear gyromagnetic or semiconductor insertions having sizes comparable to the wavelength. This is important for CAD of prospective ferrite or composite semiconductor/ferrite devices at microwave or mm-waves.

3. Numerical Simulation of the Parametric Excitation of Oscillations in Nonlinear Gyromagnetic Structure Using Bifurcation Points

The rigorous mathematical modeling of parametric oscillations in strip-slot RS loaded with a planar magnetized ferrite (Fig. 1) is based on solving the nonlinear diffraction boundary problem by the crosssections method of [8], using the decomposition algorithm on nonlinear autonomous blocks [10].

For the computational algorithm the transverse and longitudinal components of weakly nonlinear waves are used. It results a stable and computationally efficient algorithm for computing the instability of waves or oscillations in WGS or RS containing strongly nonlinear gyromagnetic media.

There are two incident electromagnetic waves: the signal wave of frequency ω_1 and the pumping wave of frequency ω_2 are incident on the input cross-sections S_1 of RS (Fig. 1). The waves are the fundamental and higher-order modes of strip-slot WGS, having magnitudes $C^+_{n(\alpha)}(\omega_1)$ and/or $C^+_{n(\alpha)}(\omega_2)$, where α is the index of the cross-sections, n are the indices of eigenwaves of strip-slot WGS [8,9].

The instability of parametric excitation process of oscillations in ferrite RS depending on the bifurcation parameters is simulated using the numerical method of bifurcation points analysis, developed by us [11]. The results of computing of the instability regions for parametric excitation of oscillations in ferrite RS by the incident pumping wave, depending on the magnitude $C_{2(1)}^+(\omega_2)$ and the normalized frequency (the signal frequency ω_1 with respect to the pumping frequency ω_2) are shown in Fig. 2. The onset and the breakdown of parametric oscillations caused the wave instability in nonlinear ferrite structure in the neighborhood of bifurcation parameters were simulated into the region of 'hard' nonlinearity taking into account constrained geometries RS, and it is represented in Fig. 2. It follows from the results of the mathematical modeling that the unstable regions for parametric excitation of oscillations in ferrite RS are near the values of the eigenfrequencies of fundamental and higher-order modes of oscillations of the strip-slot line resonator: $\omega_1 = m\omega_2/2$, m = 1, 2, 3, ... The threshold magnitude $C^+_{2(1)}(\omega_2)$ is rising steeply as m increases. The minimum threshold of $C^+_{2(1)}(\omega_2)$ is given by $\omega_1 = \omega_2/2$.



Figure 1: Resonator structure with nonlinear ferrite insert: 1, 2, 3, 4 – coupled strips of strip-slot WGS; 5 – strip-slot resonator; 6 – planar magnetized ferrite insert ($\varepsilon = 9, H_0 = 278 \text{ A/mm}; M_0/\mu_0 = 160 \text{ A/mm}; \omega_r = 3 * 10^9 \text{ Hz}; \beta = 45^\circ$); 7 – dielectric substrate ($\varepsilon = 9; \mu = 1$); 8 – point of field observation; $f_1 = 5 \text{ Hz}, f_2 = 10 \text{ GHz};$ all sizes are in mm.



Figure 2: Instability regions for parametric excitation of oscillations in nonlinear ferrite RS, depending on bifurcation parameters: $C_{2(1)}^+(\omega_2)$ – magnitude of incident pumping wave; ω_1 – eigenfrequency of fundamental modes of oscillations the strip-slot line resonator (length of the resonator = half-wave for signal wave at $f_1 = 5 \text{ GHz}$); ω_2 – frequency of pumping wave.

4. Conclusion

Using the achievements of modern mathematics in the area of vector field bifurcation theory opens new possibilities for computer analysis of the onset of nonlinear waves in WGS with bounded gyromagnetic media having a strong nonlinearity. This approach has a high likelihood of success in investigating nonlinear phenomena in new microwave/millimeter-wave ferrite devices [12] for frequency multiplexing/filtering, limiters, noise rejectors, signal-noise ratio enhancers, and pulse compressing devices.

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Mobile Wireless Communication System Antennas for 260 MHz-band

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1. Introduction

Automotive mounted antennae are often vertically orientated 1/4 wave antennae because the body of the car can be used as an effective reflective infinite ground plane. This ground plane permits the radiation pattern of a mounted 1/4 wave antenna to behave like that of a 1/2 wave antenna. The 1/4 wave antenna has a low cost and is also compact in size. This compact quality is important in automobile mounted antennae because it reduces the risk of antenna damage and also does not create difficulty when restrictions on the height of a car's profile are present. This paper presents a design of an antenna that is compact like a 1/4 wave antenna but can behave as closely as possible to a 1/2 wave antenna without the presence of an effective infinite ground plane.

The 1/4 helical antenna is well known, but this paper presents the design of a helical antenna that meets our height design requirements. Specifically, a monopole antenna with a height of under 300 mm is desired. The standing wave ratio must be under 2.0 in the frequency band of interest. The height of the helical element must remain at some constant so the pitch and diameter of the spiral were optimized. An improvement in gain was obtained by using a small finite reflection plate.



Figure 1: Mobile wireless communication system antennas for 260 MHz-Band.

2. Antenna Structure

As mentioned earlier, 1/4 wave antennae are usually used in mobile radio devices when a large ground plane is available. The efficiency of an antenna depends on the loss due to the mismatch between the impedence of the receiving antenna and the transmitter. The impedence of any portable receiving antenna can be quite variable depending on how it is mounted: for example, carried by a human or affixed to a car. Also of concern is the unpredictability of the height of the antenna above a ground plane. This creates an unstable state because of the high frequency current flowing within the standard antenna. Our antenna has been designed to deal with this instability and is more efficient than a standard 1/4 wavelength antenna.

Figure 1(a) shows the basic structure of our helical antenna as detailed in [1]. The figure shows a finite reflected plate. Figure 1(b) is the simplified model of the antenna with a helical antenna above a finite reflector plate. The center frequency was chosen to be 265 MHz. With the height H of the helical element fixed to be 220 mm, we adjust pitch P and diameter D of the spiral. Shown in Figure 2 is a 2D plot of the horizontal gain as a function of P and D. There is a region where 2 dBi gain is realizable.



Figure 2: Horizontal gain of mapping for helical antenna. A: 2 dBi more than B: 1.5 2 dBi C: 1 1.5 dBi D: 0.5 1 dBi E: 0 0.5 dBi

3. Experimental Result

The antenna can be adjusted to a particular resonant frequency to create the VSWR pattern shown in Figure (3). A good impedence match is found between 262 to 275 MHz where the VSWR is lower than 2.0 which corresponds to $-9.5 \,\mathrm{dB}$. The vertical radiation pattern of this antenna is compared with the pattern from a 1/2 wavelength antenna in Figure (4). The actual gain was about -1 to $-2 \,\mathrm{dB}$ but the horizontal radiation pattern is omni directional.



Figure 3: Characteristic of VSWR.

Figure 4: Vertical Radiation Pattern. (Dotted line: 1/2 wavelength, Solid line: this antenna)

4. Conclusion

The effectiveness of a finite reflector plate with a helical antenna is examined. There is an increase of gain in the horizontal direction and a reduction in antenna size. This type of antenna is more immune to the effects of the surrounding environment. This antenna is very practical and may be used when it is necessary to mount antennae on cars with plastic outer bodies such as ambulances. Changes in the shape of the radiating element and simplification of the feed line will be the future work on this project.

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Compact Surface-mount UWB Monopole Antenna for Mobile Applications

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Abstract—In this paper, a novel surface-mount ultra-wideband (UWB) monopole antenna with a compact size of only $12.5 \times 9 \times 1.5 \text{ mm}^3$ is obtained by folding a metal-plate onto a low-profile rectangular-box foam base. By carefully adding a matching slit on the upper side of metal-plate, the antenna can achieve good impedance matching over a very wide bandwidth of about 7.97 GHz (3.03–11.0 GHz, defined by 2:1 VSWR). Experimental results of a constructed prototype of the proposed antenna are presented.

1. Introduction

In the recent years, short range and high data rate wireless communication is applied in multimedia device. UWB radio technology can meet these requests. Planar antennas have many advantages, such as low profile, small size and easy to fabricate, which are suitable for portable devices. There are several UWB planar antenna designs, including planar metal-plate antenna [1], half-disk antenna [2], and planar horn antenna [3], which have been reported.

In this paper, a novel surface-mount UWB monopole antenna, which is suitable for metal stamping processing and low fabricating cost, is presented. The proposed antenna has a compact structure, which makes it easy to fit in any possible margin within the housing of a mobile/hand-held wireless device, thus leading to an internal UWB antenna.

2. Antenna Design

Figure 1 shows the proposed UWB monopole antenna mounted at the front surface of a 0.8 mm thick FR4 substrate ($\varepsilon_r = 4.4$). The ground plane (length 40 mm and width 60 mm) printed on the back surface of the FR4 substrate can be considered as the system circuit board of a wireless access point. The proposed UWB antenna is easily constructed by folding a metal plate onto a rectangular form base of compact size $12.5 \times 9 \times 1.5 \text{ mm}^3$.



microstrip transmission line rectangular foam base





Figure 2: Planar structure of the unfolded metal plate (dashed lines on the metal plate are the stamping line).

Figure 2 shows the detailed dimension of the metal plate. The size of the metal plate is $15.5 \times 9 \text{ mm}^2$ with a matching slit of $1 \times 3 \text{ mm}$ near the antenna feed. The lowest resonant frequency and the highest resonant frequency of proposed antenna were mainly controlled by length and width of the metal-plate. Broadband matching technique is the key point in UWB antenna design, by carefully adding a matching slit on the upper side of metal-plate, the antenna can achieve a good impedance matching over a very wide bandwidth. With the matching slit, the current distribution on the surface of the radiation conductor can be altered. By carefully tuning the width and length of the matching slit, a very wide band impedance matching (defined by 2:1 VSWR) from 3.03 GHz to 11.0 GHz is obtained. This result is mainly due to the asymmetrically antenna structure which will lead to asymmetrically current path and it is helpful to achieve broadband impedance matching. Thus, the mechainism of the matching slit is similar to asymmetrically feed mentioned in [4].

The radiation energy of the proposed antenna is activated by the antenna feed via the microstrip transmission line. To accomplish the impedance matching between the metal plate and the microstrip transmission line, there are trapezoid-shaped metallic strip formed between the metal plate and the microstrip transmission line.



Figure 3: Measured return loss of the proposed antenna.



Figure 4: Measured radiation patterns at 3.5 and 7.0 GHz.

3. Experimental Results

Figure 2 shows the measured return loss for the proposed antenna. From the results, it clearly indicates that the impedance bandwidth, defined by 2:1 VSWR, is as large as 7.97 GHz. Thus, the proposed antenna can cover the full band of DS-UWB [5].

The far-field radiation characteristic across UWB bandwidth of the proposed antenna is also studied. The far-field radiation pattern (only co-polarization is showed) of the proposed antenna at 3.5 GHz (light color line) and 7.0 GHz (deep color line) are shown as Figure 4. It is first observed that, in vertical cut of azimuthal plane (or x-z plane), the radiation pattern of antenna shows an omni-directional radiation characteristic, but apparently shift to the -x direction. It is mainly due to the ground plane effect. In the radiation pattern of horizontal cut of y-z plane, especially at 7.0 GHz, it is also seen that there are two null in $\pm y$ direction. According to what has been mentioned above, in brief, it is a monopole-like radiation pattern. The measured antenna gain against frequency is presented in Figure 5. Across the impedance bandwidth in 3.0 to 10.0 GHz, the measured antenna gain increases from about 1.7 to 3.3 dBi with increasing frequencies. Measured peak antenna in the lower band and the higher band of DS-UWB is about 2.4 dBi and 3.2 dBi, respectively.



Figure 5: Measure peak antenna gain against frequency of the proposed antenna.

4. Conclusions

A compact surface-mount UWB chip antenna, which mainly constructed by stamping the metal plate has been fabricated and studied. Results indicate that the constructed prototype showed a very wide-impedance bandwidths covering the lower and higher bands of DS-UWB. Good antenna gain in the operation bands is also obtained. In addition, the antenna has a compact structure, which makes it easy to fit in any possible margin within the housing of a mobile/hand-held wireless device, thus leading to an internal UWB antenna. The small size UWB antenna is also suitable for surface-mountable fabrication process. Thus it can effectively reduce the overall manufacturing cost.

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Reliability and Availability of GPS Measures in Airport Landing Systems

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Abstract—In the last decades, the modern airports have considerably increased the traffic to manage. Consequently, the requirement of technological solutions in order to control vehicles and airplanes in relevancy areas (such as parking area, taxiways and runways) is increased. Among various considered solutions, the more efficient ones use Global Positioning System receivers to establish location of moving objects into the airport. Thanks to this solution, it is possible to increase both the efficiency in runway usage and the safety in ground movements. Nevertheless navigational systems like Galileo or GLONASS cannot be considered as "high availability systems". For examples, positioning service can be interrupted by unintentional radiofrequency interferences, or terrorist attacks through techniques known as "antenna jamming" and "code contamination". In this paper, these problems are analyzed in the context of airplane ground control, and a secure system is suggested. Especially, technical solutions are adopted in navigational routines using Global Positioning System receivers, in order to improve reliability and security, and above all to guarantee a narrow range of variation for positioning for the whole necessary time.

1. Introduction

The most critical phase in a flight is surely the landing; therefore it is necessary to increase the safety of the aerial transportations, above all civil ones, by usage of some tools which aids the pilot during the landing phase, especially in condition of low visibility. The landing instrumentation used to assist pilots or integrated in an automatic landing systems offers different performances (precision, reliability, low latency of calculation) which are compatible with specific climatic conditions. Just to be able to well classify the different levels of risk, the International Civil Aviation Organization (ICAO) defines three categories of visibility for landing civil aircraft [1]:

- 1. Category I—Decision Height not lower than 200 ft and Runway Visual Range (RVR) not less than 1800 ft with appropriate runway lighting; Decision Height (DH) is the height above the runway at which the landing must be aborted if the runway is not in sight;
- 2. Category II—DH not lower than 100 ft and RVR not less than 1200 ft; the pilot must see the runway above the DH or abort the landing,
- 3. Category III This category is subdivided into:
 - IIIA: DH lower than 100 ft and RVR not less than 700 ft;
 - IIIB: DH lower than 50 ft and RVR not less than 150 ft;
 - IIIC: Zero visibility, no DH or RVR limits.

In comparison to the traditional systems for landing assistance (i. e., Instrument Landing Systems, INS, and the Microwave Landing Systems), Satellite Landing Systems (and particularly the Global Positioning System, GPS) seem to answer in a more suitable way to the requisite of precision and inexpensiveness for all the categories of employment mentioned above. Nevertheless the GPS technology has some problems related to the nature of the used signal and to the data transmission protocol from satellites to receiver. In fact, GPS receivers are susceptible to attacks exploiting interference techniques on the spread-spectrum signals (i. e., jamming), such as Denial of Service (DoS) attacks; their purpose is to make unusable a determined service, i. e., GPS service, for a particular time interval. Moreover, a hacker can modify the C/A and P codes so that the position calculated by the receiver is not correct (spoofing attack). Therefore, it is necessary to integrate the GPS system with other devices in order to eliminate these problems, which constitute an enormous limit for the adoption of the Global Position System during the landing phase. In this paper, an hybrid GPS/INS navigational system is proposed in order to avoid spoofing and jamming attacks, and to increase the precision of GPS positioning. In section 2,

jamming attacks and our GPS/INS solution approach are described; subsequently, section 3 gives a panorama of spoofing attacks and analyses our mixed protocol to avoid "code contamination"; finally, at section 4, some conclusions are pointed out.

2. Jamming Attacks and Proposed Solution Approach

The GPS signal has low power and is vulnerable to interference. The dangerous means of interference go from cheap, expendable, low-power jammers which can be widely distributed across an area of conflict, to medium and high-power ground and air-based jammers which can deny usage of GPS over hundreds of miles. The interruption of GPS positioning service by a jamming attack is particularly simple to be caused during landing phase of an airplane: in fact it is sufficient to send an interference signal on a defined location and in a defined temporal window. Since a landing assistance tool must have as principal characteristic a high availability degree for the whole period of employment, a low cost solution to the jamming vulnerability has been integrated into the GPS, the so called Inertial Navigation System (INS) [3].

INS is accomplished by an Inertial Measurement Unit (IMU) which integrates the output of a set of sensors in order to compute position, velocity, and attitude. Sensors used are gyros and accelerometers. Gyros determine angular velocity respect to inertial space, while accelerometers evaluate linear acceleration respect to an inertial frame. Integration is a simple process; difficulties are due to various encountered coordinate frames, sensor errors, and system noise. INS suffers of drift velocity errors constantly accumulated during time; therefore, an INS which operates during an appreciable length of time must be updated periodically with new positioning information. This can be accomplished by using an external navigation reference, such as GPS. An integrated GPS/INS system has advantages in terms of output rate, reliability, and accuracy. In fact:

- it is autonomous and does not rely on any other external aids or visibility conditions, and maintains the availability of navigation solution during GPS outages caused by interference, jamming, and so on;
- an optimal mixing of INS and GPS informations reduces the effect of GPS errors; therefore GPS accuracy is improved by integrated solutions;
- INS provides the full navigation state without differentiation (6 degrees of freedom, 3 translational and 3 rotational); GPS signals could be used to determine accelerations by differentiation or attitude by techniques;
- INS provides the navigation solution in real time (i.e., without latency) at rates higher than one achievable from a GPS receiver.

The integration between the two navigation systems with complementary characteristics is possible thanks to the use of a Kalman Filter. Kalman Filter is a recursive algorithm designed to compute corrections to a system based on external measurements. The corrections are weighted according to the filter's actual estimate of the system error statistics. The derivations of the filter equations require some knowledge of linear algebra and stochastic processes. The filter equations can be unwieldy in an algebraic point of view. Fortunately, the operation of the filter can be understood in fairly simple terms. All that is required is an understanding of various common statistical measures. Kalman filtering is an extremely effective and versatile procedure for combining noisy sensor outputs to estimate the state of a system with uncertain dynamics. Kalman Filter exploits a powerful synergism between the Global Positioning System (GPS) and Inertial Navigation System (INS). This synergism is possible, in part, because the INS and GPS have very complementary error characteristics. Shortterm position errors of INS are relatively small, but they have an unbounded degradation on time. GPS position errors, on the other hand, are not so good on short term, but they do not degrade with time. The Kalman filter is able to take advantage of these characteristics in order to provide a common, integrated navigation implementation with better performances than both GPS and INS ones. Kalman filter is able to combine a GPS system, having position uncertainty in the order of tens of meters, with INS system, having position uncertainty which degrades at kilometers per hour (INS); the achieved results is the so called Differential GPS (DGPS) system having position uncertainties in the order of centimeters up to meters. A key role performed by the Kalman filter is the statistical combination of GPS and INS information in order to track drifting parameters of the sensors in the INS. Therefore, the INS can provide enhanced inertial navigation accuracy during GPS signal losses; then, the improved position and velocity estimated by INS can be used to make faster the reacquisition of GPS signal.

Our proposed system uses the DGPS, because in case of jamming the initial state of INS has to be the most exact as possible. In Figure 1, a block model of the general system GPS/INS for landing help is illustrated.



Figure 1: Integrated GPS/INS system for secure landing: it use modality of DGPS, GPS Master on the runway airport and rover over aircraft. The communication link for security is a Wireless LAN.

The master station calculates corrections to the pseudoranges of code and phase and sends them to the rover station; it applies corrections to its observations before the calculation of the position. These operations must be performed in real time, trying to minimize the latency of the whole system. The calculation of the corrections in master station uses the following equations:

$$P_m^j = \rho_m^j + E^j + c(\Delta T_m - \Delta t') + I + T \tag{1}$$

$$\Phi_m^j = \rho_m^j + E^j + c(\Delta T_m - \Delta t') - I + T + \lambda N_m^j$$
⁽²⁾

where P_m^j and Φ_m^j are the pseudoranges of code and phase, ρ_m^j is the master-satellite distance, E^j is ephemeris error, ΔT_m and $\Delta t'$ are respectively clock errors of master and satellite, I and T are ionospheric and tropospheric delays and λN_m^j is the phase ambiguity. By positions of j^{th} satellite from the ephemeris and master station, it is possible to calculate the master-satellite distance except than ephemeris error: $\rho_m^j + E^j$. Subtracting this quantity from P_m^j and Φ_m^j , the following equations of corrections are obtained:

$$\delta P_m^j(t_i) = P_m^j - \rho_m^j - E^j = c(\Delta T_m - \Delta t') + I + T \tag{3}$$

$$\delta \Phi_m^j(t_i) = \Phi_m^j - \rho_m^j - E^j = c(\Delta T_m - \Delta t') - I + T + \lambda N_m^j$$
(4)

The master station also calculates the variation of corrections for each epoch:

$$\delta \widetilde{P}_m^j(t_i) = (\delta P_m^j(t_i) - \delta P_m^j(t_{i-1})) / \Delta t \tag{5}$$

$$\delta \bar{\Phi}_m^j(t_i) = (\delta \Phi_m^j(t_i) - \delta \Phi_m^j(t_{i-1})) / \Delta t \tag{6}$$

The main calculation is the orbit determinations, which are normally drawn by the ephemeris broadcast both in the master and in the rover. Nevertheless, it is rather expensive, if it has to be repeated each second; while it is possible to calculate more quickly the orbits using the ephemeris in SP3 format. In conclusion, using DGPS with the master station on runway and a rover on airplane, and providing airplane with the typical INS instrumentation (accelerometer, gyroscope), a GPS/INS navigation system guarantees good performances in case of jamming attack. In fact, position and speed informations retrieved by INS are satisfactory to complete the landing, while GPS stops the increasing of position error calculated by INS.

3. Spoofing Attacks and Proposed Solution

The spoofing attacks are more difficulties to be realized in comparison to jamming, but at the same time they are more dangerous. In this case, in fact, the hacker replaces actual GPS data with ones compatible with the standard GPS format, inducing the pilot to consider a wrong position of airplane. Proposed solution is based on a verification system for the trajectory suggested by the system GPS/INS to the pilot. Particularly, a software/hardware computation system is placed into the control-tower; it esteems the values of parameters retrieved by GPS/INS navigation system in the following sampling instant by means of actual measures; if a strong discrepancy is obtained, the control-tower communicates to use only INS system and to disable GPS only for a few seconds, in order to avoid contemporaneous jamming attacks. An exhaustive description of system control operations can only be obtained by analyzing the timing of events which interest the airplane during the phase of landing. In Figure 2 three operational phases are underlined, during which the modules are employed to anti-spoofing control and the control-tower develops the respective assignments.

The transmission protocol communicates the coordinates at regular time intervals. In fact, the airplane communicates its position in the WGS84 reference system at time t_i , together with the instantaneous speed. Subsequently, the control-tower esteems the value of airplane coordinates at time t_{i+1} . When the new coordinates will be received from the tower, a comparison will be made with the coordinates previously esteemed, in order to discover a spoofing attack: in this case it will be communicated to the airplane to use only the INS system. Obviously, the communication channel between control-tower and airplane must be secure; indeed, it is possible to use a wireless link similar to 802.11 b protocol or superior both for data control's communication between control-tower and airplane and for DGPS data correction's communication. Even if this technology suffers of a particular type of vulnerability, the so called "man in middle", this attack is a lot difficult to effect within the times characterizing the landing phase.



Figure 2: The aircraft communicates own coordinates to the tower in order to verify the presence of a spoofing attack.

4. Conclusions

The safety of flights is a "must" above all for civil transportation. The most dangerous phase of flight is surely the landing phase, in which the navigation systems can be subjected to jamming or spoofing attacks. In this paper, GPS and INS has been analysed; they have complementary characteristics. GPS provides an estimate of position and velocity with bounded estimation error, but it suffers of problems related to signal format and data transmission protocol. GPS uses a space-to-earth signal and the power of received signal is -160 dBW. The low power level makes GPS highly susceptible to interference and a pilot may experience short-term loss of GPS signal during the landing phase. On the other hand, INS is not dependent on the external field, but it suffers of drift velocity errors constantly accumulated during time. Therefore, the integration of these two systems is a reliable tool for helping pilots in the landing phase, above all in countries with an high risk of terrorist attacks. In this paper, a so defined GPS/INS landing-aid system has been proposed in order to avoid the jamming problem. Moreover, an infrastructure between control-tower and airplanes (based at least on 802.11 b communication protocol) has been considered in order to ensure a complete reliability of our hybrid GPS/INS system and have a secure navigational system, verifying and avoiding the presence of possible spoofing attacks.

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Development of the Pulsed Direct Current Iontophoresis and Its Clinical Application

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Abstract—The skin is a primary area of body contact with the environment and is the route by which many chemicals enter the body. The delivery of drugs into and through the skin has been an important area of research for many years. Iontophoresis can be defined as the process of increasing the rate of penetration of ions into or through a tissue by the application of an external electric field across the tissue. Impedance spectroscopy was used to investigate the electrical response of skin to different ions, applied currents and fields. The stratum corneum shows two important electrical features. First, it tends to become polarized as an electrical field is applied continuously. Second, its impedance changes with the frequency of the applied electrical field. To avoid the counterproductive polarization, the current should be applied in a periodic manner, which is called pulsed direct current. The pulsed direct current generating iontophoretic delivery system was developed. In the state of on, charged molecules are delivered by the iontophoretic diffusion process into the skin. A non-parenteral method for the delivery of macromolecules was developed by using a pulse direct current mode iontophoretic technique.

1. Introduction

Medical diagnosis and in-patient drug monitoring rely upon the detection and quantitation of endogenous and exogenous bioactive chemicals. Currently, such analysis is predominantly based upon blood sampling which is achieved invasively via needle. The inconvenience and limitations of this procedure are well-known; patients, on the whole, would rather not be injected, the frequency and amount of sampling is constrained, successful intravenous access in geriatric and pediatric patients may be difficult, and there are risks to both patient and sampler. The potential benefits of alternative, noninvasive sampling procedures for chemical exposure assessment and continuous drug monitoring have been described. One approach involves collection, via the skin, of molecules circulating systemically. A major difficulty, however, with this idea is the characteristically very slow and variable passive permeation rates of chemicals across the skin. Indeed, experiments examining the outward migration of the phylline revealed little correlation between sampled amounts and drug levels in the body. Additionally, it was necessary to collect samples over extended periods of time, a significant potential drawback. To circumvent these problems, the application of iontophoresis to enhance sampling efficiency has been examined. Iontophoresis employs an electrical potential gradient to promote the penetration of ionizable molecules across the skin [1-3]. The current uses of the technique are the treatment of dermatology and aesthetic dermatology. In this paper, we have investigated the efficiency of a pulse waveform in the iontophoretic delivery of insulin to achieve blood glucose control and compared the results with a simple direct current iontophoretic delivery system.

2. Theoretical Consideration

The skin of an average adult body covers a surface area of $\sim 2 \text{ m}^2$ and receives about one-third of the blood circulating through the body. It is one of the most readily accessible organs on the human body. The skin is divided into three layers: the epidermis, the dermis, and the subcutaneous tissue. The epidermis is the outermost portion of the skin and is composed of stratified squamous epithelium. The epidermal thickness varies from 50 μ m on the eyelids to 1.5 mm on the palms and soles. The innermost layer of the epidermis consists of a single layer of cuboidal cells called basal cells. These cells differentiate and migrate towards the skin surface. The outer layer of the epidermis is called the stratum corneum, which is composed of flattened and dead cells. As they migrate to the skin surface, the cells become more stratified and finally form the cornified layer of the stratum corneum. The skin is known to produce a large impedance to charged molecules as they are driven through the skin by an applied electrical field. The electrical properties of the skin are known to be dominated by the least conductive stratum corneum. Under the influence of electric current, ionic species or charged molecules are driven across the skin, possible through the shunt pathways or intercellular spacing in the stratum corneum, since the skin is likely to be perturbed during iontophoresis, which may remove or disrupt

the intercellular lipids, resulting in the formation of artificial shunts. Iontophoresis facilitated skin permeation flux of an ionic species can be described by an equation consisting of the following three components [4–5]:

$$J = J^p + J^e + J^c \tag{1}$$

where the first term on the right-hand side is the passive skin permeation flux, given by

$$J^p = K_s D_s \frac{dC}{h_s} \tag{2}$$

the second term is electric current driven skin permeation flux

$$J^e = \frac{Z_i D_i F}{RT} C_i \frac{dE}{h_s} \tag{3}$$

and the last term is the convective flow driven skin permeation flux

$$J^c = kC_s I_d \tag{4}$$

in which K_s is coefficient for interfacial partition from donor solution to stratum corneum, C_i is donor concentration of ionic species, C_s is concentration in the skin tissue, dE/h_s is electric potential gradient across the skin, dC/h_s is concentration gradient across the skin, D_i is diffusivity of ionic species I in the skin, D_s is diffusivity across the skin, I_d is current density applied, Z_i is electric valence of ionic species I, k is proportionality constant, F is Faraday constant, T is absolute temperature, and R is gas constant. This equations describes the flux of an ion under the influence of both a concentration gradient and an electrical field.

Iontophoretic system	Drug delivery mode	
Direct current mode	Continuous drug delivery under constant intensity of	
	direct current	
Pulse current mode		
(a) Depolarizing pulse	Continuous drug delivery under constant application of	
Iontophoresis system	pulse current	
(b) Periodic iontotherapeutic	Programmed drug delivery under periodic applications	
system	of pulse current with constant intensity	
system	of pulse current with constant intensity	

Table 1: Some representative iontophoretic delivery systems.

Essentially, two types of iontophoresis have been investigated: the earlier one uses a simple direct current mode, while the more recent one utilizes the pulsed direct current mode in shown Table 1. An iontophoretic drug

delivery system is composed of four basic components including a battery, control circuitry, electrodes, and reservoirs. The component configuration is schematically illustrated in Figure 1. The battery supplies the required energy to power the control circuitry which, subsequently, controls the electric potential applied to the electrodes. Because drug delivery is roportional to the current flowing through the body, most iontophoretic transdermal drug delivery control circuits utilize some type of current source to compensate for the difference in skin resistance from person to person. A battery and a single resistor is a simple current regulator, but regulation is effective for only small resistance variations. Active current regulators automatically adjust their operating characteristics in response to a broader range of resistances and therefore are preferred for iontophoretic transdermal drug delivery applications since the resistances from person to person or from site to site can vary significantly. The electrodes of an iontophoretic device are in direct electrical contact with the reservoirs, and together they determine the electrochemical reactions that occur at the anode and cathode. These reactions are particularly important to



Figure 1: Schematic illustration of the major components of an iontophoresis system.

consider when designing iontophoretic devices for the delivery of drugs over extended periods of time. A better choice of electrode materials would be titanium or silver for the anode and chloridized silver for the cathode.

3. Materials and Methods

Tissue samples were obtained from freshly rats skin. All tissues of exactly known time history were prepared in the same way. A cylinder of diameter corresponding to the diameter of the electrode was excised from

the bulk of the test tissue. It was then inserted into a plastic tube and a microtome was used to cut tissue discs of the desired thickness equal to the distance between the electrodes. The dielectric measurements were performed using automatic swept-frequency network and impedance analyzer. The frequency range 5 Hz to 10 MHz was covered by an HP-4192A impedance analyzer. Open-ended co-axial probe were used to interface the measuring equipment with the samples in all cases. The probe is characterized by a fringing capacitance and conductance which are functions of its physical dimension and can be measured with the impedance analyzer. In addition there are stray capacitive and inductive elements that have to be nor-



Figure 2: Schematic diagram of the measuring system.

malized. Figure 2 is a schematic diagram of the experimental arrangement [6]. For each set of measurements new samples were excised from the bulk of the tissue, which was sealed to avoid tissue drying and stored at ambient temperature during the measurement session.

4. Results and Discussion

The condition inside the tissues when adding an electric field to the tissues becomes the index that the complex permittivity of the tissue is important. The real component corresponding to the permittivity and the imaginary component, known as the dielectric loss, corresponds to the dissipative loss associated with

the movement of polarisable charges in phase with the electric field. This shows that, as a dielectric dispersion is traversed by changing the frequency of measurement, the change in conductivity is directly proportional to the change in permittivity. This follows from the fact that the total energy in the field is constant and must either be stored or dissipated by the system with which it interacts. The dielectric properties of skin show considerable variability over different parts of the body. The average electrical properties of skin in the range 5 Hz-1 MHz are shown in Figure 3. An interpretation of these properties was approached via a consideration of the inhomogeneous structure and composition of skin and of the way in which this varies from the skin surface to the underlying dermis and subcutaneous tissues. The dielectric properties of skin are largely determined by the stratum corneum, which has a thickness of some $15 \,\mu\text{m}$ and consists largely of dead cells [7–9]. As for the complex permittivity, the characteristic to become minimal around 50 kHz was measured.

It may be noted that skin prossesses a relatively weak α dispersion, and this relative lack of a significant dispersion in the frequency range 1 Hz–100 kHz is plausibly ascribed to the dead nature and low conductivity of the stratum corneum. It was tentatively suggested that the origin of these dispersions lay in the stratum corneum and was associated with the relax-



Figure 3: A typical example of the dielectric constants of skin as a function of frequency.

ation of ions surrounding the corneal cells. It is worth drawing attention to the fact that the electric impedance of those parts of the skin at which the points and meridians of acupuncture are located is significantly less than that of the surrounding tissue, a fact which may be presumed to be of diagnostic value and may plausibly underlie the mechanism by which signals are transmitted around the body by means of the meridians of acupuncture. Many therapeutic and diagnostic techniques rely upon the application of electrical fields or the measurement of electrical properties. Since skin tissue often constitutes the interface between the biological and electronic parts of the system, its dielectric properties are of some interest and importance. It is worth drawing attention to the fact that the electric impedance of those ports of the skin at which the points and meridians of acupuncture are located is significantly less than that of the surrounding tissue, a fact which may be presumed to be of diagnostic value and may plausibly underlie the mechanism by which signals are transmitted around the body by means of the meridians of acupuncture. Therefore, the complex permittivity is the important index that the energy absorption which is consumed in the tissue can be discussed.

Skin manifests large impedance to charged molecules which are driven through the skin under an applied electrical field. The electrical properties of the skin are dominated by the stratum corneum which is considered to be the least conductive layer of the skin. Stratum corneum consists of multilayers of cornified cells. These electrically insulated horny cells are continuously replenished by the slow upward migration of cells from the basal cell layer of the stratum germinativum. An analogous equivalent circuit of skin impedance is shown in Figure 4. It consists of a resistor, R_{sc} , and a capacitor, C_{sc} , existing in parallel in the stratum corneum, which is then in series with the resistor, R_{vs} , in the viable skin. The magnitude of R_{sc} is rather large, and can range from 10–20 k Ω cm² in animal skin to 100–5000 k Ω cm² in human skin, while



Figure 4: Diagrammatic illustration of the analogue equivalent circuit of skin impedance.

 R_{vs} is relative small in magnitude and is in the range of $0.1-1.0 \,\mathrm{k\Omega cm^2}$. The stratum corneum shows two important electrical features: first, it is polarized by the electrical field, and second, its impedance changes with the frequency of the applied electrical field. When an electrical field with direct current is applied in a continuous manner to the skin, an electrochemical polarization develops rapidly in the capacitor. It often operates against the applied electric field and greatly decreases the magnitude of effective current across the skin.

To avoid the polarization of the stratum corneum, a pulse direct current can be used [10]. The pulse mode is a direct current voltage which periodically alternates with the "on" and "off" of the applied voltage in Figure 5. In the state of "on", charged molecules are forced into the skin and the stratum corneum soon becomes polarized; while in the state of "off", no external stimulation is present and the stratum corneum becomes depolarized. The on/off ratio controls the proportion for polarization and depolarization process in each cycle. The current output generators were built in-house. The pulsed-current unit was designed to provide adjustable peak voltages from 0 to 20 V and variable frequency from 0 to 300 kHz. The wave form used in the pulsed-current unit is also shown in Figure 5. The on/off ratio or the percent duty could be varied from 0 to 70%. This unit had 15 channels and a display that provided the average current readout for each channel. This average current readout is the current averaged over the cycle. At a setting of 40 kHz duty, for example, the duration of each pulse would be $25 \,\mu$ s, but at a setting of 30% duty, the pulse would be on for 7.5 μ s and off for 17.5 μ s. In some experiments, a pulsed-current unit with a single output channel and a fixed frequency and percent duty was used. When an ideal on/off ratio is selected, every new cycle starts with no residue polarization left in the skin from the previous cycle, i.e., the effect of polarization is eliminated.

The energy (E) required to overcome the penetration barrier, stratum corneum, can be expressed by:

$$E = \int [V(t)i(t)]dt = \int [i(t)^2 R_t(t)]dt$$
(5)

where V(t) and i(t) are the voltage and current applied respectively and R_t is the impedance of the skin. As can be seen from Eq. 5, less energy will be required to overcome the barrier when the skin impedance is reduced. This may be achieved by applying the current with proper frequency and on/off ratio. Therefore, it is essential to select optimum pulse mode parameters to attain the best facilitating effect of iontophoresis for a particular drug or a dosage form.

Diabetes mellitus is a chronic systemic disease in which the body either fails to produce or fails to respond to the glucose regulatory hormone insulin. Insulin is required in order for cells to take up glucose from the blood, and in diabetics, a defect in insulin signaling can give rise to large fluctuations in blood glucose levels unless proper management techniques are employed. Insulin, a protein hormone containing 51 amino acid residues, has a molecular weight of approximately 6,000 daltons and an extremely short biological half-life of less than 30 minutes. In healthy humans, it is secreted by beta cells in the Langerhans islet of the pancreas in response to an increase in blood glucose level to facilitate the process of glucose utilization for either energy or storage. Inpatients with diabetes mellitus, however, the capacity of the pancreas to supply insulin in response to the increase in blood glucose level is impaired. For the control of diabetes mellitus, insulin must be supplied externally by subcutaneous injection at a dose of 10–20 units three to four times a day. Experiments were conducted in a skin to study the feasibility of facilitating the transdermal delivery of insulin across the freshly excised abdominal skin of rats by applying iontophoresis with a pulsed direct current. The results demonstrated that the skin permeation rate of insulin thus applied is enhanced substantially as compared to that achieved by passive diffusion alone. The insulin data summarized in Figure 6 were generated using the iontophoresis shown in Figure 5. The use of iontophoresis results in the typical sigmoidal dependence of receptor concentration on time. The time required to achieve a steady-state receptor concentration is determined by the receptor volume and flow rate of the receptor solution. For the in vivo conditions used in the insulin study, time to achieve a steady-state concentration was about 2 hours for all currents, as shown in Figure 6.



Figure 5: Diagrammatic illustration of the transdermal periodic iontophoresis system. Diagram of a pulse waveform profile, where A is the amplitude of a current intensity (mA), B/C are the on/off ratio, D is the duration (s) of a complete cycle and so 1/D is the frequency.



Figure 6: In vivo skin permeation profiles of insulin under the iontophoresis facilitated permeation by the pulse current.

The cumulative amount of insulin permeating through the skin increases with time during the period of transdermal periodic iontotherapeutic system treatment and gradually returns to the skin permeation profile by passive diffusion after termination of the treatment. Also plotted in Figure 6 for comparison is the rate profile of insulin permeation. Analysis of the skin permeation rate profile suggests that the pulsed direct current iontophoresis facilitated transfermal transport of drug molecules consists of four phases: (i) the facilitated absorption phase, in which the skin permeation of drug molecules in enhanced by iontophoresis treatment and the skin permeation rate linearly increases with the time of treatment; (ii) the equilibrium phase, in which the skin permeation rate has reached a plateau even though iontophoresis treatment is continuously applied; (iii) the desorption phase, in which the skin permeation rate decrease lineally with time immediately after the termination of iontophoresis treatment and the drug molecules already delivered into the skin tissues are desorbed into the receptor solution; and (iv) the passive diffusion phase, in which the skin permeation rate returns to the baseline level as defined by passive diffusion. The transdermal periodic iontotherapeutic system facilitated skin permeation rate of insulin was observed to increase linearly with the current density of pulsed direct current applied, but in a non-linear manner with the duration of transdermal periodic iontotherapeutic system application time. The therapeutic response of insulin obtained in the present study indicates that, in the presence of a facilitated transport, it may be possible to topically administer high molecular weight substances,

including other peptides and/or proteins, for systemic therapy. The present work has demonstrated that the iontophoresis technique may provide a convenient means for the systemic delivery of insulin without the use of a hypodermic needle. The feasibility of optimizing the plasma concentration of the drug by either controlling the time of application and/or regulating the magnitude of current, or alternatively, by the use of a pulse current rather than a direct current, needs further investigation.

5. Conclusions

A non-parenteral method for the delivery of macromolecules such as insulin was developed by using a pulse direct current mode transdermal iontophoretic technique. The intensity of current, frequency, on/off ratio and mode of waveform were found to play an important role in the transdermal iontophoretic delivery of insulin. More extensive investigations on various aspects of this system are necessary to obtain optimum parameters of pulse direct current mode for transdermal iontophoretic delivery of dermatology and cosmetic science.

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Computation of Resonant Frequencies of Any Shaped Dielectric Resonators by CFDTD

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Abstract—Dielectric resonators (DR) have helped achieving the miniaturization of many active and passive microwave components, such as oscillators, filters and antennas. Nowadays they are used widely in mobile telecommunications and optical instruments such as optical couplers and filters. To design such components, designers must have the knowledge of predicting the shape and resonant frequency response of usable dielectric resonators. Numerical methods such as Method Of Moment (MOM), Finite Element Method (FEM) and Finite Difference Time Domain (FDTD), are useful tools for simulating those problems. The MOM & FEM are usually in frequency domain and we need to inverse a large matrix to solve the problem. Fortunately, FDTD is in time domain and by one run, we can have a large bandwidth response of our system. We have prepared a program code for determining resonant frequencies of DRs by using Conformal Finite Difference Time Domain (CFDTD) which is used for curved surfaces such as cylindrical and spherical shapes in Cartesian coordinate. In this paper, first, we present the simulation of resonant frequencies of cylindrical dielectric resonator which its results can be compared with reference results. In proceeding, computation of resonant frequency response of a thin dielectric spherical layer that can be useful for improving the achievement of Whispering Gallery Modes, which are produced in spherical DRs, will be considered.

1. Introduction

Dielectric resonators are used widely in mobile telecommunications and optical instruments, such as antennas, filters and couplers. Knowledge of predicting the shape and resonant frequency response of usable dielectric resonators is very important for engineers to design these telecommunication systems. Finite Difference Time Domain method is one of the numerical methods and even the most useful one for determining resonant frequencies of DRs, because by one run, we can have a large bandwidth response of our system.

In this work, we have prepared a code program for determining resonant frequencies of DRs by FDTD method, and for cases with curved surfaces such as cylindrical and spherical shapes we have used Conformal FDTD to reduce the error introduced by staircasing of surfaces that are not precisely aligned with major grid planes. To achieve such a goal, we have used the method suggested by Ref. [1] for simple structures. We have expanded that method for Yee cells which contain three or more layers of dissimilar dielectrics. We have used this method for simulation of thin layer dielectric curved surfaces, such as shelled spheres, that can be useful for improving the achievement of Whispering Gallery Modes.

The analytic method for computation of resonant frequencies of any spherical layers, which is used for verification of numerical method, is discussed in section 2. The CFDTD method applied for computation of resonant frequencies of any shaped DRs is in section 3 and finally in section 4, results of simulating are compared with analytic method and Ref. [4].

2. Analytical Computation of Resonant Frequencies of Multi Spherical Shells

In this section we only consider a dielectric sphere with a single coating layer. We use the Mie theory to compute the resonant frequencies. According to Ref. [2], we introduce two vectors \mathbf{M} and \mathbf{N} which correspond to TE and TM spherical modes.

$$\mathbf{M} = \mathbf{M}_e + j\mathbf{M}_o \tag{1}$$

$$\mathbf{N} = \mathbf{N}_e + \mathbf{j}\mathbf{N}_o \tag{2}$$

where

$$\mathbf{M}_{omn} = \frac{m\hat{a}_{\theta}}{\sin(\theta)} \mathbf{Z}_n(\beta r) \mathbf{P}_n^m(\cos(\theta)) \cos(m\varphi) - \hat{a}_{\varphi} \mathbf{Z}_n(\beta r) \frac{\partial}{\partial_{\theta}} \mathbf{P}_n^m(\cos(\theta)) \sin(m\varphi)$$
(3)

$$\mathbf{M}_{emn} = -\frac{\hat{m}\hat{a}_{\theta}}{\sin(\theta)} \mathbf{Z}_n(\beta r) \mathbf{P}_n^m(\cos(\theta)) \sin(m\varphi) - \hat{a}_{\varphi} \mathbf{Z}_n(\beta r) \frac{\partial}{\partial_{\theta}} \mathbf{P}_n^m(\cos(\theta)) \cos(m\varphi)$$
(4)

$$\mathbf{N}_{omn} = \hat{a}_r \frac{n(n+1)}{\beta r} \mathbf{Z}_n(\beta r) \mathbf{P}_n^m(\cos(\theta)) \sin(m\varphi) + \hat{a}_\theta \frac{1}{\beta r} \frac{\partial}{\partial r} [r \mathbf{Z}_n(\beta r)] \frac{\partial}{\partial \theta} \mathbf{P}_n^m(\cos(\theta)) \sin(m\varphi) + \hat{a}_\varphi \frac{m}{\sin(\theta)} \frac{1}{\beta r} [r \mathbf{Z}_n(\beta r)] \frac{\partial}{\partial \theta} \mathbf{P}_n^m(\cos(\theta)) \cos(m\varphi)$$

$$\mathbf{N}_{emn} = \hat{a}_r \frac{n(n+1)}{\beta r} \mathbf{Z}_n(\beta r) \mathbf{P}_n^m(\cos(\theta)) \cos(m\varphi) + \hat{a}_\theta \frac{1}{\beta r} \frac{\partial}{\partial r} [r \mathbf{Z}_n(\beta r)] \frac{\partial}{\partial \theta} \mathbf{P}_n^m(\cos(\theta)) \cos(m\varphi)$$
(5)

$$-\hat{a}_{\varphi}\frac{m}{\sin(\theta)}\frac{1}{\beta r}[r\mathbf{Z}_{n}(\beta r)]\frac{\partial}{\partial\theta}\mathbf{P}_{n}^{m}(\cos(\theta))\sin(m\varphi)$$
(6)

In these equations, Z_n indicates the spherical Bessel or Hankel functions correspond to the direction of propagating wave. Also P_n^m is Associate Legendre function. In proceeding we use substitution indicated by Eq. (7):

$$\hat{Z}_n(\beta r) = \beta r Z_n(\beta r) \tag{7}$$

According to Fig. 1 we can expand electromagnetic fields by two vectors M and N:

$$\mathbf{E}^{i} = \sum_{n=1}^{\infty} \alpha_{n} \left[\mathbf{M}_{oln}^{(1)}(\beta_{3}, \mathbf{r}, \theta, \varphi) + j \mathbf{N}_{eln}^{(1)}(\beta_{3}, \mathbf{r}, \theta, \varphi) \right]$$
(8)

$$\mathbf{H}^{i} = \frac{j}{\eta_{3}} \sum_{n=1}^{\infty} \alpha_{n} \left[\mathbf{N}_{oln}^{(1)}(\beta_{3}, \mathbf{r}, \theta, \varphi) + j \mathbf{M}_{oln}^{(1)}(\beta_{3}, \mathbf{r}, \theta, \varphi) \right]$$
(9)

 \mathbf{E}^{i} and \mathbf{H}^{i} are incident plane wave electromagnetic fields. Also in these equations β is wave number, η is characteristic impedance and coefficient α_{n} defined as:

$$\alpha_n = (-j)^n \frac{2n+1}{n(n+1)} \tag{10}$$

Scattered and transmitted waves in each layer can be expanded as Eqs. (11) to (18):

$$\mathbf{E}^{s_3} = \sum_{n=1}^{\infty} \left[A_n^{s_3} \mathbf{M}_{oln}^{(4)}(\beta_3, \mathbf{r}, \theta, \varphi) + j B_n^{s_3} \mathbf{N}_{oln}^{(4)}(\beta_3, \mathbf{r}, \theta, \varphi) \right]$$
(11)

$$\mathbf{H}^{s_3} = \frac{j}{\eta_3} \sum_{n=1}^{\infty} \left[A_n^{s_3} \mathbf{N}_{oln}^{(4)}(\beta_3, \mathbf{r}, \theta, \varphi) + j B_n^{s_3} \mathbf{M}_{eln}^{(4)}(\beta_3, \mathbf{r}, \theta, \varphi) \right]$$
(12)

$$\mathbf{E}^{t_2} = \sum_{n=1}^{\infty} \left[A_n^{t_2} \mathbf{M}_{oln}^{(1)}(\beta_2, \mathbf{r}, \theta, \varphi) + j B_n^{t_2} \mathbf{N}_{eln}^{(1)}(\beta_2, \mathbf{r}, \theta, \varphi) \right]$$
(13)

$$\mathbf{H}^{t_2} = \frac{j}{\eta_2} \sum_{n=1}^{\infty} \left[A_n^{t_2} \mathbf{N}_{oln}^{(1)}(\beta_2, \mathbf{r}, \theta, \varphi) + j B_n^{t_2} \mathbf{M}_{eln}^{(1)}(\beta_2, \mathbf{r}, \theta, \varphi) \right]$$
(14)

$$\mathbf{E}^{s_2} = \sum_{n=1}^{\infty} \left[A_n^{s_2} \mathbf{M}_{oln}^{(4)}(\beta_2, \mathbf{r}, \theta, \varphi) + j B_n^{s_2} \mathbf{N}_{oln}^{(4)}(\beta_2, \mathbf{r}, \theta, \varphi) \right]$$
(15)

$$\mathbf{H}^{s_2} = \frac{j}{\eta_2} \sum_{n=1}^{\infty} \left[A_n^{s_2} \mathbf{N}_{oln}^{(4)}(\beta_2, \mathbf{r}, \theta, \varphi) + j B_n^{s_2} \mathbf{M}_{eln}^{(4)}(\beta_2, \mathbf{r}, \theta, \varphi) \right]$$
(16)

$$\mathbf{E}^{t_1} = \sum_{n=1}^{\infty} \left[A_n^{t_1} \mathbf{M}_{oln}^{(1)}(\beta_1, \mathbf{r}, \theta, \varphi) + j B_n^{t_1} \mathbf{N}_{eln}^{(1)}(\beta_1, \mathbf{r}, \theta, \varphi) \right]$$
(17)

$$\mathbf{H}^{t_1} = \frac{j}{\eta_1} \sum_{n=1}^{\infty} \left[A_n^{t_1} \mathbf{N}_{oln}^{(1)}(\beta_1, \mathbf{r}, \theta, \varphi) + j B_n^{t_1} \mathbf{M}_{oln}^{(1)}(\beta_1, \mathbf{r}, \theta, \varphi) \right]$$
(18)

where, in Eqs. (11) to (18), superscript (1) stands for spherical Bessel functions j_n and superscript (4) for spherical Hankel functions $h_n^{(2)}$, which we faced in **M** and **N** [2]. And also s and t superscripts in coefficients stand for scattered and transmitted waves in each layer.



Figure 1: A plane wave incident to the two layer spherical dielectric resonator.

Let us assume that $\mu_1 = \mu_2 = \mu_3 = \mu_0$ and $\sigma_1 = \sigma_2 = \sigma_3 = 0$, by applying the boundary conditions we will have two different matrixes for TE and TM modes, separately.

$$\begin{bmatrix} -h_n^{(2)}(\beta_3 b) & h_n^{(2)}(\beta_2 b) & j_n(\beta_2 b) & 0\\ -\hat{h}'_n^{(2)}(\beta_3 b) & \hat{h}'_n^{(2)}(\beta_2 b) & \hat{j}'_n(\beta_2 b) & 0\\ 0 & h_n^{(2)}(\beta_2 a) & j_n(\beta_2 a) & -j_n(\beta_1 a)\\ 0 & \hat{h}'_n^{(2)}(\beta_2 a) & \hat{j}'_n(\beta_2 a) & -\hat{j}'_n(\beta_1 a) \end{bmatrix} \begin{bmatrix} A_{n}^{s_3}\\ A_{n}^{s_2}\\ A_{n}^{t_2}\\ A_{n}^{t_1} \end{bmatrix} = \begin{bmatrix} \alpha_n j_n(\beta_3 b)\\ \alpha_n \hat{j}'_n(\beta_3 b)\\ 0 \end{bmatrix}$$
(19)

$$\begin{bmatrix} -h_{n}^{\lambda'}(\beta_{3}b) & \sqrt{\frac{\varepsilon_{2}}{\varepsilon_{3}}}h_{n}^{\lambda'}(\beta_{2}b) & \sqrt{\frac{\varepsilon_{2}}{\varepsilon_{3}}}j_{n}(\beta_{2}b) & 0\\ 0 & \sqrt{\frac{\varepsilon_{1}}{\varepsilon_{2}}}\hat{h}'_{n}^{(2)}(\beta_{2}a) & \sqrt{\frac{\varepsilon_{1}}{\varepsilon_{2}}}\hat{j}'_{n}(\beta_{2}a) & -\hat{j}'_{n}(\beta_{1}a)\\ 0 & \sqrt{\frac{\varepsilon_{2}}{\varepsilon_{1}}}h_{n}^{(2)}(\beta_{2}a) & \sqrt{\frac{\varepsilon_{2}}{\varepsilon_{1}}}j_{n}(\beta_{2}a) & -j_{n}(\beta_{1}a) \end{bmatrix} \begin{bmatrix} B_{n}^{s_{2}}\\ B_{n}^{t_{2}}\\ B_{n}^{t_{1}} \end{bmatrix} = \begin{bmatrix} \alpha_{n}j_{n}(\beta_{3}b)\\ 0\\ 0 \end{bmatrix}$$
(20)

The solution of these two complex equations will give us the resonant frequencies and quality factors of multilayer dielectric resonators.



Figure 2: (a) Mesh truncation of a spherical DR and (b) Mesh truncation of a thin layer spherical DR.

3. CFDTD Method

The CFDTD can reduce the error introduced by staircasing in Cartesian coordinate. The effective dielectric permittivity of dielectric material in any piece of stairs can be found by following equations [1].

$$\varepsilon_x^{eff} = \frac{\Delta x_1 \varepsilon_1 + \Delta x_2 \varepsilon_2}{\Delta x} \tag{21}$$

$$\varepsilon_y^{eff} = \frac{\Delta y_1 \varepsilon_1 + \Delta y_2 \varepsilon_2}{\Delta y} \tag{22}$$

$$\varepsilon_z^{eff} = \frac{\Delta z_1 \varepsilon_1 + \Delta z_2 \varepsilon_2}{\Delta z} \tag{23}$$

where, by applying these values in split Maxwell equations, the error of staircasing will be reduced.

In the case of thin layer surface, such as a spherical shell DR, Yee cells may contain three or more dissimilar dielectrics. In such cases, such as n different dielectric layers, we use average method.

$$\varepsilon_x^{eff} = \frac{\Delta x_1 \varepsilon_1 + \Delta x_2 \varepsilon_2 + \Delta x_3 \varepsilon_3 + \dots + \Delta x_n \varepsilon_n}{\Delta x}$$
(24)

$$\varepsilon_y^{eff} = \frac{\Delta y_1 \varepsilon_1 + \Delta y_2 \varepsilon_2 + \Delta y_3 \varepsilon_3 + \dots + \Delta y_n \varepsilon_n}{\Delta y} \tag{25}$$

$$\varepsilon_z^{eff} = \frac{\Delta z_1 \varepsilon_1 + \Delta z_2 \varepsilon_2 + \Delta z_3 \varepsilon_3 + \dots + \Delta z_n \varepsilon_n}{\Delta z}$$
(26)

Table 1: Comparison of resonant frequencies found by CFDTD simulation and Ref. [4], for a cylindrical DR with $\varepsilon_r = 38$, a = 5.25 mm and h = 4.6 mm.

Mode	Frequency (GHz)		Relative Error
	Ref. [4]	CFDTD	
$\mathrm{TE}_{01\delta}$	4.829	4.859	%0.621
$\mathrm{HEM}_{11\delta}$	6.333	6.299	% 0.536
$\mathrm{HEM}_{12\delta}$	6.638	6.694	%0.843
$\mathrm{TM}_{01\delta}$	7.524	7.487	%0.491
$\operatorname{HEM}_{21\delta}$	7.752	7.702	%0.644



Figure 3: (a) Resonant frequency response of a cylindrical DR with $\varepsilon_r = 38$, $a = 5.25 \,\mathrm{mm}$ and $h = 4.6 \,\mathrm{mm}$ found by CFDTD and (b) Plot of H field at $f = 10.1507 \,\mathrm{GHz}$.

4. Numerical Results

In this section the results of CFDTD simulation for cylindrical and thin layer spherical DRs are shown. For verification, the results are compared with analytical (exact) method or with other reference data.

4.1. Cylindrical Dielectric Resonator

Let us use cylindrical DR with following electrical and geometrical parameters: $\varepsilon_r = 38$, radius a = 5.25 mm and height h = 4.6 mm. In table 1, resonant frequencies found by CFDTD simulation and Ref. [4] are compared. Also in Fig. 3, frequency response of the cylindrical DR and plot of H field, stimulated at f=10.1507 GHz, are shown.

4.2. Thin Layer Spherical Dielectric Resonator

A thin layer spherical DR with $\varepsilon_r = 90$, external radius b = 2.54 cm and internal radius a = 0.99b is simulated with CFDTD method. We have compared our results with Mie method and it is fairly good. As it is shown in Fig. 4(b), Whispering Gallery Mode is stimulated in the thin layer spherical DR at f = 35.3214 GHz.

5. Conclusion

In this article, we have presented CFDTD simulation of dielectric resonators. A cylindrical DR has been simulated, and a good agreement with Ref. [4] is achieved. For the next simulation, a thin layer spherical dielectric resonator is considered and we faced with Whispering Gallery Modes which we have predicted by Mie method.

These simulations will show us that CFDTD method can give us very good results for simulating resonant frequencies of any shaped dielectric resonators.

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A Novel Transmission Line Model for Analyzing Bowtie Patch Antennas

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Abstract—A novel transmission line model is represented to model bowtie patch antennas. The purposed model uses two slots for modeling the radiation from patch sides. Each radiation slot is presented by parallel equivalent admittance. Also in this model mutual coupling and the effect of slots length limitation as well as the influence of the side slots on the radiation conductance are taken into account implicitly. Admittance and controlled source equations that used for rectangular patch antenna are modified.

1. Introduction

These days microstrip antennas are popular and getting more and more attention due to their excellent advantages. Depending upon the applications, microstip antennas having different geometrical shapes are used. In other hand now days researchers are interested in the design and development of compact microstip radiating elements. It can be argued that Bowtie microstip antenna is one of such compact microstrip antennas [2]. The Bowtie antenna dose not has a regular geometric shape and hence most of the analytical techniques such cavity model can not be used for antenna's parameter calculation directly. However, popular numerical techniques like method of moment (MOM), finite element method, finite-difference time -domain (FDTD) method, etc., could be used for analysis of such antennas but they are computationally expensive.



The transmission line method (TLM) is known to be reasonably accurate and have good efficiency especially in numerical calculation as well as it can be applied for modeling of antennas arrays. Therefore in this paper a novel transmission line model is presented for analysis of bowtie patch antenna of finite length, placed on a dielectric substrate as shown in Fig. 1. Also Fig. 2 shows the radiating slots which form a useful model for calculating the radiation filed of the antenna. These so-called equivalent slots consist of two main slots with a uniform distribution and four side slots with sinusoidal distribution. In order to model regular microstrip antenna with the existing transmission-line models, shape of the antenna represented by a line section terminated at both ends by radiation admittance. Also an improved TLM model for rectangular microstip antenna has been introduced by Pues abd Van de Cappelle which used voltage dependent current generators for modeling mutual coupling [1].

Our model in this paper is similar to Pues model [1] in addition we have used taper transmission line instead normal microstrip line for modeling bowtie shaped patch as shown in Fig. 3. More over as can be seen in Fig. 3 the offset of feed location has been considered by dividing the taper line into two sections.

2. Determination of Model Parameters

Figure 4 model contains of following unknowns:

- 1. The line parameters (Y_c) .
- 2. The self admittance of slots (Y_s) .

3. The mutual admittance (Y_m) .

Where each term can be define and calculated as follow:

2.1. Line Parameters

Characteristic impedance could be calculated by:

$$Z_c = \frac{\eta_0}{\sqrt{\varepsilon_e}} \frac{h}{W_e}$$
$$W_e = \frac{W + W_c}{2}$$

where

2.2. Self Admittance

In order to determine $Y_s (= G_s + jB_s)$, previous works for rectangular microstrip antennas have been examined for improving the accuracy and efficiency of our model. In this respect Pues formula [1] is used for determining the self conductance G_s .

$$G_s = \frac{1}{\pi\eta} \{ (wSi(w) + \frac{\sin w}{w} + \cos w - 2)(1 - \frac{s^2}{24}) + \frac{s^2}{12}(\frac{1}{3} + \frac{\cos w}{w^2} - \frac{\sin w}{w^3}) \}$$

And for suseptance we used Pues formula [1] as given by:

$$B_s = Y_s \tan(\beta \Delta l)$$

Where the open end effect can be calculated as:

$$\frac{\Delta l}{h} = 0.412 \frac{\varepsilon_e + 0.300}{\varepsilon_e - 0.258} \frac{W_e/h + 0.262}{W_e/h + 0.813}$$

2.3. Mutual Admittance

Accurate closed-form expressions have been derived for both the real and imaginary parts of mutual admittance $Y_m = G_m + jB_m$ for rectangular microstrip antenna by Pues [1]. In order to determine the Mutual admittance we used the Pues formula as follow:

$$G_m = K_g F_g G_s$$

 $K_g \approx 1$

where

$$F_g \approx J_0(l) + \frac{s^2}{24 - s^2} J_2(l)$$

and

$$B_m = K_b F_b B_s$$

where

$$F_b = \frac{b_m}{b_s} \approx \frac{\pi}{2} \frac{Y_0(l) + \frac{s^2}{24 - s^2} Y_2(l)}{\ln(\frac{s}{2}) + C_e - \frac{3}{2} + \frac{s^2/12}{24 - s^2}}$$
$$k_b = 1 - \exp(-0.21w)$$

3. Results

For different flare angle of bowtie patch antenna we calculated the antenna parameters as stated above then we compared results of return loss obtained from the purposed model with results obtained by Advanced Design System where uses MoM.

As seen in Fig. 5 good agreement have been achieved for different flare angles. Also the error between the two methods increase by increasing flare angle from 3% for $\alpha = 10^{\circ}$ to 5% for $\alpha = 30^{\circ}$. For small flare angle the results of purposed model have good agreements with full wave results, but as the flare angle increased the results are not in good agreement because of radiations from side slots. As this radiations are more effective in higher flare angle so for large flare angle this model are not valid anymore.



Figure 5: Return Loss for different flare angles. $\varepsilon = 4.4, h = 1.6 \text{ mm}, L = 18.75 \text{ mm}, W = 25.2 \text{ mm}$

4. Conclusion

A novel transmission line model is represented for modeling the bowtie patch antennas. Admittance and controlled source equations used in the passed for rectangular patch antennas have been modified in this model for bowtie patch antenna. This model is just valid for a moderate range of flare angles and results are reasonably accurate in that region and therefore this approach can be used as TLM for bowtie patch antenna for evaluating antenna parameters.

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Applying Oblique Coordinates in the Method of Lines

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Abstract—Oblique coordinates are introduced into the method of lines. For the purpose of analysis, suitable equations are derived. The formulas are applied to compute the transmission in a waveguide device consisting of straight waveguides connected by a tilted one. Furthermore, the band structure of a hexagonal photonic bandgap structure was computed using these oblique coordinates.

1. Introduction

The Method of Lines (MoL) [1] has been proven as an efficient tool for modeling waveguide structures in the microwave area and in optics. Depending on the structure under study various coordinate systems like Cartesian or cylindrical ones have been introduced, the latter allowing to examine e.g., VCSEL-structures or curved waveguides [2–5]. Formulas for arbitrary rectangular coordinates can be found in [6].

By introducing Floquet's theorem into the MoL [7,8] photonic crystal structures (PCs) with a square lattice could examined [9,8]. Due to the shape of these structures Cartesian coordinates were applied.

In contrast, the shape of the elementary cells in hexagonal structures is not rectangular. Motivated by papers found in the literature (e.g., [10–12]) an algorithm was developed that uses oblique coordinates. In the references given above algorithms for the TE-polarization (2D) were described. Here we will derive expressions for the full 3D-vectorial case from which the two-dimensional case can be easily derived.

The formulas were used to compute the propagation characteristic in a waveguide device, where two straight waveguides were connected with a tilted one. The results were compared with those obtained by a staircase approximation showing a very good agreement. As second application the band-structure of PCs with a hexagonal lattice was computed.

2. Theory

In this section we are going to derive the equations that can be used for analyzing devices with oblique coordinates. We will start with Maxwell's equations from which we determine the equations for the full vectorial case. Simpler formulas (i.e., for two-dimensional structures) are then derived from these expressions.

Consider the coordinate system shown in Fig. 1, which shows Cartesian coordinates and oblique ones. The relation between oblique coordinates (u, v, y) and Cartesian ones (x, y, z) is given as:

y =

$$x = u\sin(\theta) + v \tag{1}$$

$$z = u\cos(\theta) \tag{2}$$

$$y$$
 (3)



Figure 1: Tilted waveguide structure in an oblique coordinate system, and relation between the field components.

The y-coordinate is identical in both systems. Therefore, in the following, we will examine only the remaining ones. Next, we need the derivatives with respect to the u- and v-coordinate. By inverting the relations in (1)

and (2) and applying the chain rule, we obtain:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial}{\partial v}\frac{\partial v}{\partial x} = \frac{\partial}{\partial v}$$
(4)

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial}{\partial v}\frac{\partial v}{\partial z} = \frac{1}{\cos(\theta)}\frac{\partial}{\partial u} - \tan(\theta)\frac{\partial}{\partial v}$$
(5)

At interfaces between two waveguide sections the transverse components have to be continuous. These are the x- and the y-component in Cartesian coordinates. A closer look at Fig. 1 shows that the x-component is put together of the u- and v-component if oblique components are used. However, also in an oblique coordinate system we find that the x-component itself is continuous. Therefore, we will use the Cartesian components (i.e., x- and y-component) of the fields in oblique coordinates as well.

To derive suitable equations for these components, we start with Maxwell's equations:

$$\overline{\nabla} \times \vec{H} = j\varepsilon_{\rm r}\vec{E} \qquad \qquad \overline{\nabla} \times \vec{E} = -j\mu_{\rm r}\vec{H} \qquad \qquad (6)$$

where the coordinates have been normalized with the free-space wavenumber k_0 , (e.g., $\overline{y} = k_0 y$). Furthermore, the magnetic field was normalized with the free space wave impedance $\eta_0 = 120\pi \Omega$: $\vec{H} = \eta_0 \vec{H}$. Now, the derivatives with respect to x and z are replaced by those with respect to u and v and the z-component of the electric and magnetic field is substituted by the x- and y-components. This leads to the following first order differential equation system:

$$\frac{1}{\cos(\theta)}\frac{\partial}{\partial\overline{u}}\vec{F} + Q\vec{F} = \vec{0} \qquad \text{with} \qquad \vec{F} = \begin{pmatrix} E_x \\ \widetilde{H}_y \\ E_y \\ \widetilde{H}_x \end{pmatrix}$$
(7)

and

$$Q = \begin{bmatrix} -\tan(\theta)\frac{\partial}{\partial\overline{v}} & j\left(\mu_{r} + \frac{\partial}{\partial\overline{v}}\frac{1}{\varepsilon_{r}}\frac{\partial}{\partial\overline{v}}\right) & 0 & -j\frac{\partial}{\partial\overline{v}}\left(\frac{1}{\varepsilon_{r}}\frac{\partial}{\partial\overline{y}}\right) \\ j\left(\varepsilon_{r} + \frac{\partial}{\partial\overline{y}}\frac{1}{\mu_{r}}\frac{\partial}{\partial\overline{y}}\right) & -\tan(\theta)\frac{\partial}{\partial\overline{v}} & -j\frac{\partial}{\partial\overline{y}}\left(\frac{1}{\mu_{r}}\frac{\partial}{\partial\overline{v}}\right) & 0 \\ 0 & j\frac{\partial}{\partial\overline{y}}\left(\frac{1}{\varepsilon_{r}}\frac{\partial}{\partial\overline{v}}\right) & -\tan(\theta)\frac{\partial}{\partial\overline{v}} & -j\left(\mu_{r} + \frac{\partial}{\partial\overline{y}}\frac{1}{\varepsilon_{r}}\frac{\partial}{\partial\overline{y}}\right) \\ j\frac{\partial}{\partial\overline{v}}\left(\frac{1}{\mu_{r}}\frac{\partial}{\partial\overline{y}}\right) & 0 & -j\left(\varepsilon_{r} + \frac{\partial}{\partial\overline{v}}\frac{1}{\mu_{r}}\frac{\partial}{\partial\overline{v}}\right) & -\tan(\theta)\frac{\partial}{\partial\overline{v}} \end{bmatrix}$$

To solve this equation, we proceed as usual in the method of lines. We divide the structure under study in sections where the permittivity and the permeability (the latter usually being equal to one) depend only on the transverse coordinates (v, y). Then, the derivatives with respect to v and y are discretized with finite differences. This results in a system of coupled ordinary differential equations:

$$\frac{\partial}{\partial \overline{u}}\mathbf{F} + \overline{\mathbf{Q}} \mathbf{F} = \mathbf{0}$$
(8)

where the operator Q had been multiplied with $\cos(\theta)$: $\cos(\theta)Q = \overline{Q}$. By transformation to the principle axes we can decouple this system

$$\overline{Q} = T\Gamma T^{-1}$$
 $\mathbf{F} = T \overline{\mathbf{F}}$

with the solution

$$\overline{\mathbf{F}}(\overline{u}) = \exp(-\Gamma \overline{u}) \,\overline{\mathbf{F}}(0) \tag{9}$$

The eigenvectors of \overline{Q} give the electric and magnetic field distribution of the eigenmodes and the eigenvalues Γ are the corresponding propagation constants. Since we are dealing with a first order differential equation system here, the forward and the backward propagating modes are determined at the same time. Therefore, we can divide the eigenvalues/eigenvectors according to

$$\mathbf{\Gamma} = ext{diag}(\mathbf{\Gamma}_{ ext{f}}, -\mathbf{\Gamma}_{ ext{b}}) \qquad ext{with} \qquad ext{Re}(\mathbf{\Gamma}_{ ext{f}}, \mathbf{\Gamma}_{ ext{b}}) > 0 \qquad ext{ and } \qquad \mathbf{T} = \left[egin{array}{cc} \mathbf{T}_{ ext{Ef}} & \mathbf{T}_{ ext{Eb}} \ \mathbf{T}_{ ext{Hf}} & \mathbf{T}_{ ext{Hb}} \end{array}
ight]$$

Now, the next steps of analyzing complex circuits with the MoL are analogous to those in Cartesian coordinates. Therefore, we give just a short summary here. After having found the solution in the homogeneous sections, we have to consider the continuity at the interfaces. Together with boundary conditions at the input and the output of the device, we could e.g., derive transfer matrix formulas for the whole structure. However, these transfer matrix expressions are potentially unstable, because of the exponentially increasing terms. Therefore, we use scattering parameters or alternatively impedances/admittances. In both of these cases, we start at the output of our structure. When using scattering parameters we define a reflection coefficient as the ratio between the backward and the forward propagation modes:

$$\mathbf{F}_{\mathrm{b}} = \overline{m{r}} \; \mathbf{F}_{\mathrm{f}}$$

This reflection coefficient is transformed to the input of the device. We have to consider homogeneous sections and the interfaces between these sections. In a homogeneous section with the length d, we obtain for the transformation formula:

$$\overline{\boldsymbol{r}}(0) = \exp(-\Gamma_{\rm b}\overline{d}) \ \overline{\boldsymbol{r}}(\overline{d}) \ \exp(-\Gamma_{\rm f}\overline{d}) \tag{10}$$

In contrast to the analysis with Cartesian coordinates we multiply with different expressions from the left and from the right. For transforming the reflection coefficient at interfaces we can use expressions that were given in [13] for anisotropic material. Therefore, we do not repeat them here. After the input reflection coefficient has been determined, we compute the fields in opposite direction — from the input towards the output. In this way the explicit computation of the exponentially increasing terms can be avoided. The procedure with impedances/admittances is similar.

2.1. Two-dimensional Structures

The derivatives with respect to y are zero in case of two-dimensional structures. Therefore, the polarizations decouple like in the Cartesian case, and we obtain the following operators for the TE- and the TM-polarization:

$$Q_{\rm TM} = \begin{bmatrix} -\tan(\theta) \frac{\partial}{\partial \overline{v}} & j \left(\mu_{\rm r} + \frac{\partial}{\partial \overline{v}} \frac{1}{\varepsilon_{\rm r}} \frac{\partial}{\partial \overline{v}} \right) \\ j\varepsilon_{\rm r} & -\tan(\theta) \frac{\partial}{\partial \overline{v}} \end{bmatrix} \qquad \vec{F} = \begin{pmatrix} E_x \\ \widetilde{H}_y \end{pmatrix}$$
$$Q_{\rm TE} = \begin{bmatrix} -\tan(\theta) \frac{\partial}{\partial \overline{v}} & -j\mu_{\rm r} \\ -j \left(\varepsilon_{\rm r} + \frac{\partial}{\partial \overline{v}} \frac{1}{\mu_{\rm r}} \frac{\partial}{\partial \overline{v}} \right) & -\tan(\theta) \frac{\partial}{\partial \overline{v}} \end{bmatrix} \qquad \vec{F} = \begin{pmatrix} E_y \\ \widetilde{H}_x \end{pmatrix}$$

Instead of working with a coupled differential equation system for the electric and the magnetic field we could also derive a "wave equation" for one field component only. In case of the TE-polarization we obtain e.g., the following expression for E_y :

$$\left[\frac{\partial^2}{\partial \overline{u}^2} + \sin(\theta) \left(\frac{\partial}{\partial \overline{v}} + \mu_{\rm r} \frac{\partial}{\partial \overline{v}} \frac{1}{\mu_{\rm r}}\right) \frac{\partial}{\partial \overline{u}} + \cos^2(\theta) \mu_{\rm r} \varepsilon_{\rm r} + \mu_{\rm r} \frac{\partial}{\partial \overline{v}} \frac{1}{\mu_{\rm r}} \frac{\partial}{\partial \overline{v}}\right] E_y = 0 \tag{11}$$

However, to solve this equation with the MoL, we have to transform it back into a first order differential equation system. Another point should be mentioned: analytically, the wave equation (11) and the coupled equation (7) can be transformed into each other (if we introduce the expression for $Q_{\rm TE}$) and are therefore equivalent. On the other hand, a slight difference occurs in discretized form, because of the first order derivatives with respect to v. We will compare those two cases to see the influence on the results.

3. Numerical Results

As first example, we examined the concatenation of a tilted waveguide with a straight input and output waveguide. (see Fig. 2(a)). The diagonal length L was kept constant.

The transmission as function of the angle is shown in Fig. 2(b). Also shown are results that were obtained by a step approximation of the tilted section. To obtain convergent results at least 25 steps were required for $L = 20 \,\mu\text{m}$ with the staircase approximation. In case of $L = 5 \,\mu\text{m}$ this number dropped to 5. When using oblique coordinates the tilted part was examined in one step independent of the length of this section.

Also the two expressions for the oblique coordinates were compared. As can be seen all curves agree very well, the results obtained with the different formulation obtained with oblique coordinates are practically indistinguishable.



Figure 2: a) Concatenation of two straight waveguides with a tilted one; data $n_1 = 3.17$, $n_2 = 3.24$, $w = 0.8 \,\mu\text{m}$, wavelength $\lambda = 1.55 \,\mu\text{m}$, b) transmission of the fundamental mode.

Next we used oblique coordinates to determine the band structure of photonic crystals with a hexagonal lattice. The structure is shown in Fig. 3. It was taken from [14]. The Floquet modes which must be computed for this band structure were determined with the algorithm presented in [15].

The determined band-structure for the Γ -M band is presented in Fig. 4. Also shown are the values at the special points Γ and M taken from that reference. A good agreement for the TM-polarization is recognizable, the MoL-curves are slightly higher for the TE-polarization.



Figure 3: Elementary cell of a hexagonal photonic bandgap structure taken from [14]; data: r/a = 0.3, $\varepsilon_1 = 11.56$, $\varepsilon_2 = 1$.



Figure 4: Band structure of a hexagonal lattice a) TM-polarization, b) TE-Polarization.

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Nonlinear Pulse Propagation and Modulation Instability in Periodic Media with and without Defects

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Abstract—The nonlinear propagation of EMW in periodic media is of great interest due to the possibility to accumulate energy in periodic media within the stop band and, therefore, the input intensity levels for observation of nonlinear phenomena are quite low [1]. For resonant interactions, also it is possible to realize matching conditions, which are not possible in uniform media. Both resonant multiwave interactions and self-action of EMW in nonlinear periodic media have been analyzed [1, 2]. Also, a method based on a slow variation in time was proposed, which seems more adequate than others based on coupled equations [3].

The presence of defects in periodic structure leads to narrow regions of transmission within the stop-band. In this case, the application of coupled equations for counter propagating waves becomes doubtful, and a more general approach is needed [3]. Also, the influence of defects on the dynamics of modulation instability of long pulses is of a great interest. The present paper considers numerical simulations of the pointed above phenomena. For a correct description of the nonlinear dynamics, it is necessary also to take into account the wave dissipation and possible transverse diffraction.

The results of simulations demonstrated the essential influence of defects within the periodic structure on the nonlinear propagation of EM pulses, even if the carrier frequency is chosen within the stop band of the structure with a defect. This fact can be explained by a quite wide spectrum of the input pulse, and the "tail" of such a spectrum is within the transmission region due to the defect. This situation is analogous to the nonlinear propagation of short spin-dipole waves in the vicinity of the cut-off frequency [4]. The dynamics of modulation instability also changes in the presence of the defect. The diffraction can affect essentially the modulation instability dynamics.

1. Basic Equations

The case under research is the nonlinear periodic medium (OABAB...O), where A and B are dielectrics, and O is the vacuum. Each layer is assumed as isotropic and the influence of temporal dispersion is neglected here. The last consideration is valid only if the duration of the input pulse is long (> 0.1 ps). Within the structure, the presence of a singe defect is possible (such as ...ABABBBAB...). Consider almost transversely polarized EM wave, where only a single transverse component of the electric field (for instance, E_x) is dominating. A weak dependence on the radial coordinate is taken into account, to estimate an influence of diffraction. We use a slow dependence of the wave amplitude A only respect to time (essentially its variation in space, due to periodic structure of the medium), cubic non linearity and step-like dependence of the dielectric permittivity. Different spatial harmonics (both co- and counter-propagating) are included into the wave structure within the periodic lattice:

$$\frac{\partial A}{\partial t} + \frac{i\omega}{2} \left(1 + \frac{\Delta\varepsilon}{\varepsilon(z)} \right) A + \frac{i}{2\omega\varepsilon(z)} \left(\frac{\partial^2 A}{\partial t^2} + \nabla_{\perp}^2 A \right) + \gamma A = 0; \quad \Delta\varepsilon = \alpha(z) |A|^2 \tag{1}$$

Here $\Delta \varepsilon(E)$ is the change on the dielectric permittivity of the periodic medium due to the cubic nonlinearity. The scale of longitudinal spatial dependence is arbitrary, because of step-like dependence of dielectric permittivity. Then, the electric field is considered as:

$$E = \frac{1}{2}A(z,\rho,t) \times \exp(i\omega t) + c.c.$$
⁽²⁾

In equation (1), $\varepsilon = \varepsilon(z)$ is the dielectric permittivity of the periodic medium, ω is the carrier frequency $(\omega = 2\pi c/\lambda_0, \text{ with } \lambda_0 \text{ the wavelength in vacuum})$. Also the wave dissipation γ is included in this equation.

The boundary conditions at the interfaces between the layers are taken into account in Equation (1). Additionally, it is necessary to consider the boundary conditions for the tangential components of electric and magnetic fields at the input (z = 0) and at the output (z = L) of the periodic medium:

$$z = 0:$$
 $A = A_{inc} + A_{refl};$ $\frac{\partial A}{\partial z} \approx \frac{\partial A_{inc}}{\partial z} + \frac{\partial A_{refl}}{\partial z}$ (3a)

where: $A_{inc}(z, \rho, t) \approx A_{i0}(\rho, t) \times \exp(-ik_0 z)$; $A_{refl}(z, \rho, t) \approx A_{r0}(\rho, t) \times \exp(+ik_0 z)$, A_{i0} , A_{r0} are the amplitudes of incident and reflected waves, respectively; $k_0 = \omega/c$ is the wave number of the wave in vacuum. It is assumed that the amplitude of the incident wave is known.

Equation (3a) can be reduced as it is shown in the following boundary conditions:

$$z = 0: \quad \frac{\partial A}{\partial z} - ik_0 A \approx -2ik_0 A_{i0}(\rho, t) \qquad z = L: \quad \frac{\partial A}{\partial z} + ik_0 A \approx 0. \tag{3b}$$

Equation (1) and boundary conditions (3) are valid in the case of transversely wide pulses. Otherwise, it is impossible to use the approximation of transversely polarized EM wave.

2. Method of Simulations

The splitting with respect to physical factors is applied. The problem of stability of simulations is very important because the possible modulation instability is under research. The full explicit three-layer scheme is used for nonlinearity, dissipation, and longitudinal transport fractional step. Two-layer implicit scheme is utilized for the diffraction fractional step. In simulations, the one-dimensional representation of Equation (1) has been used:

$$\frac{\partial \bar{A}}{\partial \bar{t}} + \frac{i\bar{\omega}}{2} \left(1 + \frac{\Delta\varepsilon}{\varepsilon(\bar{z})} \right) \bar{A} + \frac{i}{2\bar{\omega}\varepsilon(\bar{z})} \left(\frac{\partial^2 \bar{A}}{\partial \bar{z}^2} + \frac{1}{\bar{\rho}} \frac{\partial}{\partial\bar{\rho}} \left(\bar{\rho} \frac{\partial \bar{A}}{\partial\bar{\rho}} \right) \right) + \bar{\gamma}\bar{A} = 0; \quad \Delta\varepsilon = b(\bar{z})|\bar{A}|^2 \tag{4}$$

where $\bar{z} = z/l_n$, $\bar{t} = t/t_n$, $t_n = l_n/c$; $\bar{\rho} = \rho/l_n$.

The value of $l_n = 1 \,\mu$ m has been chosen; thus, $t_n = 3.15 \times 10^{-15}$ s. Below, the lines over the one-dimensioned quantities (t, z, ρ, A) are omitted, because we use one-dimensional representation. The shape of the input pulse is:

$$A_i(t,\rho) = A_{i0} \times \exp\left(-\left(\frac{t-t_1}{t_0}\right)^4\right) \times \exp\left(-\left(\frac{\rho}{\rho_0}\right)^4\right)$$
(5)

3. Results of Simulations

The structures under simulation include 48 layers, their dielectric permittivities are $\varepsilon_A = 3.5$, $\varepsilon_B = 2.0$. A-layers are nonlinear (b = -0.1), whereas B-layers are assumed as linear (b = 0). Each layer has a length of $0.25 \,\mu$ m. A single defect is replacement of the 23^{rd} A-layer by a B-layer. The central frequency has been chosen in the region of the stop-band. In Fig. 1, the linear transmission coefficients are given for the cases of the



Figure 1: Linear transmission coefficients. a) The general picture, b) A section only. Note that the solid lines correspond to the case without defect and the dotted line is for with defect.

periodic structure without and with the single defect. The presence of the single defect causes a narrow region of transparency and a shift of the limits of the stop-band. Therefore, the most interesting regions of the central wave numbers (or frequencies) are localized near the upper limit of wave numbers ($\lambda_0 \sim 1.80 \,\mu\text{m}$). Note that in the linear case the total reflection takes place for the monochromatic input EM wave.

In Fig. 2, the nonlinear transmission coefficients of monochromatic waves are given in the structures with and without defect. In the structure with defect the transmission coefficient gets more broken dependence, in comparison with the linear case. More over, in the vicinity of the narrow transmission region, the shape of such dependence becomes chaotic-like. This fact can be explained by the accumulation of the energy of EM oscillations at the defect of the lattice.



Figure 2: Nonlinear transmission coefficients through the structure with the defect for the monochromatic wave: a) with $A_0 = 0.01$ (linear case), a) $A_0 = 0.1$, b) $A_0 = 0.12$, c) $A_0 = 0.15$, d) $A_0 = 0.2$. For a comparison, e) is for $A_0 = 0.2$ (the regular structure).



Figure 3: Propagation of the short transversely wide pulses. The carrier frequency corresponds to the wave number $\lambda_0 = 1.80 \,\mu\text{m}$. Part a) is the nonlinear propagation though the regular structure $(A_0 = 1)$; b) the same as a), but the input amplitude is 3 times smaller $(A_0 = 0.31)$; c) the same input pulse as a) $(A_0 = 1)$ but the propagation through the structure with defect.

Two different situations can occur due to the dependence on the duration of the incident pulse. The first one corresponds to relatively short pulses. The typical results of simulations are given in Figs. 3 and 4. Here the intensities of transmitted pulses are presented in the figure captions; $t_1 = 100$, $t_0 = 80$. In the case of the regular periodic structure, the nonlinear transparency phenomenon occurs at lower amplitudes of the input pulse, compare Fig. 3(a) and 3(c). In the case of the structure with the defect, the amplitude of the transmitted pulse is quite small even in nonlinear case, Fig. 3(c). The simulations of nonlinear propagation of transversely narrower pulses have demonstrated that an influence of diffraction is not expressed for the maximum of the transmitted pulse but changes weakly the rear part of it.

A comparison of Figs. 3 and 4 has demonstrated that a relatively small shift of the central frequency of the pulse changes the nonlinear dynamics essentially. Moreover, the regular periodic structures and ones with the defect possess different frequency regions for the manifestation of the nonlinear transparency phenomenon.



Figure 4: Propagation of the short, transversely wide pulses. The carrier frequency corresponds to the wave number $\lambda_0 = 1.82 \,\mu\text{m}$. Part a) is the nonlinear propagation though the regular structure $(A_0 = 1)$; b) the same as a), but the input amplitude is 3 times smaller $(A_0 = 0.31)$; c) and d) are the same input pulses as a) and b) $(A_0 = 1, A_0 = 0.31)$ but the propagation is through the structure with defect.



Figure 5: Modulation instability with and without defects. The carrier frequency corresponds to the wave number $\lambda_0 = 1.82 \,\mu\text{m}$. The maximal input amplitude is $A_0 = 1.00$. Part a) is for the regular structure, b) is for the structure with defect ($\rho_0 = 60$), c) is for the structure with defect (with an influence of diffraction, $\rho_0 = 24$).

The case of much longer incident pulses corresponds to the occurrence of modulation instability (MI). Here the general picture is somewhat different from the previous case, see Figs. 5 and 6. The parameters of the input pulses are $t_0 = 340$, $t_1 = 400$. At the output of the structure the multipeak signal occurs. Also, the role of
diffraction is essentially expressed; compare Fig.6(d) and 6(e). In the case of MI developed, the influence of temporal dispersion can be essential, and the used approximation cases its validity.

An interesting result of simulations is also the fact that the regular structure and one with the defect possess different frequency regions of observing modulation instability within the stop-band.



Figure 6: Modulation instability with and without defects. The carrier frequency corresponds to the wave number $\lambda_0 = 1.80 \,\mu\text{m}$. Part a) is for the regular structure, $A_0 = 1.0$, (transversely wide pulse, $\rho_0 = 60$); b) is for the structure with defect, $A_0 = 1.0$ ($\rho_0 = 60$); c) is for the regular structure, $A_0 = 1.41$ ($\rho_0 = 60$); d) is for the structure with defect, $A_0 = 1.41$ ($\rho_0 = 60$); e) is for the structure with defect, $A_0 = 1.41$ ($\rho_0 = 60$); e) is for the structure with defect, $A_0 = 1.41$ (ransversely narrow pulse, $\rho_0 = 24$).

4. Conclusions

The results of simulations have been demonstrated an essential influence of defects within the periodic structure on the nonlinear propagation of EM pulses, even if the carrier frequency is chosen within the stop band of the structure with the defect. This can be explained by a quite wide spectrum of the input pulse, and the "tail" of such a spectrum is within the transmission region due to the defect. The dynamics of modulation instability also changes in the presence of the defect. The diffraction can affect essentially the modulation instability dynamics.

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SAR Remote Sensing of Snow Parameters in Norwegian Areas — Current Status and Future Perspective

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Abstract—The paper presents results from a series of European and national projects on remote sensing of snow parameters. Currently, satellite borne syntethic aperture radar (SAR) data are only available at C-band frequencies. Other frequencies such as L-band or Ku-band may be favorable in several snow applications, but current C-band SAR may still be used and further developed to a more mature level. In particular, the advent of wide swath SAR data have provided frequent data sets at medium spatial resolution, that can be used to monitor snow parameters operationally.

We will present results from a snow cover area monitoring service developed for Norway and Sweden. The service, which is based on Envisat ASAR wide swath data, produces snow cover maps on average 3-4 times per week. The resulting time series gives a unique data set for studying the snow cover as it rapidly retreats during the melting season, and is of high value to hydro power companies.

Snow water equivalent (SWE) is the key parameter for hydrological applications. Norut IT has developed a technique using repeat pass interferometry to measure SWE, based on the linear relationship between the change in SWE and the change in interferometric phase. The technique has been demonstrated, but scarceness of usable interferometric baseline pairs have so far not allowed wide spread applicability of the technique.

A future SAR using Ku-band frequency as carrier will maybe solve the problem of retrieving SWE. Since backscatter at Ku-band frequency is more sensitive to SWE, it is good hope that robust SAR methods can be invented for this purpose. It will, however, be extremely important for the scientific society to validate the retrieval algorithms against in-situ data. The authors have developed an innovative validation concept using ground-penetrating radars at the same carrier frequencies as the space borne SARs to validate EO data in an efficient manner. The concept has been studied at C-band frequencies on glaciers at Svalbard, and we hope to build a similar platform for Ku-band frequencies, and will be used to validate model based retrieval algorithms.

1. Introduction

Accurate knowledge of snow properties is essential for snow hydrology. Onset of snow melt, snow covered area, snow wetness, and snow water equivalent are variables that goes into the hydrological models to predict runoff. These predictions are used for flood forecasting and production planning in hydro power plants. Currently 98% of the Norwegian electricity production comes from hydro power and approximately 50% of this comes from melted snow. With the establishment of a common Nordic power trading market in the late nineties, the power producers, power traders and the regulatory branches have developed a need for accurate prediction of run off. This has created a market for accurately derived snow products based on satellite data. The most important parameter is the distribution of snow water equivalent (SWE). Most of the power companies and trading companies in Norway run their own HVB (Bergström, 1992) hydrological models and for the satellite products to be useful the accumulated accuracy within a catchment must be 10% or better. One will also need to know the altitude distribution of snow within the catchment. The most important period is from onset of snow melt in the spring to the end of melt (April-June in Norways mountainous regions).

2. Current Status of Operational Snow Monitoring

Currently the only fully operational satellite based snow mapping in Norway is snow covered area (SCA) maps based on optical data (AVHRR). To improve the quality and both the spatial and temporal resolutions a combined method of SCA retrieval based on both SAR and optical (ASAR and MODIS) data has been developed (Solberg et al., 2004). These methods have been incorporated into a semi-automated production line for operational use and were tested operationally at Kongsberg Satellite Services in spring 2005. The main

reason for using SAR data for SCA mapping is the problem of lack of cloud free optical data. Clouds can be a very persistent problem for optical remote sensing in Norway.

For SAR data to be useful in an operational snow monitoring, the wideswath/scansar mode data has to be used to get large and frequent enough coverage. This will give a full coverage 2-3 times per week over Scandinavia.

2.1. SCA Retrieval Technique

The retrieval of SCA from SAR is based on the difference in backscatter between a reference scene in the same orbital track as the scene that will be classified, obtained during cold, dry snow conditions. We then classify wet snow based on the difference in backscatter caused by the high absorptance of the wet snow pack. A threshold of -3dB has been chosen based on comparison with field data (Nagler et al., 2000; Storvold et al., 2005). In the right panel of Figure 1 each classified pixel is assigned a quality value ranging from 0-100 depending on distance from threshold, viewing geometry, and quality of reference image (probability that the snow in the reference image is dry).

Combining the SAR data with temperature fields allows us to estimate quality of the retrieved data and make an assessment of the distribution of dry snow that is not directly detectable by SAR. The temperature fields are constructed based on more than 300 meteorological stations scattered throughout the region.

Dry snow is postulated in areas of higher elevation than the mean wet snow elevation within 20 km of the classified pixel at the same time as the air temperature is below freezing. The dry snow classification is assigned a quality number ranging from 0-70 depending on the elevation and air temperature. Figure 1 shows the differential backscatter between a cold winter reference scene and the May 5th 2004 scene covering most of the mountainous regions of South Norway (left panel), the resulting SCA map with wet and dry snow (center panel), and the corresponding confidence values (right panel).



Figure 1: The left panel shows the differential backscatter on May 1st 2004. The center panel shows classified snow covered area map for May 1st 2004. Black indicates bare ground, white-wet snow, light grey-dry snow, and grey are unclassified pixels due to forest, lakes, lay-over and shadow. The right panel show the corresponding classification confidence map. (The map resolution is 100 meter and map coordinates are given in the UTM zone 33 grid, WGS-84 datum.)

2.2. SCA Retrieval Results 2004

The SAR wideswath scenes are georeferenced within a fraction of a pixel accurcy with Norut IT's automated geocoding routine (Lauknes et al., 2005). The difference between South Norway SCA classifications based on SAR and optical data (MODIS), for the 2004 season, is shown in Table ??. Only cloud free pixels are compared and this causes the apparent large day to day variation in total SCA shown in the table. We see from Table ?? that the total snow covered area is on average approximately 4-5% larger for the Modis retrievals than for the SAR retrievals. The largest difference coinsided with a late May cold period with approximately 5 cm of new dry snow.

2.3. Current Pittfalls in SAR SCA Retrievals

The main uncertainties in todays SAR based snow covered area algorithms lies in the dry snow cover estimate,

which currently are based on air temperature and presence of wet snow from which a lower altitude boundary for the dry snow coverage is derived. This causes occasional problems in early spring when there are no wet snow present within the preset distance of the pixel that is to be classified, causing the algorithm to predict bare ground. The preset distance is choosen based on the climatic scales of the region. In the late season scattered wet snow pixels can cause false classifications of dry snow pixels during short cold spells.

The algorithm currently in use does not work in forested areas. Algorithms for estimation of snow cover in forested areas has been demonstrated in Finland (Koskinen et al., 1999) but these cannot easily be used in Norway due to the different climate regime (Norway has a costal climate with multiple freeze thaw cycles throughout the winter).

Compared with SCA retrieval from optical data, the SAR derived algorithm is binary (snow/no snow) and tend to underestimate the snowcover when pixels are partially snow covered. This is due to the high sensitivity to strong scatters (rocks and vegetation) that is often exposed early in the melt.

Table 1: Comparison between SCA classification of South Norway based on Envisat ASAR Wideswath and MODIS in spring 2004.

ASAR		MODIS		Snow Cover
Date	Snow Cover [%]	Date	Snow Cover [%]	Difference [%]
20040501	58.8	20040501	58.9	0.1
20040510	65.6	20040510	68.2	2.6
20040523	29.4	20040523	36.1	6.7
20040526	55.1	20040527	54.5	-0.6
20040528	39.2	20040527	40.0	0.8
20040529	28.5	20040530	35.7	7.2
20040531	26.1	20040531	41.8	15.7
20040601	24.9	20040601	27.0	2.1
20040601	25.9	20040602	29.0	3.1
20040604	3.0	20040603	4.3	1.3

3. Snow Water Equivalent

3.1. Repeat Pass Interferometry

Use of repeat pass interferometry can be used to detect changes in SWE between successive passes (Guneriusen et al., 2001). Main limitation on this method currently is the problem of unwrapping the phase change when the change in snow water equivalent is larger than typically 1-2 wavelengths. The equation below relates the phase change with the change in SWE (Guneriusen et al., 2001)

$$\Delta \Phi_s \approx \frac{1.6}{\cos \theta_i} k \rho \Delta Z_s = \frac{1.6k}{\cos \theta_i} \Delta SWE \quad , \tag{1}$$

where $SWE = \rho Z_s$. For currently available SAR data (C-band) and repeat cycles (35 days Envisat) this is not a feasable approach, but with the launch of ALOS L-Band SAR this method is likely to become more useful. Still there are problems with large snowfalls and with high absorption if the snow is wet. Wet snow absorption limits the usefulness of this method in the most interesting period, which is during the snow melt.

3.2. Δk Repeat Pass Interferometry

For a snow density of 0.3 kg/dm^3 , radar wavelength of 5.62 cm (Envisat), and an incidence angle $\theta_i = 23^\circ$, phase wrapping occurs at a snow depth of only 10.7 cm. Retrieval of SWE from C-band SAR can be performed using repeat pass interferometry and delta-K processing. This method was demonstrated using ERS data (Engen et al., 2004) and Envisat ASAR data (Larsen et al., 2005). Based on the delta-k principle known from the radar literature (T. Hagfors, 1961), it has been proposed to handle the phase unwrapping problem by splitting the bands of both the summer and the winter image into two subbands in the slant range dimension. This results

in two bands with slightly different carrier frequencies. By forming interferograms for each of the subbands, we get two interferograms with a different phase, due to the different carrier frequencies. This phase difference, the delta-k interferometric phase, is given by

$$\Delta \Delta \Phi_s = \frac{1.6}{\cos \theta_i} (k_2 - k_1) \Delta SWE = \frac{1.6}{\cos \theta_i} \Delta k \Delta SWE \quad , \tag{2}$$

where $\Delta k = k_2 - k_1$ is the difference in wavenumbers between the two subbands. For Envisat ASAR data with a maximum bandwith of 15 MHz, we split the band into two subbands with a carrier frequency difference of about 6.5 MHz. This results in phase wrapping at a snowdepth of about 85 m. Thus, the phase unwrapping problem is effectively avoided. The largest drawback of this method is the course resolution of the SWE product due to the need for averaging to improve the signal to noise (5 × 5 km for Envisat). In Figure 2 we show a comparison between field measurements of SWE and SWE derived from Envisat.



Figure 2: Interpolated SWE field data versus ASAR SWE result. Outliers are eliminated.

3.3. Snow Volume Backscatter

Ku-band SAR instruments have the potential to overcome some of the problems that L-X-band radars have in retrieving SWE. The main advantage of Ku-Band is the large sensitivity to dry snow. Ku-band signal has sufficient penetration in most snow packs, at the same time as the volume scattering signal is detectable for dry snow. Higher frequencies have shorter penetration depths, lower frequencies have less sensitivity. According to Shi et al., [2003] Ku-band is thus the optimal sensor frequency for estimation of SWE due to the balance of detectability vs. penetration depth. Repeat pass corrections will allow for removal of topographic and terrain effects and polarimetric measurements will enable decoupling of the snow volume scattering and snow surface scattering contributions.

4. Future Snow Monitoring

There are several new upcomming SAR missions (see Table 2) over the next couple of years that yield new possibilities in snow property retrieval and have potensial in operational snow monitoring. The introduction of polarimetric sensors as well as new frequencies opens new possibilities in particular in regard to operational retrieval of SWE.

4.1. Users and Requirements

Snow properties are particularly important for hydropower producers, power traders and regulators. Both for optimizing hydropower production and for avoiding flooding. For the data to be useful for this group the quality of the products has to be good (within 10% on a basin wide scale) and the coverage frequent, in particular in melting season (at least weekly).

Sensor	Satellite	f[GHz]/Polarization	Resolution [m]	Swath [km]
SAR	Radarsat1(1995-)	$5.3 \ \mathrm{VV}$	10, 30, 100	100-500
ASAR	Envisat (2002-)	5.3 HH,VV,HV	30,100	100-400
PALSAR	ALOS (2006-)	1.2 PP	15/100	40-350
TerraSAR-X	TerraSAR(2006-)	9.6 PP	1, 3, 16	5, 30, 100
SAR	Radarsat2(2006-)	5.3 PP	3, 10, 25, 50, 100	20-500
C-SAR	3-Constellation	$5.3 \ \mathrm{VV}$	50	350
Sentinel-1	GMES, ESA-EC	5.3 PP ?	25-50 ?	350-500 ?

Table 2: Future and current SAR sensors for research and operational use.

PP - Polarimetric; Source: H. Rott, personal correspondance

5. Conclusion

Future sensors and new retrieval methods will allow for establishment of operational monitoring of SCA and SWE, for hydrological purposes based on SAR, that will meet the user requirements on quality and coverage. Main obstacle today is lack of operational sensors, in particular for the retrieval of SWE. Scheduled missions within the next couple of years will change this. Wet snow remains a challenge preventing melt season estimates of SWE.

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Buried Cylinders Geometric Parameters Measurement by Means of GPR

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Abstract—Interactive technique that measures geometric parameters of buried cylinders in a homogeneous medium with unknown velocity of signal propagation is developed. After 3D GPR data processing, the velocity of signal propagation is estimated; radius, length and position of the cylinder are measured. Experimental data obtained by GPR "Defectoscope" are given.

1. Introduction

In paper [1] the hyperbola-fitting technique of radius estimation for subsurface cylindrical objects is presented. For that a direct least-square method for fitting hyperbola is used. Authors of paper [1] supposed that the propagation velocity is known or it can be estimated beforehand via finding a hyperbola resulting from a point reflector within the radarogram. In paper [2] they solved the questions of improving the accuracy of the interpretation of the radar returns reflected from buried cylinders by taking into account the influence of cylinder's orientation and electromagnetic radiation pattern of the antennas.

In [3] two-dimensional data obtained from orthogonal sounding of cylinders are analyzed. It is shown that the generalized Hough method can be used to measure buried pipe diameters from radar measurements and the velocity determination is best made independently from a point-like source at similar depth.

In this paper the interactive technique of 3D-data processing which allows us to estimate simultaneously signal propagation velocity in the medium and cylindrical objects parameters (orientation, radius, length and depth of occurrence) is proposed. Operator participation in data processing allows one to smooth reflected signals registration errors.

The technique is based on the frontal method of GPR 3D-data interpretation [4]. This method selects from the whole bulk of GPR data the first arrival of wave fronts from the recorded signals reflected from objects. These selected surfaces will be referred to as frontal hodographs.

This paper is development of the work in [5].

2. Measurement Technique

The technique proposed was developed during the creation of GPR data processing software of GPR "Defectoscope", designed for inspection of buildings [6]. The scan zone of this GPR is a rectangular area in the XYplane, and Z axis is directed to the probed medium. We will call the geometrical space with coordinate system XYZ as object space. During the results registration at any scanning point the value of reflected signal is fixed at discrete time moments at time axis T. This data array we will call the signal space or data cube. In the signal space by means of software tools, it is possible to select the surfaces of frontal hodographs as a function of time delay τ from antenna system coordinates (X, Y).

In Fig. 1, a cylinder with arbitrary orientation with respect to scanning plane is shown. Denote cylinder radius as r. Angle of inclination with respect to scanning plane XY of cylinder axis is denoted as φ ; its projection to the plane XY intercepts X axis at angle θ . Let us carry out a parallel shift of axes X and Y into an arbitrary point O belonging to the cylinder axis projection into scanning plane. In Fig. 1 such shift has been made. Cylinder axis intersects the scanning plane at a point with coordinates (CX, CY). The distance between point O and cylinder axis are denoted as h_0 . This distance equals the length of the perpendicular drawn from this point to the cylinder axis.

Let us consider this perpendicular as vector $\vec{h} = \{X_0, Y_0, Z_0\}$, where (X_0, Y_0, Z_0) are Cartesian coordinates of the perpendicular base, they are:

$$X_0 = -h_0 \sin \varphi \cos \theta$$
, $Y_0 = -h_0 \sin \varphi \sin \theta$, $Z_0 = h_0 \cos \varphi$.

Let us introduce the unit vector $\vec{e}_C = \{l_0, m_0, n_0\}$ directed from the considered perpendicular base along the cylinder axis in increasing depth. Coordinates of this vector are:

$$l_0 = \cos \varphi \cos \theta, \quad m_0 = \cos \varphi \sin \theta, \quad n_0 = \sin \varphi.$$



Figure 1: Geometry of data acquisition above an arbitrary located cylinder.

Then distance p(X, Y) from arbitrary point (X, Y) of scanning plane to cylinder axis equals:

$$p(X,Y) = \frac{\left| \left[\vec{q} - \vec{h}, \vec{e}_C \right] \right|}{\left| \vec{e}_C \right|}$$

where $\vec{q} = \{X, Y, \theta\}$, – radius-vector of the point (X, Y), $\left[\vec{q} - \vec{h}, \vec{e}_C\right]$ –vector product of vectors $\vec{q} - \vec{h} = \{X - X_0, Y - Y_0, -Z_0\}$ and \vec{e}_C , modulus means the length of the vector, i.e., $\left|\vec{h}\right| = h_0$.

Hence

$$p^{2}(X,Y) = [(Y - Y_{0})n_{0} + m_{0}z_{0}]^{2} + [(X - X_{0})n_{0} + l_{0}z_{0}]^{2} + [(X - X_{0})m_{0} - (Y - Y_{0})l_{0}]^{2}.$$

Substituting appropriate coordinates and simplifying, we obtain:

$$p^{2}(X,Y) = (X\sin\varphi + h_{0}\cos\theta)^{2} + (Y\sin\varphi + h_{0}\sin\theta)^{2} + \cos^{2}\varphi(X\sin\theta - Y\cos\theta)^{2}$$

Point (X, Y) belongs to the arc of ellipse with major semiaxis a and minor semiaxis b (see Fig. 1), where

 $a = p(X, Y) / \sin \varphi$ and b = p(X, Y).

This ellipse is very interesting. Coordinates of ellipse center are (CX, CY). The minor semiaxis b equals the distance from any point of ellipse to cylinder axis. The major semiaxis a is the cylinder axis projection into scanning plane. Therefore it defines angle θ . Ratio $b/a = \sin \varphi$ shows the angle inclination of cylinder axis to scanning plane.

The shortest distance R(X, Y) from arbitrary point with coordinates (X, Y) at scanning plane to the cylinder surface can be derived as:

$$R(X,Y) = \sqrt{(X\sin\varphi + h_0\cos\theta)^2 + (Y\sin\varphi + h_0\sin\theta)^2 + \cos^2\varphi(X\sin\theta - Y\cos\theta)^2} - r.$$
 (1)

Let us choose a measuring coordinate system SOU in the scanning plane which is formed by rotation of axes OX and OY by angle θ . Then OS axis will coincide with the projection of cylinder axis into the scanning plane and OU axis will be orthogonal to it. The third axis of the measuring coordinate system OW we will choose as a continuation of the perpendicular from the point O to the cylinder axis.

Then coordinates X and Y are related with measuring coordinates S and U as:

$$X = S\cos\theta - U\sin\theta \quad \text{and} \quad Y = S\sin\theta + U\cos\theta. \tag{2}$$

Section of cylinder by the UW plane is a circle with radius r and center, whose distance from point O equals h_0 . For monostatic system frontal hodograph or reflections from this circle for movement along OU axis one can obtain substituting (2) into (1) with conditions S = 0 and $R = \frac{V\tau}{2}$:

$$\frac{V\tau}{2} = \sqrt{h_0^2 + U^2} - r.$$

Let us analyze this hyperbolic hodograph. We fix the time delay T_0 of observed hyperbola vertex at point O and delays T_1 and T_2 for arbitrary coordinates U_1 and U_2 at axis U. According to the measurement results one can obtain equations set:

$$\frac{VT_0}{2} = h_0 - r; \quad \frac{VT_1}{2} = \sqrt{h_0^2 + U_1^2} - r; \quad \frac{VT_2}{2} = \sqrt{h_0^2 + U_2^2} - r$$

The solution is:

$$V = \frac{2}{\sqrt{T_2 - T_1}} \sqrt{\frac{U_2^2}{T_2 - T_0} - \frac{U_1^2}{T_1 - T_0}};$$
(3)

$$h_0 = \frac{1}{2\sqrt{T_2 - T_1}} \frac{U_1^2 (T_2 - T_0)^2 - U_2^2 (T_1 - T_0)}{\sqrt{U_2^2 (T_1 - T_0)^2 (T_2 - T_0) - U_1^2 (T_2 - T_0)^2 (T_1 - T_0)}};$$
(4)

$$r = h_0 - \frac{VT_0}{2}.$$
 (5)

It should be noted that V and h_0 do not depend on T_0 but depend on differences $(T_1 - T_0)$ and $(T_2 - T_0)$. It means that signal propagation velocity and the depth of the cylinder axis could be defined unambiguously on hyperbola form independently the hyperbola position at time axis.

However, it is impossible to use (3)–(5) due to the coordinates registration errors and errors of the operator carrying out this measurements. Even little errors could lead to significant deviations of calculated V and h_0 from true values. This is a typical incorrect problem, it is advisable to solve this problem by frontal hodograph $\tau(U)$ adjustment by means of graphical tools with the best approximation of the observed data. Similar procedures are widely used in GPR for the measurement of the signal propagation velocity on reflections from point-like objects.

Approximating hyperbola form selection can be organized by software tools via adjustment of measurable parameters V, h and r according to the equations:

for monostatic system

$$\tau(U, h_0, r, V) = 2 \frac{\sqrt{U^2 + h_0^2} - r}{V};$$
(6)

for bistatic system with base 2d (which is parallel to X axis):

$$\tau(U, h_0, r, V) = \frac{1}{V} (\sqrt{R^2 + d^2 + 2Rd\cos\psi} + \sqrt{R^2 + d^2 - 2Rd\cos\psi}), \tag{7}$$

where $h_0 = \frac{VT_0}{2} + r$; $R = \sqrt{U^2 + h_0^2} - r$; $\cos \psi = \frac{U \sin \theta}{\sqrt{U^2 + h_0^2}}$.

Similar analysis of signal space SOT section shows that in this section frontal hodograph represents segment of the cylinder high generatrix. The slope of this segment equals the value of φ angle. Denote time delays at beginning and the end of the approximating segment as T_H and T_K respectively, and the distance between points of measurements at OS axis as D, then φ can be calculated as follows:

$$\varphi = atctg\left(\frac{V(T_K - T_H)}{2D}\right)$$

The cylinder length is $L = D \cos \varphi$.

The technique developed was realized as a software module of GPR "Defectoscope" [6]. This software was approved experimentally by sounding three parallel cylindrical objects placed into a box filled with sand. Cylinders radii were 2 cm, 0.5 cm and 4 cm. Fig. 2 shows the geometrical position of these cylinders for two soundings.

Figure 3 shows data processing results obtained from these soundings. Frame 3a shows GPR data when cylinders ware located at angle $\theta = 60^{\circ}$ to X axis and angle of $\varphi = 0^{\circ}$ to the scanning plane. Frame 3b



Figure 2: Cylindrical objects position in testing soundings. (a—horizontal cylinders $\theta = 60^{\circ}$ and $\varphi = 0^{\circ}$, b—inclined cylinders $\theta = 60^{\circ}$ and $\varphi = 30^{\circ}$).

shows GPR data when $\theta = 60^{\circ}$ and $\varphi = 30^{\circ}$. Every frame shows control panel of the dialog window and four fragments. Two uppers fragments (1 and 2) show two vertical mutually perpendicular to each other time sections of GPR data cube. Down left fragment (3) shows a horizontal projection of data cube section which is cut in accordance with angles θ and φ . Fragment 4 is horizontal projection of data cube with signals exceeded the threshold specified by the operator.



Figure 3: Measurement results of unknown parameters of medium and cylinders using GPR data. (a—horizontal cylinders $\theta = 60^{\circ}$ and $\varphi = 0^{\circ}$, b—inclined cylinders $\theta = 60^{\circ}$ and $\varphi = 30^{\circ}$, 1—vertical data cube section along S axes shown in section 3 black line, 2—vertical data cube section along U axes shown in section 3 white line, 3—section which is parallel axis of cylinder, 4—horizontal projection of data cube which show signals exceeded the threshold specified by the operator).

Consider the sequence of measurements the result of which is shown in Figs. 3(a) and 3(b). Using reflections from a cylinder it is necessary to fix a point of measurements above the axis of a cylinder (see intersection of black and white lines on fragments 3 and 4). Furthermore, an interpreter chooses the direction along the axis of cylinder using reflections in horizontal section (see black line on fragment 3) and measures an angle θ . Next, an interpreter measures of signal propagation velocity V and radius of a cylinder r by means of fitting a hyperbolic curve by interactive varying of these parameters (see fragment 2). In the end the interpreter fixes inclined line by changing φ value to obtain parallel bounds of reflections of the cylinder high generatrix (see fragment 1).

All data entered by the interpreter are displayed in group of operating elements "Cylinder". Meaning of these elements is given in the following list:

Tt—depth of top of an approximating hyperbole measured in time samples;

Width—number of the samples in the approximating hyperbole of a fragment 2;

Theta—value of a corner $\boldsymbol{\theta}$ measured in degrees;

V—propagation speed for of signals measured in millimeters/nanoseconds;

h—depth of cylinder measured on a normal in centimeters;

r—estimation of radius of the cylinder in centimeters;

Phi—value of a corner φ measured in degrees;

Oblique plane—the tag, which passive condition is used at measurements of parameters of horizontal cylinders, and an active condition—at a choice of an inclined secant of a plane in parallel an axis of the cylinder; Measurement zone—a tag at which installation on a fragment 1 there are vertical borders of the measured length of the cylinder;

St—position of the left border of the cylinder measured in samples of scanning zone;

End—position of the right border of the cylinder measured in samples of scanning zone.

Measured parameters such as coordinates of the measurement point, the propagation velocity and also parameters of a cylindrical object such as radius, length and location are shown in status bar (Fig. 3). Value PSI shows direction of vector h. It equals $PSI = \pi/2 - \varphi$.

Inaccuracy of measurements of unknown parameters depends on next factors: the time discretization, the discrete location of antenna, the size of a object, insufficiently stretched "tails" of hyperbolic reflections restricted by directional diagram, GPR resolution, the small amplitude of reflections and the level of suppression of useful signals by reflections from other objects.

3. Conclusion

An interactive measurement technique developed in this paper allows the use of 3D GPR data which are reflections from a buried cylinder to define the signal propagation velocity in a medium and parameters of a cylindrical object such as radius, length, depth, azimuth angle and angle of inclination can be found.

Parameters which are found can be use both for direct interpretation of observed objects and for automatic shaping of reflecting surfaces of all objects detected in the scan area using the method in [4].

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Recent Advances in Bioelectromagnetics Research on Mobile Telephony and Health—An Introduction

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Abstract—It is not widely known that biological and health effects of radiofrequency (RF) energy have been studied for about 50 years. Currently, there are about 1500 published studies related to RF health research, covering various disciplines from biophysics to epidemiology, usually defined as bioelectromagnetics research. All these studies can be found at WHO EMF database (http://www.who.int/peh-emf/en/). Current international EMF safety guidelines, established by the International Commission on Nonionizing Radiation Protection (ICNIRP) and IEEE, are based on this extensive research database.

Recent advances in multidisciplinary bioelectromagnetics research addressing mobile telephony and health issue have significantly increased our knowledge about fundamental scientific questions in this area. Improved dosimetry and exposure design have made it possible to conduct well-controlled biomedical experiments. Several carefully conducted theoretical biophysical analyses have also increased our understanding about the responses of cell macromolecules to RF energy. However, inconsistent molecular biological findings have raised questions whether the observed changes are real and whether they have any significance on human health. In this regard, part of the biomedical research community has forgotten a fundamental rule that an observed effect cannot be considered established if it has not been independently replicated and confirmed by other researchers. RF energy—cancer link has been rejected by recent carefully conducted animal studies. However, inconsistent epidemiological findings and misinterpretation of epidemiological data continue to create confusion in mobile telephony—cancer debate in many countries. Studies on other health endpoints than cancer have not either been able to confirm any adverse health effects in humans, such as effects on central nervous system (CNS) at low RF exposure levels. The weight of scientific evidence shows that RF energy does not cause adverse health effects in humans below the internationally accepted RF exposure guidelines, such as established by ICNIRP and IEEE.

1. Introduction

By the end of year 2005, it has been estimated that there will be about two billion mobile subscriptions, and by the year 2010 the amount will increase up to about three billion. Huge advances have been made in the research and development in the wireless communications technology during the past two decades. However, the rapidly increased use of mobile phones and establishment of mobile base station networks has led to concerns that RF energy could possibly cause some unexpected adverse health effects in humans. It has been suggested, for example, that mobile phone use induces brain tumors or promotes brain cancer development, or have other unknown effects on central nervous system. These concerns have led to extensive media debates and also—sometimes—hasty sciencepolitical decisions to initiate extensive biomedical research programs in several countries around the world.

There has been an extensive research effort to investigate the effects of RF energy on human health. The research has been ongoing for about 50 years and has produced a large database, such as the one coordinated by WHO EMF project. When analyzing this extensive research database, it is essential to understand what the weight of scientific evidence tells us about biological and health effects following RF exposure instead of looking at outcomes of single studies. The objective of this paper is to summarize the current research conclusions related to bioelectromagnetics research on mobile telephony and health. This review will not cover all the research findings in detail but will highlight three important questions: 1) is RF energy from mobile telephony able to cause adverse effects in human central nervous system; 3) are so-called "non-thermal" biophysical interactions possible at mobile telephony frequencies.

2. The Radiofrequency (RF) Database

The WHO database on biological and health effects of RF energy is extensive and global. It comprises more than 2500 scientific publications from countries around the world. About 1000 of these are reviews, engineering

studies and non-peer-reviewed articles. As shown in Table 1, almost 1500 published papers in the database satisfy criteria for use as a basis to assess the possible public health impacts of exposure to RF fields. Table 1 shows the number of entries in the database for each of the following types of scientific studies on RF fields: epidemiological, human, animal, and cellular studies. In addition, there are about 300 studies are estimated to be published in the near future including 213 ongoing studies and 90 reported-but-not-published studies.

Although all peer-reviewed studies in the RF database (Table 1) are considered relevant to the mobile phone issue, there are also a large number of studies in this database related to mobile telephony frequencies as shown in Table 2. In this table, the number of studies in each of the four types of scientific investigations is shown. There are 673 studies listed in the database using mobile telephony-specific signals, and 412 of these have been completed. All of the literature in the RF database is available to the public on the WHO website shown in Tables 1 and 2.

Table 1: Peer-reviewed papers describing biological and health effects of RF exposure.

• All studies are listed on the WHO web site under "citation listings": http://www.who.int/pehemf/research/database/en/

Research Study Type	Ongoing	Reported but not Published	Published
Epidemiology	39	7	215
Human Studies	61	18	139
Animal Studies	54	33	717
Cellular Studies	59	32	376
Totals	213	90	1447

Table 2: Mobile telephony relevant studies in the WHO database.

Research Study Type	Ongoing	Reported but not Published	Published
Epidemiology	34	5	64
Human Studies	57	17	82
Animal Studies	47	23	170
Cellular Studies	52	26	96
Totals	190	71	412

• These studies are listed on the WHO web site: http://www.who.int/peh-emf/research/database/en/

3. RF Energy and Cancer

Today there seems to be a some kind of overreliance on what can be expected from epidemiological studies. This has particularly become evident when epidemiological studies related to mobile telephony and health have been misinterpreted in massmedia. It is often falsely interpreted that correlation between two factors, such as mobile phone use and cancer, means also that there is a cause-effect relationship. This relationship does not appear plausible when analyzing critically scientific data, both qualified epidemiologic and laboratory animal data.

Recent reviews of the published epidemiology studies [1-3] have not been able to establish a link between RF exposure and cancer. Many of the epidemiological studies have had serious problems in experimental design and exposure assessment. More reliable data will be available when a current large multi-centre case-control study (INTERPHONE), directed by the International Agency for Research on Cancer (IARC), will be completed during year 2006. The weight of evidence from the epidemiological studies indicates no adverse health effects and this conclusion is strongly supported by results from long-term animal cancer studies, many of which have well-defined RF exposure data useful for risk analysis [4]. The weight of scientific evidence of the long-term animal cancer studies indicates no effect on survival or body weight at exposure levels less than 4 W/kg, which

is regarded as the exposure threshold for adverse effects in animals. These results provide strong evidence that RF exposure does not cause life shortening diseases or general toxicity at exposure levels within ICNIRP limits which are set well below the adverse effect threshold of 4 W/kg [4].

A large amount of research has also focused on possible genotoxic effects in vitro following RF energy exposure although it is widely accepted that RF energy quanta are not capable of causing molecular damage in cell macromolecules, such as in DNA. Vijayalaxmi and Obe [5] have reviewed the scientific literature pertaining to the genotoxicity of RF energy in somatic cells, with the specific endpoints of DNA strand breaks, chromosomal aberrations, micronuclei formation, and sister chromatid exchanges. From their examination of 53 studies, the authors conclude that the weight of evidence shows that RF EMF is not genotoxic, and that many of the studies reporting positive results may have had experimental deficiencies. Meltz [6] has reviewed studies focusing on cancer-related bioeffects in mammalian cell systems and concludes that the weight of evidence available indicates that, for a variety of frequencies and modulations, low RF energy exposure levels do not cause genotoxic effects.

The bioelectromagnetics science community has also intensively debated whether RF fields are capable of causing other specific molecular biological effects than genotoxic which could be related to cancer. Main focus has been on the reports claiming that RF energy is able to interfere with the heat shock protein (HSP) metabolism [7]. It has been speculated that the reported effects are due to "non-thermal mechanisms". However, the explanations have remained vague because of lack of plausible biophysical interaction mechanism explaining the molecular biological effects which have not either been successfully replicated in other laboratories [8]. Cotgreave [9] concludes in his review paper that issues concerning the risks to human tissues from RF emissions in vivo are still clouded by a number of inconsistencies and controversies in the literature with respect to HSP response, which must be clarified by novel research. Moreover, the use of high-throughput screening techniques (HTST) such as proteomics or transcriptomics to "identify possible molecular targets" of RF energy are still very immature and are currently not useful for RF health risk assessment.

4. RF Energy and Central Nervous System

Intensive discussions—both scientific and non-scientific—have been ongoing about the potential effects of mobile telephony signals on human central nervous system. It has, for example, been proposed that RF exposure alters important physiological functions in the brain such as brain electrical activity, sleep and blood flow [10, 11].

In a review by D'Andrea et al., [12] the authors conclude: ... "the diverse methods and experimental designs as well as lack of replication of many seemingly important studies prevents formation of definitive conclusions concerning hazardous nervous system health effects from RF exposure. The only firm conclusion that may be drawn is the potential for hazardous thermal consequences of high-power RF exposure."

It has also been proposed that mobile phones may affect the human cognitive performance [13, 14]. However, replication studies with improved methodology [15], including better statistical design, have failed to replicate the original findings. An important methodological point appears to be inclusion of sufficient amount of subjects in the experiments to avoid false positive data when a large number of psychophysiological endpoints are investigated.

It has also been speculated that children with still a developing nervous system would be more vulnerable to RF emissions from mobile phones. This is not supported by scientific facts. From the exposure point of view, carefully conducted theoretical dosimetry studies have shown that there is no evidence for a correlation between energy absorption and head size [16, 17]. Other factors such as shape of the head, tissue distribution and antenna position are more important factors affecting specific absorption rate (SAR). "Child issue" is not either supported by biomedical evidence. Recent well-designed human experimental studies have found no significant differences in cognitive performance as measured by reaction time and accuracy in children exposed to RF fields typically used in mobile telephony [18, 19].

5. RF Biophysical Interaction Mechanisms

The bioelectromagnetics science community has for several years debated whether there would other RF biophysical interaction mechanisms than thermal. Unfortunately, even fundamental research findings in this field are often overlooked in speculative debates. A thermal mechanism depends only on the amount of energy absorbed and thus its frequency dependence is predictable. The amount of energy absorbed will depend on the electrical properties of the tissue and the geometrical interaction with the biological object, both of which will cause well-established frequency variations. There is no modulation dependence for a thermal mechanism. A non-thermal mechanism, on the other hand, would be expected to exhibit frequency dependent responses,

modulation dependent responses or both. The current 400 mobile telephony studies cover a wide range of frequencies and modulations and do not support the hypothesis that there is frequency dependent or modulation dependent response. This conclusion is further magnified by several biophysical analyses and reviews showing that other biophysical mechanisms than thermal are not plausible at mobile phone frequencies.

Foster and Repacholi [20] have concluded: "Modulation introduces a spread of frequencies into a carrier waveform, but in nearly all cases this spread is small compared to the frequency of the carrier. Consequently, any nonthermal (field-dependent) biological effects related to modulation must result from interaction mechanisms that are fast enough to produce a response at radiofrequencies. Despite considerable speculation, no such mechanisms have been established. Existense of "non-thermal interactions at radiofrequencies are not either supported by rigorous biophysical analyses of Pickard [21] and Adair [22].

A special target for discussion has been the DNA molecule and whether RF energy would be capable of causing vibrational modes in this macromolecule and thereby leading, for example, to molecular damage. Even fundamental physics shows that this mechanim does not appear plausible since the RF photon quantum energy is far too low to cause breaks in chemical bonds and/or conformational changes in macromolecules such as in DNA and proteins. Prohofsky [23] has shown in a theoretical study that that absorption of RF energy below several hundred GHz would not be resonantly absorbed into an intramolecular mode for macromolecules such as DNA. The absorption would be into bulk modes of the material in which the molecule is embedded. The thermalization of the RF energy would be primarily to this bulk material, rather than to a single molecule.

6. Conclusions

The weight of scientific evidence of the epidemiological and long-term animal cancer indicates that long-term RF exposures do not induce tumors or promote cancer development. Studies on other health endpoints than cancer have not either been able to establish any adverse health effects in humans, such as effects on central nervous system (CNS) at low RF exposure levels. Theoretical biophysical studies to date and lack of replicable biological effects strongly suggest that the only plausible interaction mechanism at mobile telephony frequencies and emission levels is thermal. The weight of scientific evidence shows that RF energy does not cause adverse health effects in humans below the internationally accepted RF exposure guidelines, such as established by ICNIRP.

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Epidemiologic Assessment of Cancer Risk from Mobile Phone Use: Where Are We?

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Abstract—More than a dozen epidemiological studies have addressed the possible risk of cancer associated with mobile phone use. Overall, the evidence is reassuring, as risk estimates are close to unity and confidence interval relatively narrow. However, most studies have been based on relatively small number of long-term users. When the analysis was restricted to long-term use of mobile phones, some indication of increased risk was found for acoustic neurinomas. Also, effect related to use on the same side as where the tumor was diagnosed could not be excluded. Despite the substantial volume of research some increase in risk cannot be ruled out at the moment. Knowledge could be further advanced by improving exposure assessment rather than increasing the number of case-control studies. Prospective cohort study is a gold standard in epidemiology and would substantially advance our understanding of the possible health effects of radiofrequency electromagnetic fields emitted by mobile phones.

1. Introduction

When new factors (exposures) are introduced or identified that have the potential to affect human health, multidisciplinary evaluation of possible health impact is required. Risk assessment involves hazard identification, exposure assessment and risk estimation. Hazard identification entails discovery of harmful potential, with its possible target for toxicity. Exposure assessment includes describing the occurrence of the agent, pathways and distribution in the population. Risk estimation comprises identification of mechanism of effect and evaluation of dose-response.

In this review, we summarize the findings from epidemiological studies. In addition, weaknesses in published studies are considered and some suggestions for improved assessment given.

2. Methods

We review the epidemiological evidence regarding cancer risk from mobile phone use. The evidence from studies conducted at individual level is summarized by means of meta-analysis, i. e., quantitative synthesis of results by obtaining a pooled estimate from published results. The pooled results is obtained by weighting the individual estimates with the inverse of the variance (obtained from confidence intervals), which is a measure of precision (amount of information). Consistency of results is evaluated by tests for heterogeneity. When heterogeneity is present, a random effects model is used. If no heterogeneity is found, a fixed effects model is used, assuming that all results represent the same global distribution of values. No such assumption is involved in random effects model.

3. Results

In ecological studies, brain tumor incidence and mortality have been related to mobile phone use at population level, without being able to assess if tumors have occurred in mobile phone users or not. Analyses regarding four Nordic countries showed no obvious increase in benign [1] or malignant intracranial tumors [2] parallel with increasing mobile phone coverage. However, in some subgroups including the oldest age groups and incidence of glioblastoma increase during the late 1990's was reported.

A total of 14 epidemiological studies on mobile phone use and cancer have been published by late 2005. Twelve have been case-control studies and they have included a total of more than 5000 cases with intracranial tumors. The total number of exposed cases is more than 1800 (corresponding to exposure prevalence of 1/3). In the two cohort studies the total number of brain tumor cases is much smaller, only 160. A further limitation of the latter has been relatively short follow-up, only one year in the US cohort and three years on average in the Danish study. This review will therefore focus on case-control studies, which also have an additional strength in more detailed exposure assessment.

Overall, there is substantial evidence indicating that (ever or regular) mobile phone use is not associated with the risk of intracranial tumors. The pooled overall OR from all studies is 1.09, 95% CI 0.86–1.38. For all malignant tumors, consisting mainly of glioma/astrocytoma, the pooled odds ratio from nine studies is very close to unity, with a narrow confidence interval (OR=1.02, 95% CI 0.77–1.37). Pooled odds ratio for benign brain tumors, mainly meningiomas, from eight studies is actually below unity (OR=0.89, 95% CI 0.75–1.06). For acoustic neurinoma (vestibular schwannoma) little indication of risk overall is found based on seven studies.

Eight studies have compared analog (NMT) and digital (GSM) network. Both showed some increase (OR 1.2–1.3), but neither was statistically significant.

Among subjects who have used a mobile phone for at least five years, a slightly elevated risk of borderline significance is found (OR=1.11, 95% CI 0.99–1.26). This was mainly due to acoustic neurinoma (OR=1.5, 95% CI 1.2–2.0). No clear indication of increased risk was found for malignant tumors (OR=1.1, 95% CI 0.9–1.3) or meningioma (OR=0.9, 0.8–1.1).

Ipsilateral use (mobile phone on the same side where the tumor was diagnosed) was associated with some indication of a slightly increased risk (OR=1.4, 95% CI 0.9–2.0)

When the groups with longest cumulative calling time were combined from different studies (using various cut-points), an odds ratio below one was obtained (OR=0.91, 95% CI 0.74–0.60).

4. Discussion

The number of studies conducted and number of subjects included in epidemiologic studies are relatively large. However, quality of evidence should also be considered. Most studies so far have relied on self-reported extent of mobile phone use as principal exposure measure of exposure. However, the limitations of such approach are evident. Whether or not a person is a regular mobile phone user can probably be reliably assessed. Yet, for construction of quantitative exposure-effect relationship, much more detailed information is required. Validation studies carried out indicate that the precision of self-reported use in terms of number of calls or cumulative call duration is only adequate (correlation coefficients between reported and recorded use 0.5–0.7 for both number and duration of calls) [3–5]. Furthermore, there is tendency to systematically overestimate amount of use (reported call duration up to 2–3 times the recorded value). Additional uncertainty arises from the fact that cumulative calling duration is only a proxy measure for the exposure of interest, energy absorbed in the target tissue from the radiofrequency electromagnetic field.

Random error, if non-differential, i. e., similar among cases and controls, is likely to attenuate any effect of exposure and therefore hinder detection of possible association. In addition to random error, systematic error (bias) is likely to affect the results of epidemiologic studies.

No studies have been published addressing possible information bias, i.e., differential error among cases and controls. Typically, recall bias, based on less complete reporting of exposure among controls, tends to overestimate any effect on outcome. In brain tumors, it is possible that the disease or its treatment, or anxiety following diagnosis may affect the recall and cognitive function of cases, diminishing accuracy of reporting. Also, proxy respondents are used more commonly for cases with malignant tumors than controls, which is likely to affect the quality of information.

The results of a recent study conducted in Finland [6] showed that non-participants were less likely to use mobile phone than study participants. This applied to both cases and controls. Selection bias resulted in apparent protective effect of mobile phone use. It may also distort the shape of dose-response.

These methodological weaknesses are inherent for retrospective exposure assessment. Epidemiological risk assessment is unlikely to improve from simply increasing the volume of research. A cohort study where concurrent mobile phone use is assessed would likely achieve substantially improved accuracy. Another advantage would be the possibility to risk of several health outcomes such as disease incidence and mortality (not only cancer, but also neurological, cerebrovascular and psychiatric disease), as well as 'soft' end-points, including symptoms and well-being. However, such study would require resources as recruitment of a very large number of subjects (probably > 100,000) is needed with follow-up for at least 10 years.

5. Conclusion

Currently, the factor limiting our knowledge of possible carcinogenic effect of mobile phone use is no longer the volume of evidence, but quality of epidemiological studies conducted. Improved knowledge could be gained by conducting prospective cohort studies, rather than increasing the number of case-control studies.

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NSA Calculation of Anechoic Chamber Using Method of Moment

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Abstract—NSA characteristics of an anechoic chamber were calculated by using the MoM (Method of Moment), and they were compared with those calculated by the FDTD method as well as the measured results in fully anechoic chamber. Next, we calculated of an anechoic chamber with complicated shapes. We found that first imitating a wave absorber using wire meshes with a limited electric conductivity is effective by controlling the wire interval. The NSA characteristics calculated by the MoM are as well as those calculated by the FDTD method. The NSA characteristics calculated by the MoM also agree with the measured results. And NSA characteristics of an anechoic chamber with complicated shapes were able to be calculated by using the MoM. Based on the results, it is confirmed that the MoM can be used to calculate NSA characteristics for an anechoic chamber.

1. Introduction

An anechoic chamber is a test room specially designed for completely shielding out external disturbances and for suppressing the echo caused by internal electromagnetic waves. The anechoic chamber for the radiated emission measurement specified by CISPR is usually evaluated by NSA (normalized site attenuation), which is a measure of the transmission characteristics between the standard antennas set inside the anechoic chamber. NSA is usually analyzed by using the ray tracing method [1], but the analytical accuracy of the NSA deteriorates in the low frequency band below several hundreds MHz because the ray tracing method approximates an electromagnetic wave as an optical ray. However, the FDTD (finite difference time domain) method is an analytical method that treats electromagnetic waves purely as waves. Therefore, the FDTD method can analyze NSA characteristics in the low frequency band below several hundreds MHz. The NSA of the anechoic chamber using the FDTD method was obtained by Holloway [2] and Takiguchi [3].



Figure 1: Anechoic chamber of analysis schedule.

Figure 2: Modeling of transmitting and receiving antennas.

The purpose of this research is to construct and calculate an anechoic chamber that can measure below 30 MHz because we will measure the characteristics of a PLC (power line communication) system that operates between 2 to 30 MHz. Therefore the ray tracing method is not suitable for this calculation. The FDTD method is suitable, but it cannot be used to calculate the NSA of an anechoic chamber with a cross section of a non-rectangular shape, as shown in Fig. 1. The non-rectangular shape is preferable because internal resonance of an anechoic chamber with such a cross section is lower than one with a rectangular cross section. Another way to calculate for the fully electromagnetic wave method is MoM (method of moment), which can calculate the NSA of an anechoic chamber with a non-rectangular cross section.

In this paper, we report on our comparison of calculations of the NSA of an anechoic chamber using the MoM and the FDTD method. In addition, we report on calculations of the NSA of an anechoic chamber with complicated shapes (pentagonal anechoic chamber) using the MoM. First, the NSA calculation of the

antenna factor of transmitting and receiving antennas is presented. Second, we describe the modeling method of the anechoic chamber when we calculate the NSA characteristics using MoM. Third, we describe calculation results of NSA characteristics of the anechoic chamber is provided. Fourth, the calculation results obtained by the MoM are compared with those obtained by the FDTD method. Almost the same results were obtained. Finally, we calculated Characteristics of pentagonal anechoic chamber and showed that NSA characteristics of the pentagonal anechoic chamber are better than cuboid anechoic chamber.

2. Antenna Factor

Both transmitting and receiving antennas are set in the chamber to measure the NSA characteristics and then a modeling of antenna is required in a simulation. A half-wave dipole antenna was used by Takiguchi [3] for measuring the NSA characteristics of the anechoic chamber as transmitting and receiving antennas. Therefore, we have to model the half-wave dipole antenna by using the MoM. The modeled antennas are shown in Fig. 2. In Fig. 2, we modeled the transmitting antenna by putting the transmitting voltage (V_s) and the resistance (R_s) in a gap between two elements of the antenna. In Takiguchi's study [3], V_s was 1 V, and both R_s and R_r were 50 Ω , and the diameter of an antenna element was 7 mm.

Next, we examined the antenna factor modeling of a half-wave dipole antenna. The antenna factor AF represents the ratio of the electric field (abbreviated E-field) of an incident electromagnetic wave arriving at an antenna to the voltage induced by the incident wave between the antenna elements, as shown in (1).

$$AF = 20\log_{10}\left(\frac{E_0}{V_r}\right) \tag{1}$$

Where E_0 is the E-field strength at the antenna, and V_r is the induced voltage of the antenna. The E-field strength distribution near the antenna needs to be uniform when measuring the antenna factor. It is examined how many distances between the transmitting and receiving antennas is needed when calculating the antenna factor by using the MoM. Because the E-field strength at the receiving antenna does not depend on the frequency, we calculated the dependence of the E-field strength on the distance between the transmitting and receiving antennas at a frequency of 30 MHz as shown in Fig. 3. In Fig. 3, the difference in the E-field strength is negligible beyond 400 m. Therefore, we set the distance between the transmitting and the receiving antennas to 1 km.

A comparison of the calculation results of the antenna factor using the MoM with the antenna factor from Takiguchi's study is shown in Fig. 4 [3], and their antenna factors clearly agree and have sufficient accuracy. Therefore, we used these calculated values.



Figure 3: Dependence of E-field strength on the distance between transmitting and receivng antennas.

Figure 4: Calculation results of antenna factor by using MoM.

3. Modeling of Anechoic Chamber Using MoM

3.1. Modeling of Anechoic Chamber's Walls

The anechoic chamber had electromagnetic wave absorbers set on the walls and ceiling to suppress internal echo within the chamber. Two kinds of anechoic chamber were used, fully anechoic and semi anechoic.

In the fully anechoic chamber, all of the walls, the ceiling, and the floor are covered by wave absorbers, as depicted in Fig. 5 However, in the semi anechoic chamber, the walls and ceiling are covered with the wave

absorbers, but the floor is covered metal plates instead. Therefore, we had to imitate the wave absorbers to calculate the NSA characteristics for these anechoic chambers. We used a wire mesh with limited electric conductivity to imitate the wave absorbers for this study, as shown in Fig. 6.





Figure 5: Construction of fully anechoic chamber.



We chose the anechoic chamber used by Takiguchi [3] as a calculation object. The dimensions of the chamber are W = 6.24 m, L = 6.96 m and H = 5.96 m. The walls, ceiling, and floor are composed of wire meshes with intervals of W = 0.567 m, L = 0.633 m, and H = 0.542 m, as shown in Fig. 6, and the wire meshes also have diagonal wires. The interval of the wire meshes was a bit large comparing with the wavelength of the electromagnetic wave sent from the transmitting antenna to the receiving one. It should be small at a higher frequency, especially around 100 MHz, which was the highest frequency for this calculation. To make segment length $\lambda/10$ or less, the number of segments set here is 8706 in fully anechoic chamber and 7202 in semi anechoic chamber. The calculation model of the fully anechoic chamber is shown in Fig. 6, but in the one for the semi anechoic chamber, the floor is modeled using a perfect ground.

3.2. Modeling of the Wave Absorber Reflection

The wave absorber imitates the wire mesh with a limited electric conductivity, which was calculated from the wave absorber's reflection coefficient. In this paper, we made reflecting plate and calculated reflection wave to transmission wave using MoM and obtained the limited electric conductivity corresponding to the amount of reflection. Figure 7 shows the calculation model of reflection. We set the distance between the transmitting antenna and the reflecting plate to 1 km. The reflection level is controlled by changing limited electric conductivity. The calculation method of the reflection is used space standing wave method [4]. This method is calculated from maximum and minimum electrical field strength (E_{max} , E_{min}), as shown in (2)

$$|\dot{\Gamma}| = \frac{1-\rho}{1+\rho} \quad \rho = \frac{E_{max}}{E_{min}} \tag{2}$$

Where Γ is reflection coefficient, and ρ is standing wave ratio. In (2), ρ was adjusted by changing limited





Figure 8: Reflectivity of wave absorber.

electric conductivity, and Γ was set as well as wave absorber's reflection by changing ρ . From here onwards, wave absorber was imitated by wire mesh with limited electric conductivity corresponding to wave absorber's

reflection. In this paper, we calculated limited electric conductivity corresponding to reflection from 30 MHz to 100 MHz.

3.3. Reflectivity of the Wave Absorber

Figure 8 shows the reflectivity of the wave absorber measured by the rectangular coaxial air-line method. The reflectivity of the wave absorber is 20 dB above 70 MHz and reaches 30 dB near 200 MHz. For this calculation (30 to 100 MHz), it is supposed that a pyramidal foamed ferrite is not effective. But we have only the characteristics of synthesis of pyramidal foamed ferrite tile. Therefore, we calculated using this characteristic.

4. NSA Calculation of Anechoic Chamber

4.1. Calculation Method of NSA

NSA represents the transmission characteristics between the transmitting and receiving antennas on the test site, and it is calculated by (3) [3]

$$NSA = V_{DIRECT} - V_{SITE} - (AF_T + AF_R) - \Delta NSA \ (dB) \tag{3}$$

where V_{DIRECT} is the received voltage of the measuring receiver when the connecting cables for a signal generator and the receiver are connected directly and V_{SITE} is the maximum receiving voltage of the measuring receiver when the cables are connected to the transmitting and receiving antennas and when the receiving antenna is swept between 1 and 4 m. AF_T and AF_R are the antenna factor of the transmitting and receiving antennas. Δ NSA is the correction value by direct coupling between the antennas, including image coupling between them through the metal ground plane. In this calculation, we used a half-wave dipole antenna at 30 MHz because this kind of dipole antenna was used in calculating the NSA with the FDTD method [3], and our intention is to compare the MoM and the FDTD method. In using MoM, Vsite are measured at 5points as shown Fig. 9 and fully anechoic chamber, Δ NSA is zero [3].



Figure 9: Position of transmitting and receiving antennas.

4.2. NSA Characteristics for Fully Anechoic Chamber

Figure 10 shows the calculation results for the NSA characteristics in the fully anechoic chamber where the MoM is used, and (a) is the vertical polarization, and (b) is the horizontal one. Curves from 1-1' to 5-5' are the calculation results using MoM for antenna position as shown Fig. 10. In addition, the calculation results using the FDTD method and the measured results obtained from Takiguchi's study [3] are shown in Figure 10. The NSA characteristics calculated by the MoM in the frequency range from 30 to 100 MHz is similar to the measured results. The NSA calculation results using MoM hardly depend on various antenna positions. Especially, NSAs for 2-2' and 3-3' reached almost the same value as well as NSAs for 4-4' and 5-5'. However, the calculated NSA using the FDTD method does not agree with that using MoM. But, it is thought that the calculation accuracy using MoM is better than that using FDTD, because the humps existing the measured value is also appeared in NSA using MoM as shown Figure 10(a). NSA using MoM from 30 MHz to 60 MHz is fitted to the measured value more than NSA using FDTD. As the results, it is clear that the NSA characteristics calculated by using the MoM is more accurate than that calculated by using the FDTD method.

4.3. NSA Characteristics for Semi Anechoic Chamber

We also calculated the NSA characteristics of the semi anechoic chamber as shown in Fig. 11. In Fig. 11, (a) shows the vertical polarization, and (b) shows the horizontal polarization. In addition, for the semi anechoic chamber, the Δ NSA from the VCCI [5] was used. The NSA characteristics calculated by using the MoM for the semi anechoic chamber agree with the measured results. The difference among the calculation results using MoM for various antenna positions was not so many. The measured results of the NSA characteristics in the semi anechoic chamber have humps, and the calculated results using the MoM also appeared though it was not plenitude. Regardless, it is clear that the NSA characteristics calculated by using the MoM agrees with the measured results.



Figure 10: NSA characteristics of the fully anechoic chamber.



Figure 11: NSA characteristics of semi anechoic chamber.

4.4. Characteristics of Pentagonal Anechoic Chamber

Next, we calculated the characteristics of the pentagonal anechoic chamber, as shown in Fig. 12, using the MoM. The base area in the pentagonal anechoic chamber is given as the same that in the rectangular anechoic chamber as shown in Fig. 6, and heights of the both chambers are also set as the same value. The reason is to calculate only the modification effect by changing the shape of the anechoic chamber from rectangular to pentagonal. The modeling method of an echoic chamber is similar to that in the case of Fig. 6, and the calculation method of antenna factor for the shortened dipole antenna is also similar to that in the case of Fig. 7. In this paper, we estimated the fully anechoic chamber using the NSA because we could not calculate the Δ NSA when calculating the NSA using a shortened dipole antenna.

Figure 12 shows the calculation results of the NSA characteristics in the fully anechoic chamber. In Fig. 13, "Theory" was calculated as shown in (4).

$$NSA = 20\log_{10}(D) - 20\log_{10}(F) + 32(dB)$$
(4)

Where, D is the distance from transmitting to receiving antenna (m) and F is frequency (MHz). (4) is analysis of geometrical optics in free space.

In the vertical polarization as shown in Fig. 13(a), the NSA characteristics appeared to be high in the frequency range from 60 to 80 MHz comparing with the Theory. But the maximum difference value between the Theory and the calculation results using MoM is 2 dB, and this value exists in a permissible value. In the horizontal polarization, the NSA characteristics appeared to be high from 30 to 70 MHz, and become almost the same values as the Theory from 60 to 100 MHz. But the maximum difference value between theory and calculation using MoM is 2 dB. The NSA characteristics of both vertical and horizontal polarizations in the pentagonal anechoic chamber are better than that in the rectangular anechoic chamber as shown in Fig. 10. As

a result, it is confirmed that the NSA characteristics can be improved by changing to a pentagonal shape in a fully anechoic chamber.



Figure 12: Outline of anechoic chamber.



Figure 13: Characteristics of fully anechoic chamber for the pentagonal anechoic chamber.

5. Conclusion

In this paper, we calculated the NSA characteristics of an anechoic chamber using the MoM and compared them with the characteristics using the FDTD method as well as the measured results. The following items were clear.

- 1 Imitating a wave absorber using wire meshes with limited electric conductivity is effective by controlling the wire interval.
- 2 The NSA characteristics calculated by using the MoM are more accurate than those using the FDTD method.
- 3 The NSA characteristics calculated by using the MoM agree with the measured results.
- 4 Calculation of NSA characteristic using MoM is available in anechoic chamber with compricated shapes.

Future tasks are improvement of calculation accuracy and NSA calculation for the semi anechoic chamber with a pentagonal base.

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An Effective Inversion Method Based on the Padé via Lanczos Process

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Abstract—In this paper we present a nonlinear effective inversion method based on the Padé Via Lanczos process (PVL process). The method finds so-called effective medium parameters of some inhomogeneous object by minimizing an objective function which describes the discrepancy between the scattered field produced by an inhomogeneous object and the scattered field produced by a homogeneous one. This minimization procedure can be carried out by inspection, since the scattered field produced by homogeneous objects can be computed very efficiently using the PVL process. The constant medium parameters of the homogeneous object for which the objective function is minimum are the effective medium parameters we are looking for. A number of numerical experiments are presented in which we illustrate the performance of the method.

1. Introduction

We consider a two-dimensional configuration that is invariant in the z-direction. The position vector in the transverse xy-plane is denoted by x. An object, with known support \mathbb{S}^{obj} , is located in vacuum and is characterized by a conductivity $\sigma(\mathbf{x})$ and a permittivity $\varepsilon(\mathbf{x})$. The object is illuminated by E-polarized waves which are generated by a line source of the form

$$J_z^{\text{ext}}(\mathbf{x},\omega) = f(\omega)\delta(\mathbf{x} - \mathbf{x}^{\text{src}}),\tag{1}$$

where $f(\omega)$ is the source signature, and the delta function on the right-hand side is the Dirac distribution operative at $\mathbf{x} = \mathbf{x}^{\text{src}}$. The source is located outside the object ($\mathbf{x}^{\text{src}} \notin \mathbb{S}^{\text{obj}}$), and the incident electric field strength generated by the line source is given by

$$E_z^{\rm inc}(\mathbf{x},\omega) = \gamma_s H_0^{(1)}(k_0 |\mathbf{x} - \mathbf{x}^{\rm src}|), \qquad (2)$$

where $H_0^{(1)}$ is the zero-order Hankel function of the first kind, $\gamma_s = i\omega\mu_0 f(\omega)/4$, and k_0 is the wave number of vacuum.

The total electric field strength is measured at some receiver location $x^{rec} \in S^{obj}$ and since the incident electric field is known, the scattered electric field strength at the receiver location is known as well. We denote this scattered field by E_z^{sc} . In what follows we assume that this field does not vanish at the receiver location.

The full inversion problem consists of retrieving the conductivity $\sigma(\mathbf{x})$ and permittivity $\varepsilon(\mathbf{x})$ of the object from the measured electric field strength. In our effective inversion method, however, we follow a different approach. We act as if the object is homogeneous and try to find position-independent medium parameters for which the scattered field at the receiver location matches the true scattered field using a well-defined objective function.

Let us be more precise. Introducing the contrast coefficient of the homogeneous object as

$$\zeta(\omega) = \tilde{\varepsilon}_{\rm r} - 1 + i \frac{\tilde{\sigma}}{\omega \varepsilon_0} \tag{3}$$

where $\tilde{\varepsilon}$ and $\tilde{\sigma}$ are position-independent, we have for the scattered field at the receiver location the integral representation

$$\tilde{E}_z^{\rm sc}(\mathbf{x}^{\rm rec},\omega) = \frac{ik_0^2}{4}\zeta(\omega)\int_{\mathbf{x}'\in\mathbb{S}^{\rm obj}} H_0^{(1)}(k_0|\mathbf{x}^{\rm rec}-\mathbf{x}'|)\tilde{E}_z(\mathbf{x}',\omega)dA.$$
(4)

This so-called data equation relates the scattered field at the receiver location to the contrast coefficient and the total electric field inside the object. This total field is unknown, but we do know that it satisfies the object equation

$$\tilde{E}_{z}(\mathbf{x},\omega) - \frac{ik_{0}^{2}}{4}\zeta(\omega) \int_{\mathbf{x}'\in\mathbb{S}^{\mathrm{obj}}} H_{0}^{(1)}(k_{0}|\mathbf{x}-\mathbf{x}'|)\tilde{E}_{z}(\mathbf{x}',\omega)dA = E_{z}^{\mathrm{inc}}(\mathbf{x},\omega) \quad \text{with} \quad \mathbf{x}\in\mathbb{S}^{\mathrm{obj}}$$
(5)

This object equation is an integral equation of the second kind for the total electric field strength E_z for a given value of the contrast coefficient.

Discretizing the object and data equation on a uniform grid using square discretization cells with side lengths δ is standard and we do not discuss it in this paper. We only give the final forms of the discretized data and object equations, and refer to [1] for details on the discretization process.

After the spatial discretization procedure we obtain the discretized data equation

$$\iota^{\rm sc}(\zeta) = \gamma_{\rm r} \zeta \mathbf{r}^T \mathbf{u},\tag{6}$$

where $\gamma_{\rm r} = i(k_0\delta)^2/4$, **r** is a receiver vector, and **u** is a vector containing the expansion coefficients of the total electric field inside the object. Furthermore, the discretized object equation for the homogeneous object is given by

$$(I - \zeta G)\mathbf{u} = \mathbf{u}^{\mathrm{inc}} \tag{7}$$

where I is the identity matrix, and matrix G is a square and symmetric (but not a Hermitean) matrix with (scaled) Green's function values as its entries. Since matrix G results from a discretization of a convolution operator on a uniform grid, we can compute its action on a vector very efficiently using the Fast Fourier Transform (FFT). Finally, the vector \mathbf{u}^{inc} is a vector consisting of incident electric field strength values. This vector can be written in the form $\mathbf{u}^{\text{inc}} = \gamma_s \mathbf{s}$ where \mathbf{s} is such that $\mathbf{s} = \mathbf{r}$ if the source and receiver locations coincide. Using the latter form for the incident field vector in the discretized object equation, solving this equation for the total field \mathbf{u} , and substituting the result in the discretized data equation, we arrive at

$$u^{\rm sc}(\zeta) = \gamma \zeta \mathbf{r}^T (I - \zeta G)^{-1} \mathbf{s},\tag{8}$$

with $\gamma = \gamma_{\rm r} \gamma_{\rm s}$. If we compute the scattered field $u^{\rm sc}$ equation (8) directly, we have to solve a forward problem for each new value of ζ . Such a procedure can be computationally intensive and it turns out that it can be avoided using the Padé Via Lanczos (PVL) process. We briefly describe this process in the next section.

2. The Padé via Lanczos Process

We first define our domain of interest. Let $\tilde{\varepsilon}_{r;max}$ and $\tilde{\sigma}_{max}$ be a priori given upper bounds for the constant medium parameters. Then our domain of interest is defined as

$$\mathbb{T} = \{ \zeta \in \mathbb{C}; 0 \le \operatorname{Re}(\zeta) \le \tilde{\varepsilon}_{r;max} - 1, 0 \le \operatorname{Im}(\zeta) \le \tilde{\sigma}_{\max} / (\omega \varepsilon_0) \},\$$

since we require that $\tilde{\varepsilon}_r \geq 1$ and $\tilde{\sigma} \geq 0$. We now compute [k - 1/k]-Padé approximations for the scattered field u^{sc} around an expansion point $\zeta_0 \in \mathbb{T}$ by performing k iterations of the two-sided Lanczos algorithm (see [2]). Matrix factorization is required for any nonzero expansion point and computing such a factorization is expensive (although it has to be computed only once). However, no such factorization is needed if we take $\zeta_0 = 0$ as an expansion point. Only matrix-vector products with matrix G are required in this case and, as we have mentioned above, such products can be computed efficiently using FFT. We therefore construct [k - 1/k]-Padé approximations for the scattered field u^{sc} around the expansion point $\zeta_0 = 0$ by performing k iterations of the two-sided Lanczos algorithm using the source and receiver vectors s and r as starting vectors. We denote the resulting Padé approximation by u_k^{sc} . The crux of the matter is that to evaluate this approximation for each $\zeta \in \mathbb{T}$, we need to solve a k-by-k tridiagonal system and k is typically much smaller than the order of the original discretized object equation. Assuming now that k is such that essentially

$$u_k^{\mathrm{sc}}(\zeta) = u^{\mathrm{sc}}(\zeta) \quad \text{for all} \quad \zeta \in \mathbb{T}$$

we can conclude that we have an efficient way of evaluating the scattered field for all ζ -values of interest.

3. The Effective Medium Parameters

The effective medium parameters follow from minimizing an objective function defined over the domain of interest. More precisely, the effective medium parameters are defined as those parameters for which the objective function

$$F_1(\zeta) = \frac{|E_z^{\rm sc} - u_k^{\rm sc}|^2}{|E_z^{\rm sc}|^2} \tag{9}$$



Figure 1: Two-dimensional test configuration.

attains a minimum in our domain of interest \mathbb{T} . If multiple frequency data $E_z^{\rm sc}(\omega_1), E_z^{\rm sc}(\omega_2), \dots, E_z^{\rm sc}(\omega_N)$ is available, we look for those medium parameters for which the multiple frequency objective function

$$F_N(\zeta) = \sum_{n=1}^{N} \omega_n \frac{|E_z^{\rm sc}(\omega_n) - u_k^{\rm sc}(\omega_n)|^2}{|E_z^{\rm sc}(\omega_n)|^2}$$
(10)

is minimum. In the above equation, the weights ω_n satisfy $\sum_{n=1}^{N} \omega_n = 1$. Notice that in the multiple frequency case we have to apply the PVL process for each frequency separately. Moreover, for multiple frequencies the domain of interest on which all the PVL approximations match the true scattered field due to a homogeneous object is taken to be the domain \mathbb{T} which corresponds to the lowest frequency of operation. Minimizing the objective functions can be carried out by inspection since we have a very efficient way of computing the scattered fields $u_k^{\rm sc}(\omega_n)$. Finally, we mention that we cannot guarantee that the effective medium parameters are unique. The objective function may have multiple minima on the domain of interest and each minimum gives a set of effective medium parameters for the object. However, usually we can overcome the nonuniqueness of the effective medium parameters by including more a priori information, or by performing additional experiments at different frequencies while keeping the source/receiver unit fixed.

4. Numerical Results

We illustrate our effective inversion approach using the two-dimensional configuration shown in Figure 1. A square block with side lengths ℓ is located in a vacuum domain. The block has an inner and an outer part and each part has its own constant medium parameters. Specifically, the outer part has a conductivity σ_1 and a relative permittivity $\varepsilon_{r;1}$, the inner part a conductivity σ_2 and a relative permittivity $\varepsilon_{r;2}$. Obviously, the block is homogeneous if $\sigma_1 = \sigma_2$ and $\varepsilon_{r;1} = \varepsilon_{r;2}$. Finally, the source/receiver unit is located a distance $\ell/2$ above the object and the source and the receiver are located 2 cm apart.

In our first example, we operate at a frequency of 36 MHz, and take $\ell = \lambda_{36}$, where λ_{36} is the free-space wavelength corresponding to the operating frequency of 36 MHz. The block is homogeneous with $\sigma_1 = \sigma_2 =$ 7.5 mS/m and $\varepsilon_{r;1} = \varepsilon_{r;2} = 5$. For the maximum conductivity and maximum relative permittivity we take $\tilde{\sigma}_{max} = 10 \text{ mS/m}$ and $\tilde{\varepsilon}_{r;max} = 6$, respectively. The domain of interest is discrerized on a 50-by-50 grid (leading to 2500 forward problems solved by PVL in less than a second on a notebook with a 1.6 GHz Pentium M processor) and the objective function F_1 on this domain of interest is shown in Figure 2 (left). We observe that the true conductivity and permittivity of the object are recovered. However, a number of additional minima are present near the $\tilde{\varepsilon}_r$ -axis. To remove these minima we add two more frequency measurements, namely, one at a frequency of f = 30 MHz and one at f = 42 MHz. The objective function F_3 for these two frequencies and the frequency of 36 MHz is shown in Figure 2 (right), where we have taken $\omega_n = 1/3$ for n = 1, 2, 3. Clearly, the multiple minima have disappeared and a single minimum remains. In addition to using multiple frequency data, we could also change the source and receiver locations. This latter option is not considered in this paper, however.



Figure 2: Base 10 logarithm of F_1 (left) and base 10 logarithm of F_3 (right) on the domain of interest.



Figure 3: Base 10 logarithm of F_3 for the $\lambda_{36}/4$ -block (left) and the λ_{36} -block (right).

We now apply our effective inversion method to inhomogeneous blocks. Two blocks of different sizes will be considered. The first block has a side length $\ell = \lambda_{36}/4$ and the second one a side length of $\ell = \lambda_{36}$. The outer part of the two blocks has a conductivity $\sigma_1 = 3.0 \text{ mS/m}$ and a relative permittivity $\varepsilon_{r;1} = 3$, while the medium parameters of the inner part are $\sigma_2 = 5.0 \text{ mS/m}$ and $\varepsilon_{r;2} = 5$. For both blocks the area of the inner part is 50% of the total area of the block. Using the same three frequencies as in the previous examples, we obtain the objective functions as shown in Figure 3. The minimum for the $\lambda_{36}/4$ -block is located at an acceptable location in the domain of interest, but for the large block the effective medium parameters are smaller than the smallest medium parameters of the block. This result is unexpected. We therefore carried out an additional number of experiments and all these experiments indicate that for inhomogeneous objects it all depends on the size of the object and the sizes of the perturbations with respect to a constant contrast function. This latter function may be large, but the perturbations cannot be "too large". Finding a condition that tells us for which contrast perturbations the proposed method gives reliable results is a topic we are presently investigating. In addition, we want to know how this condition changes if the data is perturbed (by noise, for example) given the magnitude of the data perturbations.

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QN Inversion of Large-scale MT Data

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Abstract—A limited memory quasi-Newton (QN) method with simple bounds is applied to a 1-D magnetotelluric (MT) problem. The method is used to invert a realistic synthetic MT impedance dataset, calculated for a layered earth model. An adjoint method is employed to calculate the gradients and to speed up the inverse problem solution. In addition, it is shown that regularization stabilizes the QN inversion result. We demonstrate that only a few correction pairs are enough to produce reasonable results. Comparison with inversion based on known L-BFGS-B optimization algorithm shows similar convergence rates. The study presented is a first step towards the solution of large-scale electromagnetic problems with a full 3-D conductivity structure of the Earth.

1. Introduction

Quasi-Newton (QN) methods have become a very popular tool for the numerical solution of electromagnetic (EM) inverse problems (see [8,7]). The reasoning behind it is that the method requires calculation of gradients only, while at the same time avoiding calculations of second-derivative terms. However, even with the gradients only, the QN methods may require excessively large computational time if the gradients are calculated straightforwardly. An effective way to calculate the gradients is delivered by an adjoint method (see [11, 4]). Also, for large-scale inverse problems the limited memory QN methods have to be applied, since their requirements for storage are not so excessive as for other QN methods. In this paper, as a first step to solving the 3-D EM case, we have applied a limited memory QN method for constrained optimization to solve 1-D magnetotelluric (MT) problems. This optimization method is an extension of previous work [9]. As distinct from this earlier work we implement the Wolfe conditions to terminate the line search procedure, as was recommended in [3]. First we present a simple review of the limited memory QN method for inversion of 1-D MT data. Then, we demonstrate the efficiency of our inversion on a synthetic, but realistic numerical example, along with a comparison with inversion based on the L-BFGS-B optimization method introduced by [3]. The results presented are encouraging and suggest that the method has the potential to handle the more geophysically realistic 3-D inverse problem.

2. 1-D MT Inversion

In the frame of the magnetotelluric method both the electric and magnetic fields are recorded. These fields are then processed to calculate the observed impedance dataset. This dataset is finally inverted to derive a distribution of electrical conductivity in the earth.

Thus, for 1-D MT inversion a layered earth model is considered and conductivities of the layers are sought. This problem is usually solved by minimization of the following objective function

$$\varphi(\mathbf{m},\lambda) = \varphi_d(\mathbf{m}) + \lambda \varphi_s(\mathbf{m}) \xrightarrow{\mathbf{m}} \min, \qquad (1)$$

where $\varphi_d(\mathbf{m}) = \frac{1}{2} \sum_{j=1}^M \alpha_j |Z_j - d_j|^2$ is the data misfit. Here $\mathbf{m} = (m_1, ..., m_N)^T$ is the vector consisting of the electrical conductivities of the layers; superscript T means transpose; N is the number of the layers; $Z_j(\mathbf{m})$ and d_j are the complex-valued modeled and observed impedances at the *j*-th period (j = 1, ..., M) respectively; $\alpha_j = \frac{2}{M} \epsilon_j^{-2} |d_j|^{-2}$ are the positive weights; ϵ_j is the relative error of the impedance $Z_j(\mathbf{m})$ and λ is a Lagrange multiplier. Our choice of λ is a simple variant of an algorithm presented in [6]. As prescribed by the regularization theory (see [12]), the function (1) has a regularized part (a stabilizer) $\varphi_s(\mathbf{m})$. This stabilizer can be chosen by many ways, and moreover, the correct choice of $\varphi_s(\mathbf{m})$ is crucial for a reliable inversion. However, this aspect of the problem is out of the scope of this paper. We consider the following stabilizer (see [5]) $\varphi_s(\mathbf{m}) = \sum_{i=2}^{N} \left(\frac{m_i}{m_i^0} - \frac{m_{i-1}}{m_{i-1}^0}\right)^2$, where $\mathbf{m}^0 = (m_1^0, ..., m_N^0)^T$ is an initial guess model. It is of importance that, as the

conductivities $m_i (i = 1, ..., N)$ must be non-negative and realistic, the optimization problem (1) is subject to bounds

$$l_i \le m_i \le u_i,\tag{2}$$

where l_i and u_i are the lower and upper bounds, respectively and $l_i \ge 0$ (i = 1, ..., N).

A limited memory quasi-Newton method. We notice that the problem posed in (1)–(2) is a typically constrained optimization problem with simple bounds. To solve this problem Newton-type iterative methods are commonly applied. However, most of these methods are not applicable to large-scale optimization problems because the storage and computational requirements become excessive. To overcome this, a limited memory quasi-Newton method has been developing (see [10] for a good introduction). Let us now describe our implementation of such a technique.

At each iteration step k the search direction, vector $\mathbf{p}^{(k)}$, is calculated as $\mathbf{p}^{(k)} = -\mathbf{G}^{(k)}\mathbf{g}^{(k)}$, where the symmetric matrix $\mathbf{G}^{(k)}$ is an approximation to the inverse Hessian matrix and $\mathbf{g}^{(k)}$ is the gradient $\mathbf{g} = (\frac{\partial \varphi}{\partial m_1}, \ldots, \frac{\partial \varphi}{\partial m_N})^T$ calculated at $\mathbf{m} = \mathbf{m}^{(k)}$. An explicit expression for the matrix $\mathbf{G}^{(k)}$ is given in [10, p. 225, formula (9.5)]. It is important that the matrix $\mathbf{G}^{(k)}$ is stored implicitly using n_{cp} correction pairs $\{\mathbf{s}^{(n)}, \mathbf{y}^{(n)}\}$ $(n = k - n_{cp}, \ldots, k - 1)$ previously computed as $\mathbf{s}^{(n)} = \mathbf{m}^{(n+1)} - \mathbf{m}^{(n)}$, $\mathbf{y}^{(n)} = \mathbf{g}^{(n+1)} - \mathbf{g}^{(n)}$. The main idea behind this approach is to use information from only the most recent iterations and the information from earlier iterations is discarded in the interests of saving storage. In [10, p. 225] it is advocated that n_{cp} between 3 and 20 may produce satisfactory results.



Figure 1: Comparison of LMQNB inversions for 2, 5 and 20 correction pairs.

The next iterate $\mathbf{m}^{(k+1)}$ is then found as $\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} + \alpha^{(k)} \mathbf{p}^{(k)}$, where the step length $\alpha^{(k)}$ is computed by an inexact line search procedure. This procedure finds a step length that delivers an adequate decrease in the objective function φ along the search direction $\mathbf{p}^{(k)}$. Let us demonstrate how in our implementation we provide the positive definiteness of the matrix $\mathbf{G}^{(k)}$, required to guarantee the descent direction $\mathbf{p}^{(k)}$. When the vectors $\mathbf{s}^{(k-1)}$ and $\mathbf{y}^{(k-1)}$ satisfy the curvature condition $\mathbf{s}^{(k-1)T}\mathbf{y}^{(k-1)} > 0$, it can be shown that the matrix $\mathbf{G}^{(k)}$ is positive definite. This condition is guaranteed to hold if we use the Wolfe conditions (see [10]) to terminate the line search. But the Wolfe conditions may not be reached inside the feasible region defined within the bounds set by equation (2). In this case, we modify $\mathbf{s}^{(k-1)}$ as prescribed in [9, p. 1513] to guarantee that matrix $\mathbf{G}^{(k)}$ is positive definite.

Alternative ways to deal with such boundary constrained QN optimization can be found in [3].

Speeding up the solution. As presented above, at each iteration step k the inverse problem solution requires calculating the gradient $\mathbf{g}^{(k)}$. However for large-scale problems (when N is large) a straightforward calculation may be prohibitive in terms of computational time. One can significantly speed up the calculation by using an adjoint method (see [11, 4]). We have applied such a method to the 1-D MT case. The derivatives

are numerically calculated as $\frac{\partial \varphi_d}{\partial \sigma_i} = -Re\left(\sum_{j=1}^M \alpha_j \Gamma_{ij} \left(\overline{Z_j} - \overline{d_j}\right) Z_j^2\right)$, where $i = 1, \ldots, N$, the sign Re means the real part of its complex argument and all the coefficients Γ_{ij} are found by solving a single adjoint forward

problem. Thus, our calculation of the gradient requires the solution of a single forward and adjoint problem. This approach may be extended with some effort to the 3-D case. It is also noteworthy that for the 1-D MT case the gradient can also be calculated using the chain-rule (see [5]).

We have therefore implemented the limited memory QN method with simple bounds (hereinafter, referred as LMQNB), which is described above. It should be noted here that our implementation differs from that of [9], in that the LMQNB uses the Wolfe conditions to terminate the line search.

3. Model Examples

Let us study on a synthetic 1-D MT example the convergence rate of the LMQNB inversion for a various number n_{cp} of correction pairs. A 7-layered earth model (see Fig. 3) was compiled from the models (see [1]) derived from a seafloor MT and a global GDS long-period dataset collected in the North Pacific Ocean. To complicate the inversion process we subdivided the three upper layers in this model (to depth of 394 km) into 197 equally thick sublayers (N = 197). For this 201-layered model we inverted the impedance $d_j = Z_j(\mathbf{m})$ calculated at M = 30 periods from 10 s to 10800 s.



Figure 2: Comparison of inversions based on the LMQNB (1) and L-BFGS-B (2) optimisations with $n_{cp} = 5$ and $\lambda = 320$.



Figure 3: The conductivity models obtained from inversion based on the LMQNB optimization with $n_{cp} = 5$ for $\lambda = 0$ and $\lambda = 320$. The true model is shown by the solid line and the initial guess by the dashed line.

In addition, we added 0.5% random noise to the impedance data. The relative error ϵ_j (see α_j on page 1) of the impedance was taken as 0.01. A 10 ohm-m uniform half-space was used as an initial guess $\mathbf{m} = \mathbf{m}^0$. The convergence rate curves are shown in Fig. 1 as a function of the total number (n_{fg}) of function φ and gradient

g evaluations for $\lambda = 320$. It is surprising that so small a number of pairs $(n_{cp} = 5)$ can be chosen sufficiently to get a relatively reasonable result.

Let us again consider the 201-layered model and the dataset, described in the previous example. In our next example (see Fig. 2) we present a comparison of two solutions of a 1-D MT inverse problem. The first solution is based on the optimization method offered in this paper and the second one exploits the L-BFGS-B optimization code from [3]. The comparison is presented for $n_{cp} = 5$ correction pairs and for $\lambda = 0$ and $\lambda = 320$. Both solutions converge in a similar way and also produce similar models. The inversion results are presented in Fig. 3.

4. Conclusion

This paper described a limited memory QN method applied to solve a 1-D MT inverse problem. The method is also valid for large-scale problems, and may be equally applied for 1-D, 2-D, or 3-D MT cases. In the numerical examples presented we have demonstrated that a few correction pairs are enough to obtain reasonable inversion results. To speed-up the inversion the adjoint method has been applied to calculate the gradients. The non-trivial problem of such a calculation of gradients in the 3-D MT case is presented in a companion paper [2]. Another finding of our numerical experiments is that the LMQNB solution converges similarly to the solution based on the L-BFGS-B method introduced in [3].

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2.5D Algorithm for Tomographic Imaging of the Deep Electromagnetic Geophysical Measurement

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Abstract—We present a 2.5D inversion algorithm for the interpretation of electromagnetic data collected in a cross-well configuration. Some inversion results from simulated data as well as from field measurements are presented in order to show the efficiency and the robustness of the algorithm.

1. Introduction

Electromagnetic methods are essential tools for the appraisal of a reservoir because of their sensitivity to the resistivity (conductivity) which is a function of the fluid saturation. One of the traditional electromagnetic techniques for well logging is the induction single-well measurement. This technique is employed both as a wireline technology and as a measurement while drilling to estimate near well-bore resistivity. This induction logging measurement has a sensitivity of up to a few meters from the well and is a function of the separation between the transmitter and receiver and frequency of operation.

To reach deeper into the reservoir, a cross-well electromagnetic induction technology was developed, see Wilt et al., [6] and Spies and Habashy [4]. The system operates very similar to the single-well logging tool however with transmitter and receiver deployed in separate wells. During a cross-well survey the receivers are lowered into one well, initially to the bottom of the survey-depth interval. Then the transmitter is lowered into the second well and is moved to log the entire survey-depth interval. During logging the transmitter broadcasts electromagnetic signals at a number of pre-prescribed frequencies while at the receiver well these signals are recorded. After the transmitter run is completed the receiver array is moved to the next depth station in the survey interval and the process is then repeated until the entire depth interval has been covered. After the data set has been collected, an inversion process is applied to convert the electromagnetic signals to a resistivity distribution map of the region between the wells. Furthermore, since most of the survey involves only two wells, one can usually assume in the inversion that the geometry is 2D (the resistivity distribution is invariant along the direction perpendicular to the plane containing the wells).

This inverse process is one of the most challenging parts of the effort to make this cross-well technology work since it requires one to solve a full nonlinear inverse scattering problem, which is usually ill-conditioned and non-unique. Moreover, when the number of the model parameters to be inverted is large, the inversion can be very time-consuming.

In order to carry out the inversion within a reasonable time, we employ a finite-difference code as a forward simulator. In this forward code the configuration is numerically discretized using a small number of cells determined by the optimal grid technique, see Ingerman et al. [3]. The resulting linear system of equations representing the discretized forward problem has to be solved in each inversion step. To solve this system, we use a LU decomposition method that allows us to obtain the solution for all transmitters simultaneously. Furthermore, in order to be able to use the optimal grid without sacrificing accuracy we use an anisotropic material averaging formula. All these features help in reducing the computational time for constructing sensitivity kernel and for calculating the data misfit.

For the inversion method, we employ a constrained Gauss-Newton minimization scheme (see Habashy and Abubakar [2]) where the inverted model parameters are forced to lie within their physical bounds by using a nonlinear transformation procedure. We further enforce a reduction in the cost function after each iteration by employing a line search method. To improve on the conditioning of the inversion problem, we use two different regularizers. The first is a traditional L_2 -norm regularizer, which allows a smooth solution. The second is the so-called weighted L_2 -norm regularizer, which can provide a sharp reconstructed image, see van den Berg and Abubakar in [5]. The trade-off parameter which provides the relative weighting between the data and the regularization part of the cost function is determined automatically to enhance the robustness of the method. We will present results from simulated data as well as from field measurements to demonstrate the capabilities of the developed algorithm.

2. Methodology

We consider a general discrete nonlinear inverse problem described by the operator equation

$$\overline{\mathbf{d}}^{\text{obs}} = \overline{\mathbf{S}} \,(\overline{\mathbf{m}}),\tag{1}$$

where $\overline{\mathbf{d}}^{\text{obs}} = [d_1^{\text{obs}} d_2^{\text{obs}} \dots d_J^{\text{obs}}]^T$ is the vector of measured data and $\overline{\mathbf{S}} = [S_1 S_2 \dots S_J]^T$ is the vector of data computed for the model parameters $\overline{\mathbf{m}} = [m(x_q, z_r), q = 1, 2, \dots, Q; r = 1, 2, \dots, R]$, where x_q and z_r denote the center of the 2D discretization cell. We use a lexicographical ordering of the unknowns to map the 2D array indices to 1D column indices $(q, r) \to R \times (q - 1) + r$. The superscript T denotes the transpose of a vector. We assume that there are J number of data points in the experiment and that the configuration can be described by $I = Q \times R$ model parameters. In this cross-well electromagnetic problem the data are the component of the magnetic field which is parallel to the borehole axis. The unknown model parameter $m(\overline{\mathbf{r}}) = \sigma(\overline{\mathbf{r}})/\sigma_0$ is the normalized conductivity where σ_0 is a constant conductivity. In the implementation σ_0 is chosen to be the average of the initial model used in the inversion.

We pose the inversion as the minimization problem. Hence at the n^{th} iteration we reconstruct $\overline{\mathbf{m}}_n$ that minimizes

$$\Phi_n(\overline{\mathbf{m}}) = \phi^d(\overline{\mathbf{m}}) + \lambda_n \phi_n^m(\overline{\mathbf{m}}), \tag{2}$$

where ϕ^d is a measure of data misfit:

$$\phi^{d}(\overline{\mathbf{m}}) = \frac{\sum_{j=1}^{J} \left| W_{j,j} [d_{j}^{\text{obs}} - S_{j}(\overline{\mathbf{m}})] \right|^{2}}{\sum_{j=1}^{J} \left| W_{j,j} d_{j}^{\text{obs}} \right|^{2}},$$
(3)

in which $|\cdot|$ denotes the absolute value and $\overline{\mathbf{W}}$ is a diagonal matrix whose elements are the estimates of the standard deviations of the noise. The symbol λ denotes the regularization parameter and ϕ^m is a measure of the variation in the geometrical configuration:

$$\phi_n^m(\overline{\mathbf{m}}) = \int_D \mathrm{d}\overline{\mathbf{r}} \, b_n^2(\overline{\mathbf{r}}) \bigg\{ \big| \nabla[m(\overline{\mathbf{r}}) - m^{\mathrm{ref}}(\overline{\mathbf{r}})] \big|^2 + \delta_n^2 \bigg\},\tag{4}$$

where $\nabla = [\partial_x \partial_z]^T$ denotes spatial differentiation with respect to $\bar{\mathbf{r}} = [x \, z]^T$, and the weight $b_n^2(\bar{\mathbf{r}})$ is given by

$$b_n^2(\mathbf{\bar{r}}) = \frac{1}{\int_D d\mathbf{\bar{r}} \left| \nabla [m_n(\mathbf{\bar{r}}) - m^{\text{ref}}(\mathbf{\bar{r}})] \right|^2 + \delta_n^2}$$
(5)

for the L_2 -norm regularizer and

$$b_n^2(\overline{\mathbf{r}}) = \frac{1}{V} \frac{1}{\left|\nabla[m_n(\overline{\mathbf{r}}) - m^{\text{ref}}(\overline{\mathbf{r}})]\right|^2 + \delta_n^2} \tag{6}$$

for the weighted L_2 -norm regularizer introduced in van den Berg and Abubakar [5]. The symbol $V = \int_D d\bar{\mathbf{r}}$

denotes the volume of the computational domain and $\overline{\mathbf{m}}^{\text{ref}}$ is the known reference model. Note that for the L_2 -norm regularizer the weight $b_n^2(\overline{\mathbf{r}})$ is independent of the spatial position $\overline{\mathbf{r}}$. The δ_n^2 is a constant which is chosen to be equal to: $\delta_n^2 = \phi^d(\overline{\mathbf{m}}_n)/(\Delta_x \Delta_z)$, where Δ_x and Δ_z are the widths of the discretization cell. The regularization parameter λ is determined automatically using the technique described in Habashy and Abubakar [2].

To solve (2) we employ a Gauss-Newton minimization approach. At the n^{th} iteration we obtain a set of linear equations for the search vector $\overline{\mathbf{p}}_n$ that identifies the minimum of the approximated quadratic cost function, namely,

$$\overline{\overline{\mathbf{H}}}_n \cdot \overline{\mathbf{P}}_n = -\overline{\mathbf{g}}_n,\tag{7}$$

where

$$\overline{\overline{\mathbf{H}}}_{n} = \overline{\overline{\mathbf{J}}}_{n}^{T} \cdot \overline{\overline{\mathbf{W}}}^{T} \cdot \overline{\overline{\mathbf{W}}} \cdot \overline{\overline{\mathbf{J}}}_{n}^{T} + \lambda_{n} \overline{\overline{\mathcal{L}}}(\overline{\mathbf{m}}_{n}),$$
(8)

$$\overline{\mathbf{g}}_{n} = \overline{\overline{\mathbf{J}}}_{n}^{T} \cdot \overline{\overline{\mathbf{W}}}^{T} \cdot \left[\overline{\mathbf{d}}^{\text{obs}} - \overline{\mathbf{S}}(\overline{\mathbf{m}}_{n})\right] - \lambda_{n} \overline{\overline{\mathcal{L}}}(\overline{\mathbf{m}}_{n}) \cdot \overline{\mathbf{m}}_{n}, \tag{9}$$

in which

$$\overline{\mathcal{L}}(\overline{\mathbf{m}}_n) \cdot \overline{\mathbf{m}}_n = \nabla \cdot [b_n^2(\overline{\mathbf{r}}) \, \nabla_{m_n}(\overline{\mathbf{r}})]. \tag{10}$$

In (8) and (9), $\overline{\overline{\mathbf{J}}}_n = \overline{\overline{\mathbf{J}}}(\overline{\mathbf{m}}_n)$ is the $J \times I$ Jacobian matrix and is given by the following expression:

$$J_{j,i;n} = \eta \frac{\partial S_j(\overline{\mathbf{m}}_n)}{\partial m_{i;n}}, \qquad \eta = \frac{1}{\sum_{k=1}^J |W_{k,k} d_k^{\text{obs}}|^2}.$$
(11)

This Jacobian matrix is calculated using an adjoint formulation, which only needs an extra forward problem solution at each Gauss-Newton search step. In this extra forward problem solution the roles of the transmitters and receivers are interchanged. However since we are using a 2.5D forward code with a LU decomposition solver, we need only one forward call to calculate both the data misfit and to generate the Jacobian matrix. Note that the use of the direct solver is possible, since we reduced the number of grids outside the inter-well region by employing the optimal grid technique in Ingerman et al. [3]. Furthermore, in order to be able to use the optimal grids without scarifying accuracy we use an anisotropic homogenization technique.

Since the size of the Hessian matrix $\overline{\mathbf{H}}_n$ is large, we solve the linear system of equations (7) using a linear iterative method. To that end we first rewrite equation (7) as follows:

$$\overline{\overline{\mathcal{K}}} \cdot \overline{\mathbf{p}}_n = \overline{\mathbf{f}},\tag{12}$$

where $\overline{\overline{\mathcal{K}}} = \overline{\overline{\mathbf{H}}}_n$ and $\overline{\mathbf{f}} = -\overline{\mathbf{g}}_n$. Since $\overline{\overline{\mathcal{K}}}$ is a self adjoint matrix, we employ a Conjugate Gradient Least Square (CGLS) scheme to solve this linear system of equations. This CGLS scheme starts with the initial values:

$$\overline{\mathbf{r}}^{(0)} = \overline{\mathbf{f}} - \overline{\overline{\mathcal{K}}} \cdot \overline{\mathbf{p}}_n^{(0)}, \ \text{ERR}^{(0)} = \frac{||\overline{\mathbf{r}}^{(0)}||}{||\overline{\mathbf{f}}||},\tag{13}$$

where $\overline{\mathbf{p}}_n^{(0)} = \overline{\mathbf{p}}_{n-1}$. Next, we compute successively for $N = 1, 2, \ldots$,

$$\begin{aligned} A^{(N)} &= \langle \overline{\mathbf{r}}^{(N-1)}, \, \overline{\mathcal{K}} \cdot \overline{\mathbf{r}}^{(N-1)} \rangle, \\ \overline{\mathbf{u}}^{(N)} &= \overline{\mathbf{r}}^{(N-1)}, \, N = 1, \\ &= \overline{\mathbf{r}}^{(N-1)} + \frac{A^{(N)}}{A^{(N-1)}} \overline{\mathbf{u}}^{(N-1)}, \, N > 1, \\ B^{(N)} &= ||\overline{\overline{\mathcal{K}}} \cdot \overline{\mathbf{u}}^{(N)}||^{2}, \\ \overline{\mathbf{p}}_{n}^{(N)} &= \overline{\mathbf{p}}_{n}^{(N-1)} + \frac{A^{(N)}}{B^{(N)}} \overline{\mathbf{u}}^{(N)}, \\ \overline{\mathbf{r}}^{(N)} &= \overline{\mathbf{f}} - \overline{\overline{\mathcal{K}}} \cdot \overline{\mathbf{p}}_{n}^{(N)}, \, \text{ERR}^{(N)} = \frac{||\overline{\mathbf{r}}^{(N)}||}{||\overline{\mathbf{f}}||}, \end{aligned}$$
(14)

where $||\overline{\mathbf{u}}|| = \sqrt{\langle \overline{\mathbf{u}}, \overline{\mathbf{u}} \rangle}$ denotes the L_2 -norm of a vector. This CGLS iteration process stops if the relative error ERR^(N) reaches a prescribed value, or when the total number of iterations N exceeds a prescribed maximum.

After the search vector $\overline{\mathbf{p}}_n = \overline{\mathbf{p}}_n^{(N)}$ has been obtained, the unknown model parameters are updated as follows:

$$\overline{\mathbf{m}}_{n+1} = \overline{\mathbf{m}}_n + \nu_n \overline{\mathbf{p}}_n,\tag{15}$$

where ν_n is a scalar constant parameter to be determined by a line search algorithm. In the implementation we always try first the full step, i.e., $\nu_n = 1$, and check if it reduced the value of the cost function Φ_n . If not, we backtrack along the Gauss-Newton step until we have an acceptable step. Since the Gauss-Newton step is a descent direction for Φ_n , we are guaranteed to find an acceptable step. In this procedure ν_n is selected such that:

$$\Phi_n(\overline{\mathbf{m}}_n + \nu_n \overline{\mathbf{p}}_n) \le \Phi_n(\overline{\mathbf{m}}_n) + \alpha \nu_n \delta \Phi_{n+1}, \tag{16}$$

where $0 < \alpha < 1$ is a fractional number, which is set to be quite small, i.e., α to 10^{-4} , so that hardly more than a decrease in cost function value is required (see Dennis and Schnabel [1]). The parameter $\delta \Phi_{n+1}$ is the rate of decrease of $\phi(\mathbf{\overline{m}})$ at $\mathbf{\overline{m}}_n$ along the direction $\mathbf{\overline{p}}_n$ and is given by:

$$\delta \Phi_{n+1} = \frac{\partial}{\partial \nu} \Phi_n(\overline{\mathbf{m}}_n + \nu \overline{\mathbf{p}}_n) \bigg|_{\nu=0} = \overline{\mathbf{g}}_n^T \cdot \overline{\mathbf{p}}_n.$$
(17)
If, at the $(n+1)^{\text{th}}$ iteration, $\nu_n^{(m)}$ is the current step-length that does not satisfy the condition (16), we compute the next backtracking step-length, $\nu_n^{(m+1)}$, by searching for the minimum of the cost function assuming a quadratic approximation in ν . Hence $\nu_k^{(m+1)}$ for $m = 0, 1, 2, \ldots$ is given by:

$$\nu_n^{(m+1)} = \frac{-0.5 [\nu_k^{(m)}]^2 \delta \Phi_{(n+1)}}{\Phi_n(\overline{\mathbf{m}}_n + \nu_n^{(m)} \overline{\mathbf{p}}_n) - \Phi_n(\overline{\mathbf{m}}_n) - \nu_n^{(m)} \delta \Phi_{n+1}}.$$
(18)

In general, it is not desirable to decrease $\nu_n^{(m+1)}$ too much since this may excessively slow down the iterative process. To prevent this slow down, we set $\nu_n^{(m+1)} = 0.1\nu_n^{(m)}$ if $\nu_n^{(m+1)} < 0.1\nu_n^{(m)}$ (but with ν_n not to decrease below 0.1, i.e., $\nu_{\min} = 0.1$ to guard against a too small value of ν) and then proceed with the Gauss-Newton step.

To impose *a priori* information of maximum and minimum bounds on the unknown parameters, we constrained them using a nonlinear transformation of the form:

$$m_i = \frac{m_i^{\max} + m_i^{\min}}{2} + \frac{m_i^{\max} - m_i^{\min}}{2}\sin(c_i),$$
(19)

where m_i^{\max} and m_i^{\min} are upper and lower bounds on the physical model parameter m_i . It is clear that $m_i \to m_i^{\min}$, as $\sin(c_i) \to -1$ and $m_i \to m_i^{\max}$, as $\sin(c_i) \to +1$. This nonlinear transformation will force the reconstruction of the model parameters to lie always within their prescribed bounds. Formally by using this nonlinear transformation we should be updating the auxiliary unknown parameters c_i instead of the model parameters m_i . However by using the relation $p_i = q_i dm_i/dc_i$ where q_i is the Gauss-Newton search step with respect to c_i , we obtain the following relationships between the two successive iterates $m_{i,n+1}$ and $m_{i,n}$ of m_i :

$$m_{i,n+1} = \frac{m_i^{\max} + m_i^{\min}}{2} + \alpha_n \sin\left(\frac{\nu_n p_{i,n}}{\alpha_n}\right) + \left(m_{i,n} - \frac{m_i^{\max} + m_i^{\min}}{2}\right) \cos\left(\frac{\nu_n p_{i,n}}{\alpha_n}\right),\tag{20}$$

where $\alpha_n = \sqrt{(m_i^{\text{max}} - m_{i,n})(m_{i,n} - m_i^{\text{min}})}$.

The iteration process will be terminated if one of the following conditions occurs: (1) The misfit $\phi^d(\overline{\mathbf{m}}_n)$ is within a prescribed tolerance factor; (2) The difference between the misfit at two successive iterates n is within a prescribed tolerance factor; (3) the difference between the model parameters $\overline{\mathbf{m}}$ at two successive iterates n is within a prescribed tolerance factor; (4) The total number of iterations exceeds a prescribed maximum.

3. Numerical Example

As a test example we employ a model shown in Fig. 1(a). This model was originally used to study a CO_2 injection operation and is employed here as it includes smoothly varying dipping stratigraphy as well as sharp boundaries and deviated wells. The background model shown in Fig. 1(b) is obtained using single-well logs interpolated between the two wells. The hypothesized CO_2 injection region is shown in red in Fig. 1(a). The change between the true model and the background model is shown in Fig. 1(c) given in percentage difference (%). The data are collected using 41 transmitters and 41 receivers. The locations of the transmitters and receivers are denoted by 'T' and 'R' in Fig. 1. Thus we have 1681 complex-valued data points. After generating the synthetic data, we corrupted the data with random white noise that corresponds to 2% of the maximum amplitude of all data points. The inversion domain is from x = -30 m to x = 350 m and z = 950 m to z = 1250 m and is discretized into cells of dimensions 5 m by 5 m, hence the total number of unknown model parameters is 4636.

First we run our inversion algorithm using the L_2 -norm regularizer given in (4) and (5). As the initial estimate we use the background model given in Fig. 1(b). Using this regularization term, the scheme took 15 iterations to converge. Figs. 1(d) and 1(e) show the percentage difference between the inverted resistivity and the background resistivity. The image obtained using the L_2 -norm regularizer is shown in Fig. 1(d). The image obtained in this case has the appearance of a spatially smoothed version of the model changes in Fig. 1(c). Next we rerun our inversion code, however now we use the weighted L_2 -norm regularization term given in (4) and (6). The inversion results after 19 iterations are shown in Fig. 1(e). By using the weighted L_2 -norm regularizer we obtain a significant improvement in the reconstruction of the geometry and the amplitude of the change due to the CO_2 injection. Finally we note that one iteration of the scheme takes only 180 seconds on a PC with a Pentium IV 3.04 GHz processor.



Figure 1: The resistivity distribution of the true model (a), of the initial model (b), the changes between (a) and (b) given in percentage (c), the inverted resistivity plotted as the change with respect to the model in (b) obtained using a L_2 -norm regularizer (d) and a weighted L_2 -norm regularizer (e).

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Abstract—The limited-memory quasi-Newton optimization method with simple bounds has been applied to develop a novel fully three-dimensional (3-D) magnetotelluric (MT) inversion technique. This nonlinear inversion is based on iterative minimization of a classical Tikhonov-type regularized penalty functional. But instead of the usual model space of log resistivities, the approach iterates in a model space with simple bounds imposed on the conductivities of the 3-D target. The method requires storage that is proportional to $n_{cp} \times N$, where the N is the number of conductivities to be recovered and n_{cp} is the number of the correction pairs (practically, only a few). This is much less than requirements imposed by other Newton type methods (that usually require storage proportional to $N \times M$, or $N \times N$, where M is the number of data to be inverted). Using an adjoint method to calculate the gradients of the misfit drastically accelerates the inversion. The inversion also involves all four entries of the MT impedance matrix. The integral equation forward modelling code x3d by Avdeev et al. ([1, 2]) is employed as an engine for this inversion. Convergence, performance and accuracy of the inversion are demonstrated on a 3D MT synthetic, but realistic, example.

1. Introduction

Limited memory quasi-Newton (QN) methods are becoming a popular tool for the numerical solution of three-dimensional (3-D) electromagnetic (EM) large-scale inverse problems ([11,7]). The reason is that the methods require calculation of gradients of the misfit only, while at the same time avoiding calculations of second-derivative terms. They also require storing merely several pairs of so-called correction vectors that dramatically diminish the storage requirements. A more complete review on this subject may be found in [4].

In this paper we apply a limited memory QN optimization method with simple bounds (hereinafter, referred to as LMQNB) to solve the 3–D magnetotelluric (MT) inverse problem. In section 2 we briefly describe the setting of the inverse problem, as well as some key features of our implementation, referring the reader to the paper [3] for details.

In section 3, we develop the theory and basic equations for the calculation of gradients of the misfit. We demonstrate that the calculation of gradients at a given period is equivalent to only two forward modellings and does not depend on the number of conductivities to be recovered. The mathematical details of the approach are not presented here except the key formula (3), which is central to the method.

In section 4 we demonstrate how our inversion practically works on a synthetic, but realistic numerical example. This example includes a tilted conductive dyke in a uniform half-space (see [17]). The results presented are encouraging and suggest that the inversion may be successfully applied to solving realistic 3-D inverse problems with real MT data.

2. 3-D MT Inversion

Let us first consider a 3-D earth conductivity model discretized by N cells, such that $\sigma(\mathbf{r}) = \sum_{k=1}^{N} \sigma_k \chi_k(\mathbf{r})$,

where $\chi_k(\mathbf{r}) = \begin{cases} 1, \mathbf{r} \in V_k \\ 0, \mathbf{r} \notin V_k \end{cases}$, V_k is the volume occupied by k-th cell and $\mathbf{r} = (x, y, z)$. In the frame of MT inversion conductivities σ_k (k = 1, ..., N) of the cells are sought. This is a typical optimization problem, such that $\varphi(\sigma, \lambda) \xrightarrow[\sigma]{} \min$, with a penalty function φ given as

$$\varphi(\sigma, \lambda) = \varphi_d(\sigma) + \lambda \varphi_s(\sigma), \tag{1}$$

where $\varphi_d = \frac{1}{2} \sum_{j=1}^{N_S} \sum_{i=1}^{N_T} \alpha_{ji} tr[\overline{\mathbf{A}}_{ji}^T \mathbf{A}_{ji}]$ is the data misfit. Here $\sigma = (\sigma_1, \dots, \sigma_N)^T$ is the vector consisting of the electrical conductivities of the cells; hereinafter superscript T means transpose and the upper bar stands for the complex conjugate; N is the number of the cells; N_S is the number of MT sites, $\mathbf{r}_j = (x_j, y_j, z = 0)$, where $j = 1, \dots, N_S$; N_T is the number of the frequencies ω_i , where $i = 1, \dots, N_T$; the 2×2 matrices \mathbf{A}_{ji} are defined as $\mathbf{A}_{ji} = \mathbf{Z}_{ji} - \mathbf{D}_{ji}$, where $\mathbf{Z}_{ji} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}_{ji}$ and $\mathbf{D}_{ji} = \begin{pmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{pmatrix}_{ji}$ are matrices of the complex-valued

predicted $\mathbf{Z}(\mathbf{r}_j, \omega_i)$ and observed $\mathbf{D}(\mathbf{r}_j, \omega_i)$ impedances, respectively; $\alpha_{ji} = \frac{2}{N_S N_T} \epsilon_{ji}^{-2} \left(tr[\overline{\mathbf{D}}_{ji}^T \mathbf{D}_{ji}] \right)^{-1}$ are the positive weights, where ϵ_{ji} is the relative error of the observed impedance \mathbf{D}_{ji} ; and λ is a Lagrange multiplier. The sign $tr[\cdot]$ introduced above means the trace of its matrix argument, which is defined as $tr[\mathbf{B}] = B_{xx} + B_{yy}$, for any $\mathbf{B} = \begin{pmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{pmatrix}$. As prescribed by the Tikhonov regularization theory [15] the penalty function φ of (1) has a regularized part (a stabilizer) $\varphi_s(\sigma)$. This stabilizer can be chosen in different ways. However, this aspect of the problem is out of the scope of this paper. It is of importance that, as the conductivities σ_k ($k = 1, \ldots, N$) must be non-negative and realistic, the optimization problem (1) is subject to the bounds

$$\mathbf{l} \le \sigma \le \mathbf{u},\tag{2}$$

where $\mathbf{l} = (l_1, \ldots, l_N)^T$ and $\mathbf{u} = (u_1, \ldots, u_N)^T$ are respectively the lower and upper bounds and $l_k \ge 0$ $(k = 1, \ldots, N)$.

Optimization method. We notice that problem (1)-(2) is a typically optimization problem with simple bounds (see [12]). To solve this problem we apply the limited memory quasi-Newton method with simple bounds. Our implementation of this method is described in a companion paper [3], which demonstrates the application of the method to the 1-D problem. At each iteration step l, we find the search direction $\mathbf{p}^{(l)}$ as $\mathbf{p}^{(l)} = -\mathbf{G}^{(l)}\mathbf{g}^{(l)}$, where $\mathbf{g}^{(l)} = (\frac{\partial \varphi}{\partial \sigma_1}, \ldots, \frac{\partial \varphi}{\partial \sigma_N})^T$ is the gradient vector and $\mathbf{G}^{(l)}$ is an approximation to the inverse Hessian matrix, that is updated at every iteration using the limited memory BFGS formula (see [12], formula (9.5), p.225). The next iterate $\sigma^{(l+1)}$ is then found as $\sigma^{(l+1)} = \sigma^{(l)} + \alpha^{(l)} \mathbf{p}^{(l)}$, where the step length $\alpha^{(l)}$ is computed by an inexact line search. What is crucial in this approach it is that it requires 1) relatively small storage proportional to $n_{cp} \times N$, where n_{cp} is the number of the correction pairs, and 2) only the calculation of gradients rather than the time-consuming sensitivities and/or the Hessian matrices.

Calculation of gradients. To derive derivatives $\frac{\partial \varphi_d}{\partial \sigma_k}$ we apply an adjoint method. This method uses the EM field reciprocity and has been applied previously to calculate the sensitivities ([16, 9]) and for forward modelling and inversion ([6, 13, 11, 5]). Let us now describe our implementation of such a technique.

It can be proven with some effort that

$$\frac{\partial \varphi_d}{\partial \sigma_k} = Re \left\{ \sum_{i=1}^{N_T} \int_{V_k} tr \left[\mathbf{u}_i^T \mathbf{E}_i \right] dV \right\},\tag{3}$$

where $tr[\mathbf{u}_{i}^{T}\mathbf{E}_{i}] = u_{x}^{(1)}E_{x}^{(1)} + u_{y}^{(1)}E_{y}^{(1)} + u_{z}^{(1)}E_{z}^{(1)} + u_{x}^{(2)}E_{x}^{(2)} + u_{y}^{(2)}E_{y}^{(2)} + u_{z}^{(2)}E_{z}^{(2)}$, the sign *Re* means the real part of its argument and the superscript 1 or 2 denotes polarization of the source \mathbf{J}_{i} . By definition, 3×2 matrices

$$\mathbf{E}_{i}(\mathbf{r}) = \begin{pmatrix} E_{x}^{(1)} & E_{y}^{(1)} & E_{z}^{(1)} \\ E_{x}^{(2)} & E_{y}^{(2)} & E_{z}^{(2)} \end{pmatrix}_{i}^{T} \text{ and } \mathbf{u}_{i}(\mathbf{r}) = \begin{pmatrix} u_{x}^{(1)} & u_{y}^{(1)} & u_{z}^{(1)} \\ u_{x}^{(2)} & u_{y}^{(2)} & u_{z}^{(2)} \end{pmatrix}_{i}^{T} \text{ satisfy the following equations} \\ \nabla \times \nabla \times \mathbf{E}_{i} - \sqrt{-1}\omega_{i}\mu\sigma(\mathbf{r})\mathbf{E}_{i} = \sqrt{-1}\omega_{i}\mu\mathbf{J}_{i}, \qquad (4)$$

$$\nabla \times \nabla \times \mathbf{u}_i - \sqrt{-1}\omega_i \mu \sigma(\mathbf{r}) \mathbf{u}_i = \sqrt{-1}\omega_i \mu \left(\mathbf{j}_i^{ext} + \nabla \times \mathbf{h}_i^{ext} \right), \tag{5}$$

where
$$\mathbf{j}_{i}^{ext} = \sum_{j=1}^{N_{S}} \alpha_{ji} \mathbf{p}^{T} \overline{\mathbf{A}}_{ji} (\mathbf{H}_{ji}^{-1})^{T} \delta(\mathbf{r} - \mathbf{r}_{j}), \mathbf{h}_{i}^{ext} = -\frac{1}{\sqrt{-1}\omega_{i}\mu} \sum_{j=1}^{N_{S}} \alpha_{ji} \mathbf{p}^{T} \mathbf{Z}_{ji}^{T} \overline{\mathbf{A}}_{ji} (\mathbf{H}_{ji}^{-1})^{T} \delta(\mathbf{r} - \mathbf{r}_{j}), \mu \text{ is the magnetic}$$

permeability, δ is the Dirac's delta-function and $i = 1, \dots, N_T$. Here $\mathbf{p} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ is the projection matrix, 2×2 matrices \mathbf{A}_{ji} , \mathbf{Z}_{ji} are previously explained and the 2×2 $\mathbf{H}_{ji} = \begin{pmatrix} H_x^{(1)} & H_y^{(1)} \\ H_x^{(2)} & H_y^{(2)} \end{pmatrix}_{ji}$ is composed of

the magnetic fields calculated at the *j*-th MT site and at the *i*-th frequency. The key formula (3) practically means that computational loads for calculating gradient $(\frac{\partial \varphi}{\partial \sigma_1}, \ldots, \frac{\partial \varphi}{\partial \sigma_N})^T$ are equivalent to those for the solution of $2 \times N_T$ forward problems using Eq. (4) to find \mathbf{E}_i and of $2 \times N_T$ adjoint problems using Eq. (5) to find \mathbf{u}_i for all $i = 1, \ldots, N_T$. Straightforward calculation of the gradient would require solution of $2 \times N_T \times (N+1)$ forward problems.

The approach described is quite general. It is not limited to magnetotellurics only, but can be applied to a variety of EM problems.

3. Model example

Let us demonstrate on a numerical example how MT inversion allows conductivity to be recovered. In Fig. 1 we present a model including a tilted 3 ohm-m dyke embedded into a 100 ohm-m half-space. The dyke is located at depth 200 to 700 m and it consists of 5 shifted adjacent blocks of $200 \times 800 \times 100 \text{ m}^3$ size each. Our modeling domain comprises of $N_x \times N_y \times N_z = 16 \times 24 \times 8$ rectangular prisms of $100 \times 100 \times 100 \text{ m}^3$ size that cover the dyke and the some part of the surroundings. Notice that the volume lies at depths of 100-900 m.

The inversion domain coincides with the modeling domain. This means that N = 3072 conductivities σ_k (k = 1, ..., N) of the prisms need to be recovered. The x3d forward modeling code described in ([1, 2]) was used as an engine for inversion to solve the forward and adjoint problems given in Eq. (4) and (5). It also was used to calculate 2×2 matrices \mathbf{D}_{ji} of "observed" impedances at $N_T = 4$ frequencies of 1000, 100, 10 and 1 Hz. The impedances were computed at $N_S = 168$ sites r_j $(j = 1, ..., N_S)$ coinciding with the nodes of a homogeneous $n_x \times n_y = 12 \times 14$ grid, where 100 m is the distance between adjacent nodes.

In addition, the number of the correction pairs n_{cp} was chosen as 6, and the relative error ϵ_{ji} of the impedance was taken as 0.05. A 100 ohm-m uniform half-space was used as an initial guess. In Fig. 1 we also present the convergence of the inversion along with a set of 3-D models recovered at various iterations. It should be mentioned, however, that during inversion we did not use the stabilizer φ_s at all; the Lagrange multiplier λ was assigned a zero value. Instead, we assigned the lower conductivity limits of Eq. (2) as $l_k = 0.005$ ($k = 1, \ldots, N$). In other words, resistivities $\rho_k = 1/\sigma_k$ of the cells were constrained from above by a value of 200 ohm-m. This turned out to play a similar role to that of regularization. It should be noted also that without putting



Figure 1: Convergence of the inversion for a 3-D model of a 3 ohm-m dyke in a uniform 100 ohm-m half-space. The left-upper panel presents the misfit and cpu time vs the iteration number. Other panels present images of the initial guess, the true model, as well as the models obtained at various stages of inversion. Number of iterations is given in upper-left corner of each panel.

constraints on l_k the iteration method without a stabilizer (i.e., when $\lambda = 0$) stagnates, when the misfit φ_d drops to a value of 1.3 and it fails to produce a good conductivity image (not presented here).

4. Conclusion

In this paper we have developed a novel approach to 3-D MT inversion. The most essential part of our derivation is that we developed and implemented the adjoint method to derive explicit expressions for the calculation of the gradients of the misfit. Our development is quite general and is not limited to magnetotellurics alone. It can be applied to a variety of EM problems, such as marine controlled-source EM etc. With a synthetic MT example, we have obtained the first promising results of convergence of our solution. The method still needs further development to become a user-end product of universal value to the EM community.

Further work will be concentrated on adapting various types of regularization techniques, and introducing the static shift into the penalty function (1). It is also planned to apply our inversion scheme to an experimental data set. However, previous examples from other 3-D MT inversion software developers (see [8, 10, 14, 17]) indicate that successful verification of the inversion technique even on a single practical dataset is a complex task and may take some time.

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Contrast Source Inversion of 3D Electromagnetic Data

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Abstract—This paper discusses the full vector three-dimensional inverse electromagnetic scattering problem. We consider the determination of the location and the electromagnetic composition of an inhomogeneous bounded object in a homogeneous embedding from measurements of the scattered electromagnetic wavefield, when the object is illuminated by a known single frequency wavefield. To solve this large-scale nonlinear inversion problem, we apply the so-called multiplicative regularized contrast source method.

1. Introduction

In this MR-CSI method we reconstruct the complex permittivity contrast and the so-called contrast sources (the product of the contrast and the fields) by minimizing a cost functional in which the residual errors in the field equations occur. This minimization is carried out in an alternating way. In each iterative step we update the contrast and the contrast sources each using one conjugate gradient step so that the total computational complexity of the method is equal to the complexity of solving only two forward problems. By operating in this manner solving a full three-dimensional vector nonlinear inverse scattering problem is feasible. Further this method is equipped with total variation type regularization. This regularization is included as a multiplicative constraint, so that the regularization parameter needed in the minimization process is determined automatically. The multiplicative type of regularization handles noisy as well as limited data in a robust way without the usually necessary a *priori* information. We illustrate the performance by presenting some inversion results from 3D electromagnetic experimental data. Further, we discuss an inversion method to invert not only the complex electric contrast but also the magnetic contrast of a three-dimensional object. The contrast source inversion is extended by introducing both the electric contrast sources and the magnetic contrast sources. Further, an extended cost functional is introduced in which the residual errors in both the electric and magnetic field equations occur. Additionally, the multiplicative regularization is extended such that the spatial variations of both the electric and magnetic contrast are minimized. Since in [1] we tested the algorithm for a heterogeneous object that has intermingled electric and magnetic contrast, we will present a numerical example with disjoint electric and magnetic contrast.

2. Inversion Algorithm

The Multiplicative Regularized Born Inversion (MRCSI) consists of an algorithm to construct sequences $w_j = \{w_{j,n}\}$ and $\chi = \{\chi_n\}$ which iteratively reduce the value of the cost functional,

$$F_n = [F_S + F_{D,n}]F_n^R = \left[\frac{\sum_j \|u_j^{\text{sct}} - G_S w_j\|_S^2}{\sum_j \|u_j^{\text{sct}}\|_S^2} + \frac{\sum_j \|\chi u_j^{\text{inc}} - w_j + G_D w_j\|_D^2}{\sum_j \|\chi_{n-1} u_j^{\text{inc}}\|_D^2}\right] \frac{1}{V} \int_D \frac{|\nabla \chi|^2 + \delta_n^2}{|\nabla \chi_{n-1}|^2 + \delta_n^2} dx$$

where

 $[G_S w_j](\bar{x}) = \int_D g(\bar{x}, \bar{x}') w_j(\bar{x}') dv(\bar{x}'), \ \bar{x} \in S, \ \text{and} \ [G_D w_j](\bar{x}) = \int_D g(\bar{x}, \bar{x}') w_j(\bar{x}') dv(\bar{x}'), \ \bar{x} \in D.$ The subscripts D and S indicate that the observation point \bar{x} lies either in D, a bounded domain containing the scattering object,

and S indicate that the observation point x lies either in D, a bounded domain containing the scattering object, or S, a domain disjoint from D on which the scattered field u_j^{sct} , j = 1, ..., J, is measured for each known incident field u_j^{inc} . The symbol V denotes the volume of the domain D. Further, $\| \bullet \|_S$ and $\| \bullet \|_D$ denote the norms on $L_2(S)$ and $L_2(D)$. Further, g denotes the Green function of the background medium, while χ denotes the contrast with the background medium. For the steering parameter δ_n^2 we choose progressively decreasing values in such a way that, for given contrast sources, the cost functional F_n as a function of the contrast χ , remains convex during all iterations. We relate this parameter directly to the decreasing object error $F_{D;n-1}$. The structure of the cost functional is such that it will minimize the regularization factor F_n^R with a large weighting parameter in the beginning of the optimization process, because the value of $F_S + F_{D;n-1}$ is still large, and that it will gradually minimize more and more the error in the data and object equations when the value of F_n^R has reached a nearly constant value equal to one. If noise is present in the data, the data error term F_S will remain at a large value during the optimization and therefore, the weight of the regularization factor will be more significant. Hence, the hindering character of noise will, at all times, be suppressed in the reconstruction process, but at the cost of decreased resolution. This minimization is carried out in two alternate steps. For given contrast, χ_{n-1} , the contrast sources are updated via conjugate gradient directions of the cost functional, while for given contrast sources, $w_{j,n}$, the contrast is updated via a preconditioned conjugate gradient direction of the cost functional.

3. Integral Operators for Electric Contrast

Firstly we consider the 3D electromagnetic inversion problem, where the scattering object only has electric contrast χ^E with respect to its embedding. Then we deal with electric contrast sources

$$\bar{w}_j^E(\bar{x}) = \chi^E(\bar{x})\bar{E}(\bar{x}), \quad \text{where} \quad \chi^E(\bar{x}) = \frac{\sigma'(\bar{x})}{\sigma'_b} - 1,$$

with complex conductivity, $\sigma'(\bar{x}) = \sigma(\bar{x}) - i\omega\varepsilon(\bar{x})$, for the inhomogeneous object, and the complex conductivity, $\sigma'_b = \sigma_b - i\omega\varepsilon_b$, for the homogeneous embedding. The scalar Green function is given by

$$g(\bar{x}, \bar{x}') = \frac{\exp(ik_b|\bar{x} - \bar{x}'|)}{4\pi|\bar{x} - \bar{x}'|}, \quad k_b = \sqrt{(i\omega\mu_b\sigma_b')}.$$

In this 3D case the field function u_j^{inc} to be replaced by the incident electric field vector \bar{E}_j^{inc} and the scattered field data u_j^{sct} has to be replaced by either the measured scattered electric field vector \bar{E}_j^{sct} for an electric dipole receiver or the measured scattered magnetic field vector \bar{H}_j^{sct} for a magnetic dipole receiver. The governing integral operators become

$$G_S w_j := \begin{cases} [k_b^2 + \nabla \nabla \bullet] \bar{A}_j^E & \text{for an electric dipole receiver,} \\ \sigma'_b \nabla \times \bar{A}_j^E & \text{for a magnetic dipole receiver,} \end{cases}$$

and

$$G_D w_j := [k_b^2 + \nabla \nabla \bullet] \bar{A}_j^E, \qquad \text{where } \bar{A}_j^E(\bar{x}) = \int_D g(\bar{x}, \bar{x}') \bar{w}_j^E(\bar{x}') dv(\bar{x}').$$

We will illustrate the performance of this type of inversion scheme by presenting some inversion results from 3D electromagnetic experimental data.

4. Integral Operators for Electric and Magnetic Contrast

Secondly, we consider the 3D electromagnetic inversion problem. where the scattering object has both electric contrast χ^E and magnetic contrast χ^H with respect to its embedding. In addition the electric contrast sources, we also deal with the magnetic contrast sources

$$\bar{w}_j^H(\bar{x}) = \chi^H(\bar{x})\bar{H}(\bar{x}), \quad \text{where} \quad \chi^H(\bar{x}) = \frac{\mu(\bar{x})}{\mu_b} - 1.$$

with permeability, $\mu(\bar{x})$, for the inhomogeneous object, and permeability, μ_b , for the homogeneous embedding. In this 3D case the cost functional has to be extended to the following form

$$\begin{split} F_n \ &= \ [F_S^E(\bar{w}_j^E, \bar{w}_j^H) + F_{D,n}^E(\bar{w}_j^E, \bar{w}_j^H, \chi^E)] \frac{1}{V} \int_D \frac{|\nabla \chi^E|^2 + (\delta_n^E)^2}{|\nabla \chi_{n-1}^E|^2 + (\delta_n^E)^2} dv \\ &+ [F_S^H(\bar{w}_j^E, \bar{w}_j^H) + F_{D,n}^H(\bar{w}_j^E, \bar{w}_j^H, \chi^H)] \frac{1}{V} \int_D \frac{|\nabla \chi^H|^2 + (\delta_n^H)^2}{|\nabla \chi_{n-1}^H|^2 + (\delta_n^H)^2} dv \end{split}$$

where

$$F_S^E(\bar{w}_j^E, \bar{w}_j^H) = \frac{\sum_j \|\bar{E}_j^{\text{sct}} - G_S^E(\bar{w}_j^E, \bar{w}_j^H)\|_S^2}{\sum_j \|\bar{E}_j^{\text{sct}}\|_S^2} \quad \text{for an electric dipole receiver,}$$

$$F_{S}^{H}(\bar{w}_{j}^{E}, \bar{w}_{j}^{H}) = \frac{\sum_{j} \|H_{j}^{\text{sct}} - G_{S}^{H}(\bar{w}_{j}^{E}, \bar{w}_{j}^{H})\|_{S}^{2}}{\sum_{j} \|\bar{H}_{j}^{\text{sct}}\|_{S}^{2}} \quad \text{for a magnetic dipole receiver,}$$

and

$$\begin{split} F_{D,n}^{E}(\bar{w}_{j}^{E},\bar{w}_{j}^{H},\chi^{E}) &= \frac{\sum_{j} \|\chi^{E}\bar{E}_{j}^{\mathrm{inc}}-\bar{w}_{j}^{E}+G_{D}^{E}(\bar{w}_{j}^{E},\bar{w}_{j}^{H})\|_{D}^{2}}{\sum_{j} \|\chi_{n-1}^{E}\bar{E}_{j}^{\mathrm{inc}}\|_{D}^{2}},\\ F_{D,n}^{H}(\bar{w}_{j}^{E},\bar{w}_{j}^{H},\chi^{H}) &= \frac{\sum_{j} \|\chi^{H}\bar{H}_{j}^{\mathrm{inc}}-\bar{w}_{j}^{H}+G_{D}^{H}(\bar{w}_{j}^{E},\bar{w}_{j}^{H})\|_{D}^{2}}{\sum_{j} \|\chi_{n-1}^{H}\bar{H}_{j}^{\mathrm{inc}}\|_{D}^{2}}, \end{split}$$

The governing integral operators become

$$G_S^E(\bar{w}_j^E, \bar{w}_j^H) = [k_b^2 + \nabla \nabla \bullet] \bar{A}^E + i\omega\mu_b \nabla \times \bar{A}^H, \quad G_S^H(\bar{w}_j^E, \bar{w}_j^H) = \sigma_b' \nabla \times \bar{A}^E + [k_b^2 + \nabla \nabla \bullet] \bar{A}^H,$$

for observation points in the data domain S,

and

$$G_D^E(\bar{w}_j^E, \bar{w}_j^H) = [k_b^2 + \nabla \nabla \bullet] \bar{A}^E + i\omega \mu_b \nabla \times \bar{A}^H, \quad G_D^H(\bar{w}_j^E, \bar{w}_j^H) = \sigma_b' \nabla \times \bar{A}^E + [k_b^2 + \nabla \nabla \bullet] \bar{A}^H,$$

for observation points in the data domain D,

where

$$\bar{A}^{E}(\bar{x}) = \int_{D} g(\bar{x}, \bar{x}') \bar{w}_{j}^{E}(\bar{x}') dv(\bar{x}') \quad \text{and} \quad \bar{A}^{H}(\bar{x}) = \int_{D} g(\bar{x}, \bar{x}') \bar{w}_{j}^{H}(\bar{x}') dv(\bar{x}')$$

The minimization of the extended cost functional is carried out iteratively in three alternate steps:

- For given electric contrast χ^E_{n-1} and magnetic contrast χ^H_{n-1}, the contrast source vector {w^E_j, w^H_j} is updated via conjugate gradient directions of the cost functional.
 For given electric contrast sources w^E_{j,n} and magnetic contrast sources w^H_{j,n}, the electric contrast χ^E is updated via preconditioned conjugate directions.
 For given electric contrast sources w^E_{j,n} and magnetic contrast sources w^H_{j,n}, the magnetic contrast χ^H is updated via preconditioned conjugate directions.
- updated via preconditioned conjugate directions.



Figure 1: 3D scattering object in a $3\lambda \times 3\lambda \times 3\lambda$ vacuum domain with disjoint electric and magnetic contrast.



Figure 2: Exact (left) and reconstructed (right) of the real part of the electric contrast, at 30 different horizontal planes.



Figure 3: Exact (left) and reconstructed (right) of the imaginary part of the electric contrast, at 30 different horizontal planes.



Figure 4: Exact (left) and reconstructed (right) magnetic contrast, at 30 different horizontal planes.

As numerical example we have simulated electromagnetic field data from an object in a vacuum domain with size of $3\lambda \times 3\lambda \times 3\lambda$. One part of the object is "E"-shaped and has only electric contrast given by the complex contrast function $\chi^E = 1 + 1i$, while the other part is "M"-shaped and has only magnetic contrast given by the real contrast function $\chi^H = 1$ (see Figure 1). We use vertical magnetic dipoles both as transmitters and as receivers. In an area above the domain under investigation, 30 transmitters have been located, viz., at the vertical position $x_3 = -0.1\lambda$ In an area below the domain under investigation, 30 receivers have been located, viz., at the vertical position is subdivided into $30 \times 30 \times 30$. This means that we have 27000 unknown complex-valued electric contrast points plus 27000 unknown real-valued magnetic contrast points. The reconstruction results are given in Figures 2–4. Although the number of unknowns is much larger than the number of known data points, we observe a very good reconstruction in the horizontal plane through the middle of the "E"-shaped object (see Figure 3) and the middle of the "M"-shaped object (see Figure 4), while the reconstruction deteriorates towards the top and bottom part the objects.

We conclude that the present extended form of the MR-CSI method enables the full nonlinear 3D inversion of large scale electromagnetic data.

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Abstract—We consider a perfectly conducting plane with a local cylindrical perturbation illuminated by a monochromatic plane wave. The perturbation is represented by a random function assuming values with a Gaussian probability density. For each realization of the stochastic process, the spatial average value over the width of the modulated zone is zero. The mean value of the random function is also zero. Without any deformation, the total field is the sum of the incident field and the reflected field. For a locally deformed plane, we consider — in addition to the incident and reflected plane waves — a scattered field. Outside the modulated zone, the scattered field can be represented by a superposition of a continuous spectrum of outgoing plane waves. The method of stationary phase leads to the asymptotic field, the dependence angular of which is given by the scattering amplitudes of the propagating plane waves. Using the first-order small perturbation method, we show that the real part and the imaginary part of scattering amplitudes are uncorrelated Gaussian stochastic variables with zero mean values and unequal variances. Consequently, the probability density for the amplitude is given by the Hoyt distribution and the phase is not uniformly distributed between 0 and 2π .

1. Introduction

The problem of electromagnetic wave scattering from random surfaces continues to attract research interest because of its broad applications. The three classical analytical methods commonly used in random roughsurface scattering are the small-perturbation method, the Kirchhoff method and the small slope approximation [1–5]. The electromagnetic analysis of rough surfaces with parameters close to the incident wavelength requires a rigorous formalism. Numerous method based on Monte Carlo simulations are available for 1D and 2D random rough surfaces [6, 7]. Most of research works focus on the determination of coherent and incoherent intensities. There is not such a voluminous literature on the statistical distribution of scattered field [3]. In this paper, we derive the statistical distribution in the far field zone from the first-order small perturbation method in the particular case of perfectly conducting 1D random rough surface illuminated by an $E_{//}$ polarized monochromatic plane wave.

2. The Random Surfaces under Consideration

The geometry of the problem is depicted in Fig. 1. The rough surface is represented in Cartesian coordinates by the equation $y = a_0(x)$ and consists of a small cylindrical random perturbation with length L and zero mean



Figure 1: The slightly rough surface.

 $(\langle a_0(x) \rangle = 0)$ in a perfectly conducting plane y = 0. Each realisation can be described by the following equation

$$a_0(x) = a(x) - m \quad \text{if } |x| \le \frac{L}{2}$$

$$a_0(x) = 0 \text{ outside} \tag{1}$$

where

$$m = \frac{1}{L} \int_{-L/2}^{+L/2} a(x) dx$$
 (2)

a(x) is a random function assuming values distributed normally with zero mean and variance σ_a^2 . Here it's important to distinguish the spatial average m from the statistical mean $\langle a(x) \rangle$. Insofar $\langle a(x) \rangle = 0$, we have $\langle m \rangle = 0$. The random process is assumed stationary with a Gaussian statistical correlation function

$$R_{aa}(x) = \sigma_a^2 \exp\left(-\frac{x^2}{l_c^2}\right) \tag{3}$$

where l_c is the correlation length.

3. The Scattering Amplitudes in the Far Field Zone

The surface is illuminated under incidence θ_i by an z-polarized monochromatic plane wave $E_i \vec{u}_z$ of wavelength λ . The Oz-electric component of field is

$$E_i(x,y) = \exp(-j\alpha_i x + j\beta_i y) \tag{4}$$

where

$$\alpha_i = k \sin \theta_i; \quad \beta_i = k \cos \theta_i; \quad k = 2\pi/\lambda \tag{5}$$

The time-dependence factor $\exp(j\omega t)$ where ω is the angular frequency is assumed and suppressed throughout. The total electric field above the rough surface is the sum of the incident field E_i , the field reflected E_r by the plane without deformation (an infinite perfect mirror) and the scattered field E_d .

$$E_t(x,y) = E_i(x,y) + E_r(x,y) + E_d(x,y)$$
(6)

where

$$E_r(x,y) = -\exp(-j\alpha_i x - j\beta_i y) \tag{7}$$

Above the highest point on the surface, the scattered field can be represented by a superposition of a continuous spectrum of outgoing plane waves, the so-called Rayleigh integral [5].

$$E_d(x,y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{C}(\alpha) \exp\left(-j\beta(\alpha)y\right) \exp(-j\alpha x) d\alpha$$
(8)

with

$$\beta = \sqrt{k^2 - \alpha^2}, \quad \text{Im}\,\beta < 0 \tag{9}$$

In the far-field zone, the Rayleigh integral is reduced to the only contribution of the propagating waves ($\alpha \leq k$). The method of stationary phase leads to the asymptotic field [8]

$$E_d(r,\theta) \approx \sqrt{\frac{k}{2\pi r}} \hat{C}(k\sin\theta)\cos\theta\exp(-jkr)\exp\left(j\frac{\pi}{4}\right)$$
(10)

The angular dependence in the far field zone is given by the function $\hat{C}(\alpha) \cos \theta$ and becomes identified with the propagating wave amplitudes of the continuous spectrum (8) with $\alpha = k \sin \theta$ [9, 10]. Let us recall that the normalized bistatic scattering coefficient $\sigma(\theta)$ is defined by the power scattered per unit angle $d\theta$ normalized with respect to the flux of incident power through the modulated region

$$\sigma(\theta) = \frac{1}{P_i} \frac{dP_d}{d\theta} = \frac{|\hat{C}(k\sin\theta)|^2 \cos^2\theta}{\lambda L\cos\theta_i}$$
(11)

For a random process, the scattered field is a random function of position (r, θ) but the scattering amplitude $\hat{C}(\alpha)$ is a random function of the observation angle θ only [10]. The scattering amplitude can be written as the sum of an average amplitude $\langle \hat{C}(\alpha) \rangle$ which gives the coherent far-field from (11) and a fluctuating amplitude which leads to the incoherent far-field. The first order small perturbation method applied to the Rayleigh integral (8) and the Dirichlet boundary condition gives an approximation of the scattering amplitudes [1,2]

$$\hat{C}(\alpha) = -2j\beta_i \int_{-L/2}^{+L/2} a_0(x) \exp\left(+j(\alpha - \alpha_i)x\right) dx$$
(12)

Making a change of variable $\gamma = \alpha - \alpha_i$, real part $\hat{C}_r(\alpha)$ and imaginary part $\hat{C}_i(\alpha)$ of scattering amplitudes can be expressed as

$$\hat{C}_r(\gamma) = +2\beta_i \int_{-L/2}^{+L/2} a(x)\sin(\gamma x)dx$$
(13)

$$\hat{C}_i(\gamma) = -2\beta_i \left(\int_{-L/2}^{+L/2} a(x) \cos(\gamma x) dx - mL \sin c(\gamma L/2) \right)$$
(14)

where $\sin c(x) = \sin x/x$. It can be noticed that the scattering amplitude is zero in the specular direction $\gamma = 0$. \hat{C}_r and \hat{C}_i are obtained from mathematical linear operations applied to the Gaussian random function a(x). Consequently, \hat{C}_r and \hat{C}_i are also quantities distributed with Gaussian probability densities.

4. The Statistical Distribution of Scattering Amplitudes

4.1. The Incoherent Intensity

From (13) and (14), we derive $\langle \hat{C}(\gamma) \rangle = 0$. Consequently, the coherent density is zero. Moreover, after some extensive mathematical manipulations, we deduce the variances

$$r = \langle \hat{C}_{r}^{2} \rangle = 4\beta_{i}^{2} \int_{0}^{+L} (L-x) \Big[\cos \gamma x - \sin c \big(\gamma (L-x) \big) \Big] R_{aa}(x) dx$$
(15)

$$s = \langle \hat{C}_{i}^{2} \rangle = 4\beta_{i}^{2} \int_{0}^{+L} (L-x) \Big[\cos \gamma x + \sin c \big(\gamma (L-x) \big) \Big] R_{aa}(x) dx - 4\beta_{i}^{2} \sin c (\gamma L/2)$$
$$\Big[\sin c (\gamma L/2) \int_{0}^{+L} x R_{aa}(x) dx + 2 \int_{0}^{+L} (L/2-x) \sin c \big(\gamma (L/2-x) \big) R_{aa}(x) dx \Big]$$
(16)

where the statistical correlation function $R_{aa}(x)$ is given by (3).

The variances depend on the width L of the modulated zone. But, outside the specular reflection zone, if L goes to infinity, the variances of the real and imaginary parts become identified. Using (11), (15) and (16), we obtain the incoherent intensity $I_f(\theta) = \langle \sigma(\theta) \rangle$

$$I_f(\theta) = \frac{\langle \left| \hat{C}(k\sin\theta - k\sin\theta_i) \right|^2 \rangle \cos^2\theta}{\lambda L\cos\theta_i} \quad \text{with} \langle \left| \hat{C}(\gamma) \right|^2 \rangle = \langle \hat{C}_r^2 \rangle + \langle \hat{C}_i^2 \rangle \tag{17}$$

We note that the incoherent intensity is not proportional to the surface power spectrum.

4.2. Probability Densities of the Amplitude and Phase

Random quantities $A = \hat{C}_r(\alpha)$ and $B = \hat{C}_i(\alpha)$ are distributed normally with zero mean values and unequal variances r and s. Moreover, we show that they are uncorrelated. Consequently, they are independent and we can write:

$$p_{AB}(a,b) = p_A(a)p_B(b) = \frac{1}{2\pi\sqrt{rs}} \exp\left(-\frac{a^2}{2r} - \frac{b^2}{2s}\right)$$
(18)

where $p_{AB}(a, b)$ is the two-dimensional normal distribution of $\hat{C}_r(\alpha)$ and $\hat{C}_i(\alpha)$. Transforming to polar coordinates,

$$A = M\cos\psi; \qquad B = M\sin\psi \tag{19}$$

we obtain the required distributions for the modulus M and the phase ψ :

$$p_M(m) = \int_{0}^{2\pi} p_{M\psi}(m,\varphi) m \, d\varphi = \frac{m}{\sqrt{rs}} \exp\left(-\frac{m^2}{4r} - \frac{m^2}{4s}\right) \tag{20}$$

$$p_{\psi}(\varphi) = \int_{0}^{+\infty} p_{M\psi}(m,\varphi)m \, dm = \frac{1}{2\pi} \frac{\sqrt{rs}}{s\cos^2\varphi + r\sin^2\varphi}$$
(21)

These formulas show that $p_M(m)$ is the Hoyt distribution [3] and that the phase is not uniformly distributed between 0 and 2π . Nevertheless, outside the specular reflection zone and if L goes to infinity, $p_M(m)$ is reduced to the Rayleigh distribution and the phase is uniformly distributed.

5. Results

Figure 2 gives the incoherent intensity for a Gaussian random profile having a modulation length $L = 24\lambda$, a rms height $h = 3\lambda/100$ and a correlation length $l_c = 2\lambda$. We can note the zero value of $I_f(\theta)$ in the specular direction ($\theta = \theta_i = 30^\circ$). Outside the specular zone, the comparison with results obtained by the C method [10] is good. The dashed curve and the solid curve show the values obtained by (15) and by the C method.



Figure 2: Incoherent intensity for a Gaussian random profile.



Figure 3: Amplitude and phase distributions.

Figure 3 show the values of the Hoyst distribution and the phase distribution (given by (20) and (21), respectively) for an observation angle $\theta = 10^{\circ}$. The comparison with the normalized histogram obtained by a Monte-Carlo simulation with 10000 surface realizations is good.

6. Conclusion

We have derived the statistical distribution in the far field zone from the first-order small perturbation method in the particular case of perfectly conducting 1D random rough surfaces illuminated by an $E_{//}$ polarized monochromatic plane wave. We have shown that the real part and the imaginary part of scattering amplitudes are uncorrelated Gaussian stochastic variables with zero mean values and unequal variances. The probability density for the amplitude is given by the Hoyt distribution and the phase is not uniformly distributed between 0 and 2π . Comparisons with statistical observation over 10000 surfaces confirm the result. This approach can be extended to dielectric random rough surfaces illuminated by a polarized plane wave $E_{//}$ or $H_{//}$. The generalization of these results to slightly rough surface with an arbitrary statistical height distribution with an arbitrary correlation function is in progress.

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Dispersion Characteristics of Coplanar Waveguides at Subterahertz Frequencies

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Abstract—We present experimental and simulated studies of the dispersion characteristics of coplanar waveguides (CPWs) at subterahertz frequencies. Two types of CPWs were studied, those with wide ground planes and those with narrow ground planes. In both cases, simulations closely followed the experimental results, thus giving us a basis for implementing the simulations in circuit-design models for a wide range of such waveguides.

1. Introduction

Coplanar waveguide (CPW) structures are commonly used in high-speed circuits and interconnect. Although the wave-propagation characteristics of CPWs have been well studied [1–11], only recently has this work been extended to the terahertz range for different geometries [6–9] and compared with experimental results [10, 11]. While our previous work focused on the attenuation characteristics [10, 11], in this paper we report results on the dispersion characteristics to complete the study. Our work is based on comparing experimental data with simulations that make use of full-wave analysis, allowing for direct verification of the validity of the latter. The effects of ground-plane width and lateral line dimensions have also been analyzed.

2. Background

CPWs are a family of transmission lines consisting of a center conductor strip and two ground conductor planes with variable widths. All three conductors are placed on the same side of a dielectric substrate, as shown in Figure 1.



(a) CPW with wide ground planes.

(b) CPW with narrow ground planes.

Figure 1: Cross-section of CPWs being investigated.

Two classes of CPWs were studied. The first contains ground planes at least 10 times wider than

the center conductor or the conductor spacing and closely approximates an ideal CPW structure with infinitely wide ground planes. It should be pointed out that, for this class of transmission lines, closedform analysis is in principle possible with the use of conformal mapping [12]. The second class uses ground planes with the same width as that of the center conductor, representing a practical geometry used in integrated circuits. For the latter category, theoretical studies are restricted to numerical simulations.

When an electromagnetic wave propagates on a CPW, the electric fields above conductors experience the permittivity of the air, while those below conductors experience the permittivity of the substrate. The effective permittivity thus takes on a value between that of the air and substrate. When the frequency of propagating wave increases, the effective permittivity approaches that of the substrate, as the density of electric field lines below the



Figure 2: Effective permittivity of a CPW with wide ground planes. See Section 3 for geometry and electrical parameters.

conductors gets higher. The difference in the effective permittivity at different frequencies results in a modal dispersion that can be described with a frequency-dependent propagation constant:

$$\beta = 2\pi f \cdot 10^9 \cdot \frac{\sqrt{\varepsilon_{eff}}}{c} \tag{1}$$

where f is the frequency, c is the speed of light in free space, and ε_{eff} is the frequency-dependent effective permittivity. An example of ε_{eff} is demonstrated in Figure 2 [1].

Figure 2 shows that, with increasing frequency, the effective permittivity increases and a steep step is located at the position where the lowest-order surface-wave mode starts to interact with the CPW mode. For CPWs with infinitely wide ground planes, the TM₀ mode enters first and, at higher frequencies, this mode and higher-order modes contribute to the increase of ε_{eff} . For the case of narrow ground planes, the lowest-order surface-wave mode that can be supported is TE₀ mode [6]. Since the entry frequency of TM₀ mode is lower than that of TE₀ mode, CPW with wide ground planes supports more surface-wave modes and therefore suffers higher dispersion than CPW with narrow ground planes in the 100's GHz range.

3. Experiment and Simulation

In our work, the CPWs were fabricated on 500- μ m-thick semi-insulating GaAs substrates. Gold was evaporated on the substrate and formed the transmission line patterns using a "lift-off" process [13]. The thickness of gold conductor was measured as t = 290 nm. Each set of CPW had a center conductor and conductor spacing with a width of S = W = 50 or $10 \,\mu$ m. The ground plane was chosen as $G = 500 \,\mu$ m for CPW with wide ground planes or as the lateral line dimension for CPW with narrow ground planes. With a testing method utilizing the non-uniform illumination of photoconductive switches together with electro-optic sampling, the broadband description of CPW characteristics has been obtained [11, 13]. The signal propagating along the CPW was measured in the time domain and then converted to the frequency domain by Fourier transform [13].

For comparison with the experiments, we used the software package Sonnet Suites to simulate the transmission lines. The simulation makes use of a modified method of moments based on Maxwell's equations to perform a three dimensional full-wave analysis of predominantly planar structures [14] and returns the values of the scattering matrices. The geometry and electrical parameters used for simulation are the same as those in the experiments. The simulated scattering matrices are then converted to propagation constant and, in turn, to the effective permittivity by (1).

4. Results and Discussion



Figure 3: Simulated and experimental effective permittivity of CPWs with a 50- μ m center conductor. $\varepsilon_{eff,sim-wg}$ and $\varepsilon_{eff,exp-wg}$ refer to simulated and experimental effective permittivity of the wide-ground CPW, respectively. $\varepsilon_{eff,sim-ng}$ and $\varepsilon_{eff,exp-ng}$ refer to simulated and experimental effective permittivity of the narrow-ground CPW, respectively.

Figure 3 shows the subterahertz effective permittivity of CPWs with a 50- μ m center conductor as a function of the frequency. We present the simulated and experimental effective permittivity of CPWs with both wide and narrow ground planes for comparison. There is a good agreement except for the several peaks on the simulated curves. These peaks are remnants of the poles in the Green's function used in the Sonnet Suites and correspond

to the sequential entry of the surface-wave modes. Although the poles are removed one by one in the final results, some oscillations remain and are an unavoidable artifact.



Figure 4: Simulated and experimental effective permittivity of CPWs with a 10- μ m center conductor. $\varepsilon_{eff,sim-wg}$ and $\varepsilon_{eff,exp-wg}$ refer to simulated and experimental effective permittivity of the wide-ground CPW, respectively. $\varepsilon_{eff,sim-ng}$ and $\varepsilon_{eff,exp-ng}$ refer to simulated and experimental effective permittivity of the narrow-ground CPW, respectively.



Figure 5: Simulated and experimental effective permittivity of CPWs with narrow ground planes. The lateral line dimensions are 50 μ m and 10 μ m, respectively. $\varepsilon_{eff,sim-50}$ and $\varepsilon_{eff,exp-50}$ refer to simulated and experimental effective permittivity of the CPW with a 50- μ m center conductor, respectively. $\varepsilon_{eff,sim-10}$ and $\varepsilon_{eff,exp-10}$ refer to simulated and experimental effective permittivity of the CPW with a 10- μ m center conductor, respectively.

An important observation of Figure 3 is that one clearly sees that the CPW with narrower ground planes returns a lower effective permittivity and reduced dispersion in both the experimental and simulated data. The reduced ground-plane width gives rise to a reduction in coupling between the CPW mode and surface-wave modes, which consequently decreases dispersion along the CPW.

We also investigated dispersion characteristics of CPWs with narrower lines. Figure 4 shows the subterahertz effective permittivity of CPWs with a 10- μ m center conductor. Again, it can be seen that CPW with narrow ground planes encounters lower effective permittivity than CPW with wide ground planes. Dispersion is slightly improved in CPW with narrow ground planes.

To examine the effect of lateral line dimension, we combine the narrow-ground CPW data into Figure 5. It is clearly seen that the effective permittivity of CPW with a 10- μ m center conductor is lower than that of CPW with a 50- μ m center conductor and the overall dispersion is much less.

5. Conclusion

In summary, we present experimental and simulated dispersion characteristics of CPWs with wide and narrow ground planes. The simulation results agree well with the experimental data up to subterahertz frequencies. It is shown that CPW with narrow ground planes suffers lower dispersion than CPW with wide ground planes. Furthermore, dispersion can be reduced by reducing the lateral line dimension of the CPW. Combining our previous studies on the attenuation characteristics [10, 11] and the current work on dispersion, we conclude that in the frequency range where radiation effects dominate (100's GHz for the lines considered in this report), the narrow-ground CPWs perform better in both aspects.

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A Multi Conductor Transmission Line Model for the Evaluation of the Rotor Shaft Voltages in Adjustable Speed Drive Motors

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Abstract—The use of switching devices, such as IGBTs, characterised by high switching frequencies and very low switching times in new generation pulse width modulation (PWM) inverters has increased the efficiency and performances of Adjustable Speed Drives (ASDs) for industrial and traction applications. However, such systems may be affected by disadvantages like over voltages at the motor terminals, when long cables are used between the drive, and the generation of rotor shaft voltage, due to the capacitive couplings in the motor (between the windings and the rotor and between the rotor and the stator). The shaft voltage may cause the breakdown of the lubricating film in the bearings. The resulting impulsive currents, by damaging the bearing elements shorten the component life, which in turn seriously affects the ASD reliability. For this reason, it is of great importance to develop numerical models able to predict the shaft voltage so as to estimate the currents flowing through the bearings. Several works, based on either concentrated or distributed circuit models, have been proposed for the evaluation of the shaft voltage magnitudes for several motors sizes. However, the results obtained by such approaches suffer from approximations and simplifications in the considered circuit model. Therefore, in the present paper, a numerical model able to accurately predict the shaft voltage in high power induction motor for traction applications fed by a PWM inverter is presented. The windings of the motor are modelled by a multi conductor transmission line (MTL), whereas the cables between the source and the motor are described by a single transmission line. The effect of wave propagation and reflection and of the frequency-dependent distributed losses is considered by using a time-domain equivalent circuit to represent the MTL. A semi-analytical method, based on the perturbation theory of the spectrum of symmetric matrices, is adopted. The parameters of the MTL are obtained either analytically or numerically by using a commercial software (Maxwell® by Ansoft). The effects of the rise time of the input voltage together with the length of the cables are considered.

1. Introduction

A drive system composed of a traditional induction motor matched to a pulse-width modulated (PWM) inverter can overcome the limitations of induction motors operating directly on line voltage, consisting essentially in a nearly constant, unadjustable output speed and in a small starting torque, drawing a large starting current. By feeding the motor with a variable ac voltage and a variable ac frequency, thus obtaining an adjustable-speed drive (ASD), most requirements of modern drives can be satisfied. More recently, such systems have reached a great diffusion, which can be mostly associated to the rapid development of new switching devices, such as the insulated gate bipolar transistor (IGBT), which have led to increased efficiency, performance and controllability in medium voltage, medium power induction motor applications, such as traction, cranes for port operation, etc.

Unfortunately, the output voltage from the inverter is not purely sinusoidal and, in particular, steep front pulses can be generated in correspondence to the commutations of the switching devices. Slew rates (dV/dt) of about $2500 \text{ V}/\mu\text{s}$ can be produced, resulting in overvoltages at the motor terminals and in critical stresses of the motor electrical insulation which can lead to a sensible reduction of the life-time of the machine [1].

Furthermore, it should be considered that such pulses can excite the capacitive coupling between the stator and the rotor, resulting in shaft voltages, even 20 times larger than those observed when feeding the motor with a pure sinusoidal waveform. The shaft voltages may cause the breakdown of the lubricating film in the bearings. The resulting impulsive currents, by damaging the bearing elements can shorten the component life, which in turn seriously affects the ASD reliability. For design purposes, it seems, therefore, of great importance to provide the electrical engineers with a model able to predict the voltage distribution in the motor windings and evaluate the electrical stresses on the rotor shafts as a function of many geometrical and electrical parameters, such as the stator insulation dimension, the length of feeding cable, shape of the applied voltage, insulating material permittivity etc.

The model developed by Melfi et al., [2] is based on a representation of the motor winding as a lumped network; in order to preserve its physically distributed nature, the parasitic coupling between the stator and the rotor has been modelled over a range of frequencies. Also Lipo et al., [3] have used an equivalent lumped parameter π -network to describe the parasitic coupling phenomenon. They remark that the parasitic coupling circuits are the same as transmission line circuits, but a distributed parameter circuit is not suitable for a simplified analysis of the bearing currents.

The present paper is dedicated to the illustration of a machine model based on the representation of the motor winding as a connection of multiconductor transmission lines. The model is able to predict the voltages across the rotor shaft, taking into account the main phenomena occurring along the lines, such as the propagation and the reflection, together with the time dispersion introduced by the losses, eventually dependent on the frequency. The solution technique is accurately described in many papers [4]; it consists of a semi-analytical method based on the perturbation theory of the spectrum of symmetric matrices. The MTL lines are described by their characteristic **R**, **L**, **C**, and **G** per unit length matrices, that is, in the Laplace domain by the longitudinal impedance $\mathbf{Z}(s) = \mathbf{R} + s\mathbf{L}$ and transverse admittance $\mathbf{Y}(s) = \mathbf{G} + s\mathbf{C}$.

The authors explicitly remark that the paper is dedicated to the illustration of the model and its potentiality but, at present, not to the estimation of the shaft voltages in the motor operating conditions. In fact, as discussed in the following sections, the results of the simulations have been obtained by feeding the machine not with a typical three-phase inverter output voltage, but providing a single-phase ramp voltage with variable slew rate. It is the author's opinion that the findings are still extremely significant since they can describe the effects of slew rate (dV/dt) of the input voltage together with the length of the feeding cable adopted. Simulations in real operating conditions, together with experimental verifications will be the subject of future works.

In the following, section 2 is dedicated to an illustration of the basic model, together with a brief description of the solution technique; in section 3 the results of the numerical simulations are illustrated; the last section contains remarks, comments and proposals for the future activity.

2. The Model

A schematic representation of the model is reported in Fig. 1. An ideal ramp voltage generator is connected through a feeder cable to the motor. The stator winding is represented by a form wound stator coil, composed of conductors of rectangular cross section; it faces the rotor iron laminations; the rotor is connected to a pair of bearings represented in the picture by their equivalent capacitance C_{b1} and C_{b2} .

The system can be studied (Fig. 2) as single transmission line, representing the cable, connected to four multiconductor transmission lines in series placed in the slot and overhang regions of the machine. Further details can be found in [5] by Lupò et al., The MTL are composed of n conductors; the n-th conductor represents the rotor iron.



Figure 1: Schematic representation a motor phase.

Figure 2: MTL model of the machine.

The multiconductor line can be studied in the time-domain by means of a 2n-ports representation (Fig. 3) described by Eqs. (1) and (2) [4]:

$$\begin{cases} \mathbf{i}_{0}(t) = \int_{0^{-}}^{t^{+}} \mathbf{Y}_{\mathbf{c}}(t-\tau) \mathbf{v}_{0}(\tau) d\tau + \mathbf{j}_{0}(t) \\ \mathbf{i}_{d}(t) = \int_{0^{-}}^{t^{+}} \mathbf{Y}_{\mathbf{c}}(t-\tau) \mathbf{v}_{d}(\tau) d\tau + \mathbf{j}_{d}(t) \end{cases}$$
(1)

$$\begin{cases} \mathbf{j}_{0}(t) = \int_{0^{-}}^{t^{+}} \mathbf{P}(t-\tau) [-2\mathbf{i}_{d}(\tau) + \mathbf{j}_{d}(\tau)] d\tau \\ \mathbf{j}_{d}(t) = \int_{0^{-}}^{t^{+}} \mathbf{P}(t-\tau) [-2\mathbf{i}_{0}(\tau) + \mathbf{j}_{0}(\tau)] d\tau \end{cases}$$
(2)

The impulse responses $\mathbf{Y}_{\mathbf{c}}(t)$ and $\mathbf{P}(t)$ are defined as:

$$\begin{cases} \mathbf{Y}_{\mathbf{c}}(t) = \mathbf{L}^{-1}[\mathbf{Y}_{\mathbf{c}}(s)] = \mathbf{L}^{-1}\left[\sqrt{\mathbf{Z}^{-1}(s)\mathbf{Y}^{-1}(s)}\mathbf{Y}(s)\right] \\ \mathbf{P}(t) = \mathbf{L}^{-1}[\mathbf{P}(s)] = \mathbf{L}^{-1}\left[\exp\left[-d\sqrt{\mathbf{Y}(s)\mathbf{Z}(s)}\right]\right] \end{cases}$$
(3)

where $\mathbf{Y}_{\mathbf{c}}(s)$ is the characteristic admittance and $\mathbf{P}(s)$ is the propagation function.

 $\mathbf{Y}_{\mathbf{c}}(t)$ and $\mathbf{P}(t)$ can be found as a sum of their principal part, i.e., the parts containing terms as Dirac pulses, and a remainder evaluated by performing a numerical inverse transform. The solution can be achieved by means of a recursive approach since at time instant t the state variables $\mathbf{j}_0(t)$ and $\mathbf{j}_d(t)$ are known because they depend on the values assumed at time instant (t - T) by themselves and by the currents, where T is the propagation time delay.



Figure 3: 2n-ports representation of a MTL.



Figure 4: Bearings' voltages $V_{b1}(t)$ and $V_{b2}(t)$.

3. Results of Numerical Simulation

The numerical simulations have been carried out on a traction motor characterised by 9 conductors per slot. The applied voltage has a maximum value Emax=750 V and a variable slew rate chosen in the interval $(0.5 \div 2.0 \text{ kV}/\mu \text{s})$; the length Lc of the feeder cable varies between 5 m and 15 m. The equivalent capacitances Cb₁ and Cb₂ are chosen equal to 5 nF. Prior to the numerical simulation the p.u. length matrices, **C** and **L** have been evaluated with the software packing Maxwell® by solving, respectively, an electrostatic and a magnetostatic problem.



Figure 5: Peak value of Vb1 vs dV/dt and Lc.



Figure 7: Peak value of Vb1 and Vb2 vs. dV/dt.



Figure 6: Peak value of Vb2 vs dV/dt and Lc.



Figure 8: Peak value of Vb1 and Vb2 vs. Lc.

As an example of the results obtained, in Fig. 4 the time evolutions of the voltage $V_{b1}(t)$ and $V_{b2}(t)$ across the two bearings are reported when $dV/dt = 1 \text{ kV}/\mu \text{s}$ and Lc = 10 m. Voltage $V_{b2}(t)$ is slightly delayed with respect to $V_{b1}(t)$, due to a propagation delay of about 40 ns.

Since the main parameter influencing the breakdown phenomena in the lubricating film in the bearings is the maximum amplitude Vm of the voltage, in Figs. 5 and 6 the peak values of V_{b1} and V_{b2} are reported as a function of the slew rate and the cable length.

In particular, it is evident that critical situations can be reached with long cables and high slew rates: the peak voltages can be almost 5 times higher with $Lc = 15 \text{ m} - \text{dv}/\text{dt} = 2.0 \text{ kV}/\mu\text{s}$ if compared with Lc = 5 m and $\text{dv}/\text{dt} = 0.5 \text{ kV}/\mu\text{s}$. Furthermore, the peak voltages can be different for the two bearings and, as a consequence, the breakdown phenomena can occur only in one bearing.

Such a difference is amplified when the bearings' capacitances are not equal, for instance when bearings of different type are installed (bearings produced by diverse manufacturers, standard or insulated bearings, new or aged bearings, etc.). In fact, when introducing in the simulations two values for such capacitances (Cb1=4nF;

Cb2=5 nF), the difference in the peak values of Vb1 and Vb2 is strongly evident, as shown in Figs. 7 and 8. In particular, the dissimilarity between peaks of Vb1 and Vb2 grows with the slew rate and the length of the feeding cable.

4. Conclusions

The present paper describes an equivalent MTL model able to predict the shaft voltages in high power induction motor for traction applications fed by a PWM inverter. By feeding the motor with an ideal ramp voltage generator, it is possible to derive significant information on the voltages across the motor bearings; their peak values strongly depend on the slew rate of the applied voltage and on the length of the connecting cable. The effect of different bearings' capacitances has also been evidenced. Further work is in progress in order to introduce a typical three-phase inverter output voltage and to clarify the effect of other parameters like the amplitude of the applied voltage, the electrical characteristics of the cables, the geometrical parameters of the machine.

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Abstract—The transducers studied herein allow for frequency-selective measurement of mm-wave power parameters. Frequency selectivity is assured by a monocrystalline hexagonal ferrite resonator (HFR). The HFR is in direct contact with a semiconductor element (SE)—an unpackaged Hall-element, or a chip transistor (or diode). Power absorbed by the HFR at the ferromagnetic resonance converts to heat, and the heat flux penetrates through the current-carrying SE. A number of thermo/electro/magnetic phenomena accompany the Hall-effect in a semiconductor and cause a voltage in addition to the Hall-effect voltage. The conversion coefficient of a transducer is analyzed using the power balance equation. Some experimental results using the designed power transducers in the 8-mm waveband are presented.

1. Introduction

Many applications require an adequate information about mm-wave (30–300 GHz) spectra of signals, e.g., spectral power density, peak power of pulse signals, integral power in the given frequency band, width and central frequency of spectrum [1, 2]. Equipment for measuring power parameters of electromagnetic signals contains a primary measuring transducer and a secondary processing/display unit. A primary transducer in this case converts the energy associated with electromagnetic oscillations into a different form of energy (e.g., thermal, mechanical, etc.), or into voltage which is convenient to register and quantify. Most of microwave and mm-wave power meters and detectors are not frequency-selective. Their application for "fine" spectra measurements needs additional high-Q filters and cumbersome calibration of the receiving path. Heterodyne-type spectrum analyzers and measuring receivers typically have numerous parasitic channels of reception, which is an especially difficult problem for the analysis of mm-wave spectra of signals of middle and higher intensity levels (more than 1 mW of continuous power).

The transducers proposed herein allow for frequency-selective measuring of mm-wave power parameters. Frequency selectivity is assured by incorporating a monocrystalline hexagonal ferrite resonator (HFR) with a narrow ferromagnetic resonance (FMR) line. An advantage of an HFR is its high intrinsic field of magnetic crystallographic anisotropy, so it does not need massive bias magnetic systems for achieving FMR [2]. In the transducer, the HFR is in direct contact with a semiconductor element (SE). The SE may be an unpackaged Hall-element (HE) slab, or chip transistor (or diode). The mm-wave power absorbed by the HFR at the FMR converts to heat, and the heat flux from the HFR penetrates through the body of the current-carrying SE. Since this structure is in the bias magnetic field, there are a number of thermoelectric, thermomagnetic, galvanomagnetic, and thermoelectromagnetic phenomena, along with the Hall-effect in the SE. In fact, there are over 560 different known effects accompanying the Hall-effect [3]. These phenomena cause a voltage in addition to that of the Hall-effect. This happens only when the frequency of the mm-wave signal falls into the ferromagnetic resonance curve of the HFR, thereby assuring frequency selectivity of power conversion.

2. Mathematical Model for the Conversion Coefficient of the Transducer

The conversion coefficient of a frequency-selective power transducer is defined as a ratio of the amplitude of the converted signal to the power of the input microwave signal at the given frequency [1],

$$K_p = \Delta V / P(f_0). \tag{1}$$

If the thermal coefficient of voltage in a semiconductor element

$$K_T = \Delta V / \Delta T \tag{2}$$

is known, then the conversion coefficient of a transducer operating on the basis of thermal effects is

$$K_p = K_T \Delta T_{\text{stat}} / P(f_0), \tag{3}$$

where ΔT_{stat} is the stationary temperature increase in the system "HFR-semiconductor element".

Consider the case when a microwave oscillation of power $P(f_0)$ acts on the HFR continuously, and the HFR absorbs this power due to the FMR. Inside the HFR there is a constant source of heat, and the surface temperature of the HFR remains constant. Suppose that heat radiation is absent. Let us also neglect semiconductor heating when current flows in it, and further assume there is no difference in the temperature of the contacts (no thermal electromotive force). The result of the semiconductor heating is the variation in the charge carrier mobility, which leads to the variation of the thermal coefficient K_T . Then the equation for thermal balance can be written in terms of power,

$$P_{ds} = P_{FS} + P_{SM} + P_{FA} + P_{SA},\tag{4}$$

where $P_{ds} = \alpha P(f_0)$ is the mm-wave power absorbed by the ferrite at the FMR (α is the absorption coefficient); P_{FS} is the heat power transferred by the ferrite to the semiconductor element (conduction heat exchange); P_{SM} is the heat power transferred from the semiconductor element to the metal contacts (conduction heat exchange); P_{FA} is the heat power transferred by the HFR surface to air (convection heat exchange); and P_{SA} is the heat power given to air by the semiconductor (convection heat exchange).

The Cauchy problem describing the transient thermal regime in the system "HFR-SE" with a heat source inside the ferrite is analogous to that considered in [4] for a YIG bolometer,

$$C_{\Sigma}\frac{dT}{dt} + \Psi_{T\Sigma}T = P_{dbs}; \quad T(0+) = T_r,$$
(5)

where C_{Σ} [J/K] is the total heat capacity of all the elements of the thermal system, and $\Psi_{T\Sigma}$ [W/K] is the total heattransfer factor of all the system. The initial condition is the room temperature T_r . The solution of the Eq. (5) is an exponential function

$$T(t) = T_r + \Delta T_{stat} (1 - e^{-t/\tau_{\Sigma}}), \tag{6}$$

where $\Delta T_{stat} = P_{ds}/\Psi_{T\Sigma}$ is the increase of the stationary temperature, and $\tau_{\Sigma} = C_{\Sigma}/\Psi_{T\Sigma}$ is the response time of the system.

The absorption coefficient α depends on the HFR rate of coupling with the transmission line where it is placed. This coupling is described by the coupling coefficient η_c [5], and in turn, it depends on the geometry of the transmission line or waveguide, operating mode structure, the point where the HFR is situated, and the physical parameters of the HFR (its resonance line width ΔH , saturation magnetization M_S , anisotropy field H_A , and orientation of the HFR crystallographic axis in respect with the bias magnetic field H_0), as well as on the detuning $|f_{res} - f_0|$ of the FMR frequency from the mm-wave signal carrier frequency. The absorption coefficient can be obtained through solving the self-matched field problem and electromagnetic power balance equation, as described in [1,5]. From this analysis, the absorption coefficient relates to the coupling coefficient as

$$\alpha = 2|\eta_c|/|1 + \eta_c|. \tag{7}$$

For a transmission line or waveguide operating with only a single mode having transverse microwave magnetic field components $h_{x,y}$, and with the bias magnetic field for the HFR directed along z-direction, the coupling coefficient can be calculated as $\omega\mu_0 V_{f_{(z)}} = et_1 2$

$$\eta_c = j \frac{\omega \mu_0 v_f}{2N_1} (\chi_{11}^{\text{ext}} h_x^2 + \chi_{22}^{\text{ext}} h_y^2), \tag{8}$$

where $\chi_{11,22}^{ext}$ are the complex diagonal components of the external magnetic susceptibility tensor for an ellipsoidal (general case) HFR [1,5] for any arbitrary orientation of the HFR crystallographic axis with respect to the bias magnetic field. In (8), V_f is the volume of the HFR, and N_1 is the norm of the corresponding transmission line or waveguide mode, as calculated in [5].

3. Calculations and Experimental Data

The calculations were performed for a uniaxial monocrystalline HFR resonator made of M-type Ba ferrite doped with Ti and Zn ions. It was placed in a metal waveguide with a cross-section of 7.2 mm× 3.4 mm, in the point with the right circular polarization of the mm-wave magnetic field. The HFR in this case was a spheroid with the axes 0.585 mm×0.557 mm. Its magnetic parameters were the following: the field of crystallographic magnetic anisotropy was $H_A = 11.3$ kOe, saturation magnetization was $4\pi M_S = 3.5$ kG, and the unloaded resonance line width was $\Delta H = 31.1$ Oe. The density of the hexagonal ferrite was $\rho_f = 4900$ kg/m³; the specific heat was $c_f = 1100$ J/(kg.K); and thermal conductivity was $\lambda_f = 4.1$ W/(m.K). The measured input average power of the mm-wave continuous signal at the frequency $f_0 = 39.5$ GHz was $P(f_0) = 60$ mW; the HFR absorbed 5 dB at the FMR ($P_{dss}^{maas} = 41.1$ mW).

The Hall-element (HE) X511 (Russia) measured $1.5 \text{ mm} \times 2.0 \text{ mm} \times 0.1 \text{ mm}$. It was made of a monocrystalline InSb, the density was $\rho_s = 5770 \text{ kg/m}^3$, the specific heat was taken as $c_s = 700 \text{ J/(kg.K)}$; the thermal conductivity

was $\lambda_s = 18 \text{ W/(m.K)}$; and the thermal coefficient of voltage K_T was 1.5 mV/K (according to the technical passport for the X511). An active region of contact with the HFR was assumed to be 0.01 mm^2 . Heat transfer coefficients for natural convection (room temperature $T_r = 20^{\circ}C$ and normal atmosphere pressure of 760 mm of mercury) for both HFR and SE are about $30 \text{ W/(m^2 \cdot K)}$.

The total heat transfer factor was calculated as $\Psi_{T\Sigma} = \Psi_{aaw} + \Psi_{aand} = 1.2 \cdot 10^{-3} \text{ W/K}$. The total heat capacity is $C_{\Sigma} = C_f + C_s = 0.9 \cdot 10^{-3} \text{ J/K}$. The calculated response time of the system is $\tau_{\Sigma}^{calc} = 750 \text{ ms}$, and the calculated stationary temperature increase in the system is $\Delta T_{stat}^{calc} = 25^{\circ}C$. The transition time for the temperature increase is $t_{stat} = 4.6 \cdot \tau_{\Sigma} = 3.45 \text{ s}$. The calculated voltage at the SE is $\Delta V^{calc} = 37.5 \text{ mV}$, and the conversion coefficient is $K_p^{calc} = 0.625 \text{ V/W}$. The measured data are $\Delta T_{stat}^{mas} = 21^{\circ}C$ and $\tau_{\Sigma}^{mas} \approx 1 \text{ s}$.

4. Design and Experimental Data of the Frequency-selective Thermal Transducers

4.1. HFR-two Hall-elements

The structure of the frequency-selective electronically tunable power transducer is shown in Fig. 1. It contains two identical Hall elements: one is inside the waveguide, having direct contact with the HFR, and the second is on the outer side of the waveguide. Both transducers are exposed to the same uniform bias magnetic field. The Hall elements are connected so that it is possible to obtain a differential signal at the comparator. When there is no FMR absorption, the signals of the Hall elements are the same and correspond to the pure Hall-effect voltage in the given magnetic bias field. When the HFR is at the FMR, it heats up because of the resonance absorption, and there is an additional voltage that is induced on the contacts of the internal Hall element. Its value is proportional (with the coefficient K_P) to the average power of the mm-wave signal at the resonance frequency of the HFR, through the equation $\Delta V_H = K_p P_{av}(f_{res})$.



Figure 1: Schematic of the transducer based on the HFR and two Hall-elements.

When experimental structures based on the HFR and semiconductor elements were exposed to nanosecond pulses with pulse repetition frequency of 1 kHz, an off-duty factor of 2, and an average power of 10–100 mW, the mechanism of interaction was mainly inertial, of a thermal nature. Low-inertial effects, such as the magnetoresistive effect, the microwave Hall-effect in semiconductor, direct electromagnetic field detection by the SE due to variation in the magnetic flux from the HFR are negligibly small at average power levels of a few dozen mW. This inertial mechanism of interaction between the HFR and the mm-wave field is determined by heating of the HFR at the FMR power absorption and the corresponding heat flux acting on the Hall element.

Suppose that the total voltage induced in the semiconductor plate carrying a current I placed in a transverse magnetic field H_{0z} consists of a Hall-effect voltage V_H and a number of additional terms, corresponding to the most important effects accompanying the Hall-effect [3]:

$$V = V_H + V_{neq} + V_{mr} + V_{temf} + V_E + V_{NE} + V_{PNE} + V_{RL} + V_{PRL}$$

$$\tag{9}$$

In (9), V_{neq} is the non-equipotentiality voltage, V_{mr} is the magnetoresistive voltage, V_{tenf} is the thermoelectromotive force voltage, V_E is the Ettingshausen galvanomagnetic voltage, V_{NE} is the Nernst- Ettingshausen thermomagnetic voltage, V_{PNE} is the Peltier-Nernst-Ettingshausen thermomagnetic electrothermal/ thermogalvanomagnetic voltage, V_{RL} is the Righi-Leduc thermomagnetic voltage, and V_{PRL} is the Peltier-Righi- Leduc electrothermal/thermogalvanomagnetic voltage. In the proposed design, the contribution of V_H and V_{nn} is compensated by the second Hall-element. The voltages V_{neqv} and V_{tenf} are independent of the bias magnetic field, and can be taken into account and compensated. It is impossible to separate the remaining five contributions. However, the Nernst-Ettingshausen effect might be dominant. It is a thermomagnetic effect, and appears as a transverse voltage with respect to the current I flowing in the semiconductor slab, assuming the latter is in the magnetic field and is affected by the heat flux.

In the experiment, two Hall elements were used. The first one was an X511 characterized by $R_{in} = 2.0$ Ohms, $R_{\alpha t} = 1.6$ Ohms, $I_{qpr} = 100$ mA, and having a thermal sensitivity of 1.05 V/(A·T). The second was X211. The characteristics of the X211 differ only in the output resistance ($R_{\alpha t} = 1.9$ Ohms) and the thermal sensitivity 1.38 V/(A·T). The slope with respect to the magnetic field is $S = \Delta V / \Delta H = 10^{-2} \text{ mV/Oe}$ for both Hall-elements. The minimum measured magnetic field for both Hall elements was 0.1 Oe. Identical operation of both Hall-elements was assured by proper choosing of their operation currents.

In the transducer, the first Hall-element was placed in the rectangular waveguide with cross-section 7.2 mm × 3.4 mm, in the point of the circular polarization of the mm-wave magnetic field. The off-resonance loss factor in the section was 1.1 dB, and the standing wave ratio in this section was SWR=1.2. The HFR was the same as discussed above. The FMR absorption was 5 dB. Fig. 2 shows the resonance dependence of the differential signal ΔV as a function of the applied bias magnetic field H_{z0} for a continuous mm-wave signal at $f_0 = 40.7$ GHz. The minimum stable measured signal was about $10 \,\mu$ W. The conversion coefficient was $K_P^{mens} = 0.6 \,\text{V/W}$, which is close to the calculated value ($K_P^{alc} = 0.625 \,\text{V/W}$). The discrepancy can be explained by the mm-wave loss in the section of the waveguide. The 50% alcohol solution of the glue BF-2 (Russia) was used to fix the HFR on the unpackaged HE, and the resonance absorption in a high-Q ferrite resonator could have decreased by about 1 dB due to the glue. The simplifying assumptions in the model, such as neglecting the heat loss on metal contacts, might also adversely influence the accuracy of computations. Also, the reference input data for the parameters of a transmission line, ferrite resonator, and InSb HE might have some tolerance. Furthermore, instrument error in the mm-wave measurements might yield another 1 dB of uncertainty.

4.2. HFR—Unpackaged Chip Diode or a Transistor

The Hall-element contacting with the HFR was replaced by a chip transistor (CT) used as a diode. The voltage thermal sensitivity of the CT (2TC398A-1 manufactured in Russia) was 3.0 mV/K, which is higher than



Figure 2: Resonance dependence of the converted voltage (HFR-2 Hall elements).



Figure 3: Linear volt-watt characteristics for transducers.

the voltage thermal sensitivity of the HE. The HFR anisotropy field was $H_A = 10.6$ kOe, and the FMR line width was $\Delta H = 30$ Oe. The HFR absorbed 3 dB of power at the FMR ($f_0 = 40.7$ GHz, $P(f_0) = 60$ mW). The conversion coefficient is 1.2 V/W, which is two times greater than that of the transducer "HFR-2 Hall elements". The minimum measured signal was about 1μ W. The shortcoming of the transducer is the presence of a "pedestal" at the level of 1 mV due to the off-resonance heating of the semiconductor element directly from the mm-wave signal power. However, this "pedestal" can be removed by a calibration in the off-resonance regime. A further improvement of the transducer can be realized by using a more thermosensitive semiconductor element. Linear volt-watt characteristics of the transducers with two Hall-elements and the chip transistor are shown in Fig. 3.

5. Conclusion

The transducers based on a high-Q hexagonal ferrite resonator in direct contact with a thermosensitive semiconductor element allow for frequency-selective measurement of mm-wave power parameters over a wide frequency range. The physical mechanisms of power conversion are analyzed, and it is shown that the conversion coefficient of a transducer can be calculated using the equation of power balance. To increase the sensitivity and conversion coefficients of a transducer based on the approach, it is necessary to use an HE with a higher thermal coefficient of voltage, assure the best possible heat contact between the HFR and the semiconductor element (increase the surface of their contact, for example, using a disk HRF), and employ a microvoltmeter with higher sensitivity to register smaller converted signals.

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Wave Propagation in Grounded Dielectric Slabs with Double Negative Metamaterials

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Abstract—In this paper, the wave propagation in a grounded dielectric slab with double negative (DNG) metamaterials is studied. Dramatically different evanescent surface modes (electromagnetic fields exponentially decay both in air and inside the slab) are observed. They are highly dependent on medium parameters. An infinite number of complex surface modes are found to be existing which have proper field distribution in the air region. The investigations on the Poynting vectors show that they do not carry away energy in both transverse and longitudinal directions.

1. Introduction

The guided dielectric slab with a DNG medium has been studied by several groups. Various novel properties are observed: [1] and [2] found that there are special regions for TM (transverse magnetic) modes where two different propagation constants exist. [3] theoretically considered the properties of a planar two-layered waveguide, whose one layer is a double positive (DPS) medium and the other is a DNG medium. Super slowwaves with extremely short wavelengthes were found whose fields exponentially decay from the interface of the two slabs inside both layers. These guided modes, termed as evanescent surface modes, were also found by [4] and [5], respectively. P. Baccarelli and his colleague suggested the concept of surface wave suppression that ensures the absence of both ordinary and evanescent surface modes. This is very attractive in view of taking DNG medium as a potential substrate candidate to reduce edge diffraction effects and enhance radiation efficiency for microstrip antennas [6].

However, so far as the authors are aware no study on the complex modes and Poynting vectors has been reported. This makes the mode spectra of DNG media unpleasantly incomplete. In this paper, the authors focus on the properties of the evanescent surface modes and the complex modes, both of which belong to the proper mode spectra of the grounded dielectric slab with a DNG medium. It is found that the evanescent surface modes are highly dependent on the medium parameters and an infinite number of complex modes exists which have exponentially decaying fields in the air region. They are termed complex surface modes. The study on the Poynting vectors shows that they have zero power flows in both transverse and longitudinal directions.

2. Eigen Equations and Graphical Solutions

The structural setup of interest here is a grounded dielectric slab of thickness d (see Figure 1). Region one is a DNG medium and region two is air. It is well known that to ensure a positive stored energy in the dielectric layer, passive DNG media must be dispersive [7]. However, for simplicity we assume that they are isotropic, losseless, and non-dispersive. This assumption is found to be acceptable since a small dispersion of ϵ and μ can satisfy the constraints.



Figure 1: Geometry structure of a grounded dielectric slab with DNG medium ($\epsilon_{r1} < 0, \mu_{r1} < 0$).



Figure 2: Graphical solutions for TE and TM modes. Solid lines in the first and fourth quadrants represent (1) or (2); solid lines in the second quadrant represent (4) or (5); dashed line in the first and fourth quadrants represents (3); dashed line in the second and third quadrants represents (6). The medium parameters are: $\epsilon_{r1} = -2.5, \mu_{r1} = -0.5, \epsilon_{r2} = 1, \mu_{r2} = 1.$

Using the well-known transverse resonance method [8], the eigen equations for ordinary $(\gamma_{y1} = jk_{y1})$ real modes are:

$$\frac{\mu_{r2}}{\mu_{r1}}(k_{y1}d)\cot(k_{y1}d) = -\alpha_{y2}d \quad \text{for TE}$$

$$\tag{1}$$

$$\frac{\epsilon_{r2}}{\epsilon_{r1}}(k_{y1}d)\tan(k_{y1}d) = \alpha_{y2}d \qquad \text{for TM}$$
(2)

$$(k_{y1}d)^2 + (\alpha_{y2}d)^2 = (k_0d)^2(\epsilon_{r1}\mu_{r1} - \epsilon_{r2}\mu_{r2})$$
(3)

The eigen equations for evanescent $(\gamma_{y1} = \alpha_{y1})$ real modes are:

$$\frac{\mu_{r2}}{\mu_{r1}}(\alpha_{y1}d)\coth(\alpha_{y1}d) = -\alpha_{y2}d \quad \text{for TE}$$
(4)

$$\frac{\epsilon_{r2}}{\epsilon_{r1}}(\alpha_{y1}d)\tanh(\alpha_{y1}d) = -\alpha_{y2}d \quad \text{for TM}$$
(5)

$$(\alpha_{y2}d)^2 - (\alpha_{y1}d)^2 = (k_0d)^2(\epsilon_{r1}\mu_{r1} - \epsilon_{r2}\mu_{r2})$$
(6)

where $k_0^2 = \omega^2 \mu_0 \epsilon_0$. γ_{y1} , γ_{y2} are the *y*-direction wave constants of the two layers. Their relationship to the longitudinal wave constant (*z*-direction) γ is written as:

$$\gamma_{yi}^2 = -k_0^2 \epsilon_{ri} \mu_{ri} - \gamma^2 \qquad (i = 1, 2)$$
(7)

Graphical representations of the above equations are shown in Figure 2. The mode index notation here follows [9]. Notice that in the first and second quadrants, α_{y2} is positive and the fields exponentially decay in the air region (proper); in the third and fourth quadrants, α_{y2} is negative and the fields exponentially increase in the air region (improper). The x-axis is divided into two segments. The right half is for $k_{y1}d$ and the fields in the dielectric layer are sine/cosine standing waves (ordinary), while the left half is for $\alpha_{y1}d$ and the fields in the dielectric layer are exponentially distributed (evanescent). Therefore, the intersection in the second quadrant represents the proper evanescent surface mode, which does not exist for a DPS medium.

Another important difference for a DNG medium that can be read from Figure 2 is that the ordinary surface mode solutions are no longer monotonic. It is clear from the subfigure in the left corner of Figure 2(a) that there are two intersections as the radius of the dashed circle decreases, which corresponds to a decrease of frequency. Once the circle has only one tangential point with the solid line, further decreasing frequency will cause this mode to be cutoff. The same thing happens to TM modes in Figure 2(b) in a more obvious way. These two possible modes have two different power flow distributions. One has more power flowing in the air region than in the dielectric region, making the total power flow in the same direction as the phase velocity. The other is in the opposite way and displays a backward property. More details on the Poynting vectors are addressed in Section 4.



Figure 3: Two possible dispersion curves for TE proper surface modes (solid lines) and TE improper leaky modes (dotted lines). The dashed line, representing $\sqrt{\epsilon_{r1}\mu_{r1}}$, is the watershed for evanescent surface mode and ordinary surface modes.

3. Evanescent Surface Mode

As stated in Section 2, the proper evanescent surface mode does exist with a DNG medium. It is the intersection in the second quadrant. The normalized effective dielectric constant $\epsilon_{eff} = (\beta/k_0)^2$ for evanescent surface mode is larger than both $\epsilon_{r1}\mu_{r1}$ and $\epsilon_{r2}\mu_{r2}$. Therefore the transverse propagation constant in the dielectric layer $\gamma_{y1} = \sqrt{-k_0^2 \epsilon_{r1} \mu_{r1} - \gamma^2} = k_0 \sqrt{\epsilon_{eff} - \epsilon_{r1} \mu_{r1}}$ is a pure real number. The electromagnetic fields are no longer sine/cosine standing waves, but have the form of $Ae^{-\alpha_{y1}y} + Be^{\alpha_{y1}y}$.

It is found, however, that the dispersion curves for evanescent surface modes are very complicated, and they are highly dependent on the medium parameters. Figure 3 shows two dispersion diagrams for TE₁ mode with different medium parameters. The dispersion curves represent the intersection points of the dashed line and the first solid branch in Figure 2(a), including the part in the second quadrant. The solid line in Figure 3 is for proper modes, while the dotted line is for improper mode, which is the set of intersections in the fourth quadrant in Figure 2(a). The dashed lines in both figures depict the value of $\sqrt{\epsilon_{r1}\mu_{r1}}$. They are the watersheds by which one can tell the evanescent surface mode from ordinary ones.

In Figure 3(a), the evanescent surface mode has low cutoff frequency. As the frequency increases, the ordinary surface mode becomes an evanescent surface mode and its effective dielectric constant, ϵ_{eff} , keeps increasing. In Figure 3(b), however, the situation is reversed. The evanescent surface mode has a high cutoff frequency above which it becomes the ordinary surface mode. At the low frequency range, the evanescent surface mode has an extremely large ϵ_{eff} , which decreases rapidly as the frequency increases. One can refer to the subfigures of Figure 3 to check the validations. The reason for such dramatically different dispersion curves is that with DNG metamaterials, one can not only make ϵ and μ simultaneously negative but also let their absolute values be less than one [5]. From (1) and Figure 2(a), it is easy to see that the crossing point of the first solid branch TE₁ with the x-axis is fixed at ($\pi/2$, 0), while the crossing point with the y-axis noted as 'A' in Figure 2(a) is (0, $|\mu_{r2}/\mu_{r1}|$). With a conventional DPS medium, μ_{r1} is always equal to unity, or slightly greater or smaller than unity as in the case of paramagnetic or diamagnetic materials. With metamaterials, however, μ_{r1} is not confined near unity any more and the intercept with the y-axis may change a lot. This change affects the possible intersections of the first solid line and the dashed line in Figure 2(a) and finally results in dramatically different dispersion curves.

4. Complex Surface Modes and Poynting Vectors

It is well known that the complete proper mode spectra for a DPS dielectric slab include discrete surface modes and continuous radiation modes, both of which are real modes [8]. With a DNG medium, however, it is proved by the authors that the complex roots of the eigen equations are exclusively on the top Riemann sheet [10]. These solutions, termed complex surface waves, form another set of proper modes since they have exponentially decaying fields in the air region and satisfy the boundary conditions at infinity. Unlike real surface modes, complex surface modes have high cutoff frequencies below which they exist.



Figure 4: Dispersion diagrams for all modes. Solid line is for normalized β of the proper modes. Dashed line is for normalized α of the proper modes. Dotted line is for normalized β of the improper modes. The medium parameters are: $\epsilon_{r1} = -2.5, \mu_{r1} = -2.5, \epsilon_{r2} = 1, \mu_{r2} = 1$.

Figure 4 shows the dispersion diagrams for both TE and TM modes, including evanescent, ordinary, and complex surface modes. Also included are improper leaky modes drawn as dotted lines. When the frequency is much lower than the first cutoff frequency of the real modes, all complex modes exist with very high normalized α and β . As the frequency increases, β/k_0 tends to decrease rapidly within a very narrow frequency range; after that it increases slowly till its cutoff frequency. Notice it is not monotonic and the value of β/k_0 can be less than unity, which is a notable difference compared with evanescent and ordinary surface modes. The curve of α/k_0 , however, monotonically decreases very fast as the frequency increases. At the cutoff point, α reaches zero and β becomes the starting point of the real mode. The real surface mode bifurcates into two branches from this point. One branch has an increasing β/k_0 as the frequency goes high, while the other has a decreasing β/k_0 , which will reach unity shortly. This property is expected from Figure 2. Further increasing frequency makes β/k_0 of the second branch begin to rise. However, it is no longer a proper mode.

It is found that the complex surface modes have zero power flows [10]. To derive the Poynting vector for complex modes, γ_{y1} , γ_{y2} , and γ are assumed to be:

$$\begin{aligned} \gamma_{y1} &= a + jb \\ \gamma_{y2} &= u + jv \\ \gamma &= \alpha + j\beta \end{aligned} \tag{8}$$

The Poynting vector is written as

$$S_z^{\rm TE} = \frac{1}{2} E_x H_y^* = \frac{|A|^2}{2} \begin{cases} S_{z1}^{\rm TE}, & \text{for } 0 < y < d \\ S_{z2}^{\rm TE}, & \text{for } y \ge d \end{cases}$$
(9)

where A is the electric field intensity and S_{z1}^{TE} and S_{z2}^{TE} are as follows:

$$S_{z1}^{\text{TE}}(y,z) = \frac{\beta + j\alpha}{2\omega\mu_{r1}} e^{-2\alpha z} [\cosh(2ay) - \cos(2by)]$$
(10)

$$S_{z2}^{\text{TE}}(y,z) = \frac{\beta + j\alpha}{2\omega\mu_{r2}} e^{-2u(y-d) - 2\alpha z} [\cosh(2ad) - \cos(2bd)]$$
(11)

Figure 5 shows the dispersion diagram and the integral results of Poynting vector for the TE₃ mode. In Figure 5(a), only the complex mode exists (branch 'A') when the frequency is lower than the cutoff frequency of the real surface mode. The zero power flow in z-direction in Figure 5(b) shows that the complex surface mode does not carry away any energy. As the frequency increases, the real surface mode begins. The top branch (branch 'B') of the real mode carries a negative power flow and shows backward properties. When a waveguide operates in this mode, its fields are largely confined inside the dielectric layer. The bottom branch (branch 'C') of the real mode carries a positive power flow and its fields extend far away in the air region. Further increasing frequency causes the fields in the air region to decay more slowly, and eventually reach infinity. At that point, the radiation boundary conditions are violated and the mode becomes improper.



Figure 5: Dispersion diagram and the power flow in z-direction for TE modes. 'A' is for complex surface mode; 'B' is for top branch of the real surface mode; 'C' is for bottom branch of the real surface mode. The medium parameters are: $\epsilon_{r1} = -2.5$, $\mu_{r1} = -2.5$, $\epsilon_{r2} = 1$, $\mu_{r2} = 1$.

5. Conclusion

In this paper, an investigation on the mode properties of a grounded dielectric slab with a DNG medium has been dealt with. The graphical method is used to find the possible real roots. Dramatically different dispersion curves of evanescent surface modes are observed, showing that they are very sensitive to the material parameters. It is found that there is an infinite number of complex surface modes with a DNG medium and they do not carry away energy. Although the considered medium here is idealized and currently cannot be realized, the results of this paper still unveil some exotic properties as well as potential applications of the metamaterials.

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Detection of Groundwater by Ground Penetrating Radar

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Abstract—The application of ground probing radar (GPR) to detection of groundwater from relatively deep aquifers in a desert environment is addressed and processing techniques to improve the detectability of a weak signal in noise and interference are reported. The study is based on simulated images from structures that have the potential of storing groundwater one of which is the buried valley structure.

To increase the signal to noise ratio to achieve a reasonable probability of detection and false alarm, various processing schemes are possible, typically employing analogue, binary (double threshold) and digital processing. Different system architectures are compared to improve detectability. Automatic detection and classification by artificial neural networks is tried to classify geologic subterranean features for aiding and speeding the process and to overcome lack of experts on the field.

1. Introduction

Figure 1 shows how losses increase with depth. Losses include attenuation, spreading losses and loss due reflection coefficient from the buried interface. The bottom layer is considered to be saturated soil giving a reflection coefficient of 0.25. Losses that depend on external influences and not on system parameters are lumped together and called external losses, they are expressed as follows.

$$Loss = \frac{\lambda^2 \sigma |\rho|^2 e^{-4 \alpha R}}{(4 \pi)^3 R^4}$$
(1)

where λ is the wavelength, σ is the scattering cross section, ρ is the reflection coefficient and R is the interface depth. For a planar interface the scattering cross section is given as $\sigma = \frac{\pi \lambda R}{4}$, which is the first Fresnel zone [1].



Figure 1: A monogram showing how losses increase as functions of depth and attenuation.

2. System comparison

Different detection techniques are compared. The processors that are compared are digital, binary and step frequency.

2.1. Digital Pulsed System

This is a pulsed radar having a digital processor assuming the use of swept gain amplifier which compensates for attenuation due to the range of each scatterer.

The system gain before digitisation is chosen so that interference and noise would not exceed the input range of the ADC full scale ratio (FSR) for an acceptable time duration (implying a large probability that the interference is within the FSR). Assuming that the input to the ADC is Rayleigh distributed. If the FSR to be equal to 10 σ implies saturation for only 1% of the time. The amplification factor satisfying the 1% saturation criterion is 48 dB (for a FSR=10 V) and 23 dB (for a FSR of 0.54 V).

The noise level without amplification is -64 dBm and the interference power is -34 dBm and if the bandwidth is increased to 20 MHz (a pulse duration of $0.05 \,\mu$ s) the noise will be -61 dBm and interference will be -31 dBm. It is seen that interference is the dominant factor and therefore the time required to increase the output signal to noise ratio for the digital processing would be equivalent to that of an analogue system. Therefore, an 8 bit ADC would probably be adequate.

FSR, V	Quantisation noise for an ADC with the number of bits in dBm			applied gain dB	amplified	amplified interference dBm
	the number of bits in abin			gam uD	noise ubii	interference ubii
	8 bits	10 bits	12 bits			
10	-9	-21	-33	48	-16	13.2
0.54	-34.3	-46	-58	23	-41	-12.3

Table 1: Quantisation noise and amplified input noise and interference.

2.2. Pulsed Radar System with Binary Integration

The binary integrator has a threshold device that generates 1's or 0's depending on whether the input to the device has exceeded a certain threshold voltage or not. After detection noise alone has a Rayleigh probability density function and the sum of the signal and noise has a Rician probability density function.

The next summing device taking the input from the threshold device will count the number of 1's obtained from a collecting a set of pulses. If the total exceeds a certain number (k out of m) which is a type of (digital) threshold, a target is declared. The input to the threshold device has a signal to noise ratio defined as the ratio of signal voltage to standard deviation of the noise and is given the symbol a, [1].

The signal to noise ratio of the quantised video is defined as $\rho = \frac{p_s - p_n}{\sqrt{p_n(1-p_n)}}$, where p_n is the probability of obtaining a quantised pulse (binary 1) due to noise alone and p_s is the probability of obtaining a quantised pulse when the signal is present. The signal to noise ratio of integrated video is $SNR_{out} = \sqrt{m}\rho$ where m is the number of integrated pulses. For an input signal to noise ratio of - 37 dB (his is when assuming the radar has a transmitted power of 10 w and a pulse duration of $0.05 \,\mu$ s) $\rho = 8 \times 10^{-5}$ for input $SNR = -37 \,\text{dB}$, $m = \left(\frac{SNR_0}{\rho}\right)^2$, making the number of pulses that are needed for integration to be about 4×10^9 . The output SNR being 7 dB. The time to collect data is about 22 hours for a prf of 50 kHz.

The choice of the second threshold k out of m is

$$k = SNR_{out}\sqrt{m}p_n(1-p_n) + mp_n + \frac{1}{2}$$
(2)

Therefore, k is 8×10^8 and so if the number of 1's exceeds 8×10^8 then a target is declared.

2.3. A Step Frequency Processor.

The radar system transmits a sinusoid and measures the magnitude and the phase angle of the received signal. It does this for a group of sinusoids forming the spectral components of the time domain signal that we want to synthesise and then an IDFT is performed to obtain the reflected signal in the time domain. This must be the point of comparing data obtained by the step frequency and the pulsed radar systems. At the input to the display device the value of signal to noise ratio must 7 dB to have the same probabilities of detection and false alarm as that of the pulsed system. The step frequency (SF) radar can be operated with an instantaneous narrow bandwidth making the input noise and interference to remain low and therefore quantisation noise may become dominant.

2.3.1. A radio Frequency Digitising System

Figure 4 presents a step frequency system digitising the signal at radio frequency.

A step frequency radar system with a system noise factor of 10 is considered, a transmitted power of 1 W, a bandwidth of the preselector filter of 1 kHz, a pulse duration of 1 ms and a burst repetition frequency of 1 kHz. The number of frequency spectral samples is 40 (20 MHz effective bandwidth with a frequency step of 0.5 MHz). The signal to noise ratio at point A is the thermal noise is kTFB = -134 dBm and the atmospheric noise (being 30 dB above thermal) is -104 dBm. Interference power is -75 dB (not allowing for low interference bands) and if low interference bands are utilised, it would be about -115 dBm (the power spectral density of interference in these bands is about -145 dBm/Hz).
The number of bursts that has to be collected and integrated is determined by the need to bring the signal to noise ratio at point C to that is required at point D. Considering an ADC having 8 bits and an FSR of 10 V, the signal to noise ratio at point C would be -71 dB. The number of bursts that are needed for integration is about 6.4×10^6 . The integration is coherent and so it is assumed to have an improvement that is $\propto N$. If the FSR is made 0.54 V with an 8 bit ADC and no gain is applied, the number of bursts that are needed for integration are 2×10^4 and the total time to collect all the data is about 13 minutes.

The signal to noise, interference and quantisation ratio at point B is SNIQR = $\frac{\Pr \cdot G}{(Na+I) \cdot G + Nq}$. where Na is atmospheric noise power, I is interference power, Nq is quantisation noise power and G is gain.

For FSR = 10 V, the gain that may be applied before clutter saturates the ADC is about 7 dB. The signal to total noise ratio (total including noise interference and quantisation noise) is -63 dB, the number of pulses to be integrated is about 9×10^5 . If the ADC has 10 bits, the time would be about 40 minutes and for a 12 bit ADC the time would be about 3 minutes. It seems that quantisation noise is higher than input noise and interference in the case of a SF radar.

2.3.2. A Proposed System

The radar system presented in Figure 4 digitises at the radio frequency which is possible at HF but if the radar operates at higher frequencies it may be difficult. There are many designs to make the ADC's work at lower frequencies by implementing a mixer to down convert the frequency either to DC or to other IF's higher than zero, all of which suffer from the inherent drawbacks of the analog components. A proposed system avoiding these problems and does not require the ADC's to work at excessively high frequencies is given in Figure 5. The system is composed of four ADC's working in an interleaved manner. The timing of sampling between the samplers is T/4, T is the period of the received signal.



Figure 2: A pulsed digital processor.



Figure 3: A pulsed system with a binary integrator.



Figure 4: A step frequency processor with RF digitisation.



Figure 5: Schematic diagram of the proposed radar system.

2.3.3. Measurement of Amplitude and Phase Angle of a Sinusoid

The magnitude and phase of the received signal is gives by the following equations.

Magnitude =
$$\frac{\sqrt{I^2 + Q^2}}{2}$$
 and the Angle = $\tan^{-1}\left(\frac{-Q}{I}\right)$ (3)

Figure 6 show a simulated traces and an image as obtained by the radar.



Figure 6: a) Signals after averaging repeated samples. (m is the number of averaged samples) and b) The radar image of a buried channel.

3. Artificial Neural Networks

Samples of the simulated images that were fed to the ANN's for classification are shown in Figure 7. The size of the images is 80 by 50 pixels. The structures are those of, mostly, a buried valley having different cross section shapes one of which is having a saturated zone. Another image is of a buried dome structure.



Figure 7: Samples of some images of the geological features that are used for the ANN's.

A backpropagation artificial network is used fro image classification. Results provided are for the conditions of number of neurons in the middle layer being 10, sum square error is 0.1 and the SNR is 10 dB. It is found that the ANN classifier gives very high success rate.

4. Summary and Discussion

It is seen that binary integration takes longer time than analogue but it has a small word length making it simpler to implement and being low cost. Because of its simplicity, it may be possible to operate the binary integration scheme at higher prf's, allowing the time of data collection to be reduced. The Step frequency radar may operate with powers that are lower than those needed for a pulsed radar and takes shorter time to acquire the data. For SF radar, the averaging of many samples of the signal increases the SNR coherently. ANN's are useful in classifying the subterranean images.

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GPR Ground Bounce Removal Methods Based on Blind Source Separation

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Abstract—In this paper, the ground bounce (GB) removal methods based on Blind Source Separation (BSS) for land mine detection using ground penetrating radar (GPR) are investigated. These methods include an Independent Component Analysis (ICA) based method and Blind Instantaneous Signal Separation (BISS) based method. First, a modified ICA based method is presented. In this method, a fully automatic eigenimage based Independent Components (ICs) selection strategy combined with a non-homogeneous detector (NHD) is introduced. A BISS based method is also proposed for the GB removal. This method can be applied in various environments as ICA, but it has much fewer number of extracted components than ICA's, but has much fewer number of components to extract, therefore less computation load is required. Experimental results show that the proposed methods exhibit good performance.

1. Introduction

Downward looking GPR has been considered a viable technology for land mine detection [1]. For GPR with the antenna positioned very close to the ground surface, the reflections from the ground surface, i. e., the GB, are very strong and can much dominate the weak returns from shallowly buried plastic mines. Hence, one of the key challenges of using GPRs for landmine detection is to remove the GB as completely as possible without altering the landmine return.

The literature suggests a number of clutter (whose dominant contributor is GB) reduction methods, such as parametric system identification [2, 3], wavelet packet decomposition [4], subspace techniques [5, 6], and simple mean subtraction [7]. However, many of these fail to detect shallowly buried landmines, mostly because of the statistical nature of the clutter, e.g., the ground surface is not perfectly flat nor even relatively smooth. The other problem is that many of the methods use reference signals to estimate the signature of a landmine. These reference signals are used to remove signals based on how they relate to the reference. This will lead to improper target signal cancellation when the reference signals are selected inadequately. For subspace techniques, the GPR signals are decomposed into clutter and landmine signals by selecting principal components (PCs) and independent components (ICs) reasonably. These methods can be more robust and lead to the best results for GB removal. But automatic selection strategy for PCs and ICs is the key problem, and reduction for computational load is an attractive work.

In this paper, we present an NHD-based modified ICA algorithm with automatic selection strategy for PCs and ICs. To reduce the computational load, we also apply NHD to BISS to determine the number of components to be extracted.

2. Data Description

Consider a stepped frequency GPR system moving in the along-track direction. Let $x_p(\omega_n)$ denote the data collected at the *pth* scan for the *nth* stepped frequency, $b_p(\omega_n)$ denote ground bounce in $x_p(\omega_n)$, where $\mathbf{x}_p = [x_p(\omega_1)x_p(\omega_2)\cdots x_p(\omega_N)]^T$ is called A-scan data vector (for impulse GPR radar, this is the data vector expressed in the frequency domain), $\mathbf{b}_p = [b_p(\omega_1)b_p(\omega_2)\cdots b_p(\omega_N)]^T$ represents the ground bounce vector, and $\mathbf{X} = [x_1x_2\cdots x_p]$ represents the B-scan data matrix. As the mutual coupling of antennas can be removed by prior measurement or estimation, the received data vector at the *pth* scan can be simplified as

$$\mathbf{x}_p = \mathbf{r}_p + \mathbf{b}_p + \mathbf{e}_p \tag{1}$$

where \mathbf{r}_p denotes reflected signal form target, and \mathbf{e}_p denotes un-modeled noise. We also set up a sliding window for modified GLR-based HND [8], which is composed of a guard area of length N_1 and local area of length N_2 in the along-track direction[8,9].

3. Modified ICA Based Method

3.1. Temporal ICA

The temporal ICA is one of the subspace techniques to remove GPR GB. The received signal is considered as the linear mixture of the independent components (ICs) [5,6], and the GB is removed by reconstructing received signal with ICs corresponding to landmine target and target-like objects. ICA algorithm is processed in two steps.

The first step is the pre-processing, which includes data centralization $(\mathbf{x}_{m,j} = \mathbf{x}_j - (1/P) \sum_{i=1}^{P} \mathbf{x}_i, j = 1 \sim P)$

and whitening. The whitening is realized by PCA ($\mathbf{X}_m = [\mathbf{x}_{m,1} \cdots \mathbf{x}_{m,P}]$, $\mathbf{X}_1 = \mathbf{X}_m^T$, $\mathbf{Y} = \tilde{\mathbf{U}}^T \mathbf{X}_1$), where $\mathbf{y} = [y_1 y_2 \cdots y_{L_1}]^T$ is constructed by L_1 selected PCs, and \mathbf{U} is the projection matrix for \mathbf{X}_1 projected in a subspace spanned by eigenvectors of L_1 selected PCs. Then, we consider \mathbf{Y} as the input of ICA defined as

$$\mathbf{Y} = \mathbf{AS} = [\mathbf{a}_1 \mathbf{a}_2 \cdots \mathbf{a}_{L_1}] [s_1 s_2 \cdots s_{L_1}]^T = \sum_{i=1}^{L_1} a_i s_j^T$$
(2)

$$\mathbf{X}_1 = \tilde{\mathbf{U}}\mathbf{Y} = \tilde{\mathbf{U}}\mathbf{A}\mathbf{S} = \mathbf{W}\mathbf{S} = [\mathbf{w}_1\mathbf{w}_2\cdots\mathbf{w}_L]\mathbf{S} = \sum_{i=1}^{r} \mathbf{s}_i\mathbf{w}_i^T$$
(3)

where **W** is called matrix of eigenimages, and **S** is the ICs. After selecting K target and target-like ICs $s_{o,i} = s_j (i = \mathbf{l} \sim K, j = \mathbf{l} \sim L_1)$ and correspondent eigenimages $\mathbf{w}_{o,i} = \mathbf{w}_j (i = \mathbf{l} \sim K)$, the GB removal output is $\hat{\mathbf{X}}_I = \sum_{i=1}^K \mathbf{s}_{o,i} \mathbf{w}_{o,i}^T$. The key problem for ICA is how to select PCs for PCA and ICs for signal reconstruction.

3.2. PCs and ICs Selection Strategy

The PCs and ICs reflect the time-domain information and the eigenimages can be considered as the spatial steering vectors correspondent to them. The result of the NHD describes the buried position of the targets and target-like objects. So we can select PCs and ICs automatically according to the consistency between the eigenimage and the output of the modified GLR-based HND [8].

4. BISS Based Method

4.1. BISS

The ICA will be very computational demanding if the number of source signals is large [10–12]. After PCA, $L_1 \ll P$, but $L_1 \gg M$ (the number of targets and target-like objects). Obviously, ICA extracts much more signal the sources than that need by signal reconstructing. Fortunately, BISS overcomes somewhat this difficulty. The spirit of the BISS is to recover only a small subset of sources from a large number of sensor signals. For GB removal, if the number of targets and target-like objects is prior known, source signals not more than M are needed to be extracted.

Like the ICA, the first step of BISS is pre-processing. Then, the small subset of targets signals \mathbf{S}_t is extracted from \mathbf{Y} as $\mathbf{S}_t = \mathbf{H}\mathbf{Y}$ (4)

where \mathbf{H} is the separating matrix, and the GB removal output is

$$\hat{\mathbf{X}}_{\mathbf{BISS}} = \mathbf{W}_{\mathbf{t}} \mathbf{S}_{\mathbf{t}} = \tilde{\mathbf{U}} \mathbf{H}^{\mathbf{T}} \mathbf{S}_{\mathbf{t}}$$
(5)

The presented BISS algorithm is gradient-based algorithm that optimizes three different criteria: Maximum Likelihood (ML), Minimum Entropy (ME) and Cumulants based index. The algorithm based ML can be explicitly computed only when the sources densities are known. It needs to approximate the activation function for ME, although it is not necessary to know the source densities. The most robust approach is the cumulant-based algorithm, since it can be realized without approximations and not dependent on the density of sources [11].

4.2. Determination of M and Cumulant Order

There are two important parameters to be conformed for cumulant-based algorithm [13]: the number of extracted signals M and the order of the cumulant. Since the location of target, target-like object, and the homogeneity of GB can be detected by modified GLR based NHD, the value of M can be prior determined, and the order of cumulant should be chosen according to the statistical nature of GPR data.

5. Experiment Result

The GPR data is obtaied from Vrije Universiteit Brussel (VUB) [14]. The experiment was performed in wet clay mixed with small rocks. An area of x = 50 cm by y = 196 cm was scanned with a scanning step of 1 cm in each direction. There were irregularities with a maximum of 10 cm between the highest and the lowest

point. The antenna head was placed at 5 cm above the highest point, and the scan was done horizontally. In the following examples, the target is a plastic anti-personal mine (PMA-1·PMA-3), big stone and curving U shape copper strip, the distribution of buried object is shown in Figure 1.



Figure 1: Distribution of buried objects.

Figure 2 shows the output of the modified GLR base NHD. Using this result, the number and position of the targets (and target-like objects) can be determined.



Figure 3: Comparison of ground bounce removal performances. (a) raw data, (b) ICA, (c) NHD based ICA, (d)BISS based 3rd order cumluant, (e) BISS based 4th order cumluant, (f) BISS based 3rd and 4th order cumluant, (g) BISS based 3rd, 4th, 5th, and 6th order cumluant.

The performance of the improved ICA and BISS based method are showed in the Figure 3. Figure 3(a) is the original received data of the GPR. It can be seen that the targets are obscured by the ground bounce. Figure 3(b) and Figure 3(c) show the results of ICA and NHD based ICA, respectively. Figure 3(d) \sim (g) shows the results of BISS based cumluants with different orders. It can be seen that there are almost no difference among these four results, so the third order cumluant is enough.

5. Conclusion

In this paper, we present NHD-based ICs selection method. ICA can be realized automatically with this selection strategy. We also apply the BISS in the GB removal combined with NHD to determine the number of extracted signal sources. The experimental results show that these two methods have excellent performance in GB removal, and the BISS based method reduces the computational load greatly.

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A High-order Finite Element Method for Electrical Impedance Tomography

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Abstract—Electrical impedance tomography (EIT) is a non-invasive imaging technique where a conductivity distribution in a domain is reconstructed from boundary voltage measurements. The voltage data are generated by injecting currents into the domain. This is an ill-conditioned non-linear inverse problem. Small measurement or forward modeling errors can lead to unbounded fluctuations in the reconstructions. A forward model describes the dependence of the noiseless voltage data on the conductivity distribution. The present work focuses on applying the high-order finite element method (p-FEM) for forward modeling. In the traditional version of the finite element method (h-FEM), the polynomial degree of the elements. In the p-version, in contrast, the polynomial degree is increased and the mesh size is kept constant. In many applications of the finite element method the performance of the p-version is better than that of the h-version. In this work, it is proposed that the p-version provides more efficient tool for EIT forward modeling. Numerical results are presented.

1. Introduction

The electrical impedance tomography (EIT) problem is to reconstruct an unknown conductivity distribution σ in an object Ω from a set of noisy voltage measurements performed on the boundary $\partial\Omega$ This problem was first introduced in 1980 by Calderón [1]. At the present, EIT has numerous applications. These include detection of tumors from breast tissue [5], measuring brain function [8], imaging of fluid flows in process pipelines [10], and non-destructive testing of materials [13]. For a review on EIT, see Cheney *et al.*, [2].

In the present version of electrical impedance tomography, a current pattern $I = (I_1, I_2, \ldots, I_L)$ is injected into a two dimensional domain Ω through a set of contact electrodes e_1, e_2, \ldots, e_L placed on the boundary $\partial \Omega$. The injected currents induce a potential field u in the domain and a electrode voltages $U = (U_1, U_2, \ldots, U_L)$. The measurement data are gathered by injecting a set of linearly independent current patterns and measuring the corresponding electrode voltages. The conductivity distribution in Ω is to be reconstructed from these voltage measurements. This is a non-linear ill-conditioned inverse problem: small errors in the measurements or in the forward modeling can produce large errors in the reconstructions.

The focus of this paper is in efficient forward modeling. A forward model describes the dependence of the noiseless voltage data on the conductivity distribution. The complete electrode model by Somersalo et al., [9] and its simulations through the traditional finite element method (h-FEM) and the high-order finite element method (p-FEM) are considered. According to the complete electrode model, the potential distribution in the domain and the voltages on the electrodes can be determined by solving an elliptic boundary value problem. Finite element simulation of this forward model has been described by Vauhkonen [12]. In the h-version of FEM, the polynomial order p of the element shape functions is relatively low and the discretization error is reduced by decreasing the element size h. In the p-version, in contrast, the polynomial order is increased and the mesh size is kept constant. Processes where either the mesh is refined or the polynomial degree is increased are called h- and p-extensions, respectively. Both extension processes increase the dimension of the finite element space which is denoted by N. Combinations of h- and p-extensions are called hp-extensions (hp-FEM). Descriptions of h-, p- and hp-versions of FEM are given e. g., in a book by Szabo and Babuska [11].

This work presents numerical results on performances of h- and p-extensions in finite element simulation of the complete electrode model. The motivation for this study is that the solution of the complete electrode model equations can be very smooth in the interior part of Ω and that in finite element computations, it is typical that p-extensions are very efficient in problems with smooth solutions. For example, when the Poisson equation $\Delta u = f$ in a two-dimensional domain Ω with zero boundary conditions on $\partial\Omega$ has a smooth solution and uniform mesh refinement is used, the finite element solution u_h satisfies the inequality $||u-u_h||_{H^1(\Omega)} \leq Ch_p$, where h is the mesh size, p is the polynomial degree, C is some constant, and $H^1(\Omega)$ denotes the corresponding Sobolev space norm. Since in two dimensions the dimension of the finite element space N grows at the rate $O(p^2/h^2)$, one can deduce from the inequality, that as a function of N the error $||u-u_h||_{H^1(\Omega)}$ cannot converge slower in *p*-extensions than in *h*-extensions. For detailed description on h-, p- and hp-convergence, see Gui and Babuska [4].

2. Finite Element Simulation of the Complete Electrode Model

In the complete electrode model, the effective contact impedance between the electrode e_l and the boundary is characterized by the number $z_{\ell} > 0$. The electrode voltages U induced by the current pattern I can be found by solving the elliptic boundary value problem described by the equation

$$\nabla \cdot (\sigma \nabla u) = 0 \tag{1}$$

in the domain Ω , by the boundary conditions

$$\sigma \frac{\partial u}{\partial n}\Big|_{\partial\Omega \setminus \cup e_{\ell}} = 0, \quad \int_{e_{\ell}} \sigma \frac{\partial u}{\partial n} \, dS = I_{\ell} \quad \text{and} \quad \left(u + z_{\ell} \sigma \frac{\partial u}{\partial n}\right)\Big|_{e_{\ell}} = U_{\ell}, \qquad \ell = 1, \, 2, \, \dots, L, \tag{2}$$

on $\partial\Omega$ and by Kirchoff's current and voltage laws $\sum_{\ell=1}^{L} I_{\ell} = 0$ and $\sum_{\ell=1}^{L} U_{\ell} = 0$. According to Somersalo *et al.*, [9], with certain assumptions made on the domain and on the conductivity distribution, there exists a unique pair $u \in H^1(\Omega)$ and $U \in \mathbb{R}^L$ that satisfies the weak formulation of this problem. The finite element solution of these equations is the pair

$$u_{FE} = \sum_{i=1}^{N} \alpha_i \varphi_i$$
 and $U_{FE} = \sum_{i=1}^{L-1} \beta_i (e_1 - e_{i+1}),$ (3)

where $\varphi_1, \varphi_2, \ldots, \varphi_N$ are the shape functions of the finite element space and e_1, e_2, \ldots, e_L are the standard basis vectors of \mathbb{R}^L . The coefficients $\alpha_1, \alpha_2, \ldots, \alpha_N$ and $\beta_1, \beta_2, \ldots, \beta_N$ can be found by solving the linear system of equations Ax = b. The entries of the vectors x and b are given by $x_i = \alpha_i$ and $b_i = 0$ if $i \leq N$, otherwise $x_i = \beta_{i-N}$ and $b_i = (e_1 - e_{i+1-N})^T I$. The system matrix entries are given by

$$A_{i,j} = \begin{cases} \int_{\Omega} \sigma \nabla \varphi_i \cdot \nabla \varphi_j \, dx dy + \sum_{\ell=1}^{L} \frac{1}{z_\ell} \int_{e_\ell} \varphi_i \varphi_j \, ds, & \text{if } 1 \le N \quad \text{and} \quad j \le N, \\ -\frac{1}{z_1} \int_{e_1} \varphi_i \, ds + \frac{1}{z_{j+1-N}} \int_{e_{j+1-N}} \varphi_i \, ds, & \text{if } i \le N \quad \text{and} \quad j > N, \\ \frac{1}{z_1} \int_{e_1} ds + \frac{\delta_{i,j}}{z_{j+1-N}} \int_{e_{j+1-N}} ds, & \text{if } i > N \quad \text{and} \quad j > N. \end{cases}$$
(4)

where $\delta_{i,j}$ is the Kronecker delta.

3. Hierarchic Shape Functions for *p*-extensions

In the standard *p*-version of the finite element method, the shape functions used in *p*-extensions are hierarchic. In this context, the term hierarchic means that the set of shape functions of polynomial order p is in the set of shape functions of order p + 1, and the number of shape functions which do not vanish at the vertices and the sides of the elements is minimal. Hierarchic shape functions are constructed by using Legendre polynomials

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n, \qquad n = 0, 1, \dots$$
(5)

Due to the orthogonality properties of these polynomials, hierarchic shape functions are well-suited for computer implementation and have very favorable properties from the point of view of numerical stability [11].

In the one-dimensional case, the standard element is the interval [-1,1]. For this element, the onedimensional hierarchic shape functions of polynomial order p are defined as

$$N_1(\xi) = \frac{1-\xi}{2}, \quad N_2(\xi) = \frac{1+\xi}{2}, \quad N_n(\xi) = \phi_{n-1}(\xi), \qquad n = 3, 4, \dots, p+1,$$
(6)

where ϕ_n is defined as $\phi_n(\xi) = \sqrt{n - 1/2} \int_{-1}^{\xi} P_{n-1}(\xi) dt$. These are organized to two categories. The first one is formed by the polynomials N_1 and N_2 , that are called the nodal shape functions, the external shape functions, or the vertex modes. The higher order polynomials $N_3, N_4, \ldots, N_{p+1}$ form the second category. These vanish at the endpoints of the interval [-1, 1] and they are called the bubble functions, the internal shape functions, or the internal modes.

The two-dimensional quadrilateral standard element is the square $[-1, 1] \times [-1, 1]$. The corresponding twodimensional hierarchical shape functions of polynomial order p are products of one-dimensional shape functions.

$$N_{n,m}(\xi,\eta) = \frac{1}{4}(1+(-1)^n\xi)(1+(-1)^m\eta), \qquad n = 1,2, \qquad m = 1,2, \qquad m = 1,2, \qquad N_{n,m}^{(0)}(\xi,\eta) = \phi_n(\xi)\phi_m(\eta), \qquad n = 2,3,\dots,p, \qquad m = 2,3,\dots,p, \qquad N_n^{(1)}(\xi,\eta) = \frac{1}{2}(1-\eta)\phi_n(\xi), \qquad n = 2,3,\dots,p, \qquad n = 2,3,\dots,p, \qquad N_n^{(2)}(\xi,\eta) = \frac{1}{2}(1-\xi)\phi_n(\eta), \qquad n = 2,3,\dots,p.$$

$$(7)$$

These are organized to three categories: vertex modes $N_{n,m}$, internal modes $N_{n,m}^{(0)}$, and side modes $N_n^{(1)}$, $N_n^{(2)}$. In this work, only quadrilateral elements are used. Construction of hierarchical shape functions for triangular elements has been described e.g., in [11].



Figure 1: The square shaped domain, the locations of the 16 electrodes, and the coarsest mesh $(h_0 = 1/9)$ used in the computations.

4. Numerical Experiments

Numerical experiments were performed concerning performances of h- and p-extensions in FEM simulation of the complete electrode model. In these computations, the domain Ω was the unit square $[0, 1] \times [0, 1]$ and the conductivity distribution σ in Ω was identically one. Sixteen electrodes with equal contact impedances $z_1 = z_2 = \ldots = z_\ell = 1$ were placed evenly on the boundary (Fig. 1). All the contact impedances were assumed to be equal to one. The generated voltage data constisted of L-1 electrode voltage vectors $U^{(1)}, U^{(2)}, \ldots, U^{(L-1)}$ induced by pair drive [7] current patterns $I^{(1)}, I^{(2)}, \ldots, I^{(L-1)}$ such that $I_k^{(k)} = 1$ and $I_{k+1}^{(k)} = -1$ and all other entries are zero. In each of these current patterns, the two current injecting electrodes were located next to each other. The finite element method was used both in data generation and simulation. Each finite element mesh used in these computations consisted of equal-sized square shaped elements as illustrated in Fig. 1. In data simulation, bilinear and hierarchic shape functions were used in h- and p-extensions, respectively. One hextension process and three p-extension processes were executed (Table 1). In these processes, elements of sizes $h = h_0, 2^{-1}h_0, \ldots, 2^{-7}h_0$ with $h_0 = 1/9$ and polynomial orders $p = 1, 2, \ldots, 8$ were employed. The growth of the dimension of the finite element space is reported in Table 1. In data generation, the size and the polynomial order of the elements were $h = 2^{-3}h_0$ and p = 8. A vector containing all the generated data is denoted by U_{EX} and a vector containing the simulated electrode voltages is denoted by \mathbf{U}_{FE} . Accuracy of the simulation is measured in ℓ^2 -norm by the relative error

$$RE = ||U_{EX} - U_{FE}||_2 / ||U_{EX}||_2.$$
(8)

5. Results and Discussion

Figure 2 illustrates the convergence of the relative error (8) in the h- and p-extension processes. The relative error is plotted against the dimension of the finite element space on $\log_{10}-\log_{10}$ scale. The results show that p-convergence rate is faster than the rate of h-convergence.

Table 1: The executed *h*- and *p*-extension processes: *h*- and *p*-values and finite element space dimensions. In data generation, the size and the polynomial order of the elements were $h = 2^{-3}h_0$ and p = 8 (down right corner).

index	type	<i>h</i> -values	<i>p</i> -values	finite element space dimensions							
(a)	h	$h_0, 2^{-1}h_0, \dots, 2^{-7}h_0$	1	100	361	784	1369	2116	3025	4096	5329
(b)	p	h_0	$1, 2, \ldots, 8$	100	280	460	721	1063	1486	1990	2575
(c)	p	$2^{-1}h_0$	$1, 2, \ldots, 8$	361	1045	1729	2737	4069	5725	7705	10009
(d)	p	$2^{-3}h_0$	$1, 2, \ldots, 7$	1369	4033	6697	10657	15913	22465	30313	



Figure 2: The relative error (8) in the executed h- and p-extension processes (a), (b), (c) and (d) plotted against the dimension of the finite element space on \log_{10} - \log_{10} scale. The straight line represents the h-extension process (a). The three curved lines from left to right represent the p-extension processes (b), (c) and (d) respectively. The dashed lines show the h-convergence rate in the cases where p = 2, 3, 4, 5, 6 or 7.

In finite element computations, p-extensions are often motivated by the fact that the solution is smooth whereas h-extensions are favorable in the case of non-smooth solutions [11]. According to Evans [3], the interior potential distribution $u \in H^1(\Omega)$ determined by the complete electrode model is smooth provided that the conductivity distribution is smooth. However, it is important to point out that the potential distribution is not smooth in the vicinity of the boundary, since according to the boundary conditions (2) the normal derivative $\partial u/\partial n$ is discontinuous on $\partial \Omega$. Consequently, it is possible that near the boundary the performance of hextensions can be better than that of p-extensions. It is also important to note that electrical impedance tomography involves a variety of applications, e. g., detection of tumors, where the conductivity is a non-smooth or a discontinuous function. A local discontinuity in the conductivity distribution, e. g., a tumor, causes local non-smoothness of the interior potential distribution in the vicinity of the discontinuity [3]. This means that the structure of the conductivity can affect the performance of h- and p-extensions in different parts of the domain. In future work it would be interesting to explore performances of different hp-extension processes with different conductivity distributions. For example, whether a priori information about the conductivity distribution can be used when designing hp-extensions could be an issue in electrical impedance tomography.

From the computational point of view, one important difference in h- and p-extensions is that in p-extensions a lot more computational effort is spent on numerical integration when constructing the system matrix (4) due to the high polynomial order of the shape functions. Electrical impedance tomography involves reconstruction methods, e.g., Markov chain Monte Carlo sampling [6], where efficient forward modeling in terms of computation time is essential, because the forward model equations have to be solved numerous times during the reconstruction process. Another interesting future consideration would be whether there are computationally tractable ways to obtain system matrices needed in EIT reconstruction, e.g., whether *a priori* knowledge about the conductivity distribution can be used when constructing a system matrix.

6. Conclusion

In this work, the *p*-version of the finite element method was applied to simulation of the complete electrode model. The motivation for this study was that the solution of the complete electrode model equations can be smooth in the interior domain and that it is typical that the *p*-version is very efficient in problems with smooth solutions. It was shown numerically by using the unit square that the performance of the *p*-version is better than that of the *h*-version when uniform mesh refinement is used. Since the solution of the complete electrode model equations is non-smooth in the vicinity of the boundary, an important topic for the future work is to explore the performance of the *h*-version of FEM. From the computational point of view, one characteristic difference in *h*- and *p*-versions of FEM is that in *p*-version a lot more computational effort is spent on construction of a system matrix. Another important future consideration is to find computationally tractable ways to obtain system matrices needed in EIT reconstruction. It is also an important issue whether *a priori* knowledge about the conductivity distribution can be used when designing a *p*-FEM implementation to be used in EIT reconstruction.

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Arbitrary Lagrangian Eulerian Electromechanics in 3D

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Abstract—We present results from an effort to couple the equations of electromagnetic diffusion with the equations of arbitrary Lagrangian-Eulerian (ALE) hydrodynamics. The electromagnetic diffusion equations are discretized using a novel mixed finite element method coupled with a generalized Crank-Nicholson time differencing scheme. At each discrete time step, electromagnetic force and heat terms are calculated and coupled to the hydrodynamic equations in an operator split approach. We present preliminary results from a fully coupled electromechanical simulation as well as results concerning advection techniques for electromagnetic quantities.

1. Introduction

We are interested in the simulation of electromechanical devices and magnetohydrodynamic events in three dimensions. Our primary goal is a numerical method that solves, in a self-consistent manner, the equations of electromagnetics (primarily statics and diffusion), heat transfer (primarily conduction), and non-linear mechanics (elastic-plastic deformation, and contact with friction). In this paper, we focus on the numerical discretization of electromagnetic diffusion in an arbitrary Lagrangian-Eulerian (ALE) fashion for the purposes of computing $\vec{J} \times \vec{B}$ forces for mechanical (or hydrodynamic) calculations and $\vec{J} \cdot \vec{E}$ Joule heating terms for thermal calculations.

The equations of electromagnetic diffusion can be derived from the full wave Maxwell equations by making the good conductor approximation (i.e., ignoring displacement current), which is standard practice in magnetohydrodynamic (MHD) formulations. For conducting materials moving with a velocity \vec{v} with respect to a fixed Eulerian (or laboratory) frame, we can derive the so called dynamo equation (also known as the hydromagnetic equation) in terms of magnetic flux density

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left(\frac{1}{\sigma} \vec{\nabla} \times \frac{1}{\mu} \vec{B}\right) + \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right) \tag{1}$$

In the Eulerian description the velocity \vec{v} is a function of time t and position \vec{x} . In the Lagrangian (or material) description (which we will designate with a "prime" symbol), the flow is described by following the position $\vec{x}(\vec{x}',t)$ of the material point that started at position \vec{x}' at t = 0. In functional form, we have

$$\vec{x}' = \vec{x}'(\vec{x}, t); \quad \vec{x} = \vec{x}(\vec{x}', t)$$

To convert between the two representations, we define the Jacobian matrix as

$$J_{i,j} = \frac{\partial x'_j}{\partial x_i} \tag{2}$$

As shown in [1], the following quantities are invariant with respect to the Lagrangian-Eulerian representations

It is well known that differential arc length and surface area elements transform according to

$$d\vec{x}' = J^T d\vec{x} \tag{4}$$

$$d\vec{a}' = |J|J^{-1}d\vec{a} \tag{5}$$

As a consequence, the electric field intensities and magnetic flux densities must transform inversely to maintain the invariance property of (3)

$$\vec{E}' = J^{-1}(\vec{E} + \vec{v} \times \vec{B}) \tag{6}$$

$$\vec{B}' = \frac{1}{|J|} J^T \vec{B} \tag{7}$$

The dynamo equation in the Lagrangian frame is therefore

$$\frac{d\vec{B}'}{dt} = -\vec{\nabla}' \times \left(\frac{1}{\sigma}\vec{\nabla}' \times \frac{1}{\mu}\vec{B}'\right) \tag{8}$$

In a typical ALE hydrodynamic calculation, an *operator split* method is employed where all calculations are performed on a Lagrangian mesh (i.e., a mesh that moves with the materials). When the Lagrange motion of the mesh causes significant mesh distortion, that distortion is corrected with an equipotential relaxation of the mesh, followed by a 2nd order monotonic remap of mesh quantities. This remap is equivalent to an advection of material through the mesh. In our proposed ALE formulation of MHD, we will employ an operator-split method with three distinct steps:

- *Electromagnetic Diffusion*–Solve the dynamo equation in the Lagrangian frame at one discrete time step for fixed materials.
- Lagrangian Motion–Move mesh nodes according to $\vec{J'} \times \vec{B'}$ forces assuming a $\frac{d\vec{B'}}{dt} = 0$ "frozen flux" condition.
- *Eulerian Advection*–Only required if mesh is relaxed, advect (or transport) magnetic (vector potential) flux quantities to new mesh.

Note that the second step (effectively "dragging" the electromagnetic quantities along with the mesh during Lagrangian motion) will only work if our discretization of the electromagnetic quantities satisfies the invariance relation of (3) (see also [2]). In the Eulerian advection step of the calculation, the computed electromagnetic degrees of freedom must be "remapped" or "advected" in a way which preserves a discrete divergence-free property of the magnetic flux density with minimal magnetic energy loss.

2. Numerical Formulation

The divergence-free (or solenoidal) nature of the magnetic flux density, $\vec{\nabla}' \cdot \vec{B}' = 0$, implies that $\vec{B}' = \vec{\nabla}' \times \vec{A}'$ where \vec{A}' is a magnetic vector potential. This in turn implies that the electric field in the Lagrangian frame is given by $\vec{E}' = -\vec{\nabla}' \phi' - \frac{\partial}{\partial t} \vec{A}'$, where ϕ' is an electric scalar potential. Using the gauge condition $\vec{\nabla}' \cdot \sigma \vec{A}' = 0$, we can reformulate the dynamo equation (8) in terms of potentials as

$$\vec{\nabla}' \cdot \sigma \vec{\nabla}' \phi' = 0 \tag{9}$$

$$\sigma \frac{d\vec{A}'}{dt} = -\vec{\nabla}' \times \frac{1}{\mu} \vec{\nabla}' \times \vec{A}' - \sigma \vec{\nabla}' \phi'$$
(10)

Note that this formulation has an additional elliptic PDE (9) to solve for the scalar potential. A key advantage of this formulation is that voltage, which is often the only known quantity for electromechanical engineering applications, appears explicitly in the equations as an essential boundary condition for the elliptic solution of (9). To compute force and heat terms, we define the *secondary variables* in terms of the potentials as

$$\vec{B}' = \vec{\nabla}' \times \vec{A}' \tag{11}$$

$$\vec{J'} = \sigma \vec{E'} = -\vec{\nabla}' \phi' - \frac{d}{dt} \vec{A'}$$
(12)

Finally, there are divergence constraints on both the primary and secondary fields, namely

$$\vec{\nabla}' \cdot \sigma \vec{A}' = 0 \tag{13}$$

$$\vec{\nabla}' \cdot \vec{B}' = 0 \tag{14}$$

To discretize the potential formulation in the Lagrangian frame, we apply the mixed finite element methods (FEM) of [3] which are based on the properties of differential forms and have been shown to preserve discrete divergence-free properties and to maintain accuracy in secondary variables (e.g., \vec{J} and \vec{B}) even when computed from potentials. Most importantly, the discrete vector fields transform identically to (6) and (7), thereby preserving the invariance property of (3).

In our proposed ALE formulation the scalar potential will be discretized on mesh nodes (i.e., a discrete 0form field), the vector potential will be discretized on mesh edges (i.e., a discrete 1-form field) and the secondary variables \vec{B} and \vec{J} will be discretized on mesh faces (i.e., discrete 2-form fields) as follows

$$\phi' \approx \sum_{i=1}^{n} v_i W_i^0 \tag{15}$$

$$\vec{A'} \approx \sum_{i=1}^{n} a_i \vec{W}_i^1 \tag{16}$$

$$\vec{B}' \approx \sum_{i=1}^{n} b_i \vec{W}_i^2 \tag{17}$$

$$\vec{J'} \approx \sum_{i=1}^{n} j_i \vec{W}_i^2 \tag{18}$$

where W^l denotes a discrete *l*-form basis function. In [3], various mass, stiffness, derivative and discrete Hodge matrices are defined. Given these matrices, the fully discrete form of the potential diffusion equation is given in [3] by applying a Generalized Crank-Nicholson method to obtain

$$\mathbf{S}^0 \mathbf{v}_{n+\alpha} = \mathbf{f}_{n+\alpha}^0 \tag{19}$$

$$(\mathbf{M}^{1}(\sigma) + \alpha \Delta t \mathbf{S}^{1}(\mu^{-1}))\mathbf{a}_{n+1} = (\mathbf{M}^{1}(\sigma) - (1-\alpha)\Delta t \ \mathbf{S}^{1}(\mu^{-1}))a_{n} - \Delta t \mathbf{D}^{01}\mathbf{v}_{n+\alpha}$$
(20)

where $\alpha \in [0,1]$ is weighting parameter which determines the type of integration such that

 $\alpha = \begin{cases} 0 & \text{Explicit, 1st Order Accurate Forward Euler} \\ 1/2 & \text{Implicit, 2nd Order Accurate Crank Nicholson} \\ 1 & \text{Implicit, 1st Order Accurate Backward Euler} \end{cases}$

Once the values for the primary potentials have been solved for, the discrete secondary fields can be computed as

$$\mathbf{e}_{n+\alpha} = -\mathbf{K}^{01}\mathbf{v}_{n+\alpha} - 1/\Delta t(\mathbf{a}_{n+1} - \mathbf{a}_n)$$
(21)

$$\mathbf{b}_{n+1} = \mathbf{K}^{12} \mathbf{a}_{n+1} \tag{22}$$

$$\mathbf{M}^{2}(\sigma^{-1})\mathbf{j}_{n+\alpha} = \mathbf{H}^{12}\mathbf{e}_{n+\alpha}$$
(23)

These terms are used to compute $\vec{J'} \times \vec{B'}$ forces which will accelerate the mesh nodes during the Lagrangian motion step. The discrete divergence constraints are given by

$$(\mathbf{D}^{01}(\sigma))^T \mathbf{a} = 0 \tag{24}$$

$$(\mathbf{D}^{01}(\sigma))^T \mathbf{e} = 0 \tag{25}$$

$$\mathbf{K}^{23}\mathbf{b} = 0 \tag{26}$$

and as shown in [3], these constraints are implicitly satisfied for all time, assuming the initial conditions and the source terms are divergence free.

To demonstrate a fully coupled Lagrangian calculation, we consider a numerical experiment in which a 5 KV capacitor bank is discharged into a can shaped aluminum structure (see Fig. 1). The voltage through the can (effectively an inductive and resistive load) is computed via a simple SPICE model. The resulting voltage vs. time profile is then used as an essential boundary condition for the discrete scalar potential solve of (19) which

drives the problem. An essential boundary condition of the form $\hat{n} \times \vec{A'} = 0$ is applied to the front side of the mesh while the remainder of the surface is subject to the natural boundary condition $\hat{n} \times \frac{1}{\mu} \vec{\nabla}' \times A' = 0$. A peak current of roughly 0.8 MA is generated in the can, creating a $\vec{J'} \times \vec{B'}$ force which causes the can to initially compresses (or implode). However, the force is not strong enough to cause the aluminum can to yield, and so the can effectively rings over time in an elastic response as shown in Figs. 1 and 2.



Figure 1: Snapshot of the fully coupled electromechanical simulation. In this image the aluminum can is elastically expanding after initially being compressed. The displacement has been exaggerated by a factor of 300 for visual clarity.



Figure 2: Measured pressure response in the aluminum can due to an electromagnetic force.

3. Constrained Transport Methods on Unstructured Grids

During the optional Eulerian advection phase of our operator split method, the computed electromagnetic values must be remapped (or advected). Remapping refers to the process of updating the representation of the field given a new grid. We consider only new grids which are "nearby" in the sense that only small perturbations of the grid are allowed (i.e., the mesh nodes should not travel farther than one mesh element in any one time step). This is known as the continuous remap approximation (CRA).

We propose to use the so called *constrained transport* method originally developed by [4] and later expanded by [5]. Suppose we have calculated the magnetic flux density $\vec{B'}$ in a Lagrangian time step via (22), we then have a local element representation of $\vec{B'}$

$$\vec{B}^{old} \approx \sum_{i=1}^{n} b_i^{old} \vec{W}_i^{2,old} \tag{27}$$

The degrees of freedom (DOF) b_i^{old} in this expansion carry the units of magnetic *flux*. For the special case of lowest order (p = 1) basis functions (i.e., six DOF per element), this implies that we know the magnetic flux through every face in the Lagrangian mesh (or the "old" mesh). Now in a standard ALE step, the old mesh is relaxed under the CRA to a *new* mesh. Therefore, our goal is to compute *new* values of the magnetic flux b_i^{new} which will allow us to represent the magnetic flux density on the new mesh. For the special case of lowest order (p = 1) basis functions, the discrete divergence free property is simply a statement that the 6 fluxes in the face sum to zero. The goal of *constrained* transport is to preserve this property on the new mesh.

For unstructured hexahedral grids, we can update the magnetic flux (or "vector potential flux" $\vec{A} \cdot d\vec{x}$) by effectively solving Faraday's law for a moving conductor (equivalent to magnetic transport under the "frozen-flux" condition)

$$\Phi^{new} \approx \Phi^{old} - \oint_C (\vec{u} \times \vec{B}) \cdot d\vec{l}$$
⁽²⁸⁾

where \vec{u} is the mesh displacement. Our goal now is to apply (28) in an algorithmic fashion to update the fluxes on the faces of a new mesh. A schematic representation of this process is shown in Fig. 3. It is clear from

the depiction of Fig. 3 that we can approximate the flux through a new face given the flux through the old face and a "measurement" of the time rate of change of flux (an effective voltage) along the closed circuit path Cdepicted in green. For a given face in the new mesh, the algorithm of (28) can be used to update the edge flux contributions



Figure 3: Schematic diagram depicting the relationship between magnetic flux through "old" and "new" mesh faces and the most accurate location for "measuring" the update EMF.

 $\vec{A} \cdot dx$ for each edge in the face (thereby updating the vector potential) or the total magnetic flux $\vec{B} \cdot d\vec{a}$ through the face. By construction, the new flux values will sum to zero, provided the old fluxes do so as well. In order for this algorithm to work, \vec{B} must be evaluated at the displacement vector midpoints for the discrete path integral; however, this is problematic for faced based representations of \vec{B} , since they are by construction, discontinuous along element edges. To overcome this, a "smooth" \vec{B} field must be patch recovered using a continuous vector nodal approximation.

4. Conclusions

We have presented and discussed an operator split approach for solving the coupled equations of electromechanics and magnetohydrodynamics using the novel mixed finite element methods of [3] to discretize the equations of electromagnetic diffusion. We have presented preliminary results for a fully coupled Lagrangian calculation and have discussed methods for advecting magnetic flux for ALE calculations.

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Hybrid Numerical Simulation of Micro Electro Mechanical Systems

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Abstract—In this paper a hybrid numerical approach for the simulation of micro electro mechanical systems (MEMS) is presented. A simulation model that takes into account the mechanical and the electrical effects is developed. The model is applied to an electrostatic force microscope (EFM) and simulation results are presented.

1. Introduction

Although micro electro mechanical systems (MEMS) already exist for many applications, their calculation is still difficult since in order to obtain accurate results, ofter times multi scale aspects have to be included. Furthermore the coupled mechanical and electrical behavior has to be taken into account. In our work this is achieved by dividing the model into a mechanical and an electrical part. The interaction between them is shown in Fig. 1 and can conveniently be realized by using a staggered simulation approach. The electric forces are calculated by the electrical part and passed to the mechanical part which uses them as input for the calculation of the mechanical deflection. In order to apply this approach to a two dimensional model of an electrostatic force microscope (EFM) (Fig. 2) both parts have to be defined. Therefore the components and the principle of an EFM will be explained in the following. An EFM is used to scan surfaces holding an electric potential or a charge distribution [1–3]. During the scanning process the tip at the end of the cantilever is run over the sample. The forces acting on the cantilever and the tip are determined by the electrostatic field and calculated by the electrostatic part of the model. The mechanical behavior of the cantilever is modelled using a beam model while for the region near the tip the finite element method (FEM) is used. A more detailed description of the electrical part will be given in the following.



Figure 1: Mechanical and electrical part.

2. Formulation of the Problem

The energy-related functional in the electrostatic calculation domain Ω (Fig. 2) can be written as

$$W = \int_{\Omega} (\nabla u)^2 d\Omega \qquad u \in H^1_D(\Omega) : \{ u \in H^1 | u_{|\Gamma_D} = u_0 \}$$
(1)

where $u(a_1, a_2, \ldots, a_m, x, y)$ is an approximation of the potential u(x, y). It is well known that the solution of

$$\frac{\partial W}{\partial a_i} = 2 \int_{\Omega} \nabla u \frac{\partial \nabla u}{\partial a_i} d\Omega = 0 \tag{2}$$

yields an approximative solution for the Laplace equation in Ω . In order to solve (2) numerically we shall take a closer look at the requirements in the different parts of the calculation domain Ω . Since most of the interaction between probe and sample happens at the bottom of the tip, accurate calculation results are important in this region. Therefore a numerical method which is able to deal with the high field values near the tip is required. For this reason the method of fundamental solutions (MFS) is applied in region Ω_M (Fig. 3). At a larger distance from the tip (region Ω_F) lower field values are expected, but possible nonlinearities and charge distributions in the sample require a versatile numerical method such as the finite element method (FEM). Because of the large difference in size of tip and cantilever length, FEM cannot conveniently be applied in the whole rest of the



Figure 2: Electrostatic force microscope and calculation domain.



Figure 3: Divided calculation domain.

calculation domain. Therefore the boundary element method (BEM) that only requires a mesh on the boundary is used in region Ω_B .

The solutions in the circular region Ω_M of radius R can be expanded into

$$u(\rho,\phi) = V_0 + \sum_{i=1}^m c_i \left(\frac{\rho}{R}\right)^{\frac{i\pi}{\beta}} \sin(\frac{i\pi\phi}{\beta}), \quad (\rho,\phi) \in \Omega_M \tag{3}$$

where V_0 is the electric potential and β is the outer opening angle of the tip [4]. For the choice of R and the position and the number of coupling points, the overlapping area of Ω_M and Ω_F must be small. Furthermore it must be considered that (3) is a good approximation of the potential only near the tip.

In region Ω_F linear FEM

$$u(x,y) = \sum_{j=1}^{n} u_j \psi_j(x,y), \quad (x,y) \in \Omega_F$$
(4)

is applied [5]. The solutions of both regions [6] are coupled by

1

$$u_c = V_0 + \sum_{i=1}^m c_i \sin(\frac{i\pi\phi_c}{\beta}).$$
(5)

Replacing the coupling node potentials in (4) by (5) and using the resulting potential functions in (2) leads to $\begin{pmatrix} M & P^T \end{pmatrix} \begin{pmatrix} c & c \end{pmatrix} = \begin{pmatrix} b & c \end{pmatrix}$

$$\begin{pmatrix} M & B^T \\ B & F \end{pmatrix} \begin{pmatrix} c \\ u_F \end{pmatrix} = \begin{pmatrix} b_M \\ b_F \end{pmatrix}$$
(6)

where \mathbf{M} is the matrix resulting from the MFS that is defined by

$$M_{ij} = 2\sum_{k \in n_c} \sum_{m \in n_c} \sin\left(\frac{i\pi}{\beta}\phi_k\right) \sin\left(\frac{j\pi}{\beta}\phi_m\right) \int_{\Omega_F} \nabla(\psi_k) \nabla(\psi_m) d\Omega + \begin{cases} i\pi, & i=j\\ 0, & \text{otherwise,} \end{cases}$$
(7)

$$F_{ij} = 2 \int_{\Omega_F} \nabla \psi_i \nabla \psi_j d\Omega \tag{8}$$

is the FEM stiffness matrix,

$$B_{ij} = 2\sum_{k \in n_c} \sin\left(\frac{j\pi}{\beta}\right) \int_{\Omega_F} \nabla \psi_i \nabla \psi_k d\Omega$$
⁽⁹⁾

is the FEM-MFS coupling matrix,

$$b_{Mi} = -2V_0 \sum_{k \in n_c} \sum_{m \in n_c} \sin\left(\frac{i\pi}{\beta}\right) \int_{\Omega_F} \nabla \psi_k \nabla \psi_m d\Omega \tag{10}$$

is the MFS right hand side and

$$b_{Fi} = -2V_0 \sum_{k \in n_c} \int_{\Omega_F} \nabla \psi_i \nabla \psi_k d\Omega_F - 2 \sum_{j \in n_D} \phi_j \int_{\Omega_F} \nabla \psi_i \nabla \psi_j d\Omega$$
(11)

is the right hand side resulting from the variation of the FEM potentials. Here n_c stands for the coupling nodes and n_D are the Dirichlet boundary conditions. The matrix F can be written as

$$\mathbf{F} = \begin{pmatrix} F_{NN} & F_{CN}^T \\ F_{CN} & F_{CC} \end{pmatrix}$$
(12)

where F_{NN} includes only the interaction between the nodes inside the FEM domain, F_{CC} stands for the interaction inside the coupling interface while the interaction of coupling interface and FEM domain is described by F_{CN} .

On the FEM-BEM transmission interface $\Gamma_T = \Gamma_B \cap \Gamma_F$: $u_B = u_F$ and $\frac{\partial u_B}{\partial n} + \frac{\partial u_F}{\partial n} = 0$. Using the Gauss theorem on $\Omega_{FM} = \Omega_F \cup \Omega_M$ one obtains [5]

$$\int_{\Gamma_F} \frac{\partial u_{FM}}{\partial n} v \ d\Gamma = \int_{\Omega_{FM}} \operatorname{div}(\nabla u_{FM} \cdot v) \ d\Omega = \int_{\Omega_{FM}} \Delta u_{FM} \cdot v \ d\Omega + \int_{\Omega_{FM}} \nabla u_{FM} \cdot \nabla v \ d\Omega \tag{13}$$

i.e., for all $v \in H^1_{D,0}(\Omega_{FM}) := \{ v \in H^1(\Omega_{FM}) : v |_{\Gamma_D \cap \Gamma_F} = 0 \}$

$$a(u_{FM}, v) := \int_{\Omega_{FM}} \nabla u_{FM} \cdot \nabla v \ d\Omega = \int_{\Omega_{FM}} f \cdot v \ d\Omega + \int_{\Gamma_F} \frac{\partial u_{FM}}{\partial n} v \ d\Gamma =: (f, v)_{\Omega_{FM}} + \langle \frac{\partial u_{FM}}{\partial n}, v \rangle_{\Gamma_F}$$
(14)

where u_{FM} includes u_F and c. The representation formula of the Laplace equation for the solution of u_B inside Ω_B

$$u_B(x) = \int_{\Gamma_B} \{\frac{\partial}{\partial n(y)} G(x, y) u_B(y) - G(x, y) \frac{\partial u_B}{\partial n(y)} \} d\Gamma, \quad x \in \Omega_B$$
(15)

with the fundamental solution of the Laplacian given by

$$G(x,y) = -\frac{1}{2\pi} \log |x-y|.$$
 (16)

If one computes the Cauchy data [7] u_B and $\partial u_B/\partial n$ of $u_B(x)$, one will get two boundary integral equations on $\partial \Omega_B$,

$$V\frac{\partial u_B}{\partial n} = (I+K)u_B \tag{17}$$

$$Wu_B = (I - K')\frac{\partial u_B}{\partial n} \tag{18}$$

where the boundary integral operators are defined as

$$V\psi(x) := 2\int_{\Gamma_B} G(x,y)\psi(y)d\Gamma_y, \quad K\psi(x) := 2\int_{\Gamma_B} \frac{\partial}{\partial n_y} G(x,y)\psi(y)d\Gamma_y, \quad x \in \Gamma_B$$
(19)

$$K'\psi(x) := 2\frac{\partial}{\partial n_x} \int_{\Gamma_B} G(x, y)\psi(y)d\Gamma_y, \quad W\psi(x) := -2\frac{\partial}{\partial n_x} \int_{\Gamma_B} \frac{\partial}{\partial n_y} G(x, y)\psi(y)d\Gamma_y, \quad x \in \Gamma_B$$
(20)

where the single layer potential V and the hypersingular operator W are symmetric and the double layer potential K has the dual K' [8].

Using (18) one can eliminate $\partial u_B / \partial n$ with (17). This leads to

$$Wu_B = (I - K')\frac{\partial u_B}{\partial n} = 2\frac{\partial u_B}{\partial n} - (I + K')\frac{\partial u_B}{\partial n} = 2\frac{\partial u_B}{\partial n} - (I + K')V^{-1}(I + K)u_B$$
(21)

with the Poincaré-Steklov-Operator S applied to u_B

$$Su_B := (W + (I + K')V^{-1}(I + K))u_B = 2\frac{\partial u_B}{\partial n}$$
(22)

which can be used for symmetric coupling. In variational form for all $w \in \tilde{H}^{1/2} := \{ w \in H^{1/2}(\Gamma_B) : w |_{\Gamma_D \cap \Gamma_B} = 0 \}$ holds

$$\langle Su_B, w \rangle_{\Gamma_B} = 2 \langle \frac{\partial u_B}{\partial n}, w \rangle_{\Gamma_B}.$$
 (23)

With (14) and (23) one can obtain the variational formulation

$$2a(u_{FM}, v) + \langle Su_B, v \rangle_{\Gamma_T} = 2(f, v)_{\Omega_{FM}} + 2\langle t_0, v \rangle_{\Gamma_N \cap \Gamma_F}$$
(24)

$$\langle Su_B, w \rangle_{\Gamma_B \cap \Gamma_N} = 2 \langle t_0, w \rangle \tag{25}$$

for all $(w,v) \in \tilde{H}^{1/2} \times H^1_{D,0}(\Omega_F)$ with f being the charge distribution inside Ω_F and t_0 are the Neumann boundary conditions.

The Poincaré-Steklov-Operator S cannot discretize directly because the inverse single layer potential V cannot be descritized in the usual way. For this reason without Poincaré-Steklov-Operator the problem can be rewritten as saddle point formulation. The saddle point formulation of the problem for all $(w, v, \psi) \in \tilde{H}^{1/2} \times H^1_{D,0}(\Omega_{FM}) \times \tilde{H}^{-1/2}(\Gamma_B)$

$$2a(u_{FM},v) + \langle Wu_B,v\rangle_{\Gamma_T} + \langle (I+K')\varphi,v\rangle_{\Gamma_T} = 2(f,v)_{\Omega_{FM}} + 2\langle t_0,v\rangle_{\Gamma_N\cap\Gamma_F}$$
(26)

$$\langle Wu_B, w \rangle_{\Gamma_B \cap \Gamma_N} + \langle (I + K')\varphi, w \rangle_{\Gamma_B \cap \Gamma_N} = 2 \langle t_0, w \rangle_{\Gamma_B \cap \Gamma_N}$$
(27)

$$\langle (I+K)u_B,\psi\rangle_{\Gamma_B} - \langle V\varphi,\psi\rangle_{\Gamma_B} = 0$$
(28)

If the bases are introduced as span{ v_1, \ldots, v_F } = X_F , span{ w_1, \ldots, w_F } = X_B and span{ ψ_1, \ldots, ψ_F } = Y_B , the basis functions of X_F and X_B are supposed to be ordered such that

$$\operatorname{span}\{v_1,\ldots,v_F\}=X_F\cap H^1_{D,0}(\Omega_F)$$

$$\operatorname{span}\{w_1,\ldots,w_B\}=X_B\cap H^{1/2}(\Gamma_B).$$

If the coefficients of u_{FM} and u_B are denoted by u and the coefficients of φ are denoted by φ again then this system is equivalent to the original differential equation that can be used for descritization. This system corresponds to a matrix formulation which can be written as

$$\begin{pmatrix} M & B^{T} & 0 & 0 & 0 \\ B & F_{NN} & F_{NC} & 0 & 0 \\ 0 & F_{CN} & F_{CC} + W_{CC} & W_{CN} & (K^{T} + I)_{C} \\ 0 & 0 & W_{NC} & W_{NN} & (K^{T} + I)_{N} \\ 0 & 0 & (K+I)_{C} & (K+I)_{N} & -V \end{pmatrix} \begin{pmatrix} u_{m} \\ u_{F} \\ u_{T} \\ u_{B} \\ \varphi \end{pmatrix} = \begin{pmatrix} b_{m} \\ b_{F} \\ b_{F} \\ b_{B} \\ b_{\varphi} \end{pmatrix}$$
(29)

where the subscript C means contribution from the coupling nodes and N means contribution from the noncoupling nodes. Finally the blocks W, V, K + I, and $K^T + I$ provide the coupling between the two ansatz spaces X_F and X_B . Here u_m are the MFS coefficients, u_F and u_B are the nodal potentials inside the FE domain and on the boundary of the BE domain respectively, u_T are the nodal potentials on the FE-BE coupling interface and φ are the normal components of the electric field distribution on the boundary of the BE domain. The vector b includes the corresponding boundary conditions. As the matrix in (29) is not positive definite, a specific algorithm such as the MINRES algorithm is required for the solution.



Figure 4: ALE mesh deformation.

Since the scanning process of an EFM is dynamic, the FEM mesh in Ω_F has to be changed during the calculation which is achieved by using the arbitrary Lagrangian Eulerian method (ALE) [9]. The mesh is modeled as a massless elastic which is deformed by the changing position of the cantilever and the sample (Fig. 4).

The result of a typical simulation can be seen in Fig. 6 and Fig. 7. As expected a high value of the electric field occurs at the tip. Since the coupling condition of MFS and FEM only includes the potential values (5), the electric field is not continuous on the interface. This indicates that FEM simulation results near the tip can be improved by using the coupled FEM-MFS approach presented here. A smoother transition of the electric field can be obtained by using a combination of FEM and MFS ansatz functions in region Ω_M . Fig. 5 shows the simulated potential between tip and sample obtained by using FEM and the hybrid simulation approach (R = 1, 2).



Figure 5: Comparison FEM/Hybrid Simulation.



Figure 6: Simulated electrostatic potential.

Figure 7: Simulated electrostatic field.

3. Conclusion

A hybrid numerical approach for the simulation of micro electro mechanical systems (MEMS) has been presented and applied to an electrostatic force microscope. In order to fulfill the special requirements in the different simulation regions an approach that combines FEM BEM and MFS was used to calculate the electrostatic field. ALE was applied to fit the FEM mesh to the changing boundaries. The results show the expected field distribution.

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EM Field Induced in Inhomogeneous Dielectric Spheres by External Sources

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Abstract—The electromagnetic field induced in the interior of inhomogeneous dielectric bodies by external sources can be evaluated by solving the well-known electric field integrodifferential equation (EFIDE). For spheres with constant magnetic permeability μ , but variable dielectric constant $\varepsilon(r, \theta, \varphi)$ a direct, mainly analytical solution can be used even in case when the inhomogeneity in ε renders separation of variables inapplicable. This approach constitutes a generalization of the hybrid (analytical-numerical) scalar method developed by the authors in two recent papers, for the corresponding acoustic (scalar) field induced in spheres with variable density and/or compressibility. This extension, by no means trivial, owing to the vector and integrodifferential nature of the equation, is based on field-vector expansions using the set of three harmonic surface vectors, orthogonal and complete over the surface of the sphere, for their angular (θ, φ) dependence, and Dini's expansions of a general type for their radial functions. The use of the latter has been shown to be superior to other possible sets of orthogonal expansions and as far as its convergence is concerned it may further be improved by properly choosing a crucial parameter in their eigenvalue equation. The restriction to the spherical shape is imposed here to allow use of the well-known expansion of Green's dyadic in spherical eigenvectors of the vector wave equation.

1. Introduction

The motivation for solving volume integral equations in the case of penetrable (dielectric) spheres with varying dielectric constant $\varepsilon(\vec{r})$ (the magnetic permeability is considered constant throughout) have been discussed thoroughly in a previous paper by the authors [1], dealing with the corresponding scalar problem. The mathematical difficulties of various approaches have been treated in this paper [1], particularly in connection with the advantages of the direct hybrid method proposed here and in [1,2] by the authors. A first generalization of the approach in acoustics concerned spheres with inhomogeneous density $\rho(\vec{r})$ [2] and herein a further generalization to the vector EM case is developed. In more specific terms we are concerned with the well-known electric field integrodifferential equation (EFIDE)

$$\vec{E}(\vec{r}) = \vec{E}^i(\vec{r}) + \frac{1}{4\pi} (k_0^2 + \nabla \nabla \bullet) \iiint_V (\frac{\varepsilon(\vec{r}')}{\varepsilon_0} - 1) \vec{E}(\vec{r}') \frac{e^{-ik_0 R}}{R} dV'$$
(1)

via which the EM field $\vec{E}(\vec{r})$, induced in the interior of an inhomogeneous dielectric body of volume V with varying dielectric constant $\varepsilon(\vec{r})$ (ε_0 is its free space value, while the magnetic permeability μ_0 is considered constant throughout), is evaluated [3–5]. In (1) $\vec{E}^i(\vec{r})$ is the imposed incident field, $R = |\vec{r} - \vec{r}'|$, $k_0 = \omega \sqrt{\mu_0 \varepsilon_0} = 2\pi/\lambda$, while $\exp(i\omega t)$ is the assumed time dependence. The induced interior field in V is of primary importance to questions of radiation hazards, to the setting of reliable safety field strength limits in media like living tissue, human heads exposed to nearby EM sources, etc. Following the evaluation of the induced interior field the exterior, scattered one may also be obtained by direct integration.

If V is restricted to be a sphere of radius "a", even when the inhomogeneity $\varepsilon(\vec{r})$ precludes separation of variables, a virtually analytical method can be used to solve (1) based on the possibility of expanding the free space Green's function $G(R) = e^{-ik_0 R}/4\pi R$ into an infinite series of spherical eigenfunctions of the Helmholtz equation [1, 2]. This well-known expansion, shown here in the following equation (2) for the corresponding Green's dyadic, is available only in spherical coordinates and combined with Dini-type expansions for the radial functions of the field vectors, provides a basis for a virtually analytical approach. The expansion of Green's dyadic in spherical coordinates is given on page 1875 of [8] in terms of the even/odd spherical eigenvectors of the vector Helmholtz equation. Here we use a more convenient form in terms of the complex form of these vectors as in [9]

$$\bar{\bar{G}}(\vec{r},\vec{r}) = \bar{\bar{I}} \frac{e^{-ik_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} = -\frac{ik_0}{4\pi} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \sum_{m=-n}^{n} \frac{(n-m)!}{(n+m)!} [\vec{M}_{mn}^{(1)}(k_0,r_<,\theta,\varphi) \widehat{\vec{M}}_{mn}^{(4)}(k_0,r_>,\theta',\varphi') + \vec{N}_{mn}^{(1)}(k_0,r_<,\theta,\varphi) \widehat{\vec{N}}_{mn}^{(4)}(k_0,r_>,\theta',\varphi') + n(n+1)\vec{L}_{mn}^{(1)}(k_0,r_<,\theta,\varphi) \widehat{\vec{L}}_{mn}^{(4)}(k_0,r,\theta',\varphi')]$$
(2)

This form can easily be shown to be equivalent to that of [8,9], where the definitions of the various symbols used here can be found.

2. Solution of the EFIDE

To solve the EFIDE we expand the unknown electric field in vector wave functions in the interval [0, a] in a manner analogous to that of Chew for unbounded media [9, p.397]. The calculation is facilitated by taking into account Gauss's law $\nabla \cdot \vec{D} = 0$, which leads us to write

$$\frac{\varepsilon(\vec{r})}{\varepsilon_0}\vec{E}(\vec{r}) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \sum_{l=1}^{\infty} \left[A_{mn\ell}\vec{M}_{mn\ell}(\frac{\gamma_{mn\ell}^M}{a},\vec{r}) + B_{mn\ell}\vec{N}_{mn\ell}(\frac{\gamma_{mn\ell}^N}{a},\vec{r}) \right]$$
(3)

excluding the vector \vec{L} from the expansion. Similarly we expand the incident field. Here, we have restricted the spectrum of the values of k in the definitions of the vectors $\vec{M}_{mn}(k, \vec{r})$ etc, to discrete sets of values $\gamma_{mn\ell}^M, \gamma_{mn\ell}^N, \ell = 1, 2, ...$, which have been chosen so as to construct a full orthogonal set of vectors \vec{M} and \vec{N} , respectively, over the volume of the sphere $0 \le r \le a, 0 \le \theta \le \pi, 0 \le \varphi \le 2\pi$. Moreover, all vectors $\vec{M}_{mn\ell}$ and $\vec{N}_{mn\ell}$ in those orthogonal relations are vectors of the first kind, i.e., $\vec{M}_{mn\ell} = \vec{M}_{mn\ell}^{(1)}$, with the upperscript (1) deleted throughout. We can then make use of results like

$$I(\vec{M}_{mn\ell}, \hat{\vec{M}}_{\mu\nu p}) = \int_{0}^{a} \int_{0}^{\pi} \int_{0}^{2\pi} \vec{M}_{mn\ell} (\frac{\gamma_{mn\ell}^{M}}{a}, \vec{r}) \cdot \hat{\vec{M}}_{\mu\nu p} (\frac{\gamma_{\mu\nu p}^{M}}{a}, \vec{r}) r^{2} \sin\theta dr d\theta r d = 4\pi \frac{n(n+1)}{2n+1} \frac{(n+m)!}{(n-m)!} \\ \delta_{m\mu} \delta_{n\nu} \frac{a^{3} j_{n}(\gamma_{mn\ell}^{M}) j_{n}(\gamma_{mnp}^{M})}{(\gamma_{mn\ell}^{M})^{2} - (\gamma_{mnp}^{M})^{2}} \left[\frac{\gamma_{mn\ell}^{M} j_{n}'(\gamma_{mn\ell}^{M})}{j_{n}(\gamma_{mn\ell}^{M})} - \frac{\gamma_{mnp}^{M} j_{n}'(\gamma_{mnp}^{M})}{j_{n}(\gamma_{mnp}^{M})} \right], l \neq p$$
(4)

and similar ones for the \overrightarrow{N} and \overrightarrow{L} vectors. Analogous relations for the scalar case were found in [1,2]. We can now establish full orthogonality of the set over the volume of the sphere by selecting $\gamma_{mn\ell}^M$ as the roots of the "M-eigenvalue equation" $\gamma_{mn\ell}^M = \gamma_{mn\ell}^M = \gamma_{mn$

$$\frac{\gamma_{mn\ell}^M j'_n(\gamma_{mn\ell}^M)}{j_n(\gamma_{mn\ell}^M)} \equiv t_{mn}^M (\ell = 1, 2, \ldots)$$

$$\tag{5}$$

in which t_{nm}^M may be any chosen constant. Orthogonality of the N-set over the volume of the sphere is, also, established if we choose $\gamma_{mn\ell}^N$ as the roots of the "N-eigenvalue equation"

$$\frac{[\gamma_{mn\ell}^N j_n(\gamma_{mn\ell}^N)]'}{(\gamma_{mn\ell}^N)^2 j_n(\gamma_{mn\ell}^N)} \equiv t_{nm}^N (\ell = 1, 2, \ldots)$$

$$\tag{6}$$

Finally for the L-set the corresponding $\gamma^L_{mn\ell}$ are chosen as the roots of the "L-eigenvalue equation"

$$\frac{j_n'(\gamma_{mn\ell}^L)}{\gamma_{mn\ell}^L j_n(\gamma_{mn\ell}^L)} \equiv t_{nm}^L(l=1,2,\ldots)$$

$$\tag{7}$$

Last, but not least, we must establish the orthogonality between the $\vec{L}_{\mu\nu\rho}$ and $\vec{N}_{mn\ell}$ sets, which is not assured from their angular part (θ, φ) only. However, over the volume of the sphere we have

$$I(\vec{N}_{mn\ell}, \hat{\vec{L}}_{\mu\nu p}) = \int_{V} dV' \vec{N}_{mn\ell}(k_{\ell}^{N}, \vec{r}) \cdot \hat{\vec{L}}_{\mu\nu p}(k_{p}^{L}, \vec{r})$$
$$= 4\pi \frac{n(n+1)}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{m\mu} \delta_{n\nu} \frac{a^{3}}{\gamma_{mn\ell}^{N} \gamma_{mnp}^{L}} j_{n}(\gamma_{mn\ell}^{N}) j_{n}(\gamma_{mnp}^{L})$$
(8)

and orthogonality is assured if we choose the roots of $j_n(\gamma^L_{mnp}) = 0$ for the \vec{L} vectors. We write also

$$\vec{E}(\vec{r}) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \sum_{l=1}^{\infty} \left[\Gamma_{mn\ell} \vec{M}_{mn\ell} (\frac{\gamma_{mn\ell}^{M}}{a}, \vec{r}) + \Delta_{mnl} \vec{N}_{mn\ell} (\frac{\gamma_{mn\ell}^{N}}{a}, \vec{r}) + Z_{mn\ell} \vec{L}_{mn\ell} (\frac{\gamma_{mn\ell}^{L}}{a}, \vec{r}) \right]$$
(9)

The calculation is carried out with the help of the following intermediate results

$$I(\vec{M} \cdot \bar{\bar{I}}g) = \int_{V} dV' \vec{M}_{mn\ell}(k, \vec{r}') \cdot \bar{\bar{I}}g(\vec{r}, \vec{r}') = \frac{1}{k^2 - k_0^2} \{\vec{M}_{mn\ell}(k, \vec{r}) - ik_0 a^2 [-kj'_n(ka)h_n(k_0a) + k_0 h'_n(k_0a)j_n(ka)]\vec{M}_{mn}(k_0, \vec{r})\}$$
(10)

$$I(\vec{N} \cdot \bar{I}g) = \int_{V} dV' \vec{N}_{mn\ell}(k, \vec{r}') \cdot \bar{I}g(\vec{r}, \vec{r}') = \frac{1}{k^2 - k_0^2} \{\vec{N}_{mn\ell}(k, \vec{r}) - ik_0 a^2 [kj_n(ka) \frac{1}{k_0 a} [xh_n(x)]'_{x=k_0 a} - k_0 h_n(k_0 a) \frac{1}{ka} [xj_n(x)]'_{x=ka}]\vec{N}_{mn}(k_0, \vec{r}) \} - ik_0 a^2 n(n+1) \frac{j_n(ka)h_n(k_0 a)}{akk_0} \vec{L}_{mn}(k_0, \vec{r})$$
(11)

$$I(\vec{L} \cdot \bar{\vec{I}}g) = \int_{V} dV' \vec{L}_{mn\ell}(k, \vec{r}') \cdot \bar{\vec{I}}g(\vec{r}, \vec{r}') = \frac{1}{k^2 - k_0^2} \{\vec{L}_{mn\ell}(k, \vec{r}) - ik_0 a^2 [kj_n(ka)h'_n(k_0a) - k_0 h_n(k_0a)j'_n(ka)]\vec{L}_{mn}(k_0, \vec{r})\} - ik_0 a^2 \frac{j_n(ka)h_n(k_0a)}{akk_0} \vec{N}_{mn}(k_0, \vec{r})$$
(12)

In all the above equations $k = \frac{\gamma_{mnl}^X}{a}$, with X = M or N or L respectively. After lengthy manipulations we get the system of equations

$$\Gamma_{mn\ell} = A_{mn}^{inc} T_{n\ell}^{M}(k_{0}) + k_{0}^{2} \frac{A_{mn\ell} - \Gamma_{mn\ell}}{\left(\frac{\gamma_{mn\ell}^{M}}{a}\right)^{2} - k_{0}^{2}} \\ -ik_{0}^{3} a^{2} T_{n\ell}^{M}(k_{0}) \sum_{p} \frac{\left[-\frac{\gamma_{mnp}^{M}}{a} j_{n}'(\gamma_{mnp}^{M})h_{n}(k_{0}a) + k_{0}h_{n}'(k_{0}a)j_{n}(\gamma_{mnp}^{M})\right]}{\left(\frac{\gamma_{mnp}^{M}}{a}\right)^{2} - k_{0}^{2}} (A_{mnp} - \Gamma_{mnp})$$
(13)

$$\Delta_{mn\ell} = B_{mn}^{inc} T_{n\ell}^{N}(k_{0}) + k_{0}^{2} \frac{B_{mn\ell} - \Delta_{mn\ell}}{\left(\frac{\gamma_{mn\ell}^{N}}{a}\right)^{2} - k_{0}^{2}} - ik_{0}^{3} a^{2} T_{n\ell}^{N}(k_{0}) \cdot \sum_{p} \frac{\left[\frac{\gamma_{mnp}^{N}}{a} j_{n}(\gamma_{mnp}^{N}) \frac{1}{k_{0}a} \left[xh_{n}(x)\right]_{x=k_{0}a}^{'} - k_{0}h_{n}(k_{0}a) \frac{1}{\gamma_{mnp}^{N}} \left[xj_{n}(x)\right]_{x=ka}^{'}\right]}{\left(\frac{\gamma_{mnp}^{N}}{a}\right)^{2} - k_{0}^{2}} \left(B_{mnp} - \Delta_{mnp}\right) \qquad (14)$$

where $T_{n\ell}^M(k_0)$ and $T_{n\ell}^N(k_0)$ are obviously the expansions coefficients of $\vec{M}_{mn}(k_0, \vec{r})$, $\vec{N}_{mn}(k_0, \vec{r})$ over the orthogonal sets $\vec{M}_{mn\ell}$ and $\vec{N}_{mn\ell}$, $\ell = 1, 2, ...$, respectively.

The next step is to eliminate one of the two groups of unknowns $\{A, B\}$ or $\{\Gamma, \Delta, Z\}$. Since Z are not present in the final expressions it is better to eliminate Γ, Δ , so we write

$$\vec{E} = \frac{1}{\frac{\varepsilon(\vec{r})}{\varepsilon_0}} \sum_{m,n,\ell} \left[A_{mn\ell} \vec{M}_{mn\ell} + B_{mn\ell} \vec{N}_{mn\ell} \right] = \sum_{m,n,\ell} \left[\Gamma_{mn\ell} \vec{M}_{mn\ell} + \Delta_{mn\ell} \vec{N}_{mn\ell} + Z_{mn\ell} \vec{L}_{mn\ell} \right]$$
(15)

We have thus constructed the necessary equations which, via the orthogonality relations, lead to a matrix equation for the unknowns A, B.

3. Numberical Results and Discussion

Numerical results were obtained for both radial and r-, θ - dependence of the inhomogeneities. For the simpler r-case we selected the well known case of Eaton lens, $\varepsilon(r) = (r/a)^2$ [11, 12]. We have then reproduced exactly the same results known from the literature.

We next present results for the more complicated r-, θ - case. Here we have worked without optimizing the t_{mn} values (a complicated problem) and without comparison to existing results, that are lacking in this more general case. However, a confirmation of the correctness of our procedure stems from the reproduction of our results with other random choices for t_{mn} .

For convenience, that is to obtain as many intermediate results as possible in analytic form and reduce the numerical burden, we have chosen the following function

$$\varepsilon(\vec{r}) = \frac{\varepsilon_0}{1 + 0.3 \left(\frac{r}{a}\right)^2 \cos\theta} \tag{16}$$

Here, working with $k_0 a = 2.0958$ and using as incident field a plane wave $\hat{x}e^{ikz}$ [10], we present final results for the total interior field $|\vec{E}^{tot}(r, \theta, \varphi = 0)/E_0|(E_0 \text{ is the amplitude of } \vec{E}_{inc})$ for a few particular values of θ , which is treated as a parameter in the Figure. Our results correspond to $\varphi = 0$. It turned out that we should take n = 5 terms for the M-component and n = 10 terms for the N-component. In all cases we used $\ell_T = 12$ terms and this proved to be sufficient.

The maximum value of the total interior field appears at r/a = 0.95 and $\theta = 77^{\circ}$.



Figure 1: |E| for $\varepsilon(\vec{r}) = \varepsilon_0 \left(1 + 0.3 \left(\frac{r}{a}\right)^2 \cos\theta\right)^{-1}$, $k_0 a = 2.0958$ for various θ , incident field $\hat{x}e^{ikz}$.

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Advanced Design of Phased Array Beam-forming Networks

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Abstract—Recent fundamental results [1] in the theory of linear, multi-port networks enable cost-effective, higher-reliability designs for electronically-steered phased arrays. The referenced paper documents and proves that, by including a properly designed beam-forming network, it becomes possible to feed an array and steer its beam, using a much reduced number of expensive and critical phase- and amplitude-controlled sources, while at the same time completely eliminating the adverse effects of element coupling. Those new results are based on a generalization of the classical concepts of *scalar* image impedance, and of *scalar* image-transfer function for two-port networks, to the new concepts of multidimensional image-impedance *matrix*, and of multidimensional image-transfer function *matrix* for linear multi-port networks.

1. The Price of Performance

Electronically-steered phased arrays provide unsurpassed agility and high angular resolution in beam-pointing, and the capability of adaptive, multifunction performance. Such highly desirable features are however only attained at the price of high cost, extreme complexity, and limited reliability. Indeed, electronically-steered phased arrays are almost always designed as active-aperture system, that include a large number of semiconductor devices and beamsteering control-elements, embedded in the physical array structure, and closely connected with all the array radiating elements. The phased arrays used in radar systems use transmit/receive modules (T/R), essentially tiny radar, each nested behind a radiating element, in a half-wavelength square section of the total array aperture. Because of the well-known low power-efficiency of semiconductors, a large heat-flux is developed locally, thus generating a complex cooling problem. Finally, notwithstanding technology advances the semiconductor devices and beam-steering control-elements still are the most expensive components of electronically-steered phased array, and cost-effective designs would only be attained by reducing their total number. Those cost and reliability advantages are however only attainable if the structure of the beam-forming network used establishes a pattern of *synergistic connectivity*, where *each* controlled source simultaneously feeds *all* the array elements, and *each* array element is simultaneously feed by *all* the sources (Figures 1 and 2).



Synergistic Connectivity

Figure 1: A clustered phased array providing synergistic connectivity.

$$\sum_{i=1}^{N} \vec{E}_{ej} = \sum_{i=1}^{n} \vec{E}_{si}$$

 \vec{E}_{ej} = Aperture Field \vec{E}_{si} = Aperture Field Generated by Element *j* Generated by Source *i*

Figure 2: The aperture field is a superposition of components.

2. Non-symmetric Beam-forming Network

Such cost and complexity reductions could only be feasible by including a non-symmetric, multiport beamforming network between the reduced number of active devices, and the much larger number of array radiating elements. Such beam-forming network would necessarily be non-symmetric, because of including an n-port interface on the side of the active devices, and an N-port interface on the side of the array radiating elements, with n < N (Figures 3 and 4). The use of a reduced number of beam-steering control-elements appears possible, by considering that current active apertures have the capability of creating a very large number of completely superfluous aperture distributions, that do not generate any practical radiation pattern. Also, the angular resolution of beam-steering could be without penalty reduced, by steering the beam in increments being only a fraction of the $-3 \,\mathrm{dB}$ beam-width.





Figure 3: Unconditional, bilateral image-impedance match: forward-wave, n-phase excitation, with arbitrary wave amplitudes and phases.

Figure 4: Unconditional, bilateral image-impedance match: backward-wave, *N*-phase excitation, with arbitrary wave amplitudes and phases.

3. Recent Theoretical Results

The referenced, recent fundamental results [1] in the theory of multi-port networks have been attained by introducing a generalization of the classical concept of scalar image-impedance of two-port networks, to that of image-impedance matrices for multiport networks. Similarly, the classical concept of scalar image-transfer function of two-port networks, has been generalized to that of image-transfer function matrices for multiport networks. These generalizations have made possible the design of non-symmetric beam-forming networks, that are simultaneously impedance-matched to the external environment at both interfaces, while having prescribed two-way transfer functions between two interfaces with different number of ports (n < N).

4. Image Impedance Matrices

The first fundamental new result expresses the $n \times n$ image-impedance matrix Z_{I1} for the *n*-port interface-1, and the $N \times N$ image-impedance matrix Z_{I2} for the N -port interface-2, as functions of the four different-size blocks Z_i of the $(n + N) \times (n + N)$ impedance matrix of a non-symmetric, multi-port network:

$$Z_{I1} = (I_n - Z_2 \cdot Z_4^{-1} \cdot Z_3 \cdot Z_1^{-1})^{1/2} \cdot Z_1 = (I_n - P_n)^{1/2} \cdot Z_1$$
(1)

$$Z_{I2} = (I_N - Z_3 \cdot Z_1^{-1} \cdot Z_2 \cdot Z_4^{-1})^{1/2} \cdot Z_4 = (I_N - P_N)^{1/2} \cdot Z_4$$
(2)

where the $n \times n$ matrix product P_n , and the $N \times N$ matrix product P_N are given by:

$$P_n = M_n \cdot M_N = Z_2 \cdot Z_4^{-1} \cdot Z_3 \cdot Z_1^{-1} = M_{Pn} \cdot \Lambda_{Pn} \cdot M_{Pn}^{-1}$$
(3)

$$P_N = M_N \cdot M_n = Z_3 \cdot Z_1^{-1} \cdot Z_2 \cdot Z_4^{-1} = M_{PN} \cdot \Lambda_{PN} \cdot M_{PN}^{-1}$$
(4)

The partial matrix-products M_n and M_N in the expressions Eqs. (3) and (4) are defined as:

$$M_n = Z_2 \cdot Z_4^{-1} \tag{5}$$

$$M_N = Z_3 \cdot Z_1^{-1} \tag{6}$$

and the matrix products \boldsymbol{P}_n , and \boldsymbol{P}_N are mutually related by the expression:

$$P_N \cdot (M_N \cdot M_{Pn}) = M_N \cdot (M_n \cdot M_N) \cdot M_{Pn} = M_N \cdot P_n \cdot M_{Pn} = (M_N \cdot M_{Pn}) \cdot \Lambda_{Pn}$$
(7)

By connecting external load-networks with internal impedance matrices $\mathbf{Z}_{L1} = \mathbf{Z}_{I1}$ and $\mathbf{Z}_{L2} = \mathbf{Z}_{I2}$ to the two interfaces, the two image-impedance matrices will transform to each other through the non-symmetric network:

$$Z_{I1} = Z_1 - Z_2 \cdot (Z_4 + Z_{I2})^{-1} \cdot Z_3 \tag{8}$$

$$Z_{I2} = Z_4 - Z_3 \cdot (Z_1 + Z_{I1})^{-1} \cdot Z_2 \tag{9}$$

5. The Block-traceless Scattering Matrix

Because of the bilateral impedance match so attained, the $(n + N) \times (n + N)$ scattering matrix S of the nonsymmetric network becomes *block-traceless*, with only the two rectangular blocks S_2 and S_3 being non-zero:

$$S = \begin{vmatrix} 0 & S_2 \\ S_3 & 0 \end{vmatrix} \tag{10}$$

$$S_2 = Z_2 \cdot Z_4^{-1} \cdot \left[I_N + (I_N - Z_3 \cdot Z_1^{-1} \cdot Z_2 \cdot Z_4^{-1})^{1/2} \right]^{-1}$$
(11)

$$S_3 = Z_3 \cdot Z_1^{-1} \cdot \left[I_n + (I_n - Z_2 \cdot Z_4^{-1} \cdot Z_3 \cdot Z_1^{-1})^{1/2} \right]^{-1}$$
(12)

6. Modal and Spectral Analysis

Two other fundamental new results express the modal matrix M_S , and the spectral matrix Λ_S of the *autonormalized* (normalized to the matrices Z_{I1} and Z_{I2}), *block-traceless* $(n + N) \times (n + N)$ scattering matrix S as:

$$M_S = \begin{vmatrix} M_1 & M_2 \\ M_3 & M_4 \end{vmatrix} \tag{13}$$

$$\Lambda_S = \begin{vmatrix} \Lambda_1 & 0\\ 0 & \Lambda_4 \end{vmatrix} \tag{14}$$

The modal matrix M_S has two square diagonal blocks M_1 of size $n \times n$, and M_4 of size $N \times N$, and two rectangular blocks M_2 of size $n \times N$, and M_3 of size $N \times n$, while the blocks Λ_1 and Λ_2 are $n \times n$, and $N \times N$:

$$M_1 = M_{Pn} \tag{15}$$

$$M_2 = -P_n^{-1/2} \cdot Z_2 \cdot Z_4^{-1} \cdot M_{PN} \tag{16}$$

$$M_3 = Z_3 \cdot Z_1^{-1} \cdot M_{Pn} \cdot \Lambda_{Pn}^{-1/2} \tag{17}$$

$$M_4 = M_{PN} \tag{18}$$

$$\Lambda_1 = \Lambda_{Pn}^{1/2} \cdot \left[I_n + (I_n - \Lambda_{Pn})^{1/2} \right]^{-1} = Diag(e^{-\gamma_n})$$
(19)

$$\Lambda_4 = -\Lambda_{PN}^{1/2} \cdot \left[I_N + (I_N - \Lambda_{PN})^{1/2} \right]^{-1} = Diag(e^{-\gamma_N})$$
(20)

Most remarkably, the block Λ_4 includes N - n identically-zero eigenvalues, that correspond to the N - n identically-zero eigenvalues of the spectral matrix Λ_{PN} of the matrix P_N , while the remaining n eigenvalues are equal to those in block Λ_1 , save for a sign change. The 2n non-zero eigenvalues in the spectral matrix Λ_S , and the corresponding eigenvectors, identify the two sets of n forward, and n backward, natural transmission modes of any given non-symmetric beam-forming network, while the N - n eigenvectors, that correspond to the zero-eigenvalues in block Λ_4 , span the null-space of the $n \times N$ block S_2 , and identify the natural cut-off modes of the network. These are the N - n voltage-wave a_j vectors of the N-port interface-2, for which the received $b_i = S_2 \cdot a_j$ vectors of the n-port interface-1 are all identically zero.

7. The Required Impedance Matrix

The final referenced fundamental result expresses the two square blocks Z_1 of size $n \times n$, Z_4 of size $N \times N$, and the two rectangular blocks Z_2 of size $n \times N$, and Z_3 of size $N \times n$, as functions of the two required

image impedance matrices Z_{I1} and Z_{I2} , and of the two required rectangular image-transfer function matrices \boldsymbol{S}_2 and \boldsymbol{S}_3 :

$$Z_1 = (I_n - S_2 \cdot S_3)^{-1} \cdot (I_n + S_2 \cdot S_3) \cdot Z_{I1}$$
(21)

$$Z_{2} = 2(I_{n} - S_{2} \cdot S_{3})^{-1} \cdot S_{2} \cdot Z_{I2}$$

$$Z_{3} = 2(I_{N} - S_{3} \cdot S_{2})^{-1} \cdot S_{3} \cdot Z_{I1}$$
(22)
(22)
(22)
(23)

$$Z_3 = 2(I_N - S_3 \cdot S_2)^{-1} \cdot S_3 \cdot Z_{I1}$$
(23)

$$Z_4 = (I_N - S_3 \cdot S_2)^{-1} \cdot (I_N + S_3 \cdot S_2) \cdot Z_{I2}$$
(24)

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Slotline Leaky Wave Antenna with a Stacked Substrate

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Abstract—This paper presents a new version of a leaky wave antenna based on a conductor-backed slotline with a stacked substrate. The antenna radiates due to the first order space leaky wave excited on a slotline with a wide slot. The antenna radiates into the one main beam above the substrate. The main lobe of the radiation pattern is relatively wide and there is also intensive radiation in the backward direction and below the substrate. Shaping the background metal layer produces a reflector, which reduces the parasitic radiation by 7 dB. When this background layer is larger than the substrate the radiation below the substrate is additionally reduced by 3 dB. The reflector moreover reduces the width of the main beam.

1. Introduction

All kinds of open planar transmission lines are predisposed to excite leaky waves. There are two kinds of leaky waves. Surface leaky waves radiate power into the substrate. These waves are in most cases undesirable as they increase losses, cause distortion of the transmitted signal and cross-talk to other parts of the circuit. Space leaky waves radiate power into a space and mostly also into the substrate. These waves can be utilized in leaky wave antennas.

Leaky wave antennas have been known for nearly 50 years [1]. The first microstrip line leaky wave antenna was described in [2] and its behaviour was analyzed in detail in [3]. The first slotline leaky wave antenna was reported in [4]. We have investigated space leaky waves on a slotline (SL) [5] and on a conductor backed slotline (CBSL) [6]. Based on these studies we have designed, fabricated, and measured several leaky wave antennas, as, e. g., the slotline leaky wave antenna in [7]. The drawback of this antenna is radiation into two main beams, one above and one below the substrate. For this reason we turned to the CBSL. Various designed and fabricated antennas utilizing this line [8,9] radiate only into the one main beam. This beam is rather wide with a high level of the side lobes (SLL). The antenna substrate has to be thick enough for effective radiation. From the fabrication point of view it is more convenient to use a stacked substrate. One slab is a thin commercially available substrate, while the second slab, known as the spacer, is filled with air [10].

This paper presents a slotline leaky wave antenna with a stacked substrate and conductor backing. The antenna radiates the space leaky wave of the first order. The CBSL with a wide slot and substrate consisting of two layers was analyzed by the APTL Program [11] based on the spectral domain method. The CST Microwave Studio performed optimization of an antenna feeder. The shape of the radiation pattern of the antenna has been improved by the background conductor formed into a simple reflector. This makes the main radiation lobe narrower, and considerably reduces the level of the side lobes.

2. Antenna Structure

The cross-section of the CBSL with a stacked substrate is shown in Fig. 1. The upper layer is substrate

GIL1000 1.52 mm in thickness with permittivity $\varepsilon_{r2} = 3.05$, loss factor $\mathrm{tg}\delta = 0.004$ and metallization thickness t = 0.03 mm. The bottom layer is air, so $\varepsilon_{r1} = 1$. Assuming that the slot is wide enough, this transmission line can support propagation of a space leaky wave of the first order with odd symmetry of the transversal component of the electric field parallel to the substrate. The dispersion characteristics of this wave calculated by the APTL Program [11] are plotted in Fig. 2. The phase constant β and the attenuation constant α are normalized to the propagation constant in free space k_0 . The upper dielectric layer, in Fig. 1, has the parameters



Figure 1: Cross-section of the CBSL with a stacked substrate.

stated above, the slot width is 30 mm and three different heights of the air layer $h_2=10$, 15, and 20 mm were used. The simulation in Microwave Studio showed that for h_2 lower than 10 mm unwanted modes excite between the parallel plates, and the radiation efficiency is low. For values of h_2 higher than 20 mm the parasitic radiation into the space below the backed metallization increases. The value $h_2 = 20$ mm was therefore chosen as a compromise between these two limits. The dispersion characteristic of the leaky mode on the CBSL with $h_2 = 20$ mm, Fig. 2, shows that this mode can be effectively excited from 4 to 10 GHz, as its phase constant is lower than k_0 and the attenuation constant has a reasonably low value. The phase constant slowly increases with frequency. The direction of the main lobe radiation pattern, determined by the phase constant, depends therefore slightly on the frequency.

The final antenna setup is shown in Fig. 3. The antenna is fed from a coaxial cable via a CPW terminated by a patch to transform the incident energy effectively into a space leaky wave of the first order. CBSL open end termination in the form of a wedge was used. The feeder geometry was optimized in the CST Microwave Studio for minimum return losses in the widest possible frequency band when the space leaky wave is effectively excited.



Figure 2: Normalized dispersion characteristic of the CBSL with a stacked substrate defined in the text.

The resulting frequency dependence of S_{11} measured and calculated by the CST Microwave Studio is plotted in Fig. 4. The antenna is matched from 5 GHz up to 7 GHz, when $|S_{11}| < -10$ dB. The measured and calculated antenna radiation patterns at 6 and 7 GHz are plotted in Fig. 5 and are in good accord. Angle Θ is read according to Fig. 6. The radiation patterns measured at additional frequencies are plotted in Fig. 6. This antenna has only a small difference between the level of the main lobe and side lobes (side lobe level—SLL), see Fig. 8, and relatively intensive radiation under the substrate. The level of the lobes directed under the substrate is about -13 dB comparing to the main lobe. The main lobe slightly tilts in the forward direction and the full width at half power (FWHP) of the main lobe decreases with frequency, as follows from Fig. 8.



Figure 3: The fabricated antenna with the feeder shown in detail.



Figure 4: S_{11} of the fabricated antenna.



Figure 5: Radiation patterns of the antenna from Fig. 3 at 6 and 7 GHz.



Figure 6: Orientation of angles.



Figure 7: Radiation pattern of the antenna from Fig. 3 measured at 5.75, 6, 6.5, 7, and 7.25 GHz.



Figure 8: Measured side lobe level, full width at half power, and the angle of maximum radiation of the antenna from Fig. 3.

3. Antenna with a Reflector

The radiation pattern of the CBSL antenna with a stacked substrate has one rather wide main lobe. The antenna also radiates backward and below the backed metallization, Figs. 5, 7 and 8. Its radiation pattern can be shaped effectively by adding the background metal reflector [10], Fig. 9. The layout of the feeder and of the slotwidth was left without any change. This antenna was simulated by CST Microwave Studio with the aim to reduce the side lobes with a reasonably small reflector. Finally the reflector position is 20 mm behind the substrate edge and exceeds the substrate height by 20 mm. This reflector scarcely influences the antenna input impedance. Fig. 10 compares the measured and calculated radiation patterns of the antenna with the reflector at 7 GHz. The radiation patterns measured at three different frequencies are plotted in Fig. 11. Comparing the radiation patterns in Fig. 11 and Fig. 7, we see that the antenna with the reflector has a narrower main beam and the level of both the side lobes and of the lobes directed under the substrate are reduced by 7 dB.

Making the size of the background metal layer larger than the antenna substrate further reduces the radiation below the substrate. In this way we get the antenna shown in Fig. 12. The reflector has the same geometry as in Fig. 9, the bottom conductor is enlarged by 30 mm on the side and front walls of the substrate.



(qB) 7 GHz **ω=0** -10 ш -20 -30 -40 measurement simulation -50 -180 -90 0 90 180 Θ (deg)

Figure 9: The Microwave Studio model of the antenna with a reflector.

Figure 10: Radiation pattern of the antenna from Fig. 9 at 7 GHz.



Figure 11: Radiation pattern of the antenna from Fig. 9 measured at 6.5, 7 and 7.25 GHz.



Figure 12: The Microwave Studio model of the antenna with a reflector and the background metal layer larger than the substrate by 30 mm.

The measured and calculated radiation patterns of this antenna are plotted in Fig. 13 at the frequency 6.75 GHz. The two lines fit each other well. The radiation patterns of this antenna measured at several frequencies are plotted in Fig. 14. It follows from Figs. 13 and 14 that the radiation below the substrate is reduced by 3 dB comparing to the antenna from Fig. 9 and by 10 dB comparing to the original antenna without the reflector in Fig. 3. The SLL is reduced from 6.5 to 6.75 GHz to -17 dB. The FWHP of the main beam varies around 20 deg when the frequency changes, which is considerably lower than the FWHP of the antenna from Fig. 3. The direction of the main lobe is saved. The SLL, FWHP, and the angle of maximum radiation of the antenna from Fig. 12 are plotted in Fig. 15. The plot in Fig. 15 in comparison with the plot in Fig. 8 shows the improvement of the radiation pattern when the reflector is applied.



6 GHz **0=0** 6.25 GHz -10 ш 5 GH: -20 -30 6.75 GHz -40 GHz 7 -50 -90 0 90 -180 180 ⊙ (deg)

Figure 13: Radiation pattern of the antenna from Fig. 12 at 6.75 GHz.

Figure 14: Radiation pattern of the antenna from Fig. 12 measured at 6, 6.25, 6.5, 6.75 and 7 GHz.



Figure 15: Measured side lobe level, full width at half power, and the angle of maximum radiation of the antenna from Fig. 12.

4. Conclusion

This paper presents a leaky wave antenna based on a conducto-backed slotline with a stacked substrate. This substrate consists of a thin dielectric layer and a thick air spacer. The antenna radiates a first order space leaky wave with odd symmetry only into one main beam above the substrate. This beam is tilted in the forward direction when the frequency increases. The antenna layout was optimized using the CST Microwave Studio. The antenna effectively radiates from 5 to 7 GHz. Its radiation pattern has a single main beam and the side lobes are at a level not worse than -10 dB below the maximum radiation. The radiation below the substrate is not worse than -14 dB below the maximum of the main lobe.

The additional reflector effectively shaped the radiation pattern. Two versions of the antenna with the reflector were designed with the aid of the CST Microwave Studio and then fabricated and measured. The antenna feeder and the slot layout were the same as the antenna without the reflector had. The antenna with the reflector and the ground conductor of the same size as the substrate reduces the level of the side lobes to -17 dB, and the radiation below the substrate to -20 dB. The antenna with the background conductor larger than the substrate has a level of radiation below the substrate lower by an additional 3 dB, i.e., -23 dB. The width of the main lobe is around 20 deg, which is narrower than for the antenna without the reflector. The direction of this lobe is the same for the antenna both with and without the reflector.

Acknowledgement

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Light Scattering on 2D Nanostructured Resonant Gratings

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Abstract—This paper studies nanostructured gratings made up by silver nanoparticles embedded in the dielectric, which are capable of maintaining quasi-static modes. The special emphasis is devoted to following specified types of gratings: row of periodical cylinders and square grating of spheres. The problem of a diffraction of a plane electromagnetic wave on such structures has been solved within the dipolar-interaction approximation. The frequency dependences of the refraction and absorption coefficients on the grating parameters have been obtained, analyzed and compared.

1. Introduction

Recently, a significant success has been achieved in the areas related to creation of metamaterials based on resonance metal elements, specifically, films with embedded metal nanoparticles [1–3] that are capable of sustaining high-Q-factor quasi-static modes. At the resonance frequency, the scattering cross-section of such particles exceeds their geometric sizes significantly, which yields a number of new collective optical properties when they join up in nanostructures. The most preferable, in terms of practical applications (both from the standpoint of their chemical stability and resonance characteristics), are the nanoparticles of silver and gold. The coherent effects of light scattering on plane gratings formed by cylindrical and spherical nanometer silver objects have been analyzed.

2. Diffraction on a Periodical Structures

This work studies the diffraction of a plane P-polarized wave with the form:

$$H_{y} = H_{o} \exp(-i\omega t - ik_{o}\cos\varphi z - ik_{o}\sin\varphi x)$$

$$E_{x} = -E_{o}\cos\varphi \exp(-i\omega t - ik_{o}\cos\varphi z - ik_{o}\sin\varphi x)$$

$$E_{z} = -E_{o}\sin\varphi \exp(-i\omega t - ik_{o}\cos\varphi z - ik_{o}\sin\varphi x)$$
(1)

that falls from the vacuum onto a plane grating formed by silver nanostructures (see Figure 1). Two simplest and at the same time, evidently, basic configurations of the grating are considered: the first (unidimensional) is a periodic row of cylinders with their axes lying in the plane z = 0 and oriented along the y-axis (see Figure 1a), and the second (two-dimensional) is a periodic (both along x and y) square grating of spheres with their centres in the plane z = 0 (see Figure 1b).



Figure 1: Configurations of considering gratings.

Let us assume, for the sake of simplicity, that the dielectric permittivity of the substrate, which the grating is mounted on, is close to unity, such that the environment is actually vacuum everywhere. Let the radii of the cylinders and the spheres are small as compared with the length of the incident wave λ ($a << \lambda$). Then the field scattered by these structural elements is the field of a linear dipole with its dipole momentum (per length unit) $\mathbf{P}_{cyl} = \alpha_{cyl} \mathbf{E}_D$ for the cylinder, and the field of a point dipole with its dipole momentum $\mathbf{P}_{sph} = \alpha_{sph} \mathbf{E}_D$ for


Figure 2: Polarizability coefficients of considering elements as a function of relative frequency shift $\delta \omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ ($\lambda_{res} = 325nm$ for cylinders, $\lambda_{res} = 352nm$ for spheres).

the sphere, where \mathbf{E}_D is the effective field. When radiation losses are neglected, the polarizability coefficients look as follows (see Figure 2):

$$\alpha_{cyl} = \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1} a^2 \qquad \qquad \alpha_{sph} = \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2} a^3 \qquad (2)$$

Here $\varepsilon(\omega)$ is the dielectric permittivity of the object. For silver, which is interest for us, it is described, in the range $\lambda \sim 300 - 500nm$, with good accuracy as [4]:

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega - i\gamma)} \tag{3}$$

where $\varepsilon_{\infty} = 4.7, \omega_p = 1.38 \cdot 10^{16} s^{-1}, \gamma = 2.7 \cdot 10^{13} s^{-1}.$

The effective field is the sum of the incident field and the fields of all other dipoles at the location of some segregated dipole in its absence. We propose that the following procedure should be used to find that field, which is somewhat different from the traditional procedure and, in our opinion, seems to be convenient. Taking into account that a polarized medium can be described by means of polarization currents $\mathbf{J} = \frac{\partial \mathbf{P}}{\partial t} = -i\omega \mathbf{P}$, let us pass over from dipoles to currents. The density of such currents is represented as:

$$\mathbf{J}_{cyl}(x) = -i\omega\alpha_{cyl}(\omega)\mathbf{E}_D^{cyl}\delta(z)\sum_{n=-\infty}^{+\infty}\delta(x-nd)$$
$$\mathbf{J}_{sph}(x,y) = -i\omega\alpha_{sph}(\omega)\mathbf{E}_D^{sph}\delta(z)\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\delta(x-nd)\delta(y-md)$$
(4)

where $\delta(\cdot)$ is the Dirac δ function.

Then, from the shown system of discrete currents, using the Poisson formula [5] we pass over to continuous surface dummy currents of spatial harmonics:

$$\mathbf{J}_{cyl}(x) = -i\omega\alpha_{cyl}(\omega)\mathbf{E}_D^{cyl}\frac{\delta(z)}{d}\sum_{n=-\infty}^{+\infty}\exp(i\frac{2\pi}{d}nx)$$
$$\mathbf{J}_{sph}(x,y) = -i\omega\alpha_{sph}(\omega)\mathbf{E}_D^{sph}\frac{\delta(z)}{d^2}\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\exp(i\frac{2\pi}{d}nx)\exp(i\frac{2\pi}{d}my)$$
(5)

Finding the field of the individual spatial harmonic is elementary for the tangential component of the current, and somewhat more difficult for the normal one. Note that the normal component of the electric field is actually equivalent to the tangential of the magnetic current. The effective field is further obtained by subtracting the field of the segregated dipole situated, e.g., at the origin of coordinates. Representing the latter as an integral over the same harmonics, we obtain finally the following self-consistent expressions:

$$\mathbf{E}_{D}^{cyl}(0) = \mathbf{E}_{o}(x=z=0) + \sum_{n=-\infty}^{+\infty} \tilde{\mathbf{E}}(\varkappa_{x} = \frac{2\pi}{d}n) - \frac{d}{2\pi} \int_{-\infty}^{+\infty} d\varkappa_{x} \tilde{\mathbf{E}}(\varkappa_{x})$$
$$\mathbf{E}_{D}^{sph}(0,0) = \mathbf{E}_{o}(x=z=0) + \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{\mathbf{E}}(\varkappa_{x} = \frac{2\pi}{d}n, \varkappa_{y} = \frac{2\pi}{d}m) - \frac{d^{2}}{4\pi^{2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\varkappa_{x} d\varkappa_{y} \tilde{\mathbf{E}}(\varkappa_{x}, \varkappa_{y}) (6)$$

from which the effective field is extracted as a function of the incident field. It should be noted that the second and third term in formula (6) have singularities at zero, which are mutually compensated. This should be taken into account when performing numerical calculations.

Granted that the effective field is known, it is possible to solve the set diffraction problem.

3. Results and Discussion



Figure 3: Reflection (firm line) and absorption (dash line) power coefficients of considered gratings as a function of relative frequency shift $\delta \omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ for various periods d of gratings (a/d = 0.2—blue line, a/d = 0.1—green line, a/d = 0.05—red line). The normal incidence case ($\varphi = 0$).



Figure 4: Reflection power coefficients of considered gratings as a function of relative frequency shift $\delta \omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ for various incidence angles ($\varphi = 0^{\circ}$ —red line, $\varphi = 30^{\circ}$ —green line, $\varphi = 50^{\circ}$ —blue line, $\varphi = 70^{\circ}$ —black line). Relation a/d = 0.2 is fixed.



Figure 5: Absorption power coefficients of considered gratings as a function of relative frequency shift $\delta \omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ for various incidence angles ($\varphi = 0^{\circ}$ —red line, $\varphi = 30^{\circ}$ —green line, $\varphi = 50^{\circ}$ —blue line, $\varphi = 70^{\circ}$ —black line). Relation a/d = 0.2 is fixed.

Further the results of the numerical calculations based on the formulas (6) are presented and discussed. Reflection and absorption power coefficients as a function of relative frequency shift $\delta \omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ for various grating parameters are shown in Figures 3, 4, 5. All results are given for the fixed parameter $k_{res}a = \frac{\omega_{res}}{c}a = 0.08$. Thus, the radius of cylinders *a* is equal 4.1*nm* and the radius of spheres *a* is equal 4.5*nm*. The left-hand parts of figures respond a case of the grating from cylinders, and right parts—to a case of the grating from spheres. Comparison allows to present common features and distinctivenesses.

Figure 3 convincingly shows effect of coherent interaction of separate elements at their converging. The peak value of reflection coefficients for the grating from spheres is noticeably less than for the grating from cylinders. It is stipulated by essential difference in filling factors $(f_{cyl}/f_{sph} = 4a/3d)$. Different shift of frequencies, at which the maxima of reflection coefficient for considered gratings is attained, is determined, apparently, various interactions of linear dipoles and point dipoles.

In case of oblique incidence (see Figure 4) with increase of an incidence angle, one more peak occurs and gradually grows. It is stipulated by coherent interaction dipoles oriented along the z-axis.

4. Conclusion

A two-dimensional problem of plane electromagnetic wave diffraction on a gratings consisting of resonance elements is solved in dipole-interaction approximation. A novel method of obtaining effective field expression is proposed. Reflection and absorption coefficients are found for various compositions of gratings parameters.

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Modified Equivalent Circuit Model of Microwave Filter with LTCC Technique

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Abstract—Because microwave products in the consumer electronics market are continuous developing, device and component manufacturers have to seek new advanced integration, packaging and interconnection technologies, as size, cost and performance are critical factors for the success of a microwave product.

One of the most promising integration technologies is the multilayer low temperature co-fired ceramic technology (LTCC). In this technology, passive components, such as inductors, capacitors and filters, are integrated into multilayer LTCC substrate. The purpose of this paper is to address the special method that should be considered for designing LTCC microwave filter. It is given how to get equivalent circuit about multilayer ceramic microwave filter that is striplines configuration or LC configuration, especially, modified equivalent circuit model is proposed, where the relation between the lumped parameters and physical dimension of LTCC microwave filter is discussed. The capacitance and inductance matrix of LTCC microwave filter is obtained using the fast multipole method. Finally, two microwave filters are designed by the novel design method of field-circuit and HFSS. The novel design method of field-circuit is more efficient, therefore, the designing time can be shortened.

1. Introduction

Low-temperature-cofired ceramics (LTCC) for microwave applications represent a key position in the development of future electronic products in a high frequency application for IC packaging radar, antennas and wireless technologies. The integration of passive components in LTCC is, therefore, particularly interesting in multilayers technology. Integration of passive devices in wireless application corresponds to the trend of mobilization and miniaturization with high electrical performance using conductive electrode materials such as gold, silver and copper. Several kinds of multilayer microwave devices have been developed, and some design methods and fabrication procedures reported [1–4]. Hence, they can be easily incorporated in the design of a variety of RF components such as passive components, voltage controlled oscillators (VCO_s), power amplifiers (PA_s), and mixers.

Among various passive components, people usually pay the most attention to the filter. Now a lumpedelement RF filter can be implemented in a stacked structure. Engineers usually use HFSS, which is a software employed by Ansoft used for high frequency E/M simulation, to design these passive components. One side, HFSS is an advanced simulation software, which can accurately calculate the E/M fields with every engaged point in the component. So its result is convictive. On the other side, HFSS is not very effectively because of long time waste depending on the capability of your computer and the simulation precision you want. This paper introduces the designing of fast multipole method for the passive filter. It is shown more effective than the HFSS by experiment proof-testing.

2. Getting Capacitance and Inductance Matrix Using the Fast Multipole Method (FMM)



Figure 1: Multi-condutor system in medium between two grounds.

Figure 1 shows there are M conductors between two grounds. $\therefore Q_n = \sum_{m=1}^M C_{n,m} V_m$, where Q_n is the quantity

of charge of conductor n and V_m is the electric potential of conductor m. $C_{n,m}$ represents the capacitance between conductor n and conductor m when the electric potential of conductor m is V_m and the electric potential of conductor n is 0.

Provided that S_n , $\rho_n(r')$ are, respectively, the superficial area and surface charge density of conductor n, quantities of electric charge Q_n of conductor n can be given as:

$$Q_n = \int_{S_n} \rho_n(r') dS \tag{1}$$

For every point on the conductor surface, the electric potential of the point can be drived by considering the image charges, we have

$$\Psi(r) = \frac{1}{4\pi\varepsilon_r\varepsilon_0} \left[\sum_{n=1}^M \int_{S_n} \frac{\rho_n(r')}{|r-r'|} dS + \sum_{i=1}^\infty \sum_{n=1}^M \int_{S_n^i} \frac{\rho'_{n,i}(r')}{|r-r'_{n,i}|} dS \right]; r \in S_m, m = 1, 2, \dots, M$$
(2)

where $\rho'_{n,i}(r')$ represents the *i*-th mirror of $\rho'_n(r')$, S^i_n represents the superficies of $\rho'_n(r')$.

Using moments method we can devide the superficies of the conductor to N pieces, provided that the charge on every piece is uniform:

$$\Psi_{l}(r_{l}) = \frac{1}{4\pi\varepsilon_{r}\varepsilon_{0}} \left[\sum_{k=1}^{N} \int_{T_{k}} \frac{\rho_{l}(r')}{|r_{l} - r'|} dS + \sum_{i=1}^{\infty} \sum_{k=1}^{N} \int_{T_{k}^{i}} \frac{\rho_{l,i}'(r')}{|r_{l} - r_{l,i}'|} dS \right]; r \in T_{k}, k = 1, 2, \dots, N$$
(3)

The capacitance matrix can be formed by the above sets of equations:



Figure 2: The structure of stripline configuration filter with LTCC.





Figure 4: Modified equivalent circuit.

The inductance matrix can be expressed as:

Figure 3: Traditional equivalent circuit.

$$L]_{n,m} = \varepsilon_0 \mu_0 \left[C_0 \right]_{n,m}^{-1} \tag{5}$$

With the equations based on the multipolar method we can get the capacitance matrix and the inductance matrix. we have proved that the method is convincing.

3. Modified Equivalent Circuit of BPF (Bandpass Filter)

3.1. Analysis of the Striplines Configuration Filter [5–7]

The schematic configuration for the striplines filter to be implemented is shown in Figure 2. It is consisted by three layers: T layer, M layer, and B layer. It is shown that this is a two LC resonance filter, and the configuration of two LC resonance is uniform. In the experimental filter, inductance L of resonance is presented by self-in-inductance LM of conductor M. resonate capacitance M is presented by self-in-capacitance CM of conductor M and coupling capacitance CT between conductor M and conductor T and coupling capacitance CB between conductor M and conductor B. Coupling C12 between resonance cells are consisted by the total of each coupling capacitance.

We give the modified equivalent circuit about the striplines configuration filter, as shown in Figure 2. The schematic illustration of the modified equivalent circuit is shown as Figure 4:

In traditional equivalent circuit, there is no one-to-one relationship between the striplines configuration and equivalent circuit numerical value in detail. But in modifided equivalent circuit, there is one-to-one relationship between the striplines configuration and equivalent circuit numerical value in detail.

3.2. Simulation Results

The scatterring parameter S_{11} , S_{21} can be expressed as follows:

$$IL = 10\log\frac{P_{in}}{P_L} = 10\log\frac{1}{|S_{21}|^2} = -10\log|S_{21}|^2 \text{ (dB)}$$
(6)

where $S_{11} = \Gamma_{in}$, $\rho = \frac{1+|S_{11}|}{1-|S_{11}|}$, $\Gamma_{in} = \frac{Z_{in}-Z_0}{Z_{in}+Z_0}$ Every no loss component can be expressed

Every no-loss component can be expressed as:

$$|S_{21}|^2 = 1 - |S_{11}|^2 \tag{7}$$

Through the above parameters we can get Insert Loss (IL), Bandpass (B) and VSWR. The input impedance can be obtained in following equations:

$$Z_{in} = \left((Z_L / / Z_{CT2} / / Z_{LM2} / / Z_{CM2} / / Z_{CB2}) + Z_{C12} \right) / / Z_{CT1} / / Z_{LM1} / / Z_{CM1} / / Z_{CB1}$$
(8)

The characteristic parameter of the component can be expressed in curves through following equations:

$$S_{11} = 20 \log \left| \frac{Z_{in} - Z_G}{Z_{in} - Z_G} \right| (dB)$$
(9)

$$S_{21} = 10\log(1 - |S_{11}|^2) = 10\log(1 - \left|\frac{Z_{in} - Z_G}{Z_{in} - Z_G}\right|^2)(\text{dB})$$
(10)

Now we provide a multiplayer ceramic microwave filter with the size of $2.0 \text{ mm} \times 1.25 \text{ mm} \times 0.95 \text{ mm}^3$ to certify the correctness of the method comparing HFSS simulation with the dielectric constant 27:



Figure 5: The structure of models.

The following graphs demonstrate the results respectively:

model	d1	d2	d3	d4	T_L	T_W	M_L	M_W	B_L	B_W
0	0.045	0.26	0.28	0.5	1	0.15	1	0.35	1	0.25
1	0.05	0.26	0.28	0.5	1	0.15	1	0.35	1	0.25
2	0.04	0.26	0.28	0.5	1	0.15	1	0.35	1	0.25
3	0.045	0.26	0.28	0.6	1	0.15	1	0.35	1	0.25
4	0.045	0.26	0.28	0.7	1	0.15	1	0.35	1	0.25
5	0.045	0.26	0.28	0.6	1	0.15	0.8	0.3	1	0.25
6	0.045	0.26	0.28	0.6	1	0.15	0.8	0.5	1	0.25
7	0.048	0.49	0.55	0.8	2.3	0.4	2.3	0.6	2.3	0.2
8	0.048	0.49	0.55	0.65	2	0.2	2	0.4	2	0.3
9	0.048	0.49	0.55	0.65	1	0.15	3	0.3	1	0.25
	Note: d1=d.d2=d_t.d3=d_b.d4=d_centers									

Table 1: The models physical size is stated follow(unit: mm)



Figure 6: Result using the modified equivalent circuit(left) and with HFSS(right).

Analyzing the two graphs above, several differences can be found including inset loss, stop band attenuation, band width, reflection in the input port.

4. Conclusion

The simulation results prove that modified equivalent circuit model where the relationship between the concentrate parameters and physical dimension of LTCC microwave filter is corrected. The novel design method of field-circuit has high efficiency, therefore, the designing time can be shortened.

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Design of a Metafilm-composite Dielectric Shielding Structure Using a Genetic Algorithm

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Abstract—An analytical model for a shielding structure containing both bulk composite layers and planar metafilms (MFs) made of perfect electric conductors is presented, allowing for synthesis of shielding structures using the genetic algorithm (GA) optimization. MFs can be of two different types: patch or aperture. The frequency response, specifically, transmission (T) and reflection (Γ) coefficients in a plane-wave formulation, of any MF is calculated based on polarizabilities determined by the particular pattern geometry. T and Γ of a patch-type MF are derived using the generalized sheet transition conditions (GSTC) and the Babinet's duality principle is used for aperture-type MF to map the results from the complementary problem. T and Γ for a singlelayered MF are represented in a unified matrix form for any angle of incidence. T-matrix approach is used for getting T and Γ for a multilayered structure. Any MF buried in a host dielectric can be decomposed into three types of basic elements: a host composite slab, interface between media, and an MF inside the homogeneous host medium. Each basic element is described by a corresponding T-matrix, and the total T-matrix of the stack is the sequential product of the each individual T-matrix. T and Γ of the stack can be easily derived from the total T-matrix. If there are two or more MFs, the distance between them justifies the condition of neglecting higher-order evanescent mode interactions. Then the GA is applied to engineer a structure with the desired frequency response. It helps to choose the best geometry of MF patterns, thickness of layers, and appropriate constitutive parameters of each composite layer.

1. Introduction

For many practical applications, it is desirable to develop shielding structures having specified frequency characteristics. Application of a robust and quickly converging genetic algorithm (GA) facilitates the engineering of composite materials, saving time and resources before manufacturing and testing real materials [1]. A shielding structure may consist of a single composite dielectric or a multilayered stack of composite dielectrics with given electromagnetic properties. However, composite dielectric layers alone may be insufficient for achieving the acceptable shielding effectiveness (SE) in a given frequency range. The presence of metafilms (MFs) buried in composite layers may increase SE in the frequency band of interest, or assure desirable frequency-selective effects.

In this work, a model of a shielding structure containing both bulk composite layers and MFs made of PEC has been developed analytically, and the approach is considered below. For simplicity, we only consider MFs constructed by square arrays (with the same periodicity along two orthogonal axes in the plane) buried inside a homogeneous host material.

2. Mathematical Model

2.1. Using T-matrix Approach to Analyze T and Γ of the Multilayered Structures

The T-matrix used in this model is similar to that defined in [2]. A wave-transmission system is modeled as a two-port network. The forward and backward waves at the input and output ports are related as

$$\begin{bmatrix} a_1\\b_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12}\\t_{21} & t_{22} \end{bmatrix} \cdot \begin{bmatrix} b_2\\a_2 \end{bmatrix},\tag{1}$$

where a_1 and a_2 are the incoming wave and b_1 and b_2 are outgoing waves.

The total T-matrix of N cascaded 2-port networks T_1, T_2, \ldots, T_N is the sequential product of the corresponding T-matrices,

$$T_{tot} = \begin{bmatrix} t_{11}^{tot} & t_{12}^{tot} \\ t_{21}^{tot} & t_{22}^{tot} \end{bmatrix} = T_1 T_2 \dots T_N.$$
(2)

Then T and Γ of the multilayered MF can be found as [2]:

$$T = t_{21}^{tot} = b_2/a_1 \Big|_{a_2=0} \text{ and } \Gamma = t_{11}^{tot} = b_1/a_1 \Big|_{a_2=0}.$$
(3)

Any MF buried in a host medium can be decomposed into 3 basic elements: (a) a host medium slab; (b) a medium interface, and (c) an MF inside the homogeneous host medium, as shown in Fig. 1. T-matrices will be obtained below for all these three cases. Then T and Γ of the stack can be easily obtained from (2) and (3).



2.2. Formulation for a Composite Layer

The Maxwell Garnett (MG) effective medium formulation can serve as a basis for engineering composite microwave materials [1],

$$\varepsilon_{eff} = \varepsilon_b + \frac{\frac{1}{3} \sum_{i=1}^n f_i(\varepsilon_i - \varepsilon_b) \sum_{j=1}^3 \frac{\varepsilon_b}{\varepsilon_b + N_{ij}(\varepsilon_i - \varepsilon_b)}}{1 - \frac{1}{3} \sum_{i=1}^n f_i(\varepsilon_i - \varepsilon_b) \sum_{j=1}^3 \frac{N_{ij}}{\varepsilon_b + N_{ij}(\varepsilon_i - \varepsilon_b)}},$$
(4)

where $\varepsilon_b(j\omega) = \varepsilon_{\infty b} + \chi_b(j\omega)$ and $\varepsilon_i(j\omega) = \varepsilon_{\infty i} + \chi_i(j\omega)$ are the relative permittivity of a base dielectric and of the *i*-th type of inclusions, respectively; $\varepsilon_{\infty b,i}$ are the high-frequency permittivities for the base material and inclusions of the *i*-th type, respectively; and $\chi_{b,j}(j\omega)$ are the corresponding dielectric susceptibility functions. f_i is the volume fraction occupied by the inclusions of the *i*-th type; N_{ij} are the depolarization factors of the *i*-th type of inclusions, and the index j = 1, 2, 3 corresponds to x, y, and z Cartesian coordinates. The effective permittivity of a mixture might have complex-shaped frequency dependence. As shown in [1], it can be approximated by a series of Debye-like terms with real relaxation constants τ_k and complex (in general case) coefficients A_k . The coefficients A_k can be found using the genetic algorithm (GA) optimization technique [3],

$$\varepsilon_{eff}(j\omega) = \varepsilon_{\infty eff} + \chi_{eff}(j\omega) = \varepsilon_{\infty eff} + \sum_{k=1}^{N} \frac{A_k}{1 + j\omega\tau_k}.$$
(5)

The *T*-matrix of the homogenized composite slab with a thickness of l is as in [4]:

$$T_S = \begin{bmatrix} e^{jkl\cos\theta} & 0\\ 0 & e^{-jkl\cos\theta} \end{bmatrix},\tag{6}$$

where $k = \omega \sqrt{\varepsilon_{eff} \mu}$ is the wave number of the effective composite medium, and θ is the angle of incidence.

2.3. T-matrix for an Interface of Two Media

Let two media have permittivites ε_1 , ε_2 and permeabilities μ_1 , μ_2 , respectively. The *T*-matrix [4] is

$$T_{I} = \frac{1}{\tau_{T1}} \begin{bmatrix} 1 & \rho_{T1} \\ \rho_{T1} & 1 \end{bmatrix},$$
(7)

where

$$\tau_{T1} = \frac{\eta_{T2} - \eta_{T1}}{\eta_{T2} + \eta_{T1}}, \ \rho_{T1} = \frac{2\eta_{T2}}{\eta_{T2} + \eta_{T1}}, \ \eta_{T1,2} = \begin{cases} \sqrt{\mu_{1,2}/\varepsilon_{1,2}}/\cos\theta & \text{for } TE \text{ plane wave;} \\ \sqrt{\mu_{1,2}/\varepsilon_{1,2}} \cdot \cos\theta & \text{for } TM \text{ plane wave.} \end{cases}$$
(8)

2.4. Plane Wave Formulas for Single-layered MFs 2.4.1. Extended GSTC for MFs

For an MF buried inside a homogeneous host medium with permittivity ε_{eff} and permeability μ , suppose the microscopic polarizability tensor $\overline{\overline{\alpha}}$ of the pattern is

$$\overline{\overline{\alpha}} = \begin{bmatrix} \overline{\overline{\alpha}}_{ee} & \overline{\overline{\alpha}}_{em} \\ \overline{\overline{\alpha}}_{me} & \overline{\overline{\alpha}}_{mm} \end{bmatrix}.$$
(9)

Extending the GSTC [5] for the case when there is cross-coupling between electric and magnetic polarizations of individual scatterers of MFs, the following boundary conditions for any metafilm in (xy) plane can be derived:

$$\hat{z} \times \vec{H} \Big|_{z=0^{-}}^{0^{+}} = j\omega \left[\overline{\overline{\alpha}}_{EE,t} \quad \overline{\overline{\alpha}}_{EM,t} \right] \cdot \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}_{av} - \hat{z} \times \nabla_t \left(\begin{bmatrix} \overline{\alpha}_{ME,z} \quad \overline{\overline{\alpha}}_{MM,z} \end{bmatrix} \cdot \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}_{av} \right);$$

$$\vec{E} \Big|_{z=0^{-}}^{0^{+}} \times \hat{z} = -j\omega\mu \begin{bmatrix} \overline{\overline{\alpha}}_{ME,t} \quad \overline{\overline{\alpha}}_{MM,t} \end{bmatrix} \cdot \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}_{av} - \frac{1}{\varepsilon_{eff}} \nabla_t \left(\begin{bmatrix} \overline{\overline{\alpha}}_{EE,z} \quad \overline{\overline{\alpha}}_{EM,z} \end{bmatrix} \cdot \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}_{av} \right) \times \hat{z}, \quad (10)$$

where the tensors are sub-components of the macroscopic polarizability, determined by the microscopic polarizability $\overline{\overline{\alpha}}$ and periodicity of the metafilm pattern. Assuming that the pattern periods are equal, $D_x = D_y = D$,

$$\overline{\overline{\alpha}}^{mac} = \begin{bmatrix} \overline{\overline{\alpha}}_{EE} & \overline{\overline{\alpha}}_{EM} \\ \overline{\overline{\alpha}}_{ME} & \overline{\overline{\alpha}}_{MM} \end{bmatrix} = \begin{bmatrix} D^2 \overline{\overline{I}} + \overline{\overline{\alpha}} \cdot \overline{\overline{G}} \end{bmatrix}^{-1} \cdot \overline{\overline{\alpha}}.$$
(11)

The matrix $\overline{\overline{G}} = Diag \left[-\frac{1}{4R\varepsilon_{eff}} - \frac{1}{4R\varepsilon_{eff}} \frac{1}{2R\varepsilon_{eff}} - \frac{1}{4R} - \frac{1}{4R} \frac{1}{2R} \right]$, where $R \approx 0.6956D$, according to [5].

2.4.2. T and Γ for Single-layered Patch-type MFs at the Oblique Plane Wave Incidence Consider the TE or TM plane waves as in Fig. 2.



Figure 2: TE and TM polarized plane waves.

Using the GSTC and the approach in [6], let us introduce the forward and backward vectors $\overline{C}_{TE(TM)}^+$ and $\overline{C}_{TE(TM)}^-$ as

$$\overline{C}_{TE}^{\pm} = \begin{bmatrix} 0 & 1 & 0 & \mp \cos \theta / \eta & 0 & \sin \theta / \eta \end{bmatrix}^T; \quad \overline{C}_{TM}^{\pm} = \begin{bmatrix} \cos \theta & 0 & \mp \sin \theta & 0 & \pm 1/\eta & 0 \end{bmatrix}^T, \quad (12)$$

where $\eta = \sqrt{\varepsilon_{eff}/\mu}$.

The following linear system can be derived for solving $T_{TE(TM)}$ and $\Gamma_{TE(TM)}$:

$$\begin{bmatrix} A_{1,TE(TM)} & A_{2,TE(TM)} \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} T_{TE(TM)} \\ \Gamma_{TE(TM)} \end{bmatrix} = \begin{bmatrix} A_{3,TE(TM)} \\ 1 \end{bmatrix},$$
(13)

$$A_{1,TE} = \left([U_{TE}(TM)] - [V_{TE}] \cdot \overline{\alpha}^{mac} \right) \cdot \overline{C}_{TE}^{+}; \qquad A_{1,TM} = \left([U_{TM}] + [V_{TM}] \cdot \overline{\alpha}^{mac} \right) \cdot \overline{C}_{TE}^{+};
A_{2,TE} = -\left([U_{TE}] + [V_{TE}] \cdot \overline{\alpha}^{mac} \right) \cdot \overline{C}_{TE}^{-}; \qquad A_{2,TM} = \left(-[U_{TM}] + [V_{TM}] \cdot \overline{\alpha}^{mac} \right) \cdot \overline{C}_{TM}^{-};
A_{3,TE} = \left([U_{TE}] + [V_{TE}] \cdot \overline{\alpha}^{mac} \right) \cdot \overline{C}_{TE}^{+}; \qquad A_{3,TM} = \left([U_{TM}] - [V_{TM}] \cdot \overline{\alpha}^{mac} \right) \cdot \overline{C}_{TE}^{+};$$
(14)

where the elements $A_{i,TE(TM)}$ are

$$\begin{bmatrix} U_{TE} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}; \qquad \begin{bmatrix} V_{TE} \end{bmatrix} = \begin{bmatrix} 0 & jk/(2\varepsilon_{eff}) & 0 & 0 & 0 & jk\sin\theta/2 \end{bmatrix}; \\ \begin{bmatrix} U_{TM} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \qquad \begin{bmatrix} V_{TM} \end{bmatrix} = \begin{bmatrix} j\omega/2 & 0 & 0 & 0 & 0 \end{bmatrix}.$$
(15)

2.4.3. T and Γ for Single Layered Aperture-type MFs

It is found that T and Γ cannot be calculated by directly applying the corresponding polarizabilities to the GSTC. However, this obstacle can be bypassed by solving the corresponding patch-type complementary problem, and then, using Babinet's duality principle [7,8], mapping the results into T and Γ of the aperture-type MFs [9]. The relations between T and Γ for two complementary arrays at oblique incidence are

$$T_{TE} = -\Gamma_{TM} \quad and \quad \Gamma_{TE} = -T_{TM}. \tag{16}$$

In (16), the tilde refers to the complementary structure.

T-matrix of a metafilm buried in a homogeneous host material is

$$T_{M_{TE(TM)}} = \frac{1}{T_{TE(TM)}} \begin{bmatrix} 1 & -\Gamma_{TE(TM)} \\ \Gamma_{TE(TM)} & T_{TE(TM)}^2 - \Gamma_{TE(TM)}^2 \end{bmatrix}$$
(17)

2.5. Requirement for Distance *d* between Neighboring MFs

The distance d between two neighboring MFs must be large enough for the evanescent modes to sufficiently decay and not interfere with the propagating mode. Given the ratio δ of the amplitude of the most intense high-order mode to the amplitude of the main propagating mode, the following inequality must fulfill:

$$\left|e^{jd(k-\sqrt{k^2-(2\pi/D)^2})}\right| < \delta.$$
⁽¹⁸⁾

Numerous simulations have shown that the ratio $\delta < 10\%$ is sufficient for neglecting the higher-order modes.

3. Genetic Algorithm for Synthesis of MF-Composite Shielding

Before the synthesis process, a designer should have some initial information based on a particular application of the shielding under design. The requirements for the desired frequency response of the shielding structure should be known, and an appropriate number of composite layers and the total maximum thickness of the structure, as well as the reasonable ranges of electromagnetic parameters of layers for the initial search pool should be specified.

The synthesis algorithm determines thickness and frequency dependence of the effective parameters of each layer. The GA yields a "recipe" of physical parameters (appropriate base material, aspect ratio, concentration, and conductivity of inclusions) for composite layers. The designer chooses the best solution (parameters characterizing the frequency dependence for composites, pattern geometry for MFs, and the order of layer disposition) for approximating the desired frequency response. This latter selection is based on a range of practically available ingredients with realistic parameters. Thus, the codes developed for the design of shielding structures with the desired frequency characteristic combine the Maxwell Garnett effective medium mixing rule, the described above analytical formulation, and the GA optimization procedure.

4. Computation Results

Consider the three-layer structure with two MFs as in Fig. 3(a). The parameters are the following: the slab thickness is $d_1 + d_2 + d_3 = 5$ mm, the cell period is $D_1 = D_2 = 2$ mm, the radius of the apertures in the left metafilm is $r_1 = 0.6$ mm, and the radius of the discs (or apertures) in the right metafilm is $r_2 = 0.6$ mm. The host dielectric is a composite containing carbon particles in a Teflon base $\varepsilon_b = 2.2$ (dispersion and loss are neglected). Carbon particles having conductivity of $\sigma = 1000$ S/m are shaped as cylinders with the aspect ratio a = length/diam = 50. Their volume fraction in the composite is 8%, while percolation threshold is higher than

9%. The best parameters of the composite from a shielding effectiveness point of view, and at the same time, practically available composite material components were chosen using the GA. The frequency characteristic of the composite dielectric with $\varepsilon_1(f)$ and $\mu_1 = \mu_0$ shown in Fig. 3(b) was modeled using (4) and approximated by one Debye term in (5). Fig. 3(c) shows the calculated transmission coefficient for the structure at different distances $d_{1,2,3}$. The best shielding effectiveness of the structure in the frequency range of interest is obtained with the thicknesses $d_1 = 1 \text{ mm}$; $d_2 = 3 \text{ mm}$; $d_3 = 1 \text{ mm}$ determined by the GA with two aperture MFs. Analytical and numerical simulation (using HFSS software) confirm this.



Figure 3: Multilayered structure with two different MFs buried in the composite dielectric layer.

5. Conclusions

The shielding structures containing MFs and composite dielectrics can be engineered based on the presented analytical formulas for T and Γ and using an optimization GA. T and Γ are directly related to geometries of MF patterns, constitutive parameters, concentrations, and geometries of composite material phases. This approach provides a straightforward synthesis process for desirable frequency responses.

The effective parameters of the composites are modeled by Maxwell Garnett mixing formalism. The analytical formulas for T and Γ of multilayered MF structures are obtained using (1) the generalized GSTC, (2) the Babinet's duality principle for complementary structures—aperture-type and patch-type MFs, and (3) the T-matrix cascading.

The analytical approach in this paper has an advantage over the full-wave numerical methods, since it saves computational resources for the synthesis process, and reduces the design cost prior to manufacturing.

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Statistical and Adaptive Signal Processing for UXO Discrimination for Next-generation Sensor Data

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Abstract—Until recently, detection algorithms could not reliably distinguish between buried UXO and clutter. leading to many false alarms. Over the last several years modern geophysical techniques have been developed that merge more sophisticated sensors, underlying physical models, statistical signal processing algorithms, and adaptive training techniques. These new approaches have dramatically reduced false alarm rates, although for the most part they have been applied to data collected at sites with relatively benign topology and anomaly densities. On more challenging sites, performance of even these more modern discrimination approaches is still quite poor. As a result, efforts are underway to develop a new generation of UXO sensors that will produce data streams of multi-axis vector or gradiometric measurements, for which optimal processing has not vet been carefully considered or developed. We describe a research program to address this processing gap, employing a synergistic use of advanced phenomenological modeling and signal-processing algorithms. The key foci of the program are (1) development of new physics-based signal processing approaches applicable to the problem in which vector data is available from such sensors; and (2) development of the theory of optimal experiments to guide the optimal design and deployment of the new sensor modalities. Here, we present initial results using simulated data obtained with our phenomenological models that indicate that optimal processing of features extracted from multi-axis EMI data can provide substantial improvements in discrimination performance over processing of features extracted from single-axis data.

1. Introduction

Until recently, detection algorithms have not reliably distinguished between buried UXO and clutter, leading to many false alarms. Over the last several years modern geophysical techniques have been developed that merge more sophisticated sensors, underlying physical models, statistical signal processing algorithms, and adaptive training techniques. These new approaches have significantly reduced false alarm rates, although for the most part they have been applied to data collected at sites with relatively benign topology and anomaly densities [1-5]. Most recently, blind source separation techniques have been applied to data collected on highly *cluttered* sites with commensurate reductions in the false alarm rates [6]. However, UXO clearance activities are ongoing or are planned at a wide variety of sites, and many of these activities involve complex terrain, vegetation and difficult geology, in addition to complex ordnance and clutter distributions. Moreover, most existing processing algorithms have been developed based on data associated with traditional sensors, such as CS-vapor magnetometers and coil-based electromagnetic induction sensors. Although these sensors are quite sensitive, they provide only limited information at any given survey point, as they do not illuminate the object under scrutiny along all three major axes. These limitations necessitate the use of highdensity spatial maps of anomalies, sometimes requiring multiple surveys, in order to collect data that support target localization and identification. Several investigators have noted that survey techniques that provide multi-angle illumination, while slower, improve discrimination performance [7, 8]. Others have shown theoretically or in limited field studies that sensors capable of multi-axis transmission provide data that result in improved object discrimination, as the parameters associated with the subsurface object are estimated more precisely [9].

In our previous efforts, we have successfully applied statistically-based signal processing algorithms using spatially-collected scalar EMI and magnetometer data for UXO discrimination. In one approach, probability density functions are developed that describe the statistical behavior of the data, x, obtained from a given sensor suite under the target and non-target hypotheses, denoted H_1 and H_0 , respectively. Given these probability density functions, $f(x|H_1)$ and $f(x|H_0)$, and a vector of sensor data, x, the optimal decision statistic, or likelihood ratio test (LRT), is utilized to make a decision regarding the appropriate hypothesis, this represented as $\Lambda(x) = f(x|H_1)/f(x|H_0)$. For EMI and/or magnetometer data, the vector x typically contains the parameters estimated from the data using a phenomenological model. (We discuss automatic feature selection algorithms below.) Features are estimated using a constrained search methodology. We have demonstrated that the multiple-local-minima problem associated with this search is largely mitigated using multi-axis sensor data, thus resulting in better parameter estimates and better discrimination performance [9, 10]. The probability density functions associated with the data under each hypothesis are estimated from training data.

Both the LRT and other approaches have been applied to several data sets, with outstanding results [2, 10]. Performance on several data sets indicated that these approaches performed very well when relevant training data were available. In particular, in one demonstration, these algorithms were scored in a blind test. Our goal in this current effort is to develop robust statistical signal processing techniques for data obtained from multi-axis sensors. Such processing has not previously been developed. We are considering simulated modeled on data from sensors already in development (e. g., Zonge nanoTEM system, the LBL/Morrison time-domain sensor, the USGS TMGS system, the QM MTG system, and the Oak Ridge SQUID-based system).



Figure 1: Standard deviation as a function noise variance for four different system configurations as noted in the legend. Ez indicates an excitation coil whose axis is perpendicular to the ground (z dimension), Rx, Ry, Rz denote receive coils in the x, y, and z dimension, and Rxyz denotes three receive coils, one in each of the dimensions.

2. Results

Typical electromagnetic induction (EMI) systems used for UXO detection had discrimination have two colocated coils—one used for transmitting the electromagnetic field, one used for receiving the field induced in the subsurface object. Historically, these coils have been located so that the axis of the coil is perpendicular to the ground. As noted above, recent studies have suggested that adding additional transmitter and/or receiver coils in different orientations can improve sensitivity and discrimination performance. The goal of this preliminary study was to assess the level of performance gain using simulated data, but realistic field scenarios and uncertainties.

It has been established that UXO can be adequately modeled using a dipole model [2, 4, 5, 8-10]. The imaginary resonant frequencies of the EMI resonant modes are a function of target material parameters. Thus, these features can be used after the data is inverted using the model for signal processing and classification. Imagine a cylinder coordinate system with the target's symmetry axis as z, the frequency-dependent moment can be expressed as

$$M(w) = \hat{z}\hat{z}[m_z(0) + \sum_{k=1}^{k} \frac{wm_{zk}}{w - jw_{zk}}] + (\hat{x}\hat{x} + \hat{y}\hat{y})[m_p(0) + \sum_{i=1}^{k} \frac{wm_{pi}}{w - jw_{pi}}]$$

The six target moment parameters are $m_z(0)$, $m_p(0)$, m_{zk} , m_{pi} , w_{zk} , and w_{pi} , where $m_z(0)$ and $m_p(0)$ account for the dipole moments contributed by ferrous targets. Parameters w_{zk} and w_{pi} are resonant frequencies, which are determined by the target geometry and material properties. Generally the first term in the sum, which is the principle dipole moment along each coordinate axis, is all that is needed to provide an accurate representation of the measured data from UXO and clutter.



Figure 2: ROC curves plotting probability of detection versus probability of false alarm for the case where measurement noise is included, but it is assumed that measurement position is known perfectly. Single axis and multi-axis results are shown.



Figure 3: ROC curves plotting probability of detection versus probability of false alarm for the case where measurement noise is included and fixed, but it is assumed that measurement position is uncertain. Single axis and multi-axis results are shown.

In our initial simulations, we consider a system with one transmitter at a fixed orientation and three receiver coils along three perpendicular axes: x, y, and z. We consider a simulated target whose resonant frequencies along horizontal and perpendicular directions are 463 Hz and 168 Hz. These parameters were estimated from 81 mm projectile field data. The target is located 0.5 meters from the sensor. To minimize the effect of the target orientation on conclusions based on this simulation, we use a uniform distribution for the target orientation. Using the dipole model, we calculate the electromagnetic field measured from the target when the receiver coil is located in the three different orientations and add Gaussian noise, as is normally observed from the instrument,

to the simulated field. Using our standard inversion algorithms [2, 9, 10], we obtain the estimated moments, the position and the orientation of the buried target from the simulated noisy field.

One mechanism by which to compare the performance of various system configurations is to consider the mean and standard deviations of the estimated moments/resonant frequencies as a function of the level of the Gaussian noise. Fig. 1 shows the standard deviation data for the simulated 81 mm target. Clearly, the three-axis receive coil provides better performance with increasing noise variance than any of the single axis systems.

In order to further quantify performance gain, the simulated 81 mm projectile was classified against a simulated clutter field where the clutter moments were distributed uniformly. A Bayesian classifier was used to discriminate the UXO object from the clutter using the estimated moments. Testing and training of the classifier were performed separately. Fig. 2 illustrates the classification performance achieved from three single axis and one multi-axis system. Some performance gain is obtained for the three axis system. Fig. 3 illustrates similar results, however in this case we simulated uncertainty in the position of the measurements. In this more realistic case, substantially more performance gain is observed in the case of the multi-axis system.

3. Conclusions

The results presented here indicate that a multi-axis system may potentially provide performance gain for discriminating UXO from clutter. We considered a case of discriminating an 81 mm projectile from a uniform field of clutter in the presence of additive Gaussian sensor noise. When no location uncertainty was included in the problem formulation, performance gains were small for the multi-axis sensor. However, when uncertainty in the exact location of the sensor was incorporated into the simulations, performance gain was enhanced substantially when the multi-axis sensor was utilized. This performance gain is a direct result of more accurate inversions possible with the multi-axis system.

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New GL Method and Its Advantages for Resolving Historical Difficulties

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Abstract—In this paper, we propose two types of new electromagnetic (EM) integral equation systems and their dual integral equation systems. Based on the EM integral equation systems, we propose the new GL EM modeling and inversion algorithms. Abstracts of our GL EM method based on the magnetic differential integral equation and the electrical integral differential equation have been published in PIERS 2005 in Hangzhuo. We used finite step iterations to exactly solve these integral equation systems or the EM and seismic differential integral equations in finite sub domains. The Global EM wave field is improved successively by the Local scattering EM wave field in the sub domains. In the FEM and FD method, the large matrix equation, inaccurate and complex absorption condition on artificial boundary, the cylindrical and spherical coordinate singularities, and ill posed in inversion are historical difficulties. The Born approximation is only used for low contrast material. The GL method is completely different from FEM, FD, and Born approximation. Our GL modeling and inversion resolved these historical difficulties. Only 3×3 or 6×6 small matrices need to be solved in the GL method; There is no artificial boundary for infinite domain in the GL method; In the GL method, the cylindrical and spherical coordinate singularities are resolved; Our GL method combines the analytic and asymptotic method and numerical method perfectly. It is more accurate than FEM and FD method and Born likes approximation. The GL method is available for all frequencies and high contrast materials. The GL solution has $O(h^2)$ convergent rate. If the Gaussian integrals are used, the GL field has $O(h^4)$ super convergence. The GL method is a high perform parallel algorithm with intrinsic self parallelization properties. The FEM and FD scheme of high order PDE are complicated. Fortunately, the GL method has very simple scheme or no scheme or half scheme such that it has half mesh and no mesh. The FEM and FD scheme only used Riemann integral. In the GL method, we can use both of Riemann and Lebesgue integral that induces a meshless method. We have developed software for 3D/2.5D EM, seismic, acoustic, flow dynamic, and QEM modeling and inversion. Our GL modeling and inversion are useful for geophysical and Earthquake exploration, environment engineering, nondestructive testing, steel and metal casting, weather radar, medical, Earth magnetic, antenna, and heating conductive imaging, space sciences and lunar and sun and stars EM and light exploration. The GL QEM modeling and inversion can be useful for studying micro optical physical and biophysical properties in nanometer materials and biophysics materials. The GL and AGILD method resolved the singularity difficulties at the poles in Navier-Stocks flow atmosphere simulation and Earth and Space EM field. We find GL numerical quanta for very high frequency EM field by GL simulation.

1. Introduction

The existing EM theory and analytical and numerical methods are published in many books and journals. However, there are historical difficulties in EM and other field modeling and inversion. The large matrix equation, inaccurate and complex absorption conditions on artificial boundary, the cylindrical and spherical coordinate singularities, and ill posed in the modeling and inversion are historical difficulties. The Born approximation can be only used for low contrast material. In this paper, we propose a new GL method "Global and Local field modeling and inversion" for resolving these historical difficulties. Our GL method is completely different from the FEM, FD, Born approximation methods.

We consider the EM, seismic, acoustic, quantum, flow and other field equations on finite inhomogeneous domain that is imbedded into an infinite domain. The analytical incident field and Green field in the background domain are called an initial global field. The inhomogeneous domain is divided into mesh or meshless sub domains. The global field is changing by local scattering field successively in each sub domain. The GL method processes will be finished when the Global field is passing through the all Local sub domains with inhomogeneous material. First in the world, the abstracts of our GL method have been published in Piers 2005 in Hangzhuo. [1, 4–9], and in the GL Geophysical Laboratorys reports [2–3].

The new GL method has the following advantages: (1) There is no large matrix to solve, only 3×3 or 6×6 small matrices need to be solved; (2) There is no artificial boundary for infinite domain; (3) The GL method combines the analytic and asymptotic method and numerical method consistently. It is more accurate

than FEM and FD method and Born likes approximation; (4) The GL modeling solution has $O(h^2)$ convergent rate. In particular, if the Gaussian integrals are used, the GL solution has $O(h^4)$ super convergence; (5) The cylindrical and spherical coordinate singularities are resolved; (6) It is available for all frequencies and high contrast materials; (7) the GL method has very simple or no scheme, it has half mesh or no mesh; (8) In the GL method, we can use both Riemann and Lebesgue integrals that induce meshless methods; (9) GL method can couple consistently with AGILD, FEM, and FD method; (10) The GL method is an intrinsic self parallel algorithm in parallel T3E and PC cluster.

The plan of this paper is as follows: The introduction is presented in the section 1. In the section 2, we propose the EM integral equation systems. We propose the 3D/2D GL EM modeling based on the EM integral equation system in the section 3. In section 4, we propose the 3D/2D GL EM modeling based on the EM differential integral equation and electric and magnetic field integral equations. We propose the GL EM inversion in the section 5. In section 6, we prove the fundamental theorems of the GL method. We describe advantages of the GL method in the section 7. The GL software, applications and conclusions are described in the section 8.

2. New Electromagnetic Integral Equation Systems

In this section, we propose the new EM integral equation systems as follows:

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int_{\Omega} \begin{bmatrix} E_b^J(r', r) & H_b^J(r', r) \\ E_b^M(r', r) & H_b^M(r', r) \end{bmatrix} [D] \begin{bmatrix} E(r') \\ H(r') \end{bmatrix} dr',$$
(1)

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int_{\Omega} \begin{bmatrix} E^J(r', r) & H^J(r', r) \\ E^M(r', r) & H^M(r', r) \end{bmatrix} [D] \begin{bmatrix} E_b(r') \\ H_b(r') \end{bmatrix} dr',$$
(2)

where [D] is the EM material parameter variation matrix, for the isotropy materials, [D] is 6×6 diagonal matrix with variance materials $(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)$ and $i\omega(\mu - \mu_b)$, for anisotropy materials the [D] will be 6×6 full matrix. E(r) is the electric field, H(r) is the magnetic field, $E_b(r)$ is incident electric field in the background medium, $H_b(r)$ is incident magnetic field in the background medium, $E_b^M(r', r), \ldots, H_b^M(r', r)$ are electric or magnetic background field Green tensors exciting by the electric or magnetic dipole source respectively, The integral equations (1) and (2) are the dual system of each other.

3. The 3D/2D New GL EM Modeling Based on the Electromagnetic Integral Equation System

We propose the GL EM modeling based on the EM integral equation system in this section.

- (3.1) The domain Ω is divided into a set of n mesh or meshless sub domains $\{\Omega_k\}, \Omega = \bigcup \{\Omega_k\}.$
- (3.2) In each Ω_k , we solve the EM Green tensor integral equation system based on the equations (1) and (2). By dual curl operation, the equation systems are reduced into a 6×6 matrix equations. By solving the 6×6 equations, we obtain Green tensor field E_k^J and H_k^M .
- (3.3) We improve the Global EM field $[E_k(r), H_k(r)]$ by the Local scattering field

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_{k} = \begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_{k-1} + \int_{\Omega_{k}} \begin{bmatrix} E_{k}^{J}(r',r) & H_{k}^{J}(r',r) \\ E_{k}^{M}(r',r) & H_{k}^{M}(r',r) \end{bmatrix} [D] \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{k-1} dr',$$
(3)

k = 1, 2, ..., n, successively. The $[E_n(r), H_n(r)]$ is the GL solution of the EM integral equations (1) and (2).

4. The 3D/2D GL EM Modeling Based on the EM Differential Integral Equation

4.1. The GL EM Modeling Based on the Magnetic Differential Integral Equation

Since 1995, we have proposed the magnetic field differential integral equation (MDI) in the frequency and time domain [10-13]. In this section, we propose the dual magnetic field differential integral equation of our MDI [10-13], $f(\tau + i v c) = (\tau + i v c)$

$$H(r) = H_b(r) + \int_{\Omega} \frac{(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)}{\sigma + i\omega\varepsilon} E^M(r', r) \cdot \nabla \times H_b(r') dr'.$$
(4)

Based on the equation (4), the GL magnetic field modeling is as follows:

(4.1) The step (4.1) is the same as (3.1).

- (4.2) In each Ω_k , k = 1, 2, ..., n, we solve the magnetic field differential integral equation to find $E_k^M(r', r)$ successively. By the dual curl operation, only 3×3 matrix equations need to be solved.
- (4.3) We improve the Global EM field $H_k(r)$ by the Local scattering field

$$H_k(r) = H_{k-1}(r) + \int_{\Omega_k} \frac{(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)}{\sigma + i\omega\varepsilon} E_k^M(r', r) \cdot \nabla \times H_{k-1}(r')dr', \tag{5}$$

k = 1, 2, ..., n, successively. $H_n(r)$ is the GL magnetic field solution of (4).

4.2. GL EM Modeling Based on the Electric Differential Integral Equation

We propose the GL electric field modeling based on the dual electric field differential integral equation of our EDI in 1995[10-13], $f \mu = \mu$.

$$E(r) = E_b(r) + \int_{\Omega} \frac{\mu - \mu_b}{\mu} H^J(r', r) \cdot \nabla \times E_b(r') dr'.$$
(6)

4.3. GL EM Modeling Based on the Electric and Magnetic Integral Equation

We propose the GL method based on the electric integral equation and the magnetic integral equation. Since the electric and magnetic integral equations have divergent Green kernel, a special approach for resolving the divergent singularity is developed.

4.4. GL Modeling for Quantum Field and QEM Field

We propose the GL Schordinger modeling for two hydrogen atoms and interaction between QEM field and atoms that is useful for QEM field in nanometer materials. We find GL numerical quanta for very high frequency EM field by GLQEM simulation.

5. The New GL EM Inversion

The formal logic system and experiments are base of the sciences. Most equations are forward equations. Maxwell equation and elastic equation are forward equation and are not for inversion. The EM integral equation systems (1) and (2) and equations (4) and (6) can be used for both forward and inversion. They are well posed for forward and ill posed for inversion. From essential formal logic in physics, these equations are well posed for forward and ill posed for inversion. How to build a well posed inverse equation is the main project of scientific inversion. Our new idea of the inverse formal logic and inverse experiment in physics motivates us to propose the GL inversion that is a new explicit inversion.

5.1. The GL EM Inversion GLEMI1 for Determining σ , ε , and μ

The following EM integral equation is for increments of EM parameters $\delta\sigma$, $\delta\varepsilon$, $\delta\mu$,

$$\begin{bmatrix} \delta E(r) \\ \delta H(r) \end{bmatrix}_{k} = \int_{\Omega_{h}} \begin{bmatrix} E_{k}^{L}(r',r) & H_{k}^{L}(r',r) \\ E_{k}^{M}(r',r) & H_{k}^{M}(r',r) \end{bmatrix} [\delta D]_{k} \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{k-1} dr'.$$

$$(7)$$

5.2. The GL EM Inversion GLEMI2 for Determining σ , ε

The following magnetic field differential integral equation is for increments of parameters $\delta\sigma$, $\delta\varepsilon$,

$$\delta H_k(r) = -\int\limits_{\Omega_k} \frac{(\delta\sigma + i\omega\delta\varepsilon)}{(\sigma + i\omega\varepsilon)^2} E_k^M(r', r) \cdot \nabla \times H_{k-1}(r') dr'.$$
(8)

5.3. The GL EM Inversion GLEMI3 for Determining μ

The following electric field differential integral equation is for increment of EM parameter $\delta \mu$,

$$\delta E_k(r) = -\int_{\Omega_k} \frac{\delta \mu}{\mu^2} H_k^M(r', r) \cdot \nabla \times E_{k-1}(r') dr'.$$
(9)

The suitable strong and weaker regularizing should be added to (7), (8), and (9) to control inversion being stable and reasonable resolution. In our GL EM inversion, only smaller matrices need to be solved. The resolution is dependent on the data configuration, quality and the regularizing parameter.

6. The Fundamental Theorems of the GL Method

Theorem 1. The GL EM field $[E_n(r), H_n(r)]$ from (3.1)–(3.3) is convergent to exact EM field that satisfies the EM integral equation systems (1) and (2). The GL EM field $[E_n(r), H_n(r)]$ is convergent to exact EM field that satisfies the MAXWELL EM equation in 3D or 2D. Theorem 2. The GL Magnetic field, $H_n(r)$ from (4.1)–(4.3) is convergent to the exact magnetic field, H(r) that satisfies the magnetic field differential integral equation (4). The GL EM field, $H_n(r)$ is convergent to the exact magnetic field H(r) that satisfies the exact MAXWELL EM equation.

Theorem 3. By Riemann division, the GL EM field $[E_n(r), H_n(r)]$ from (3.1–3.3) and the GL magnetic field $H_n(r)$ from (4.1–4.3) have $O(h^2)$ convergent if the trapezoid and mid point integrals are used. In particular, if the Gaussian integrals are used, the GL EM field has $O(h^4)$ super convergent rate. Proof: The theorem 1–3 have proved in [2].

7. Advantages of the GL Method

We have summarized the advantages of the GL method in the introduction. By reviewing the GL modeling and inversion in section 3, 4, and 5, we present several advantages as follows. We consider EM modeling in infinite domain that involves the finite inhomogeneous boundary domain. When we use FEM or implicit FD method to solve the problem, we need the radiation or absorption boundary condition on the artificial boundary with large enough domain. Solving the large matrix is difficult. The radiation and absorption boundary condition is complicated and inconvenience. In the EM inversion, the FEM and FD EM modeling is used in iterations. The absorption boundary errors will propagate into the internal domain, the noise is enhancing to damage the inversion. In the section 2, we propose the EM integral equation systems (1) and (2) that are equivalent to the 3D and 2.5D Maxwell EM equation in infinite domain with finite inhomogeneous domain for isotropic and anisotropic materials. Our GL EM modeling does not need any artificial boundary for solving the EM integral equation and the magnetic differential integral equation. Our GL EM modeling only needs to solve 3×3 or 6×6 small matrices, it does not need to solve any large matrix. There are $1/\rho^2$ singularity in the cylindrical coordinate and $1/r^2$, $1/\sin^2\phi$ singularities in the spherical coordinate system for Maxwell equation. These coordinate singularities are historic difficulties in FEM and FD method. In the EM integral equations (1-3)and electric and magnetic differential integral equations (4–6) for the cylindrical and spherical coordinate, the coordinate singularities are resolved. There is no coordinate singularity in the GL method. The GL modeling combines analytical and numerical methods consistent together and has super convergence. The GL method resolve many historical difficulties in traditional FEM, FD, and Bron_approximation methods



Figure 1: GL and ML Electric wave with freq. $1.6e^{6}$ Hz Figure 2: GL and ML Electric wave with freq. $1.6e^{8}$ Hz

We have created the GL method Since 2002. We have developed the seismic, EM, acoustic, flow, and Quantum field GL modeling and inversion algorithms and software. Many simulations show that the GL seismic and EM wave field has no any boundary error reflection. We have made several GL seismic and EM wave propagation movies that show the wave excited by internal sources is out going propagation perfectly without any error reflection on the boundary. Because the page limitation, we only use one dimension wave propagation to compare GL method and FEM method in the frequency domain. The absorption boundary condition is used for FEM. The numerical results show that GL wave is very accurate to match the multiple layer analytic wave for the high frequency 1.6×10^6 (Figure 1) and frequency 1.6×10^8 (Figure 2). The Figures 3 and 4 show that the FEM is fail to approximate the exact wave in the high Frequency. Our GL method and AGILD method have used in the EM stirring magnetic field simulation and obtained very accurate EM field. The GL, ML, and FEM total and scattering electric wave are shown in Figure 5 and figure 6 respectively. They show that the GL electric wave is very accurate to match to multiple layer wave, but FEM wave is not. Many 2.5D and 3D GL EM and seismic Wave show that GL modeling is accurate, fast and stable. The GL inversion is reasonable high resolution



GL1DE wave C ML1DE wave FEM1DE wave 0.8 Electric wave field in frequency 1.6e+6 (Real 0.6 0.4 02 0.2 04 -0.6 -08 0 X Variable (m)

Figure 3: GL, ML and FEM Electric wave with freq. $1.6e^6$ Hz

Figure 4: GL, ML and FEM Electric wave with freq. $1.6e^8$ Hz



domain



Figure 5: GL, ML and FEM Electric wave E(0,t) in time Figure 6: GL and ML Scat. Electric Swave SE(0,t) on time

8. GL Software and Applications and Conclusions

We develope many 3D and 2.5D GL EM, seismic, acoustic, flow, QEM modeling software and some GL EM and seismic inversion software. These GL EM softwares are useful for geophysical EM and seismic exploration; Earthquake EM and seismic exploration; Forest EM and seismic exploration; Environment; EM field in nanometer materials and superconductivity [6]; nondestructive testing imaging [5]; Airborne EM exploration; The stress and displacement analysis in dam, rock, underground structure; the EM Stirring and flow for caster [7]; GPR, radar, and weather imaging; Naiver Stocks weather simulation, etc.. Many applications show that the GL modeling is very fast, low cost and accurate. The GL inversion is stable and high resolution. The GL EM field is fast convergent to exact EM field for high frequency and contrast, while FEM method fails to simulate wave field in the high frequency. The GL method is breakthrough novel method and resolve historical difficulties. GL Geophysical Laboratory and authors have reserved all copyright and patents of 3D/2.5D/2D GL EM, seismic, flow, acoustic QEM modeling and inversion algorithms and have reserved all copyright and patents of the GL software.

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MATLAB SIMULINK Based DQ Modeling and Dynamic Characteristics of Three Phase Self Excited Induction Generator

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Abstract—In this paper, DQ-modeling approach for Transient State analysis in the time domain of the threephase self-excited induction generator (SEIG) with squirrel cage rotor is presented along with its operating performance evaluations. The three-phase SEIG is driven by a variable-speed prime mover (VSPM) such as a wind turbine for the clean alternative renewable energy in rural areas. Here the prime mover speed has been taken both as fixed and variable and results have been analyzed. The basic Dynamic characteristics of the VSPM are considered in the three-phase SEIG approximate electrical equivalent circuit and the operating performances of the three-phase SEIG coupled by a VSPM in the Transient state analysis are evaluated and discussed on the conditions related to transient occurs in the system and speed changes of the prime mover.

The whole proposed system has been developed and designed using MATLAB / SIMULINK.

1. Introduction

A wind electrical generation system is the most cost competitive of all the environmentally clean and safe renewable energy sources in the world. It is well known that the three phase self excited induction machine can be made to work as a self-excited induction generator [3, 4], provided capacitance should have sufficient charge to provide necessary initial magnetizing current [5, 6]. In an externally driven three phase induction motor, if a three phase capacitor bank is connected across it's stator terminals, an EMF is induced in the machine windings due to the self excitation provided by the capacitors. The magnetizing requirement of the machine is supplied by the capacitors. For self excitation to occur, the following two conditions must be satisfied:

- 1 The rotor should have sufficient residual magnetism.
- 2 The three capacitor bank should be of sufficient value.

If an appropriate capacitor bank is connected across the terminals of an externally driven Induction machine and if the rotor has sufficient residual magnetism an EMF is induced in the machine windings due to the excitation provided by the capacitor. The EMF if sufficient would circulate leading currents in the capacitors. The flux produced due to these currents would assist the residual magnetism. This would increase the machine flux and larger EMF will be induced. This in turn increases the currents and the flux. The induced voltage and the current will continue to rise until the VAR supplied by the capacitor is balanced by the VAR demanded by the machine, a condition which is essentially decided by the saturation of the magnetic circuit. This process is thus cumulative and the induced voltage keeps on rising until saturation is reached. To start with transient analysis the dynamic modeling of induction motor has been used which further converted into induction generator [8, 11]. Magnetizing inductance is the main factor for voltage buildup and stabilization of generated voltage for unloaded and loaded conditions. The dynamic Model of Self Excited Induction Generator is helpful to analyze all characteristic especially dynamic characteristics. To develop dynamic model of SEIG we first develop the dynamic model of three phase induction motor in which the three phase to two phase conversion has been done using Park's transformation, then all the equations have been developed. The traditional tests used to determine the parameters for the equivalent circuit model are open circuit and short circuit test. In this paper the DQ model shown in Fig. 1 has been used to obtain the dynamic characteristics and further a flux oriented controller is proposed to improve the dynamic characteristics.

2. Modeling of Self Excited Induction Generator

The equation shown is used for developing the dynamic model of SEIG

$$[V_G] = [R_G][i_G] + [L_G]p[i_G] + w_{rG}[G_G][i_G]$$
(1)

Where **p** represents the derivative w.r.t. time, $[V_G]$ and $[i_G]$ represents 4×1 column matrices of voltage and which is given as $[V_G] = \begin{bmatrix} V_{sd} & V_{sq} & V_{rd} & V_{rq} \end{bmatrix}^T$ and $[I_G] = \begin{bmatrix} i_{sd} & i_{sq} & i_{rd} & i_{rq} \end{bmatrix}^T [R]$, [L] and [G] represents 4×4



DQ MODEL OF THREE PHASE SELF EXCITED INDUCTION GENERATOR

Figure 1: DQ model of three phase induction generator.



Figure 2: Magnetizing inductance Vs magnetizing current.

matrices of resistance, generator inductance and conductance as given. Further Lm the magnetizing inductance, which can be obtained from the magnetizing curve of the machine shown in Fig. 2,

$$[L] = \begin{bmatrix} L_{sd} & L_{dq} & L_{md} & L_{dq} \\ L_{dq} & L_{sq} & L_{dq} & L_{md} \\ L_{dq} & L_{mq} & L_{dq} & L_{rq} \end{bmatrix} [G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & L_m & 0 & L_r \\ -L_m & 0 & -L_r & 0 \end{bmatrix} [L] = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix}$$

The Relation between L_m and i_m is given as

$$L_m = |\Psi_m| / |i_m| \tag{2}$$

Where $|\Psi_m|$ and i_m are the magnetizing flux linkage and magnetizing current. The equation defining L_m Vs $|i_m|$ used in the model is

$$|i_m| = 1.447 * L_m^6 - 8.534 * L_m^5 + 18.174 * L_m^4 - 17.443 * L_m^3 + 7.322 * L_m^2 - 1.329 * L_m + 0.6979$$
(3)

 L_{dq} used in matrix L represent the cross saturation coupling between all axes in space quadrature and is due to saturation. $L_{dq} = L_m + i_{md}/i_{mq} * L_{dq}$, $L_{mq} = L_m + i_{mq}/i_{mq} * L_{dq}$. It follows that above equations representing L_{dq} , L_{md} , and L_{mq} that under linear magnetic conditions, $L_{dq} = 0$ and $L_{md} = L_{mq} = L_m$, as expected. The two axes values of the total stator and rotor inductances are $L_{sd} = L_{s1} + L_{md}$, $L_{sq} = L_{s1} + L_{mq}$ and $L_{rd} = L_{r1} + L_{md}$, $L_{rq} = L_{r1} + L_{mq}$.

The above equations L_{s1} and L_{r1} are the leakage inductances of the stator and rotor, respectively. Because of saturation, L_{sd} ? L_{sq} , but it follows from previous arguments that under linear magnetic conditions $L_{sd} = L_{sq}$. Hence $L_r = L_{r1} + L_m$

The electromagnetic torque developed by the generator is given by

$$T_e = (3/4) * P * L_m * (i_{sq} \quad i_{rd} - i_{sd} \quad i_{rq})$$
(4)

Thus it sees that Eq. (1) consists of four first order equations. An induction motor is hence represented by these four first order differential equations. Because of the non-linear nature of the magnetic circuit, the magnitude of magnetizing current, I_m is calculated as

$$I_m = [(i_{sd} + i_{rd})^2 + (i_{sq} + i_{rq})^2]^{1/2}$$
(5)

Capacitor side equations are

$$p[v_{sG}] = (1/C)[i_C]$$
(6)

further

$$[i_C] = [i_{sG}] + [i_L] \tag{7}$$

Where $[v_{sG}]$, $[i_{sG}]$, $[i_L]$ Column matrices representing direct and quadrature axis component of capacitor current generator stator current and load current respectively.

Load Side Equations

$$[v_{sG}] = L_L p[i_L] + R_L[i_L] \tag{8}$$

Thus it is seen that complete transient model of the SEIG in d-q axis quasi stationary reference frame consists of equations from (1) to (8).

3. Modeling of Self Excited Induction Generator Using MATLAB SIMULINK

MATAB SIMULINK is powerful software tool for modeling and simulation and accepted globally. The equations from (1) to (8) have been implemented in MATLAB SIMULINK using different blocks. In this paper the step by step modeling of SEIG using SIMULINK has been described.



Gain1 Gain1 Te A Product2 Te Product2 Te Product2 Te Product1 isd

Figure 3: Stator and rotor dq currents.





Figure 7: Simulink model of proposed system.

Equation (1) shown has four first order differential equations, for which the solutions gives the four currents (stator d-q axis currents and rotor d-q axis currents). Further these currents are the function of constants viz. Stator and rotor Inductances and Resistances, speed, Excitation Capacitance, load resistances. And also variables like Magnetizing Inductance, Magnetizing currents, Electromagnetic torque generated, has been evaluated using (3) to (5). The constraints of non linear magnetizing inductance have been taken into accounts, the curve between Non linear magnetizing inductance vs magnetizing currents is shown in Fig. 2. The equation of this non linear graph has been obtained by curve fitting and hence sixth order nonlinear polynomial equation which is showing the relation between magnetizing inductance vs Magnetizing current. This equation has been implemented using function block in SIMULINK block sets. In Fig. 3, the SIMULINK model of stator and rotor

dq currents has been shown, Similarly Fig. 4, shows the electromagnetic torque generated. The load block in which the stator voltage determined has been shown in Fig. 5. To put the various parameters the masking of overall blocks has been done. The values and the masked blocks have been shown in Fig. 6.

4. Modeling Using MATLAB SIMULINK

The equation above described has been implemented in MATLAB / SIMULINK block sets. The equations from 1 to 7 have been implemented in subsystem "Self Excited Induction Generator" whose outputs are Torque, currents, rotor angle (theta), magnetizing current. Similarly the other blocks are Inverter, load, and a subsystem to find three phase voltages.

5. Results and Discussions

The Model has been simulated using MATLAB/SIMULINK shown in Fig. 7. The analysis has been done taking various constraints mainly-(i) assuming constant speed and no controller (Fig. 8), (ii) assuming variable speed without controller (Fig. 9), (iii) Constant speed which going to constant at 0.1 sec with controller (Fig. 10). And finally (iv) variable speed with controller (Fig. 11).



Figure 8: Electromagnetic torque, three phase voltage, current at constant wind speed.



Figure 10: Electromagnetic torque, three phase voltage, current at constant wind speed with controller.

250 Torque(Tet in Nn
50
-20 VokageVato/V

Figure 9: Electromagnetic torques, three phase voltage, current at variable wind speed without controller.



Figure 11: Electromagnetic torque, three phase voltage, current at variable wind speed with controller.

In first case the electromagnetic torque generated has been reached to steady state 0.2 second. Initially transients occur at 0.05 sec when currents goes to 10 ampere provided constant voltage. Note that load is constant all the time and for sake of simplicity resistive load has been considered. In second case the variable wind speed has been considered. To implement the variable speed in SIMULINK a repeating sequence block has been used. It has been observed that the electromagnetic torque developed has been a vibrations in steady

state. The currents in this case have some ripple in the waveform. In the third case the controller has been implemented and response has been observed. A ramp signal has been taken which further becomes constant at 0.05 sec, as constant speed starting from zero. As a result a transient has been occurring at 0.05 sec which then comes to steady state at 0.1 sec. And finally the wind speed has been taken as variable speed with controller has been implemented which result no transients has been occur at output currents but some ripples have been still remaining in currents waveform as shown in Fig. 9.

6. Conclusions

Self Excited Induction generator has been found suitable applicability for isolate applications. The estimation of non linear magnetizing inductance is the main factor of converting the Induction motor as self excitation induction generator. To develop the system as wider applicability the controller has been designed to improve the dynamic characteristics of the system. It has been shown that the transients have been removed when controller has been implemented.

7. Specifications of the Machine

 $10\,\mathrm{h.p}$ (7.5 kW), 3-phase, 4 poles, 50 Hz, 415 volts, 3.8 A Delta connection,

 $\begin{array}{l} \text{Base Voltage / Rated Voltage = 415 V} \\ \text{Base Current / Rated Current = 2.2 A} \\ \text{Rs = 1.0 ohm} \\ \text{Rr = 0.77 ohm} \\ \text{Xls = Xlr = 1.0 ohm} \\ \text{J = 0.1384 kg-m^2} \end{array}$

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Abstract—In this paper, a signal processing method considering not only linear correlation but also the higher order nonlinear correlation information is proposed on the basis of fuzzy observation data, in order to find the mutual relationship between sound and electromagnetic waves leaked from electronic information equipment. More specifically, by applying the well-known fuzzy probability to an expression on the multi-dimensional probability distribution in an orthogonal expansion series form reflecting systematically various types of correlation information, a method to estimate precisely the correlation information between the variables from the conditional moment statistics of fuzzy variables is proposed. The effectiveness of the proposed theory is experimentally confirmed by applying it to the observation data leaked from VDT in the actual work environment.

1. Introduction

Some studies on the mutual relationship between sound and electromagnetic waves leaked from electronic equipment in the actual working environment have become important recently because of the increased use of various information and communication systems like the personal computer and portable radio transmitters [1, 2], especially concerning their individual and/or compound effects on a living body. Sound and electromagnetic waves, especially, are often measured in a frequency domain under the standardized measuring situation in a reverberation room, anechoic room and radiofrequency anechoic chamber. Though these standard methods in a frequency domain are useful for the purpose of analyzing the mechanism of individual phenomena, they seem to be inadequate for evaluating total effects on the compound or the mutual relationship between sound and electromagnetic waves in complicated circumstances, such as the actual working environment. In order to evaluate universally the mutual correlation characteristics and its total image in the actual complex working environment, it is necessary to introduce some signal processing methods, especially in a time domain.

On the other hand, the actual observed data often contain fuzziness due to confidence limitations in sensing devices, permissible errors in the experimental data, and quantizing errors in digital observations. Therefore, in order to evaluate precisely the objective sound and electromagnetic environments, it is desirable to estimate the mutual relationship between sound and electromagnetic waves based on the fuzzy observations.

In this study, a signal processing method considering not only linear correlation but also the higher order nonlinear correlation information is proposed on the basis of fuzzy observation data, in order to find the mutual relationship between sound and electromagnetic waves leaked from electronic information equipment. More specifically, a conditional probability expression for fuzzy variables is first derived by applying the fuzzy probability [3] to a multi-dimensional joint probability function in a series type expression reflecting information on various correlation relationships between the variables. Next, by use of the derived probability expression, a method for estimating precisely the correlation information from various conditional moment statistics based on the observed fuzzy data is theoretically proposed. On the basis of the estimated correlation information, the probability distribution for a specific variable (e. g., electromagnetic wave) based on the observed fuzzy data of the other variable (e. g., sound) can be predicted. Finally by applying the proposed methodology to the measurement fuzzy data in an actual working environment, the effectiveness of theory is confirmed experimentally.

2. Prediction of Specific Probability Distribution from Arbitrary Fuzzy Fluctuation Factor

The observed data in the actual sound and electromagnetic environments often contain fuzziness due to several factors such as limitations in the measuring instruments, permissible error tolerances in the measurement, and quantization errors in digitizing the observed data.

In order to evaluate quantitatively the complicated relationship between sound and electromagnetic waves leaked from an identical electronic information equipment, let two kinds of variables (i. e., sound and electromagnetic waves) be x and y, and the observed data based on fuzzy observations be X and Y respectively. There exist the mutual relationships between x and y, and also between X and Y. Therefore, by finding the relations between x and X, and also between y and Y, based on the fuzzy probability [3], it is possible to predict the true value y (or x) from the observed fuzzy data X (or Y). For example, for the prediction of the pdf (probability density function) $P_s(y)$ of y from X, averaging the conditional pdf P(y|X) on the basis of the observed fuzzy data X, $P_s(y)$ can be obtained as: $P_s(y) = \langle P(y|X) \rangle_X$. The conditional pdf P(y|X) can be expressed under the employment of the well-known Bayes' theorem:

$$P(y|X) = \frac{P(X,y)}{P(X)}.$$
(1)

The joint probability distribution P(X, y) is expanded into an orthonormal polynomial series on the basis of the fundamental probability distribution $P_0(X)$ and $P_0(y)$, which can be artificially chosen as the probability function describing approximately the dominant parts of the actual fluctuation pattern, as follows:

$$P(X,y) = P_0(X)P_0(y)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{mn}\psi_m(X)\phi_n(y), A_{mn} = \langle \psi_m(X)\phi_n(y) \rangle.$$
(2)

The information on the various types of linear and nonlinear correlations between X and y is reflected in each expansion coefficient A_{MN} . When X is a fuzzy number expressing an approximated value, it can be treated as a discrete variable with a certain level difference. Therefore, as the fundamental probability function $P_0(X)$, the generalized binomial distribution with a level difference interval h_X can be chosen:

$$P_{0}(X) = \frac{\left(\frac{N_{X} - M_{X}}{h_{X}}\right)!}{\left(\frac{X - M_{X}}{h_{X}}\right)!\left(\frac{N_{X} - X}{h_{X}}\right)!} p_{X}^{\frac{X - M_{X}}{h_{X}}} (1 - p_{X})^{\frac{N_{X} - X}{h_{X}}},$$

$$p_{X} = \frac{\mu_{X} - M_{X}}{N_{X} - M_{X}}, \ \mu_{X} = \langle X \rangle,$$
(3)

where M_X and N_X are the maximum and minimum values of X. Furthermore, as the fundamental pdf $P_0(y)$ of y, the standard Gaussian distribution is adopted:

$$P_{0}(y) = \frac{1}{\sqrt{2\pi\sigma_{y}^{2}}} exp\{-\frac{(y-\mu)^{2}}{2\sigma_{y}^{2}}\},\$$

$$\mu_{y} = \langle y \rangle, \ \sigma_{y}^{2} = \langle (y-\mu_{y})^{2} \rangle.$$
(4)

The orthonormal polynomials $\psi_m(X)$ and $\phi_n(y)$ with the weighting functions $P_0(X)$ and $P_0(y)$ can be determined as [4]

$$\psi_m(X) = \{ (\frac{N_X - M_X}{h_X})^{(m)} m! \}^{-\frac{1}{2}} (\frac{1 - p_X}{p_X})^{\frac{m}{2}} \frac{1}{h_X^m} \\ \cdot \sum_{j=0}^m \frac{m!}{(m-j)! j!} (-1)^{m-j} (\frac{p_X}{1 - p_X})^{m-j} (N_X - X)^{(m-j)} (X - M_X)^{(j)}, \\ (X^{(n)} = X(X - h_X) \cdots (X - (n-1)h_X), \ X^{(0)} = 1),$$
(5)

$$\phi_n(y) = \frac{1}{\sqrt{n!}} H_n(\frac{y - \mu_y}{\sigma_y}); \text{ Hermite polynomial}$$
(6)

Thus, the predicted pdf $P_s(y)$ can be expressed in an expansion series form:

$$P_s(y) = P_0(y) \sum_{n=0}^{\infty} < \frac{\sum_{m=0}^{\infty} A_{mn} \psi_m(X)}{\sum_{n=0}^{\infty} A_{m0} \psi_m(X)} >_X \phi_n(y).$$
(7)

3. Estimation of Correlation Information Based on Fuzzy Observation Data

The expansion coefficient A_{mn} in (2) has to be estimated on the basis of the fuzzy observation data X and Y, when the true value y is unknown. Let the joint probability distribution of X and Y be P(X, Y), and the joint pdf of x and y be P(x, y). By applying fuzzy probability [3] to P(X, y), P(X, Y) can be expressed as:

$$P(X,Y) = \frac{1}{K} \int \mu_Y(y) P(X,y) dy,$$

$$(K : a \ constant \ satisfying \ the \ normalized \ condition : \ \sum_X \sum_Y P(X,Y) = 1).$$
(8)

The fuzziness of Y can be characterized by the membership function $\mu_Y(y) (= exp\{-\alpha(y-Y)^2\}, \alpha; a \text{ parameter})$. Substituting (2) in (8), the following relationship is derived.

$$P(X,Y) = \frac{1}{K} P_0(X) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} a_n \psi_m(X),$$

$$a_n = \int exp\{-\alpha(y-Y)^2\} P_0(y) \psi_n(y) dy.$$
 (9)

The conditional Nth order moment of the fuzzy variable X is given from (9) as

$$< X^{N}|Y> = \sum_{X} X^{N} P(X|Y) = \sum_{X} X^{N} P(X,Y) / P(Y)$$
$$= \sum_{X} X^{N} P_{0}(X) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} a_{n} \psi_{m}(X) / \sum_{n=0}^{\infty} A_{0n} a_{n}.$$
(10)

After expanding X^N in an orthogonal series expression, by considering the orthonormal relationship of $\psi_m(X)$. (10) is expressed explicitly as

$$\langle X^{N}|Y\rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} d_{m}^{N} A_{mn} a_{n} / \sum_{n=0}^{\infty} A_{0n} a_{n},$$
$$(X^{N} = \sum_{m=0}^{N} d_{m}^{N} \psi_{m}(X), \ d_{m}^{N}; \ appropriate \ constant).$$
(11)

The right side of the above equation can be evaluated numerically from the fuzzy observation data. Accordingly, by regarding the expansion coefficients A_{mn} as unknown parameters, a set of simultaneous equations in the same form as in (11) can be obtained by selecting a set of N and/or Y values equal to the number of unknown parameters. By solving the simultaneous equations, the expansion coefficients A_{mn} can be estimated. Furthermore, using these estimates, the pdf $P_s(y)$ can be predicted from (7).

4. Mutual Relationship between Sound and Electric Field from a VDT in Actual Working Environment

By adopting a personal computer in the actual working environment as specific information equipment, the proposed method is applied to investigate the mutual relationship between sound and electromagnetic waves leaked from a VDT under the situation of playing a computer game. In order to eliminate the effects of sound from outside, a personal computer is located in an anechoic room (cf. Fig. 1). The RMS value (V/m) of the electric field radiated from the VDT and the sound intensity level (dB) emitted from a speaker of the personal computer are simultaneously measured. The data of electric field strength and sound intensity level are measured by use of a electromagnetic field survey mater and a sound level meter respectively. The slowly changing nonstationary 600 data for each variable are sampled with a sampling interval of 1 [s]. Two kinds of fuzzy data with the quantized level widths of 0.1 [v/m] for electric field strength and 5.0 [dB] for sound intensity level are obtained. Based on the 400 data points, the expansion coefficients A_{mn} are first estimated by use of (11). Next, the 200 sampled data within the different time interval which are nonstationally different from data used for the estimation of the expansion coefficients are adopted for predicting the probability distributions of

(i) the electric field based on sound and (ii) the sound based on electric field. Membership functions of the sound level and electric field are shown in Figs. 2 and 3. The parameter α is decided so as to express the distribution of data as precisely as possible.



Figure 1: A schematic drawing of the experiment.



Figure 2: Membership function of sound level.



Figure 4: Prediction of the cumulative distribution for the electric field strength based on the fuzzy observation of sound.



Figure 3: Membership function of electric field.



Figure 5: Prediction of the cumulative distribution for the sound level based on the fuzzy observation of electric field.

The experimental results for the prediction of electric field strength and sound level are shown in Figs. 4 and 5 respectively in a form of cumulative distribution. From these figures, it can be found that the theoretically predicted curves show good agreements with experimental sample points by considering the expansion coefficients with several higher orders. For comparison, the generalized regression analysis method [4] previously reported is applied to fuzzy observation data as a trial. After paying our attention to the probability distribution without considering any membership function, the probability distribution Y can be predicted on the basis of fuzzy observation daya X. The predicted results for electric field strength and sound level are shown in Figs. 6 and 7 respectively. The theoretical curves show large prediction error to the true values as compared with the prediction results in Figs. 4 and 5. These results clearly show the effectiveness of the proposed method for application to the fuzzy observation data.



Figure 6: Prediction of the cumulative distribution for the electric field strength by use of the extended regression analysis method.



Figure 7: Prediction of the cumulative distribution for the sound level by use of the extended regression analysis method.

5. Conclusion

In this paper, a signal processing method has been proposed in order to grasp minutely and universally the mutual relationship between sound and electromagnetic waves leaked from electronic information equipment. More specifically, based on the fuzzy observation data on the sound and electromagnetic waves, a method to estimate not only the linear correlation of lower order but also the nonlinear correlation information of higher order between both variables has been derived by introducing the fuzzy probability. The validity and effectiveness of the proposed method have been confirmed experimentally by applying it to the observation data radiated from a personal computer in an actual working environment playing a computer game.

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A Single Phase Single Stage AC/DC Converter with High Input Power Factor and Tight Output Voltage Regulation

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Abstract—A single stage single switch AC/DC converter is an integration of input current shaper and a DC/DC cell with a shared controller and one active switch. The converter is applicable for digital input power supply with high input power factor and tight output voltage regulation. The focus of the topology is to reduce the DC bus voltage at light load without compromising with input power factor and voltage regulation. The concept behind this topology is direct power transfer scheme. Using special configuration of DC/DC cell does reduction of DC bus voltage and DC/DC cell works on the principle of series charging and parallel discharging. The power output of this converter can go up to 200 W.

1. Introduction

An act o dc converter is an integral part of any power supply unit used in the all electronic equipments. Also, it is used as an interface between utility and most of the power electronic equipments. These electronic equipments form a major part of load on the utility. Generally, to convert line frequency act o dc, a line frequency diode bridge rectifier is used. To reduce the ripple in the dc output voltage, a large filter capacitor is used at the rectifier output. But due to this large capacitor, the current drawn by this converter is peaky in nature. This input current is rich in low order harmonics. Also, as power electronics equipments are increasingly being used in power conversion, they inject low order harmonics into the utility. Due to the presence of these harmonics, the total harmonic distortion is high and the input power factor is poor. Due to problems associated with low power factor and harmonics, utilities will enforce harmonic standards and guidelines which will limit the amount of current distortion allowed into the utility and thus the simple diode rectifiers may not in use. So, there is a need to achieve rectification at close to unity power factor and low input current distortion. Initially, power factor correction schemes have been implemented mainly for heavy industrial loads like induction motors, induction heating furnaces etc., which forms a major part of lagging power factor load. However, the trend is changing as electronic equipments are increasingly being used in everyday life nowadays. Hence, PFC is becoming an important aspect even for low power application electronic equipments.

The objective of the paper has been in the direction of better understanding of direct power transfer scheme, closed loop simulation and analysis of proposed AC/DC converter. Emphasis of the paper has been the design of 100 W AC/DC converter with high input power factor and tight output voltage regulation without compromising with high DC bus voltage at light loading condition.

2. Power Factor Correction Techniques

In recent years, single-phase switch-mode AC/DC power converters have been increasingly used in the industrial, commercial, residential, aerospace, and military environment due to advantages of high efficiency, smaller size and weight. However, the proliferation of the power converters draw pulsating input current from the utility line, this not only reduce the input power factor of the converters but also injects a significant amount of harmonic current into the utility line [1]. To improve the power quality, various PFC schemes have been proposed. There are harmonic norms such as IEC 1000-3-2 introduced for improving power quality. By the introduction of harmonic norms now power supply manufacturers have to follow these norms strictly for the remedy of signal interference problem [2].

The various methods of power factor correction can be classified as:

- (1) Passive power factor correction techniques
- (2) Active power factor correction techniques

In passive power factor correction techniques, an LC filter is inserted between the AC mains line and the input port of the diode rectifier of AC/DC converter as shown in Figure 1. This technique is simple and rugged but it has bulky size and heavy weight and the power factor cannot be very high [1]. Therefore it is now not applicable for the current trends of harmonic norms. Basically it is applicable for power rating of lower than 25 W. For higher power rating it will be bulky.

In active power factor correction techniques approach, switched mode power supply (SMPS) technique is used to shape the input current in phase with the input voltage. Thus, the power factor can reach up to unity. Figure 2 shows the circuit diagram of basic active power correction technique [2]. By the introduction of regulation norms IEC 1000-3-2 active power factor correction technique is used now a day. There are different topologies for implementing active power factor correction techniques. Basically in this technique power factor correcting cell is used for tracking the input current in phase of input voltage such that input power factor come up to unity. Comparing with the passive PFC techniques, active PFC techniques have many advantages such as, high power factor, reduced harmonics, smaller size and light-weight. However, the complexity and relatively higher cost are the main drawbacks of this approach.



Figure 1: Passive PFC technique.



Figure 2: Active PFC technique.

The active PFC techniques can be classified as:

- (1) PWM power factor correction techniques
- (2) Resonant power factor correction techniques
- (3) Soft switching power factor correction techniques

In PWM power factor correction approach, the power switching device operates at pulse-width-modulation mode. Basically in this technique switching frequency of active power switch is constant, but turn-on and turn-off mode is variable. The advantages are simple configuration, ease of analysis and control, lowest voltage and current stress. Therefore it is extensively used in PFC circuits. For the minimization of converter size PWM technique generates significant switching loss [1].

Different topologies of PWM techniques are as follows:

- (1) Buck type
- (2) Flyback type
- (3) Boost type
- (4) Cuk' type

Figure 3 shows the buck type topology. The advantage of buck type topology is that the converter can supply a low output voltage with respect to input voltage. The disadvantages are, significant current distortion, EMI is higher because discontinuous input current so filter design is costly. It is a basic DC-DC converter and it is not used for power factor correction [1].

Figure 4 shows the flyback type topology. Its advantages are, output voltage can be higher or lower than input voltage and input and output can be isolated. The disadvantages are higher switching device voltage and current rating, input current is discontinuous so requirement of careful design of input filter, difficult to program the input current with current mode control [1].



Figure 3: Buck type topology.

Figure 4: Flyback type topology.

The boost type topology is shown in Figure 5. Advantages of this topology are current mode control is easy and less EMI so reduced input filtering requirements. The main disadvantages are more conduction loss, no isolation and output voltage is always higher than input voltage [1].

The Cuk' type is shown in Figure 6. The advantages are, input current is remain continuous even if the converter operates in discontinuous conduction mode, and output voltage can be lower or higher than the instantaneous input voltage. The disadvantages are the increased voltage and current stress on power devices, requirement of extra inductor and capacitor and isolation is not provided [1].



In the resonant converter, the voltage across a switch or the current through a switch is shaped by the resonance of inductor and capacitor to become zero at the time of turned on or off. Thus the switching loss is greatly reduced. The high power factor is achieved by the natural gain-boosting characteristic of the resonant converter. The major drawbacks are higher voltage and current stress on the power switch with respect to PWM mode and variable switching frequency employed. Figure 7 shows a PFC circuit in which a resonant converter is inserted between the input diode rectifier and the dc-dc converter. This resonant converter can be series resonant converter or a charge pump resonant network [3]. The advantage is that the current stress and voltage stress on resonant components as well as power switches are lower than the previous resonant converter [1].



Figure 7: Resonating PFC circuit.

Figure 8: Soft switching PFC circuit.

The soft-switching PFC technique combines the advantages of PWM mode and resonant mode techniques with an additional resonant network consisting of a resonant inductor, a resonant capacitor and an auxiliary switch. The AC/DC converter operates in PWM mode during most portion of a switching cycle but operates in resonant mode during the switch turn-on and turn-off intervals. As a result, the PFC circuit works at constant switching frequency and the power switch turns on and off at zero current or zero voltage conditions. Thus efficiency and power factor both improved by this technique. Figure 8 shows boost PFC circuit with a soft switching network [1].
3. Single Phase Active Power Factor Correction

Conventional off-line power converters with diode capacitor rectifier front-end have distorted input current waveform with high harmonic content. They cannot meet neither the European line-current harmonic regulations defined in the IEC1000-3-2 document nor the corresponding Japanese input-harmonic current specifications. To meet the requirements of above norms it is customary to add a power factor corrector ahead of the isolated dc/dc converter section of the switching power supply. Again another dc/dc converter is needed for output voltage regulation. Thus there are two converter is needed for single-phase active power factor correction for the requirement of high input power factor and tight output regulation. There are two approaches for single-phase active power factor correction:

- (1) Two-stage approach
- (2) Single-stage approach

Two-stage approach is commonly used approach in high power applications [4]. The block diagram of two stages PFC converter is shown in Figure 9. In this approach, there are two independent power stages. The front-end PFC stage is usually a boost or buck-boost (or flyback) converter. The dc/dc output stage is the isolated output stage that is implemented with at least one switch, which is controlled by an independent PWM controller to tightly regulate the output voltage. The two-stage approach is a cost-effective approach in high power applications; its cost-effectiveness is diminished in low-power applications due to the additional PFC power stage and control circuits.

A single-stage scheme combines the PFC circuit and dc/dc power conversion circuit into one stage. A number of single-stage circuits have been reported in recent years. Figure 10 shows the block diagram of single-stage approach. Compared to the two-stage approach, the single approach uses only one switch and controller to shape the input current and to regulate the output voltage. Although for a single-stage PFC converter attenuation of input-current harmonics is not as good as for the two-stage approach. But it meets the requirements of IEC1000-3-2 norms. Again it is cost effective and compact with respect to two stage approach [4].



Figure 9: Block diagram of two stage approach.



There are four possible combinations to obtain different single stage single switch PFC converters [5]:

- (1) Discontinuous Conduction Mode PFC + Continuous Conduction Mode DC/DC
- (2) Discontinuous Conduction Mode PFC + Discontinuous Conduction Mode DC/DC
- (3) Continuous Conduction Mode PFC + Continuous Conduction Mode DC/DC
- (4) Continuous Conduction Mode PFC + Discontinuous Conduction Mode DC/DC

4. Direct Power Transfer Scheme

Either in two-stage or single-stage of single phase PFC the input power is processed twice to reach the output. There are two functional cells known as PFC cell and DC/DC cell is used for power factor correction and output voltage regulation respectively. Figure 11 shows the power processing in typical single-phase single-stage approach by block diagram. Suppose efficiency of PFC cell is η_1 and DC/DC cell is η_2 than the output power will be

$$P_0 - P_{\rm in}\eta_1\eta_2 \tag{1}$$

Thus the efficiency of single-stage AC/DC converter will be,

$$\eta = \eta_1 \eta_2 \tag{2}$$

Thus the twice power processing approach means low conversion efficiency because it is a product of two fraction. So, advancement is needed for the improvement of conversion efficiency.

The proposed approach come into picture according to that, it is not necessary to process all input power twice to achieve well-regulated and high input power factor DC output power. In this approach some power is processed only once and remaining power processed twice to keep the total DC output power constant. Figure 12 shows the proposed new direct power transfer scheme. For this kind of power transfer, with whom some power is processed only once is called direct power transfer (DPT) scheme [6].



Figure 11: Conventional single stage approach.

Figure 12: Proposed DPT scheme.

Let k portion of power from PFC cell be directly transferred to output, and remaining (1-k) power from PFC cell is stored in intermediate bus capacitor and then processed by DC/DC cell. Based on the proposed concept, output power can be obtained by Eq. (4), (3).

$$P_0 = P_{\rm in}\eta_1\eta_2(1-k) + P_{\rm in}\eta_1k \tag{3}$$

Thus efficiency of proposed direct power transfer scheme single-stage PFC AC/DC converter is,

$$\eta = \eta_1 \eta_2 + k(1 - \eta_2) \eta_1 \tag{4}$$

Comparing Eq. (2) and Eq. (4), it is easy to say efficiency of DPT scheme is more than the efficiency of conventional single-stage scheme.

5. Direct Power Transfer Topology

The proposed DPT topology integrates flyboost PFC cell in existing single stage DC/DC cell. All the derived topologies are differentiated by only application of DC/DC cell. Different DC/DC cells are used for improving voltage regulation and reducing DC bus stress. Power unbalance between PFC stage and dc/dc stage is the inherent reason for causing high DC bus voltage stress. In order to meet the criteria of low DC bus voltage DC/DC cell used in this converter is work on the concept of "series charging, parallel discharging capacitors scheme (SCPDC)" [5]. The SCPDC means that the two energy-storage capacitors are charged in series when the switch is off and discharged in parallel when switch is on.

This topology integrates one parallel PFC cell and one parallel-series forward DC/DC conversion cell. Parallel PFC cell is basically a flyback transformer and integration of boost features. For achieving high power factor PFC block should work on DCM. Figure 13 shows the laboratory type AC/DC converter. Flyboost part is already explained in section 4.2. Here main difference in flyboost part of previous topology is unbalanced power is controlled properly, so DC bus voltage is less compared to other topologies of single stage single switch AC/DC converter.



Figure 13: Proposed AC/DC converter.



Parallel PFC cell contains; transformer T_1 , input bridge rectifier, two intermediate bus capacitors, diode D_1 and diode D_2 , and active switch S. The parallel-series forward DC-DC conversion cell contains, forward transformer T_2 , output inductor L_o , output capacitor C_o , and also bus capacitors, diodes, switch. Thus both cells share bus capacitors, the only active switch and controller. Same as other single-stage PFC topologies the two cells are operate independently. But the operation of this topology differs in other topologies in terms of parallel power flow nature and special mode of operation. For low input voltage, works as a flyback transformer and at the high input voltage works as a boost inductor.



Figure 15: Flyback mode of operation.



s t_1 t_2 t_{10} $t_{$

Figure 16: Boost mode of operation.



Figure 17: Waveforms of input voltage, input current and modes of operation.

Figure 18: Waveforms of bus voltage, output voltage and current.

Modes of Operation: PFC cell works in two modes of operation. The following discussion explains the modes of operation.

Suppose diode D_1 is conducting, applying KVL for primary side

$$|V_{\rm in}(t)| = V_o/n_1 + V_{D2} + V_{cb}$$



Figure 19: Waveforms of current of transformer T_1 and T_2 .



Figure 20: Bus voltage versus line voltage.

Where $V_{in}(t)$, is input voltage, V_o the output voltage, n_1 the turn ratio of transformer T_1 , V_{cb} the voltage across bus capacitor and V_{D2} the voltage across diode D_2 . So, $V_{D2} = |V_{in}(t)| - (V_{cb} - V_o/n_1)$

For D_2 to be conducting the condition is;

$$|V_{\rm in}(t)| \ge (V_{cb} - V_o/n_1) \tag{5}$$

Now suppose diode D_2 is conducting, the reflected voltage on secondary side will be;

$$[|V_{\rm in}(t)| - V_{cb}]n_1$$

Applying KVL on secondary side

$$[|V_{\rm in}(t)| - V_{cb}]n_1 + V_{D1} + V_o = 0$$



Figure 21: Bus voltage versus output inductance.



Figure 22: Bus voltage versus DPT.

Figure 23: DPT versus line voltage.

Where, V_{D1} is the voltage across diode D_1 . For D_1 to be conduct V_{D1} should be greater or equal to zero, so the condition is; $|V_{in}(t)| \le (V_{cb} - V_o/n_1)$ (6)

From the Eq. (5) and Eq. (6) it is clear that the diode D_1 and the diode D_2 do not conduct simultaneously. So, when D_1 will conduct D_2 will not conduct vice-versa. Thus there are two modes of operation. Figure 14 shows the operation modes of flyboost PFC cell.

Thus there are two modes of operation:

- (1) Flyback Mode
- (2) Boost Mode

Flyback Modes of Operation: it is easy to understand for the interval when line voltage $|V_{in}(t)|$ is less than $V_{cb} - n_1 V_o$ transformer T_1 work as a flyback transformer. It discharges all its energy directly to the load. Thus power transferred directly. This portion of power is processed by active switch only once. At the same time DC/DC cell will deliver some power from bus capacitors to the load to improve output voltage regulation. Operational waveform in this mode is shown in Figure 15. There are three interval of operation in flyback mode.

- (1) First Interval: When switch is on at t_o rectified line voltage is applied to the transformer T_1 . Transformer T_1 work as a flyback transformer. Power is transferred to the load at the time when switch is off. The bus capacitors voltage is applied to the inductor through the transformer T_2 for the regulation of output voltage. The special configuration of parallel-series forward conversion cell is useful for controlling DC bus voltage.
- (2) Second Interval: When switch is turned off at t_1 , energy is transferred by transformer T_1 to the load so voltage across T_1 will be n_1V_o . Freewheeling path of diode D_6 is through output inductor L_o . Transformer T_2 resetting its energy through bus capacitors by the help of diodes D_2-D_4 . Voltage across switch is clamped to sum of capacitors, C_1 , C_2 voltage. Thus the switching voltage is reduced by this configuration.
- (3) Third Interval: At t_2 all magnetizing energy of transformer T_1 is transferred to the load. Now the current of secondary winding of transformer will be zero. Transformer T_2 continues to reset through the bus capacitors, since it is fixed to bus voltage.

Boost mode of operation: When line voltage $|V_{in}(t)|$ is higher than $V_{cb} - n_1 V_o$ transformer T_1 works like a boost inductor. All magnetizing energy of both bus capacitors is discharged via D_2 and DC-DC cell delivers all output power from bus capacitors to the load. Thus power processing two times by the active switch. The

operational waveform is shown in Figure 16. Circuit operation in this mode is same as flyback mode in the first and third interval. But for the second interval it is different. In the second interval primary current of transformer T_1 will decrease. The current of output inductor freewheels through diode D_6 . Transformer T_2 is resetting through diode D_3-D_4 . The transformer action of flyback transformer is not working. This is due to reverse biased nature of flyback transformer diode D_1 . Therefore output current of secondary transformer is zero in this mode.

6. Simulation Results

The circuit shown in Figure 13 has been simulated in Matlab, the simulation results is shown in Figures 17, 18 and 19. The operating switching frequency is 50 kHz. As Figure 17 shows the input current tracks the input voltage so input power factor is almost unity. The two modes of operation, flyback mode and boost mode are clearly specified in Figure 17. As shown in Figure 18, bus voltage is fixed at 420 V; so switching stress is not high. Output voltage is almost fixed at 30 V, so it is well regulated. Simulation is done for 100 W AC/DC converter.

The Figure 19 shows currents in different cells. As the requirement is PFC cell works on DCM mode and DC-DC cell works on CCM mode, it is clearly specified in Figure 19. Secondary winding current of flyback transformer carry current only in flyback mode, in boost mode primary winding of flyback transformer works as a boost inductor.

Figure 20 shows bus voltage versus line voltage at different turns ratio. As turns ratio of transformer T_1 increases bus voltage increases almost linearly.

Figure 21 shows DC bus voltage output inductance value of forward DC-DC cell. As output inductance increases DC bus voltage increases. Again it is a function of turn ratio of flyback transformer, higher the turn ratio lowers the flyback mode, lower the direct power transfer and so lower the efficiency. But small turns ratio will results in very low bus voltage, which may cause PFC cell to operate under CCM. Since there is no active control over PFC cell, it will cause very high peak current. So, it limits the minimum turns ratio value. Hence, in order to achieve lower bus voltage, L_1 should be as large as possible, while n_1 and L_o should be as small as possible.

Figures 22 and 23 shows the direct power transfer versus bus voltage and line voltage. Higher the line voltage and bus voltage higher will be the direct power transfer.

7. Conclusion

From above discussions in this paper, it is clear that the power factor correction is being given significant importance for low power applications. Also as power electronic equipments are increasingly being used, they pose a serious problem of low order harmonics on utility side. Among various schemes available for PFC, the single stage scheme is best suited for low power application because of its cost effectiveness. But in this scheme, there is a serious limitation of high dc link voltage rise under light load condition. This problem can be addressed by using the concept of Direct Power Transfer. From the simulation and experimental results of DPT topology, it is clear that DPT is an effective way to control high dc link voltage and hence reduces the component stresses. This topology also maintains a good source power factor and a tight output voltage regulation without compromising with high DC bus voltage. Moreover, the efficiency of overall power conversion is high.

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A New Generalized Space Vector Modulation Algorithm for Neutral-point-clamped Multilevel Converters

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Abstract—Neutral-point-clamped converters are increasingly applied in industrial drive systems as they allow the use of lower voltage devices in higher voltage applications, provide reduced output voltage total harmonic distortion(THD), and can develop low common mode voltage. Several distinct modulation strategies have been proposed in the past for eliminating the common mode voltage, providing low THD output voltage or reducing the neutral point current ripple. However each of these strategies improves the performance of the converter in one view point while losing performance in the other view point. A new generalized space vector modulation technique is proposed. Analytical model and simulation results are presented.

1. Introduction

In recent years, industry has begun to demand higher power equipment, which now reaches the megawatt level. Controlled ac drives in the megawatt range are usually connected to the medium voltage network. Today it is hard to connect a single power semiconductor switch directly to medium voltage grids. For these reasons, a new family of multilevel inverters came as the solution for working with higher voltage levels. In particular, they provide low line voltage dv/dt and improved spectral characteristics of the output signals. The three level neutral-point-clamped (NPC) converters are used widely in adjustable speed a.c drive systems, providing less stress on motor winding, insulation and bearings. It is leading to an increase of the reliability and the life period of the drive systems.

Several carrier-based and space vector modulation strategies have been proposed for these converters [7]. These algorithms were designed to provide adjacent state switching action in the converter which yields the lowest possible output voltage and current total harmonic distortion. The most significant advantages of SVPWM are fast dynamic response and wide range of fundamental voltage compared with the conventional PWM. But when it is applied to the diode-clamped converter, the SVPWM strategy also has to solve the neutral-point voltage un-balance problem.

There are three main steps to obtain the proper switching states during each sample period for the SVPWM method:

- 1. Choose proper basic vectors.
- 2. Calculation of dwelling time of selected vectors.
- 3. Selection of proper sequence of pulses.

One way for calculating the time is to decompose all the vectors into real and imaginagy part [1]. Another way is to calculate the time according to reference voltage vector of each phase [3, 2].

To solve the problem of computational complexity in multilevel converter due to large number of redundant switching states, a new generalized space vector algorithm is proposed. This new space vector strategy eliminates low frequency ripple from the dc link capacitors of a three-level converter. The proposed algorithm for threeClevel converter is verified by simulation and the results are compared with the existing method, nearest three vector algorithm [4].

2. Three-phase Three-level Converter

Figure 1 presents the basic structure of a three-level neutral-point-clamped converter. Each of three legs of the converter consists of four power switches, four freewheeling diodes and two clamping diodes that limit the voltage excursions across each device to half the input dc-bus voltage.

3. Nearest Three Vector Modulation

Table 1 shows the possible switching states for the three level converter of Figure 1. There are nineteen basic space vectors for a three-level converter and they are shown in Figure 2. The zero voltage vector has three switching states (000, 111, -1-1-1). Each of the six small vectors (V_1-V_6) has two switching states and each of the middle vectors $(V_8, V_{10}, V_{12}, V_{14}, V_{16})$ and the large vectors $(V_7, V_9, V_{11}, V_{13}, V_{15}, V_{17})$ has one state respectively.



Figure 1: Basic structure of three level diode clamped converter.

Figure 2: Switching states of converter.

Table 1: Switching states of three-level converter (X = A, B, C).

STATUS	S_{1X}	S_{2X}	S_{3X}	S_{4X}
1	ON	ON	OFF	OFF
0	OFF	ON	ON	OFF
-1	OFF	OFF	ON	ON

In SVPWM technique, the reference voltage vector V_r is located in a triangle, in which three voltage vectors are corresponding to the three apexes. They are selected to minimize the harmonic components of the output line voltage.

The dwelling time of each vector should satisfy the following equations:

$$V_1 t_1 + V_7 t_7 + V_8 t_8 = V_r T \tag{1}$$

$$t_1 + t_8 + t_8 = T (2)$$

where T is the sampling period and t_1 , t_7 and t_8 are the dwelling time of V_1 , V_7 and V_8 respectively.

Following four methods are required in the nearest three vector modulation algorithm to attain the complete switching states for a multilevel converter by using SVPWM.

Step I: Decomposing of Basic Vectors

A new coordinates namely *m*-*n* coordinates can be established. The new coordinate have two axes intersecting with the angle of $\pi/3$. Only the first quadrant of the coordinate is used because the vector located in other region can be transformed to the first quadrant by clockwise rotating an angle $K^*(\pi/3)$ where, K = 1, 2, 3, 4, 5for region B, C, D, E and F respectively.



Figure 3: Space vector of the three level converter.



Figure 4: Decomposing of reference vector.

Universally a (M+1) level inverter is discussed here. As shown in Figure 4, the reference vector is decomposing into m-axis and n-axis.

$$V_{rm} = 2MV_r / (\sqrt{3}V_{dc})\sin(\pi/3 - \theta) \tag{3}$$

$$V_{rn} = 2MV_r / (\sqrt{3}V_{dc})\sin(\theta) \tag{4}$$

Step II : Selecting Three Nearest Vectors

Considering that following inequalities are satisfied by V_{rm} and V_{rn}

$$(m-1) < V_{rm} < m \tag{5}$$

$$(n-1) < V_{rn} < n \tag{6}$$

where m and n are integers. There are three possible cases:

- 1. $V_{rm} + V_{rn} < (m + n 1)$: That means V_r is located in the left bottom shadow triangle. The vectors (m-1, n-1), (m-1, n) and (m, n-1) are the nearest vectors.
- 2. $V_{rm} + V_{rn} > (m+n-1)$: That means V_r is located in the right top shadow triangle. The vectors (m-1, n), (m, n-1) and (m, n) are the nearest vectors.

3. $V_{rm} + V_{rn} = (m + n - 1)$: V_r lies at the middle line and either 1. or 2. can be choosen.

Step III : Dwelling Time Calculation

Dwelling time calculation is given here. Taking (m1, n1), (m2, n2) and (m3, n3) are three nearest vector. Corresponding dwelling time can be calculated from the following equations.

$$m_1 t_1 + m_2 t_2 + m_3 t_3 = V_{rm} * T \tag{7}$$

$$n_1 t_1 + n_2 t_2 + n_3 t_3 = V_{rn} * T \tag{8}$$

$$t_1 + t_2 + t_3 = T \tag{9}$$

Step IV: Neutral Point Potential Control

It is very important to balance neutral point potential. For balancing neutral point potential selection of proper switching sequence is necessary. For example, when V_r falls in the triangle formed by the apexes of vectors V_1 , V_7 and V_8 , the switching sequence can be selected as (100) - (10-1) - (00-1) - (01-1) or (110) - (100) - (10-1) - (00-1). The two sequences lead to the same output voltage but have the opposite effect on the neutral point voltage.

Over Modulation Control

However, there is one exception when the reference voltage vector lies outside the hexagon. In this case, the over modulation mode occures and the output line voltages distort.

(0,V (0, V m') (Vm', 0)(Vrm,0)

Figure 5: Over modulation control.

Following algorithm can be used when reference vector lies in the over modulation region,

If
$$(V_{rm} + V_{rn}) > M$$

$$V_{rm'} = V_{rm} * M / (V_{rm} + V_{rn}) \tag{10}$$

$$V_{rn'} = V_{rn} * M / (V_{rm} + V_{rn})$$
(11)





Figure 6: Redistribution of switching states in R.S.S. algorithm.

4. New Space Vector Modulation Algorithm

It has been shown that switching states where the three ac terminals are connected to the three different voltage level of dc bus are the primary cause of increased harmonic content in the dc bus currents. The presence of significant third harmonic content in the neutral point current causes a significant sizing penalties on the dc link capacitor of three level converter [6]. A new space vector modulation algorithm is proposed to eliminate the harmonic content in neutral current.

It can be deduced that the identical voltages can be generated if the duty cycle of these state is equally divided between two states that are adjacent to it and lie on the same hexagonal plane. Figure 6 shows the redistribution of switching states.

As an example, redistribution of dwelling time in Figure 6 for (1,-1,-1), (1,1,-1) and (1,0,-1) can be done as follows,

$$t_{1,-1,-1} = t_{1,-1,-1} + (t_{1,0,-1}/2)$$

$$(12)$$

$$t_{1,-1,-1} = t_{1,-1,-1} + (t_{1,0,-1}/2)$$

$$(12)$$

$$t_{1,1,-1} = t_{1,1,-1} + (t_{1,0,-1}/2)$$
(13)

$$t_{1,0,-1} = 0 (14)$$

Though we have eliminated six switching states still DC-bus utilization factor in this method is identical with nearest three vector modulation, as the hexagon and inscribe circle are equal.

5. Simulation Results

Extensive MATLAB/SIMULINK models of three level inverter systems were developed for analysing the nearest three point modulation and new proposed method on output line voltage and neutral point current. The simulation of threeClevel converter shown in Figure 1 was done under the conditions listed in Table 2.

Sampling Time	$50\mu\mathrm{Sec.}$
DC Link Voltage	$400\mathrm{Volts}$
Active Load	$1\mathrm{KW}$
Reactive Power (-ve)	$500\mathrm{VAR}$
Output Frequency	$50\mathrm{Hz}$
Modulation Index	0.7

Table 2: Simulation conditions.

1. Output Line Voltage

Figure 7 and Figure 9 are the line-line voltage of nearest three vector modulation and new proposed approach respectively. The FFT analysis (Figure 8 and Figure 10) of both the line-line voltages are also shown.







Fundamental (60Hz) = 167.5 , THD= 32.52%

Frequency (Hz)

Fundamental (60Hz) = 185.2 , THD= 46.67%



Figure 14: FFT analysis of line current.



Figure 16: FFT analysis of neutral current.

2. Line Current

Figure 11 and Figure 13 are the line currents of nearet three vector modulation and new proposed approach

respectively and their FFT analysis also shown in Figure 12 and Figure 14.

3. Neutral Point Current

FFT analysis of neutral current of nearest three vector modulation and new proposed approach are shown in Figure 15 and Figure 16 respectively. New proposed method eliminates the harmonic content in neutral point current while the nearest three vector method shows harmonics in neutral point current.

6. Conclusion

A new simple SVPWM method is proposed and verified by simulation of three-level-inverter. If number of levels increases more complexity will come in the proposed method while algorithm will be same. This paper presents a new way of implementing the proposed space vector modulation algorithm for reducing the neutral point current in the multilevel inverter.

From the FFT analysis of line voltage and neutral current it is concluded that

- 1. Low frequency harmonic content of the neutral current is zero.
- 2. Neutral point current has a zero d.c average value.
- 3. Line voltage contains slightly larger harmonics in proposed method with respect to nearest three vector modulation.

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Improved Mesh Conforming Boundaries for the TLM Numerical Method

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Abstract—The numerical simulation of wave propagation in bounded space using differential-based models generally encounter spatial discretisation problems when the boundaries of the computation space do not fall on exact multiples of the models discretisation. While the accuracy can be improved by refinement of the model, the computational load can increase exponentially, often making the problem unsolvable. There have been some previous attempts to achieve boundary conforming meshes for the TLM numerical method. This paper describes a novel approach which compares well with these methods with a significantly reduced computational load.

1. Introduction

While the Transmission Line Matrix (TLM) numerical method is becoming increasingly easy to utilise for a wider variety of electromagnetic problems, in part, due to the definition of perfectly matched loads (PMLs) [1-3], boundary conforming schemes often increase the computational load and/or complexity of the algorithm, making TLM difficult to implement for simulations of bounded regions that fluctuate rapidly along the periphery (Figure 1(a)). The dotted lines show the sections that will require extra analysis in order to meet with the boundary. While some TLM simulations can be approximated reasonably accurately with a stepped formulation (by extension/deletion of the dashed sections), in the case of most TLM simulations the errors introduced are unacceptable and refinement of the mesh is often performed, increasing both time and memory requirements. If the true distance, l_a in figure 1b could be incorporated within the simulation, the mesh would pertain much more closely to the true boundary/surface of the device being modelled.

The length of the transmission line in the TLM algorithm cannot simply be adjusted. As can be observed from Figure 1(b), a signal travelling along the full length $(\Delta x/2)$, where Δx is the spatial discretisation in the model), will, after reflecting from the surface, appear back at the boundary adjacent node at time $(k + 1)\Delta t$, the same signal travelling along the line of length l_A will appear back at the boundary adjacent node at a time less than $(k + 1)\Delta t$, but greater than $k\Delta t$ (where k is the current iteration (discrete time step) and k + 1 is the succeeding iteration). As propagating signals in TLM must all arrive in steps of the same discretised time (Δt) , this cannot be modelled directly.



Figure 1: a) Cartesian mesh of non-stepped boundary, b) boundary adjacent node.

De Cogan and de Cogan [4] have adapted existing schemes to demonstrate the application of boundaryconforming finite difference schemes for the solution of the Laplace equation. In a uniformly bounded space where h is the length of the line segments (Figure 2) we use

$$T(x+h,y) + T(x,y+h) + T(x-h,y) + T(x,y-h) + 4T(x,y) = 0$$
(1)



Figure 2: a) 2D FDTD visual representation, b) 2D FDTD irregular boundary visualisation.

In the situation where we take account of a node where we have unequal distances between nodes, given by ah, bh, ch and dh (h is the uniform line-length), the two-dimensional Laplacian becomes

$$\nabla^2 V = \frac{2}{h^2} \left[\frac{V_A}{a(a+c)} + \frac{V_B}{b(b+d)} + \frac{V_C}{c(c+a)} + \frac{V_D}{d(d+b)} - \frac{ac+bd}{abcd} V_0 \right]$$
(2)

where we consider the potentials of the four nodes, A, B, C, and D which surround the potential V_0 .

Using concepts developed in [5] it is easy to see how one might develop a boundary conforming implementation of the wave equation

$$\nabla^2 V = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} \tag{3}$$

Although FDTD is well established in electromagnetics and hybrid schemes involving finite difference are well known, there is not much evidence that such a mesh conforming scheme is widely used. Probably because time is implicitly discretised in TLM and because coincidence of arrival is such an important part of TLM algorithms, this subject has received significantly more attention. Early amongst these was the work of Jaycocks and Pomeroy [6]. However, the first really effective technique which was based on firm theoretical foundations was due to Beyer, Mueller and Hoefer [7] and this will hereafter be referred to as the *BMH* method.

This paper will start with a restatement of the BMH method. We will then present our improved formulation which represents a significant improvement in computational efficiency. Analytical results for the horn antenna [8] will be used as a benchmark against which to compare these boundary-conforming schemes against a conventional TLM model with stepped boundaries.

2. Introduction to the BMH Method

The technique proposed in [7] suggests a recursive definition to describe the arbitrary placed boundaries of the mesh. The definition we are interested in here describes an electric wall (reflection coefficient of $\rho = -1$), by adjusting the incident pulses on the line intersected by the boundary. This is given in formula (5) of the BMH paper [7] as:

$$_{k}V^{i} = \rho \frac{1-\kappa}{1+\kappa} _{k}V^{r} + \frac{\kappa}{1+\kappa} (\rho_{k-1}V^{r} + _{k-1}V^{i})$$
(4)

where $\kappa = 2l/\Delta x$, k is the discretised time step and $_{k-1}V^r {}_k V^i$ represent the reflected (scattered) and incident pulses at time k - 1 and k respectively. Figure 3 illustrates the technique graphically. This approach uses a reference plane located at $\Delta x/2$ from a surface adjacent node (broken line in Figure 3). While this is very effective, there are two important shortfalls. As observed from Figure 3, the bounding wall can only cut the transmission line segment after $\Delta x/2$, if the transmission line is intersected before this, it is necessary to remove the node, extending the line from the previous node, causing κ to become greater than 1. While analysis of this has been performed in [7] ensuring the system remains stable, another error is introduced in the system as the connections between neighbouring nodes are now missing. It appears this does not inhibit the accuracy of the technique as much as a stepped approximation would.



Figure 3: BMH model of arbitrary placed boundary.



Figure 4: a) Apparent impedance of section of length l_A , b) transformed impedance.

The large memory requirements of the recursive procedure proved particularly limiting in the simulations performed in section 4. For a stepped Cartesian mesh using a scatter-collect type algorithm it is necessary to store only 12 scatter and collect matrices (for the 3D case). To implement (4), a further 8 matrices were required to save the previous scattered and incident pulses. While it is possible to only save the data for the boundary nodes, the complexity of the algorithm is further increased.

3. Improved Conforming Boundary Description

The technique we propose avoids the need for recursion, removing the limitations of the BMH approach, while obtaining results in comparison. We begin with a load termination at some arbitrary non-discrete distance from the $\Delta x/2$ line end (Figure 4(a)). Observing the impedance looking 'down' this line from the node:

$$Z_{obs} = Z_0 \left[\frac{Z_L + Z_0 \tanh(\beta l_A)}{Z_0 + Z_L \tanh(\beta l_A)} \right]$$
(5)

where Z_0 is the intrinsic impedance of the line of length l_A , Z_L represents the load impedance, tanh is the

hyperbolic tangent function, and $\beta = 2\pi/\lambda$, where λ is the wavelength. As: $Z_0 = \frac{\Delta t/2}{C_d \Delta x/2} = \frac{\Delta t'}{C_d l_A}$, where $\Delta t'$ is the time a signal takes to traverse the line of length l_A we can replace the line of length l_A , impedance Z_0 , with another line of length $\Delta x/2$, with impedance Z_A , as shown in Figure 4(b). For the case when $\rho = 1$ i.e., $Z_L \to \infty$ Eq. (5) can be simplified to give, after transformation:

$$Z_{obs} = \frac{Z_A}{\tanh(\beta \Delta x/2)},$$

where $\Delta x/2$ is the discretisation of the model.

Assuming low frequencies $\tanh(\beta \Delta x/2) \approx \beta \Delta x/2$. The impedance transformation observed from the node must be the same before and after transformation:

$$\frac{Z_0}{\beta l_A} = \frac{Z_A}{\beta \Delta x/2}$$

$$Z_A = Z_0 \left[\frac{\Delta x}{2l_A} \right]$$
(6)

hence:



Figure 5: The two cases of impedance transformations covered by (6) and (7).

so if $l_A = \Delta x/2$ $Z_A = Z_0$ if $l_A > \Delta x/2$ $Z_A < Z_0$ case A in Figure 5 if $l_A < \Delta x/2$ $Z_A > Z_0$ case B in Figure 5

Likewise for the case when $\rho = -1$ (i.e., $Z_L \rightarrow 0$) (5) simplifies to give, after transformation:

 $Z_{obs} = Z_A \tanh(\beta \Delta x/2)$, again assuming $\tanh(\beta \Delta x/2) \approx \beta \Delta x/2$,

Equating before and after transformation:

$$Z_0\beta l_A = Z_A\beta\Delta x/2$$

$$Z_A = Z_0 \left[\frac{2l_A}{\Delta x}\right]$$
(7)

therefore:

Using both (6) and (7) to describe the boundary adjacent transmission lines for the cases when $\rho = 1$ and $\rho = -1$ respectively, causes the propagating signal to arrive back at the node at time $(k + 1)\Delta t$, appearing to have travelled to the true boundary location, while propagating on a line of length $\Delta x/2$.

This scheme will be termed as *uniform* in the analysis performed in section 4.

The nature of mesh-lines at the interface with real surfaces means that we could be dealing with line-lengths in the range $0 < l_A < \Delta x$. Our treatment of this involves expressions with either $\tanh(\beta \Delta x/2)$ or $\tanh \beta \Delta x$. The subsequent analysis assumes that $\tanh \theta \doteq \theta$ so that it is sensible to consider the error bands that are involved. In order to reduce the effects of mesh dispersion conventional TLM in two-dimensions is modelled using discretised frequency $\Delta x/\lambda \leq 0.1$, which means that we are looking at $\lambda \geq 10\Delta x$. If this is the case then

$$\tanh\beta\frac{\Delta x}{2} = \tanh\frac{2\pi}{\lambda}\frac{\Delta x}{2} = \tanh\frac{\pi}{10}$$

The difference between this and $\pi/10$ is 3.16%, a lower bound.

If $l_A \doteq \Delta x$ then our transformation requires that we have $\tanh \beta \Delta x \doteq \beta \Delta x$ so that if we persist with $\Delta x/\lambda \leq 0.1$ then there is an error of 11.37% in this assumption, an upper bound. We can deduce from this that if we operate at $\Delta x/\lambda \leq 0.1/\pi$, then the dispersion at any of the boundary-conforming transmission lines will be no different than if we had used a stepped boundary description with $\Delta x/\lambda \leq 0.1$.

4. Comparison of All Techniques

In order to test the accuracy and viability of this technique, an E-plane sectoral horn antenna has been modelled. The analytical solution to describe the radiated fields from the aperture of the horn is described in Balinis [8]. Figure 6 shows the coordinate system used to describe the dimensions of the horn. The field emitted from the E-plane (y-direction in the TLM models) is given as:

$$E_{\theta} = -j \left(\frac{a \sqrt{\pi \zeta \rho_1} E_1 e^{-j\zeta r}}{8r} \right) \left\{ -e^{j(\zeta \rho_1 \sin^2 \theta/2)} \left(\frac{2}{\pi} \right)^2 (1 + \cos \theta) F(t_1, t_2) \right\}$$
(8)

where ζ denotes the phase factor, E_1 is a constant $F(t_1, t_2) = [C(t_2) - C(t_1)] - j [S(t_2) - S(t_1)]$, $C(t_n)$ and $S(t_n)$ denote the cosine and sine Fresnel integrals:



Figure 6: 3D view of E-plane sectoral horn, analytical coordinate system.

Modelling the horn with the dimensions shown in Figure 6, inserting a point source with wavelength of 15 m at the apex of the horn, produces the radiation pattern, along the E-plane, as shown in Figure 7. This has been extracted across the aperture of the horn, from 1/4 into the aperture to 3/4 across (Figure 8), i.e., the centre half of the pattern, this is then plotted over half of the polar diagram. This will act as the benchmark against which to compare the 3D TLM solutions.





Figure 7: E-plane radiation pattern of E-plane sectoral horn antenna, analytical solution.

Figure 8: Plotted section of radiated field.

The standard TLM approach creates a 3D Cartesian mesh (for this problem, this is 81 nodes in the northsouth direction (y), 128 in the east-west direction (x) and 27 front-back (z)), however as elaborated upon in the discussion given earlier the non-discrete boundaries will become stepped approximations to the true boundary. For this problem the top (south) and bottom (north) boundaries of the mesh will become stepped. The east, west, front and back boundaries are chosen to fall on exact multiples of the models discretisation for the edges of the horn, however to allow the data propagating on the 'corners' of the horn to be included in the simulation, the mouth of the horn has been placed inside the computation space, resulting in east and west boundaries that are also stepped on the flared section of the antennas aperture (Figure 9). Using a reflection coefficient (ρ) of

 t_n represents:

-1 on all bounding surfaces of the horn and inputting a continuous sinusoidal wave of wavelength 15 nodes (m) at the apex of the horn (marked as ° in Figure 9), centred in the z direction, the results for the pattern along the aperture of the horn, in comparison to those from the analytical solution are generated in Figure 10. The patterns from the TLM models will never match the analytical solution directly due to the stepwise nature of the computation space. The boundaries are placed at the ends of the transmission lines of length $\Delta x/2$, in comparison to some TLM models which place the boundary at the node. A section of wave-guide of length 150 nodes is appended to the beginning of the model to ensure any errors from the PML have little effect on the signal propagating into the horn. This approach is also used in the BMH and uniform models. A mean error (sum of absolute differences) of 0.0583 is observed, indicating that while the technique produces considerably accurate results given the simplicity of the formulation, they are far from perfect.



Figure 9: North-south, east-west plane view of 3D E-plane sectoral horn antenna, illustrating TLM stepped formulation.





Figure 10: E-plane radiation pattern of E-plane sectoral horn antenna, analytical against stepped TLM (run for 2500 iterations).

Figure 11: E-plane radiation pattern of E-plane sectoral horn antenna, analytical solution against BMH TLM (run for 788 iterations).

The boundary conforming mesh described in [7] and analysed above was implemented as a comparison to the stepped mesh, again, placing the mouth of the horn inside the computation space, the results which were produced were a considerable improvement on those generated from the stepped mesh and are illustrated in Figure 11, this is modelled with a reflection coefficient of -1 (or electric wall in the terminology of [7]). As can also be seen the BMH technique requires considerably less iterations than those of the stepped formulation, producing results comparable with the analytical solution after only 788 iterations. The mean error recorded for this mesh was 0.0441.

When the technique that we propose here is implemented, it is clear that its memory requirements are almost identical to those of the stepped mesh. The extra computation needed at the start of the simulation to calculate the lengths of the transmission line segments meeting with the boundaries are usually performed in stepped schemes before the rounding up or down is performed, therefore the only extra computation required in the formulation is the adjustment of the impedances saved in the boundary locations of the impedance matrices. The algorithm then runs in an identical manner to the stepped system. The results produced when this technique was implemented are given in figure 12. The mean error was recorded at 0.0406. Figure 13 shows the uniform scheme in comparison to the BMH results and these are within 0.0035 units of one another, illustrating the accuracy of the new scheme with a substantially smaller computational 'footprint' than the BMH approach.





Figure 12: E-plane radiation pattern of E-plane sectoral horn antenna, analytical solution against uniform (run for 2118 iterations).





Figure 14: Difference plots of stepped, BMH and uniform TLM models against analytical solution.

Figure 14 gives a graphical view of how close the BMH and uniform models are. Due to the symmetry of the patterns, only half of the plot is shown. As can be observed, the uniform mesh is slightly closer to the analytical solution than the BMH model, while the stepped mesh, as expected, displays significant deviation.

5. Conclusion

The TLM numerical method is widely used, not only in electromagnetics, but many other fields of physics. The technique proposed in this paper gives an accurate approximation to arbitrary placed boundaries of a TLM mesh, while achieving a computational complexity and load equivalent to a normal Cartesian stepped formulation. The method has been compared with another widely used boundary smoothing scheme, illustrating its desirable properties further. The accuracy obtained from the new scheme is in tier with the previously used technique.

We propose this novel approach to model arbitrary placed boundaries of a TLM mesh that do not fall within the discretised formulation of the model. Due to the simplicity of the impedance transformations the computational requirements are practically unaltered from the stepped formulation most commonly used by engineers.

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The Spectral Expansion on the Entire Real Line of Green's Function for a Three-layer Medium in the Fundamental Functions of a Nonself-adjoint Sturm-Liouville Operator

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Abstract—We obtain a new representation for Green's function in the space R^2 of the Helmholtz equation with the coefficient representing a complex-valued piecewise constant function. We set that the coefficient in the equation depends on the one variable and represents three complex constants.

This representation is the expansion of Green's function in the fundamental functions which are bounded on the entire real line R^1 solutions of the ordinary Sturm-Liouville equations with complex coefficients.

The spectrum consists of two half-lines parallel to the real axis on the complex plane, issuing from the points characterized by the coefficient in the equation on semi-infinite intervals, and going in the positive direction of the real axis.

1. Introduction

In the present paper, we obtain a new representation ([1,2]) for the solution of a problem for a threelayer medium similar to the problem on a dipole in the space containg a plane interface between two media characterized by constant wave numbers k_i , i = 1, 2. The latter problem was considered in [3].

The new representation follows from the representation of Green's function in the form of a Fourier integral obtained by reducing an integral on the complex plane of the spectral parameter to integrals over the edges of cuts passing through the points k_i , $i = 0, 2(k_i = anst)$ characterized by the coefficient k(z) of the equation outside a finite interval [0, H] where $k(z) = k_l = anst(l = 1)$. The function k(z) represents a complex-valued piecewise constant function of a variable z on the entire real line R^1 .

2. Formulations and Equations

Green's function u(z, x) satisfies the Helmholz equation

$$\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial x^2} + k^2(z)u = -2\delta(z - z')\delta(x - x')$$
(1)

with the delta function on the right-hand side and with either the radiation condition $u \to 0$ as $r = \sqrt{z^2 + x^2} \to \infty$ [if $Im k(z) \neq 0$] or the radiation condition following from the limiting absorption principle [if Imk(z) = 0]. The function u is a bounded function in R^2 with the exception of the source position M'(z', x') where it has a logarithmic singularity. At the points z = 0, z = H of discontinuity of the function k(z), the function u(z, x) satisfies the matching conditions for the function and its normal derivative on the boundary. We consider Eq. (1) under the assumption that $z, z', x, x' \in R^1$, where R^1 is the real axis. We set

$$k(z) = \begin{cases} k_0 = const, z < 0, \\ k_1 = const, 0 < z < H, \\ k_2 = const, z > H. \end{cases}$$

where the k_l are complex constants, $k_l^2 = \varepsilon_l + j\sigma_l$, $\varepsilon_l \in R^1$, $\varepsilon_0 = \varepsilon_2, \sigma_l \ge 0$, $\sigma_0 < \sigma_1 < \sigma_2$ or $\sigma_0 > \sigma_1 > \sigma_2$, l = 0, 1, 2, and j is the imaginary unit.

To find the solution, we consider the Fourier expansion of u with respect to the variable x [of which the cofficient in Eq. (1) is independent]:

$$u = \lim_{\Lambda \to \infty} u_{\Lambda}, \qquad u_{\Lambda} = \frac{1}{\pi} \int_{-\Lambda}^{\Lambda} e^{j\alpha(x-x')} g(z, z'; \alpha) \, d\alpha.$$
⁽²⁾

The function g can be found from the equation

$$lg + \alpha^2 g = \delta(z - z'), \qquad z, z' \in \mathbb{R}^1, \tag{3}$$

where l is the differential operator

$$l\psi = -d^2\psi/dz^2 - k^2(z)\psi,$$

and has the form

$$g(z, z'; \alpha) = \frac{\psi(z_{>}, \alpha)\varphi(z_{<}, \alpha)}{w(\alpha)}.$$
(4)

Here $z_{>} = max(z, z'), z_{<} = min(z, z')$, and $w(\alpha)$ is the Wronskian of the linearly independent solutions ψ and φ of the homogeneous Eq. (3). The function g is bounded for $z, z' \in \mathbb{R}^1$ and satisfies the matching condition for the function and its derivative dg/dz at the points z = 0 and z = H.

Taking into account the representation (4) of the function g via the functions φ, ψ , and w, we find that the function g has the ramification points $\pm k_i$ in the complex plan α ([4] p. 23–25, Vol. 2 of the Russian translation).

The cuts corresponding to the ramification points k_i go along the lines

$$\alpha = \sqrt{-\mu^2 + k_i^2} = j\sqrt{\mu^2 - k_i^2}, \qquad \mu \in R^1, \, i = 0, 2.$$

We assume that $Re\sqrt{-\mu^2 + k_i^2} \ge 0$, and $Re\sqrt{\mu^2 - k_i^2} \ge 0$.

Consider the case in which $\varepsilon_0 = \varepsilon_2, \sigma_2 > \sigma_0, \sigma_0 > 0, \sigma_2 > \sigma_1 > \sigma_0$.

The Wronskian $w(\alpha) \neq 0$ on the entire complex α -plane [5, 6].

Consider the case x - x' > 0.

Using the Cauchy theorem, we reduce the integral u_{Λ} given by (2) in the upper half of the complex α -plane to two integrals over the edges of the cuts passing through the points k_i and the integral $I_{C_{\Lambda}}$ over the half-circle C_{Λ} of the radius Λ .

We have

$$u_{\Lambda}(z,x) = \sigma_{\Lambda}(z,u) + I_{C_{\Lambda}},\tag{5}$$

where

$$\sigma_{\Lambda}(z,u) = \frac{j}{\pi} \sum_{i=0,2} \int_{-M_i(\Lambda)}^{M_i(\Lambda)} \mu \frac{e^{-\sqrt{\mu^2 - k_i^2(x - x')}}}{\sqrt{\mu^2 - k_i^2}} \frac{\psi_i(z_>, \mu)\varphi_i(z_<, \mu)}{w_i(\mu)} d\mu.$$
(6)

The function $M_i(\Lambda)$, which define the limits of integration in (6), depend on Λ , and $M_i(\Lambda) \sim \Lambda$ as $\Lambda \to \infty$. The quantity $M_i(\Lambda) > 0$ occurring in (6) is the value of μ at which the right edge of the cut passing through the point k_i intersects the half-circle C_{Λ} .

The functions $\psi_i(z,\mu)$ and $\varphi_i(z,\mu)$ are linearly independent solutions of the equations

$$l\chi_i = (\mu^2 - k_i^2)\chi_i, \ \mu \in \mathbb{R}^1, \ i = 0, 2.$$
(7)

They are related to the functions $\psi(z, \alpha)$ and $\varphi(z, \alpha)$ [which are linearly independent solutions of the equation $(l + \alpha^2)\chi = 0$] by the formula $\chi(z, \mu) = \chi(z, \alpha = j(\mu^2 - k_i^2)^{1/2})$.

The Wronskian on the cuts passing through the ramification points k_i is given by the formula $w_i(\mu) = w(\alpha = j(\mu^2 - k_i^2)^{1/2}).$

The integral $I_{C_{\Lambda}}$ occuring in (5) has the form

$$I_{C_{\Lambda}} = rac{1}{\pi} \int_{C_{\Lambda}} e^{j\alpha(x-x')} g(z,z';\alpha) d\alpha,$$

where C_{Λ} is the half-circle of radius Λ centered at the point $\alpha = 0$ in the upper half-plane of the complex variable α .

We introduce the functions $\eta^l = \sqrt{\alpha^2 - k_l^2}$, $l = 0, 1, 2, Re\eta^l \ge 0$. The representations of the functions ψ_i , φ_i , and w_i can be derived from ψ , φ , and w with regard to the the fact that $Im\eta^l < 0$ in the domain lying on the left of the hyperbola $\alpha_2 = \sigma_l/(2\alpha_1)$ passing through the point k_l in the upper half-plane of the variable $\alpha = \alpha_1 + j\alpha_2$; next, $Im\eta^l > 0$ in the domain on the right of the hyperbola $\alpha_2 = \sigma_l/(2\alpha_1)$ passing through the point k_l ([4] p. 30, Vol.2 of the Russian translation). The following condition is satisfied on the cuts drawn along the hyperbolas: $\mu > 0$ on the right edge of the cut, and $\mu < 0$ on the left edge of the cut passing trough the points k_i , i=0, 2.

We have $I_{C_{\Lambda}} \to 0$ as $\Lambda \to \infty$.

Using the functions $u_0 = \psi_0$ and $u_2 = \varphi_2$, we can rewrite the function (6) as

$$\sigma_{\Lambda}(z,u) = \sum_{i=0,2} \int_{0}^{M_{i}(\Lambda)} \frac{e^{-\sqrt{\mu^{2} - k_{i}^{2}(x-x')}}}{\sqrt{\mu^{2} - k_{i}^{2}}} u_{i}(z,\mu) u_{i}(z',\mu) dp_{i}(\mu), \tag{8}$$

where $dp_0(\mu) = d\mu/a_0^0(\mu)b_0^0(\mu)2\pi$, $dp_2(\mu) = d\mu/a_2^2(\mu)b_2^2(\mu)2\pi$. a_i^i and b_i^i are coefficients connected with transmission and reflection coefficients of u_i .

In deriving (8), we represent the integral (6) as two integrals over the positive and negative semiaxis and make the change of variables $\mu' = -\mu$ in the integral over the negative semiaxis.

Passing to the limit in (5) as $\Lambda \to \infty$, we obtain the representation

$$u = \sum_{i=0,2} \int_0^\infty \frac{e^{-\sqrt{\mu^2 - k_i^2(x - x')}}}{\sqrt{\mu^2 - k_i^2}} u_i(z, \mu) u_i(z', \mu) dp_i(\mu).$$
(9)

which holds for x - x' > 0.

In a similar way, we consider the case x - x' < 0 by reducing the integral u_{Λ} with the use of the Cauchy theorem in the lower half-plane of the complex variable α to two integral over the edges of the cuts passing through the points $-k_i$.

Passing to the limit in (9) as $x \to x'$, we obtain relation (9)) with x = x'.

3. Conclusion

We have thereby obtained the definitive representation

$$u = \sum_{i=0,2} \int_0^\infty \frac{e^{-\sqrt{\mu^2 - k_i^2 |x - x'|}}}{\sqrt{\mu^2 - k_i^2}} u_i(z, \mu) u_i(z', \mu) dp_i(\mu), \tag{10}$$

which is valid for $x, x'z, z' \in \mathbb{R}^1$. This representation of Green's function u was obtained under the assumption that $\sigma_2 > \sigma_0 > 0$ ($\sigma_2 > \sigma_1 > \sigma_0$) The case $0 < \sigma_2 < \sigma_0$ ($\sigma_2 < \sigma_1 < \sigma_0$) can be treated in a similar way.

The representation (10) is the expansion of Green's function in the fundamental functions u_i , which are bounded on the entire real line R^1 solutions of the ordinary Sturm-Liouville Eq. (7) with complex coefficients. This expansion is characterized by the spectral measure, which is a diagonal matrix function with nonzero entries $p_i(\mu)$.

Equation (7) for the functions u_i indicates that the spectrum $\lambda = \mu^2 - k_i^2$, $\mu \in \mathbb{R}^1$, consists of two half-lines parallel to the real axis on the complex λ -plane, issuing from the points $-k_i^2$, and going in the positive direction of the real axis.

Passing to the limit as $\sigma_2 \rightarrow \sigma_0$, we arrive the case $\sigma_0 = \sigma_2$.

If $\sigma_2 \to 0$ and $\sigma_0 \to 0$, then we obtain the limit case $\sigma_0 = \sigma_2 = 0$. Then the spectrum belongs to real axis, and the spectrum is double for $\lambda \ge -\varepsilon_0$. In this case, the lower bound of the spectrum is limited to the number $\lambda = -\varepsilon_0$. This case is an example of the expansion of a function of the class $L_p(R^1), p > 2$, in the fundamental functions of the Sturm-Liouville operator with a real coefficient $-k^2(z)$ satisfying the Kato condition ([7]).

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Some Applications of the High-mode-merging Method

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Abstract—Waves guided along dielectric step discontinuity can be described by a multi-port network [1] and it is simplified as a two-port network with the influence of high-modes retaining [2]. These results can be used for treat dielectric strip waveguide, even more complicated structures. Some numerical results are got for a strip and a groove dielectric waveguide some kind of resonant phenomena also is obtained. Some comments on this method and some suggestions are given furthermore.

1. Introduction

A dielectric strip waveguide can be seen as a system constituted by 2 step discontinuous structure as shown in Fig. 1. As a symmetric system, it also can be treated by so-called bisection method, namely, it can be see equivalently as the result of superposition of 2 networks with short circuit(sc) and open circuit(oc) separately at the terminals of transmissions of length l/2 (Fig. 2) [1].





Figure 1: Network of a dielectric strip waveguide.

Figure 2: Bisection treatment of the dielectric strip waveguide.

This problem can be simplified by high-mode-merging method provide in [2], a two-port network in which the influence of high modes is considered has been obtained, but in present case, all Z_m in E, F, G and H of [2] ((17a-d) in [2] respectively) are replaced by

$$jZ_m tan k_0(l/2)$$
 (for sc) (1)

$$-jZ_m \cot k_0(l/2) \quad \text{(for oc)} \tag{2}$$

There are three methods to treat the problem of strip waveguide.

2. Equivalent Circuit Method

As most of microwave engineers are more familiar to the circuit language, the network is realized by a simple T circuit generally, (see Fig. 3), impedances of which, Z_a , Z_b and Z_c are related to elements of Z-matrix, $Z_{11}Z_{12}Z_{21}$ and Z_{22} by

$$Z_a = Z_{11} - Z_{12} \quad Z_b = Z_{22} - Z_{12} \quad Z_c = Z_{12} (= Z_{21})$$
(3)

(see appendix) and also be distinguished for sc and oc. In present case, we have Fig. 4(a), the impedances Fig. 4(a), the impedances looking left into the network at 2-2' plane in two cases are

$$Z_{in}^{sc} = Z_b^{sc} + \frac{Z_c^{sc}(Z_a^{sc} + Z_{in,0}^{sc})}{Z_c^{sc} + Z_a^{sc} + Z_{in,0}^{sc}} \quad \text{(for sc)}$$
(4a)

$$Z_{in}^{oc} = Z_b^{oc} + \frac{Z_c^{oc}(Z_a^{oc} + Z_{in,0}^{oc})}{Z_c^{oc} + Z_a^{oc} + Z_{in,0}^{oc}} \quad \text{(for oc)}$$
(4b)



Figure 3: The comparison of a network and a circuit.

where $Z_{in,0}^{sc}$ and $Z_{in,0}^{oc}$ are the input impedances at 1-1' plane looking left into transmission line with length l/2 in sc and oc cases, which can be got by (1a) and (1b) respectively.



Figure 4: a) T-circuit b) Γ -circuit.

The system in Fig. 4(a) also can be changed to a Γ -circuit as shown in Fig. 4(b). Obviously,

$$Z_{\Gamma,in}^{sc} = Z_{\Gamma,b} \tag{5a}$$

$$Z_{\Gamma,in}^{oc} = Z_{\Gamma,b} + 2Z_{\Gamma,m} \tag{5b}$$

Let them be equivalent, i.e., we have $Z_{T,in}^{sc} = Z_{\Gamma,in}^{sc}$ and $Z_{T,in}^{oc} = Z_{\Gamma,in}^{oc}$, then,

$$Z_{\Gamma,b} = Z_{\Gamma,in}^{sc} = Z_{T,in}^{sc} \tag{6a}$$

$$Z_{\Gamma,m} = (Z_{\Gamma,in}^{oc} - Z_{\Gamma,b})/2 = (Z_{T,in}^{oc} - Z_{T,in}^{sc})/2$$
(6b)

where $Z_{T,in}^{sc}$ and $Z_{T,in}^{oc}$ are given by (4a) and (4b) respectively. A strip dielectric waveguide can be seen as a combination of two Γ -circuits which connected back to back as shown in Fig. 5(a), then it can be reformed as Fig. 5(b). Going back to network, the elements of a strip waveguide can be got by (3a) and (3b) in opposite way.



Figure 5: (a) Combination of two Γ -circuits (b) Its deformation.

3. Equivalent Network Method

A little bit different approach is called equivalent network method (EN method). The elements of Z-matrix can be normalized as follows.

$$Z'_{11} = Z_{11}/Z_0 \qquad Z'_{12} = Z_{12}/\sqrt{Z_0 \overline{Z_0}}$$
(7a)

$$Z'_{21} = Z_{11} / \sqrt{\overline{Z_0}} Z_0 \qquad Z'_{22} = Z_{22} / \overline{Z}_0$$
(7b)

Considering the high-mode=merging method, we have

$$Z'_{11} = (\frac{F}{H})/Z_0 \qquad Z'_{12} = (E + \frac{GF}{H})/\sqrt{Z_0\overline{Z_0}}$$
 (8a)

$$Z'_{21} = (\frac{1}{H})/\sqrt{\overline{Z_0}Z_0} \qquad Z'_{22} = (\frac{G}{H})/\overline{Z}_0$$
 (8b)

Where E, F, G and H are given in [2], and the determinate of normalized Z-matrix

Det
$$Z' = Z'_{11}Z'_{22} - Z'_{21}Z'_{12} = \frac{E}{H}/(Z_0\overline{Z}_0)$$

then we can get the transfer matrix for right step discontinuity of the strip waveguide

$$A'_{r} = A' = \frac{1}{Z'_{21}} \begin{bmatrix} Z'_{11} & |Z'| \\ 1 & Z_{22} \end{bmatrix} = \begin{bmatrix} F\sqrt{\overline{Z}_{0}/Z_{0}} & -E/\sqrt{\overline{Z}_{0}\overline{Z}_{0}} \\ H\sqrt{\overline{Z}_{0}\overline{Z}_{0}} & G/\sqrt{\overline{Z}_{0}/\overline{Z}_{0}} \end{bmatrix}$$
(9)

Let voltage and current in both sides of the right discontinuity be normalized as

$$V_0' = V_0 / \sqrt{Z_0}, \ I_0' = I_0 \sqrt{Z_0}, \ \overline{V}_0' = \overline{V}_0 / \sqrt{\overline{Z}}_0, \ \overline{I}_0' = \overline{I}_0 \sqrt{\overline{Z}}_0$$

Then we have

$$\begin{bmatrix} V_0'\\ I_0' \end{bmatrix} = A_r' \begin{bmatrix} \overline{V_0'}\\ \overline{I_0'} \end{bmatrix}$$

Considering symmetry of the strip, the matrix of left step discontinuity is just the inverse matrix of one of the right step:

$$A'_{l} = (A'_{r})^{-l} (= (A')^{-l}) = \begin{bmatrix} G\sqrt{\overline{Z}_{0}/Z_{0}} & H/\sqrt{\overline{Z_{0}}\overline{Z}_{0}} \\ -E\sqrt{\overline{Z_{0}}\overline{Z}_{0}} & F/\sqrt{\overline{Z_{0}}/\overline{Z}_{0}} \end{bmatrix}$$
(10)

The uniform structure between two step discontinuities corresponds a segment of an uniform transmission line with length l, the transfer matrix of which is

$$A'_{m} = \begin{bmatrix} \cos k_{0}l & j \sin k_{0}l \\ j \sin k_{0}l & \cos k_{0}l \end{bmatrix}$$
(11)

Finally, the transfer matrix of the whole strip can be got as the continued-multiplication product:

$$A'_{strip} = A'_l A'_m A'_r = (A')^{-1} A'_m A'$$
(12)

This procedure can be shown in Fig. 6(a). For a rectangular groove dielectric waveguide, corresponding matrix, then, is

$$A'_{groove} = A'A'_{m}(A')^{-1}$$
(13)

(see Fig. 6(b)).



Figure 6: (a) Strip waveguide (b) Groove waveguide .

4. Effective Dielectric Constant Method

Besides there is also a rather rough method, in which the influence of all high-modes is neglected, that we only take the A'_m as the transfer matrix of whole strip (or groove waveguide):

$$A'_{strip} = A'_m \tag{14}$$

It's so-called effective dielectric constant (EDC) method.

5. Numerical Examples

For comparing these 3 methods, some numerical calculations have been done for some characteristics of some kinds of waveguides: Fig. 7(a)-(c) show plots of reflection and transmission coefficients (including the argument and modulus both of them) vs width of waveguide; Fig. 7(d) gives ones for loss.



Figure 7: Some numerical results.





Figure 9.

6. Conclusion

- (1) The results for all methods give similar tendency and accord one another pretty good in certain accuracy; especially results of EN method and EC method are more closed.
- (2) The EDC method is still useful in some cases because it is rather simple and easy for calculations and with clear physical meaning. A significant defect is that it can't give the loss.
- (3) There is some kind of periodic phenomena existing. The reflection coefficients, both argument and modulus, and loss are varying with the width of waveguide periodically. It is coincide with the conclusion of [3]. This phenomena can be seen as resonance, but the mechanism of it is remained to be explained further. It is also indicated that the high-mode-merging method is correct.
- (4) In high-mode-merging method, the coupling between TE modes and TM-modes has not been considered. It is also one of the defects of this theory.
- (5) In the original theory of [1], two parallel perfect conductive planes are needed. So, the waveguide discussed here is not open absolutely. If the upper one of them moved far enough, it almost can be seen as an open one approximately. If we want to remove the upper conductive plane, we'll get infinite number of continuous high modes it is a problem of continuous spectrum and is out of the topic of this paper.
- (6) The cascade network method is not confined to solve only symmetric system like single strip dielectric waveguide but also can be extended to treat some more complicated structures, such as finite periodic strip(groove) dielectric waveguide (Fig. 8), the curved surface dielectric waveguide (Fig. 9) etc.

Appendix

Consider a T-circuit as shown in Fig. 3. The transfer matrixes of Fig. 3 devices are

$$A_a = \begin{bmatrix} 1 & Z_a \\ 0 & 1 \end{bmatrix}, \quad A_c = \begin{bmatrix} 1 & 0 \\ 1/Z_c & 1 \end{bmatrix}, \quad A_b = \begin{bmatrix} 1 & Z_b \\ 0 & 1 \end{bmatrix}$$

As a whole, the transfer matrix then is

$$A = A_a A_c A_b = \begin{bmatrix} 1 + Z_a/Z_c & Z_a + Z_b + Z_a Z_b/Z_c \\ 1/Z_c & 1 + Z_b/Z_c \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where $a = 1 + Z_a/Z_c$, $b = Z_a + Z_b + Z_aZ_b/Z_c$, $c = 1/Z_c$, and $d = 1 + Z_b/Z_c$ with |A'| = ad-bc=1. Then, changing it to impedance matrix equivalently, we get

$$Z = \frac{1}{c} \begin{bmatrix} a & |A| \\ 1 & d \end{bmatrix} = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Namely, we get

$$Z_{11} = Z_a + Z_c \qquad \qquad Z_{12} = Z_{21} = Z_c \qquad \qquad Z_{22} = Z_b + Z_c$$

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Design of a Non-uniform High Impedance Surface for a Low Profile Antenna

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Abstract—Two Non-Uniform High Impedance Surfaces (NU-HIS or tapered HIS) are proposed against a Uniform HIS (U-HIS). All surfaces are one dimensional (1D) and made of parallel wires with a length a little less than $\lambda/2$ around the resonance frequency. To show the effect of the surfaces, a half wavelength dipole antenna is placed over four different surfaces, PEC, U-HIS, NU-HIS, and modified NU-HIS (MNU-HIS) while the dipole height is fixed and very close to the surface. These four EM problems are analyzed numerically by the method of moments (MoM), and the results are compared. It is concluded that MNU-HIS yields more bandwidth than NU-HIS, and also, NU-HIS yields more bandwidth than U-HIS, while overall structures in all cases have identical volumes and nearly identical gains. This effect is attributed to the decrease of sensitivity to the angle of incidence by applying non-uniformity.

1. Introduction

It is well known that a high impedance surface or specifically a hypothetical perfect magnetic conductor may be very useful in a large variety of microwave and antennas applications. Recently, electromagnetic bandgap (EBG) structures have been widely studied for their behavior as High Impedance Surface (HIS) or Artificial Magnetic Conductor (AMC). Principally, they show stop band frequencies in which the tangential magnetic fields are considerably reduced. AMC is a special member of HIS family, which is designed to imitate the behavior of a perfect magnetic conductor (PMC). In fact, the AMC condition is characterized by the frequencies where the phase of the reflection coefficient is zero, i.e., $\Gamma = +1$ [1]. In contrast, a HIS may deviate a little from this condition, sometimes yielding more flexibility in antenna design. For example, in [2], the mushroom structure played the role of a ground plane for a dipole antenna a little upper than its resonance or AMC condition. Besides, in [3] the behavior of the same structure as a reactive impedance surface (RIS) was introduced, and the idea was applied to patch and dipole antennas. Repeatedly, it has been shown that HIS structures improve antenna performance and reduce the effects of surface waves. The latter feature yields better antenna radiation pattern and less coupling between elements of an array [2, 4]. So far, some 3D [5] and 2D [1, 3] structures have been proposed to realize HISs. Current realizations of 2D HISs are based on a planar FSS at the interface of a metal-backed dielectric slab with or without vertical vias [6]. This configuration is desirable because it is low-cost and easy to integrate in practice [7]. There is a problem with most of proposed HISs, however. In fact, the shift of the resonant frequency versus the incidence angle affects the performance of most well known HISs [8,9]. To clarify this flaw, the behavior of a typical mushroom structure for different angles of incidence is presented in Fig. 1.

The curve has been obtained by Ansoft's Designer software, which is an electromagnetic solver based on MoM (equipped with Periodic MoM, PMM [1]). Generally, if the frequency bandwidth of low-profile antennas, placed near a typical HIS is within the resonance band of HIS, a significant improvement in the radiation efficiency is expected, compared to the conventional cases using PEC ground plane. However, the improvement is not always as much as desired [10]. An explanation for this behavior is that the high-impedance surface does not exhibit uniform surface impedance with respect to the different spatial harmonics radiated by an antenna, as depicted in Fig. 1. For instance, it is known that electrically small horizontal antennas radiate a large angular spectrum of TE and TM-polarized plane waves. As a result, the resonant frequency at which the effect of the magnetic wall is observed depends on the incidence angle; Therefore, the total interaction between the antenna and the HIS will be a summation of constructive and destructive effects [6].

References [1, 6, 7, 9, 11] are examples of the works concentrating completely on designing angularly stable HISs or AMCs. In all of these cases, the basic cell shape is changed and optimized, while the cell size is fixed throughout the structure (uniformly periodic structures). In the present work, we seek angular stability for HIS in order to improve antenna radiation near the surface. This is done by applying non-uniformity to a uniform



Figure 1: Behavior of a typical mushroom structure in different angles of incidence, f = 18.55 GHz, a) front view, cross section, and the relevant dimensions, b) phase of reflection coefficient versus angle of incidence obtained by Ansoft's Designer software.

HIS. Two 1D NU-HISs made of parallel wires are proposed, and their behaviors are compared with that of the uniform version (U-HIS). Then, the performance of a half wavelength dipole very close to all of these surfaces is investigated. During the process, as shown in Fig. 2, the dipole length and radius ($\approx 0.45\lambda \& \lambda/220$), the spacing of side elements from center ($\approx 0.23\lambda$), and the spacing of dipole from the lower section of the planes (PEC planes) ($\approx \lambda/12$) are kept fixed for better understanding of the influence of the surfaces alone. Because the structures are composed of wires, NEC software (NEC Win Pro. V 1.1), which is an electromagnetic solver based on MoM, is used for analysis.



Figure 2: The geometry of the dipole antenna located over a) U-HIS, b) NU-HIS, and c) MNU-HIS.

2. The Main Idea, Explanation and Verification

The underlying basis for the idea in this paper returns to an important clue from this equation [12]:

$$\frac{X}{\eta_0} = F(p, w, \lambda) = \frac{p \cos \theta}{\lambda} \left[\ln \left(\cos ec(\frac{\pi w}{2p}) \right) + G(p, w, \lambda, \theta) \right]$$
(1)

where η_0 and λ are free space wave impedance and wavelength respectively. Also, G is a correction term for large angles of incidence. The equation gives the surface impedance of parallel strips facing a TE plane wave as depicted in Fig. 3. Ignoring G in (1), the clue is that when w and λ are fixed, X can be kept stable by a proper



Figure 3: Front and side view of parallel metal strips (or equivalently wires) facing a TE plane wave impinging in different angles of incidence.



Figure 4: VSWR and input impedance of the dipole located over a) PEC ground plane, b) U-HIS, c) NU-HIS, d) MNU-HIS, e) MNU-HIS with finite wire ground plane, and f) the last design after a little tuning the dipole length and radius of side wires.

Figure 5: Gain (dB) for Fig. 2 (c), while the infinite PEC plane is replaced by the finite wire ground plane, a) E-plane, b) H-plane.

increase of p against the increase of θ . In other words, by gradually increasing p from center elements to the side ones (applying tapering), more angular stability is achievable. Note that the same effect is also attained by gradually decreasing w. But there are three problems in using such an idea. Firstly, as in Fig. 2, we have used parallel wires instead of strips; Secondly, (1) is not correct when the structure is placed near the PEC plane; Thirdly, in Fig. 3, the length of strips are infinite while those of this work are finite ($\approx \lambda/2$). The first problem is solved considering the nearly equivalent scattering properties of strip and wire as depicted in Fig. 3 and stated in [6]. As for the second, it can be said that because here we need the general (not exact) effect of tapering on X, we can foresee that even in the present condition the general behavior in (1) remains true. Finally, as for the third, it is reminded from transmission line theory that a $n\lambda/2$ slice of a transmission line represents an infinite line because the input impedance of such a line equals the load impedance. Fortunately, our numerical investigations have confirmed the correctness of the approximations and predictions above, at least for our proposed structures.

To study the effect of the idea, a half wavelength dipole antenna is numerically analyzed (by NEC) placed



Figure 6: Near electric fields on the high impedance surfaces along Y-axis, excited by the dipole in Fig. 2, X = 0, Z = 7.5 cm, f = 184 MHz, a) U-HIS (a), b) NU-HIS (b), and c) MNU-HIS (c) (upper row for amplitude and lower for phase of the fields).



Figure 7: Phase of near electric fields excited by TE plane wave (in Fig. 2, E_X) on the high impedance surfaces, along Y-axis, X = 0, Z = 7.5 cm, f = 184 MHz.

over four different surfaces, PEC, U-HIS, and NU-HIS, and modified NU-HIS (MNU-HIS) while the dipole height is fixed and very close to the surface. The proposed HISs are shown in Fig. 2 in which the dipole and the parasitic wires radii are 1 mm and 8 mm, respectively. As in Fig. 4 (a), for the dipole near the PEC plane without any parasitic wires, there is no resonance in Z_{in} . As a result, the VSWR is very poor. Deploying uniformly-placed wires ($\approx \lambda/17.5$) close to the PEC plane ($\approx \lambda/22$), as in Fig. 2 (a), a U-HIS is formed. As a result, the VSWR of the dipole will improve very much as in Fig. 4 (b). The bandwidth on VSWR (< 2, $Z_0 = 50$) is 6.3%. The curves are very similar to those in [2] and [3]. Now the non-uniformity idea emerging from (1) is applied by removing the two wires A and A' in Fig. 2 (a) and properly shifting the positions of B and B' sidewards. The best result rendering the most bandwidth is a Non-Uniform HIS (tapered HIS) shown in Fig.2 (b). Here the spacing BC is about $\lambda/13.5$. Fig. 4 (c) shows the VSWR and Z_{in} of this surface. As observed, the bandwidth increases form 6.3% to 9.3%. In the second step, considering the same point emerging from (1), it seems that also by making the center elements, C, D and C' a little denser the bandwidth may become better. Thus, using a simple optimization procedure, the spacing CD and simultaneously BC are adjusted in order to optimize the bandwidth on VSWR. The result is referred to as MNU-HIS and is shown in Fig. 2 (c). The spacing BC and CD are about $\lambda/13$ and $\lambda/29$ respectively. Fig. 4 (d) shows the related VSWR and Z_{in} . As seen, the bandwidth increases from 9.3% to 11.33%. Note that in all of these cases, the overall gain is nearly identical $(\approx 9 \text{ dB})$ while the overall structure volume is fixed (not including the PEC plane, $0.45\lambda \times 0.45\lambda \times \lambda/12$).

Up to this point, all of the presented designs used an infinite PEC plane. In the next step, this ideal plane is modeled in NEC as a real finite plane ($x \approx 0.45\lambda \& y \approx 0.63\lambda$). Therefore, the overall structure volume is $(0.45\lambda \times 0.63\lambda \times \lambda/12)$. The corresponding VSWR and Z_{in} are shown in Fig. 4 (e). As obvious, due to cutting the plane, the bandwidth deteriorates to 7.95%. To remove this descent, the radius of the side elements is tuned a little. In fact, from (1), it is deduced that gradually reducing the radius is an alternative means of improving angular stability of the surface. This tuning is done simultaneously with a little tuning of the dipole length. After tuning, the best side elements radius is 7 mm (formerly 8 mm) and dipole length is 79.6 cm (formerly fixed at 80 cm). The improved result shows 9.45% bandwidth as in Fig. 4 (f). The relevant gains in E and H-planes are depicted in Fig. 5. To give better understanding of the behavior of the surfaces, phase and amplitude of near fields excited by the dipole on the surfaces are presented in Fig. 6. In addition, the phases of near fields (on the surfaces) exited by a TE plane wave in different angles of incidence are rendered in Fig. 7. As deduced from Fig. 6, both the amplitude and phase become more stable as a result of imposing non-uniformity. In other words, in MNU-HIS the element right under the dipole and those on sides are illuminated much the same by the dipole. This is an implication for angular stability of the surface. Note that from apertures theory it is known that uniform phase and amplitude is an ideal condition yielding maximum performance. Fig. 7 is also a good indicator of angular stability of the surfaces. As observed, the phase of near fields on NUM-HIS withstands the most against increase of incidence angle. It can be concluded that the more the surface is angularly stable, the more bandwidth it renders near the dipole antenna. In other words, angular stability of the HIS, obtained through non-uniformity, improves the antenna performance.

3. Conclusion

The paper studies the effects of applying non-uniformity to a 1D uniform HIS. The proposed surfaces are made of parallel wires placed uniformly (U-HIS) or non-uniformly (NU-HIS) over a PEC ground plane. To show the effect of imposing non-uniformity, a half wavelength dipole antenna is numerically analyzed by MoM in the close vicinity of four different ground planes, PEC, U-HIS, NU-HIS, and modified NU-HIS (MNU-HIS), while the dipole height and length are kept fixed. Comparison of the results shows that MNU-HIS yields more bandwidth than NU-HIS, and also, NU-HIS yields more bandwidth than U-HIS, while all cases have identical volume and nearly identical gain. This effect is attributed to the improvement of angular stability of the surfaces caused by applying an apt non-uniformity.

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Survey on Interference Mitigation via Adaptive Array Processing in GPS

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Abstract—Due to the extremely low power, GPS signal can be easily affected by interferences. In this paper, from the point of adaptive array processing, we review the existing spatial and space-time interference suppression methods which attempt to mitigate interferences before the GPS receiver performs correlation. These methods comprise self-coherence restoral technique based on the nature of GPS signal, space-time minimum mean square error, power minimization technique, GPS multipath mitigation technique using the vertical array etc. Also we summarize their performance and applicability by analyzing all these techniques, in which some of our work and opinions are included.

1. Introduction

Global Positioning System (GPS) is a satellite-based navigation system which can provide the position, velocity and timing information for users in all weather conditions, anywhere in the world and anytime in the day. Therefore, it has been widely used in civil and military applications such as navigation by general aviation, positioning for users and so on. Because of its high precision, general acceptability and easier equipment of user's receiver, GPS will gradually become a main means of global navigation.

However, GPS signal is susceptible to interferences from either intentional or unintentional sources for the reason that it arrives at the receiver at a very low-power level, typically $20 \sim 30 \text{ dB}$ below the receiver's thermal noise level [1]. Based on that, the performance of GPS navigation and positioning degrades dramatically. Hence, one of the hot topic of using GPS is to cancel the interference as completely as possible without any distortion of desired GPS signal.

Conventional GPS suppression methods including time-domain and frequency-domain filtering techniques [1–4] employ DFT technology to suppress interference by taking out abnormal spectrum line of digital intermediate frequency signal. These methods have the advantages of easy implementation and low cost, but they can not mitigate multiple narrowband interferences as well as wideband interferences owing to its incapability of differentiating between desired signal and interference in the spatial domain. However, array signal processing techniques can efficiently suppress the above interferences according to the spatial information. Adaptive nulling technique [5] based on array antennas can adaptively place nulls in the direction of interferences, which is very popular to be used in improving the performance of GPS receiver. Unfortunately, the above method may be inadequate for broader band operation, especially when interference multipath is present. In order to solve this problem, space-time adaptive processing (STAP) techniques [6–9] are proposed in recent years. STAP can greatly increase the number of degrees of freedom under the equivalent antennas condition and thus can efficiently suppress wideband interference. So it is a trend of GPS interference mitigation.

This paper mainly discusses the existing methods based on adaptive array processing. The following section will describe some spatial techniques emphasis on self-coherence restoral technique using the nature of GPS signal. In Section 3 we firstly give a uniform data model, and then several STAP methods consisting of maximum signal-to-interference ratio algorithm, minimum mean square error, space-time Capon algorithm and power minimization technique are described. GPS multipath mitigation technique using the vertical array is given in Section 4. While, concluding remarks are given in Section 5.

2. Spatial Adaptive Processing Techniques

As demand for accurate GPS positioning, adaptive beamforming algorithm should not cause any significant distortion of the desired GPS signals when it is used to suppress interferences. Such algorithms include the direction finding algorithm [10], which is an adaptive beamforming algorithm based on estimation of the directions of arrival (DOAs) of the received signals, and directional constrained adaptive beamforming algorithm, which is based on the principle that if the dynamic of the GPS receiver is not too high and the beampattern

is not too narrow, rough coordinates of the receiver and the coordinates of the satellites in view can be used to calculate the DOAs of the signals of interest from different satellites, and so on. However, these algorithms do not fully take advantage of the GPS signal structure. So Wei Sun proposed a GPS interference mitigation method using self-coherent feature of GPS signal [11, 12].

The method considers interference suppression in GPS using spatial processing that incorporates the known temporal structure of the GPS signal. And it utilizes the replication property of the C/A-code within the navigation symbol to suppress interferences which are aperiodic or have a different periodic signal structure from that of the GPS signal. A block diagram of the proposed algorithm is shown in Fig. 1, which consists of a main channel and a reference channel. The samples in the main channel and the reference channel are processed by a beamformerwand another processor **f** respectively, where the samples of reference channel are lP chips $(P = 1023, 1 \le l < 20)$ delay of the main channel's data. These samples in the main channel are given by:

$$\mathbf{x}(n) = \mathbf{a}s(n) + \sum_{j=1}^{K} \mathbf{b}_j i_j(n) + \mathbf{v}(n)$$
(1)

where $\mathbf{x}(n)$ is the $M \times 1$ data vector, s(n) is the desired GPS signal and $i_j(n)$ is the *j*th interference, **a** and \mathbf{b}_j are $M \times 1$ steering vectors of the desired GPS signal and the jth interference respectively, and $\mathbf{v}(n)$ is the thermal noise vector. This paper provided that GPS signal, interference, and noise are uncorrelated unless special statement.

Due to the repetition of GPS signal, GPS signal samples of two channels in Fig. 1 have the same values as long as they are within the same symbol. However, the interference samples have different values because they are aperiodic or have a different periodic signal structure from that of the GPS signal. Thus the samples in the reference channel are given by:

$$\mathbf{x}(n-lP) = \mathbf{a}s(n) + \sum_{j=1}^{K} \mathbf{b}_j i_j \left(n-lP\right) + \mathbf{v}(n-lP)$$
(2)

The algorithm proposed can adaptively update the weight vectors \mathbf{w} and \mathbf{f} by maximizing the crosscorrelation between the output of the main channel and the reference channel. Accordingly, we define the following cost function:

$$C(\mathbf{w}, \mathbf{f}) = \frac{|R_{zd}|}{R_{zz}R_{dd}} = \frac{|\mathbf{w}^H \mathbf{R}_{xx}^P \mathbf{f}|^2}{|\mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}| |\mathbf{f}^H \mathbf{R}_{xx} \mathbf{f}|}$$
(3)

where

$$R_{zd} = E \left\{ z(n)d^{H}(n) \right\} = \mathbf{w}^{H} E \left\{ \mathbf{x} \left(n \right) \mathbf{x}^{H} \left(n - lP \right) \right\} \mathbf{f} = \mathbf{w}^{H} \mathbf{R}_{xx}^{P} \mathbf{f}$$

$$R_{zz} = E \left\{ z\left(n \right) z^{H}\left(n \right) \right\} = \mathbf{w}^{H} E \left\{ \mathbf{x} \left(n \right) \mathbf{x}^{H}\left(n \right) \right\} \mathbf{w} = \mathbf{w}^{H} \mathbf{R}_{xx} \mathbf{w}$$

$$R_{dd} = E \left\{ d\left(n \right) d^{H}\left(n \right) \right\} = \mathbf{f}^{H} E \left\{ \mathbf{x} \left(n - lP \right) \mathbf{x}^{H}\left(n - lP \right) \right\} \mathbf{f} = \mathbf{f}^{H} \mathbf{R}_{xx} \mathbf{f}$$
(4)

The algorithm makes full use of the nature of GPS signal and does not need any knowledge of transmitted signals or the location of the satellite. Meanwhile, it is not sensitive to steering error and robust. So the algorithm is a promising method in GPS interference cancellation.

Generally speaking, spatial adaptive processing techniques are easy to implement and convenient for calculation. But it will increase array cost for an interference consuming one degree of freedom. To solve this problem, the techniques based on space-time joint processing are proposed [7–9, 13]. They all provide more degrees of freedom via time tap than only space processing.

3. Space-time Joint Processing Techniques

STAP algorithms employ the multiple receiving elements ("space") of an antenna array and multiple temporal samples ("time") to cancel interferences. The space-time weights are realized through a tapped-delay-line behind each antenna, as shown in Fig. 2. Some scholars, such as Dr. Fante and Dr. Zoltowski, have gained some achievements in GPS interference mitigation based on STAP [7, 8, 14]. In this section, based on the fruits of their study, we give the general space-time data model for GPS interference suppression.



Figure 1: Block diagram of the interference suppression based on the self-coherence of GPS signal.



Figure 2: Block diagram of the STAP technique for GPS interference mitigation.

3.1. Data Model

The space-time data model can be written as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \sum_{j=1}^{K} \mathbf{B}_j \mathbf{i}_j(t) + \mathbf{V}(t)$$
(5)

where $\mathbf{x}(t) = [x_{11}(t) \dots x_{M1}(t) x_{12}(t) \dots x_{M2}(t) \dots x_{1N}(t) \dots x_{MN}(t)]^T$ is the received data (*M* is the number of antenna and *N* is the number of tap each antenna), $\mathbf{A} = \mathbf{I}_{N \times N} \otimes \mathbf{a}$ and $\mathbf{s}(t) = [s(t) \dots s(t - (N - 1)T)]^T$, \mathbf{B}_i and $\mathbf{i}_i(t)$ have the same structure as \mathbf{A} and $\mathbf{s}(t)$ respectively.

3.2. STAP Algorithm

3.2.1. Maximum Signal-to-Interference-plus-Noise Ratio

This approach chooses weight vectors \mathbf{w} to maximize signal-to-interference-plus-noise ratio of the output of beamformer. Accordingly, there is the following cost function:

$$\mathbf{w}_{opt} = \arg\max_{\mathbf{w}} SINR = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w}}$$
(6)

where \mathbf{R}_n is interference-plus-noise covariance matrix. Because the GPS signal strength is at least 20 dB below the thermal noise floor, \mathbf{R}_n can be estimated by averaging approximately 4MN independent samples of the received signal [15], namely:

$$\mathbf{R}_{n} \approx \mathbf{R} = E\left\{\mathbf{x}(t)\mathbf{x}^{H}(t)\right\} \approx \frac{1}{4MN} \sum_{q=1}^{4MN} \mathbf{x}\left(q\right)\mathbf{x}^{H}\left(q\right)$$
(7)

 \mathbf{R}_s is the desired GPS signal covariance matrix, which can be derived by the density of power spectrum of GPS signal. Note that this method requires information on platform attitude in order to determine \mathbf{R}_s , and meanwhile the processor is required being repeated for each GPS satellite which is used to determine user's position.

3.2.2. Minimum Mean Square Error Algorithm

This method obtains the weight vector by minimizing the mean square error between the desired GPS signal and the output of the processor in Fig. 2. Accordingly, the cost function is given by:

$$\mathbf{w}_{opt} = \operatorname*{arg\,min}_{\mathbf{w}} E\left\{ \left(s_d - \mathbf{w}^H \mathbf{x} \right) \left(s_d - \mathbf{w}^H \mathbf{x} \right)^H \right\}$$
(8)

where s_d is the desired signal. By solving this optimum question, we can find:

$$\mathbf{w}_{opt} = \mathbf{R}^{-1} \mathbf{g}_s \tag{9}$$

where $\mathbf{g}_s = E\{\mathbf{x}s_d^*\}$ is the first column of \mathbf{R}_s . Note that the processor is also repeated for each GPS satellite to calculate user's position and requires attitude information.

3.2.3. Space-time Capon Beamforming

When the direction of desired GPS satellite signal can be estimated, this algorithm minimizes the output power with attempting to preserve the gain in the desired signal direction for each of the N "tap times" of the processor in Fig. 2. This leads to the power minimization with N linear constraints:

$$\min_{\mathbf{w}} \mathbf{w}^{H} \mathbf{R} \mathbf{w}$$

s.t. $\mathbf{w}_{i}^{H} \mathbf{a} = 1, \ i = 1, \dots, N$ (10)

where $\mathbf{w} = [\mathbf{w}_1, \dots, \mathbf{w}_N]^T$, $\mathbf{w}_i = [w_{1i}, \dots, w_{Mi}]$. And (10) can be rewritten as:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}$$

$$s.t. \mathbf{w}^H \mathbf{A} = \mathbf{1}_{N \times 1}$$
(11)

Using the method of Lagrange multipliers, the solution to (11) is:

$$\mathbf{w}_{opt} = \mathbf{R}^{-1} \mathbf{A} (\mathbf{A}^H \mathbf{R}^{-1} \mathbf{A})^{-1} \mathbf{1}_{N \times 1}$$
(12)

This result is similar to the one obtained by standard capon beamforming (SCB). So the approach is called space-time capon method. Like the SCB, the performance of the approach becomes worse when steering vector error exists. Therefore, some robust STAP algorithm will be developed based on the robust capon beamforming algorithm [16].

3.2.4. Space-time Power Minimization Algorithm

Because the received GPS satellite signals are well below the thermal noise floor, this method is extraordinary efficient for GPS interference cancellation. It simply constraints the weight on the first tap of reference antenna 1 (see Fig. 2), and then minimizes the output power, namely:

$$\min_{\mathbf{w}} \mathbf{w}^{H} \mathbf{R} \mathbf{w}$$

$$s.t. \mathbf{w}^{H} \boldsymbol{\delta}_{MN} = 1$$
(13)

where $\boldsymbol{\delta}_{MN} = [1, 0, \dots, 0, \dots, 0]^T$ is the $MN \times 1$ vector. Using the method of Lagrange multipliers, the solution to (13) is

$$\mathbf{w}_{opt} = \frac{\mathbf{R}^{-1} \boldsymbol{\delta}_{MN}}{\boldsymbol{\delta}_{MN}^{H} \mathbf{R}^{-1} \boldsymbol{\delta}_{MN}} \tag{14}$$

This approach has the advantages of not requiring to know the DOA of the incoming GPS signal and implementing easily. So it is adopted by some available GPS antennas [13] to mitigate interference.

3.3. Reduced-rank STAP Technique

Because of the large dimensionality of the space-time covariance vector and weight vector, STAP techniques will lead to a larger computational burden and slower convergence. Therefore, the study on reduced-dimension techniques becomes a hot topic in recent years [8, 17]. Reduced-dimension techniques are mainly to constraint weight vector to lie in a lower dimensional subspace by the transformation matrix $\mathbf{T}_{NM \times D}$ (D < NM), namely let:

$$\mathbf{w} = \mathbf{T}\mathbf{w}_r \tag{15}$$

so (13) can be rewritten as:

$$\min_{\mathbf{w}} \mathbf{w}_{r}^{H} \mathbf{T}^{H} \mathbf{R} \mathbf{T} \mathbf{w}_{r}$$
s. t. $\mathbf{w}_{r}^{H} \mathbf{T}^{H} \boldsymbol{\delta}_{MN} = 1$
(16)

the solution to (16) is

$$\mathbf{w}_{r} = \frac{(\mathbf{T}^{H}\mathbf{R}\mathbf{T})^{-1}\mathbf{T}^{H}\boldsymbol{\delta}_{MN}}{\boldsymbol{\delta}_{MN}^{H}\mathbf{T}(\mathbf{T}^{H}\mathbf{R}\mathbf{T})^{-1}\mathbf{T}^{H}\boldsymbol{\delta}_{MN}}$$
(17)

where the dimension of $\mathbf{T}^{H}\mathbf{R}\mathbf{T}$ is $D \times D$, which is less than the one of \mathbf{R} . This leads to lower computational complexity and rapid convergence. We can obtain the matrix \mathbf{T} by techniques such as the cross-spectral metric (CS) or principal-components (PC). But both techniques are quite computational burden since it is necessary to generate the eigenvectors of covariance matrix before finding \mathbf{T} .

Fortunately, Dr. Zoltowski proposed a reduced-dimension STAP technique based on multistage nested wiener filter (MSNWF) [17]. This technique accomplishes the reduced-dimension processing via the innovative multistage wiener filter and does not require computing the inversion of \mathbf{R} . Thus it can reduce computational complexity and improve the speed of convergence compared with CS and PC.
4. GPS Signal Multipath Mitigation Techniques

The error due to GPS signal multipath is an important factor of positioning error. At the present time, the common techniques for multipath mitigation mainly include DLL and MEDLL. Both techniques change a standard receiver structure, so their compatibility is very poor. In [18] Dr. Stoica proposed a multipath mitigation algorithm based on the vertical array, which suppress multipath interference before correlation without changing the receiver structure.

The above method assumes that the directions of arrival of the GPS multipath signals are approximately known relative to the direction of arrival of the GPS signal, which is possible in GPS vertical array. When the GPS multipath signals exist, the data model received by GPS vertical array is given by:

$$\mathbf{x}(t) = \mathbf{a}s(t) + \sum_{q=1}^{Q} \mathbf{a}_q \beta_q s(t) + \sum_{j=1}^{K} \mathbf{b}_j i_j(t) + \mathbf{v}(t)$$
(18)

(18) can be rewritten as:

$$\mathbf{x}(t) = (\mathbf{a} + \mathbf{V}\beta)s(t) + \sum_{j=1}^{K} \mathbf{b}_j i_j(t) + \mathbf{v}(t)$$
(19)

where the matrix **V**'s range space is a good approximation of the one spanned by the GPS multipath signals, β is an unknown vector whose elements equal to the ratios between the GPS multipath signals and the GPS signal. According to literature [16], **a** and β can be determined by solving the following problem:

$$\min_{\mathbf{a},\boldsymbol{\beta}} (\mathbf{a} + \mathbf{V}\boldsymbol{\beta})^H \mathbf{R}^{-1} (\mathbf{a} + \mathbf{V}\boldsymbol{\beta})$$

s. t. $\mathbf{a} = \mathbf{B}\mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^2 \le \varepsilon$ (20)

let **G** be a basis of the null space of \mathbf{V}^{H} , so (20) can be rewritten as:

$$\min_{\mathbf{a},\beta} \mathbf{a}^{H} \mathbf{G} \left(\mathbf{G}^{H} \mathbf{R} \mathbf{G} \right)^{-1} \mathbf{R}^{-1}$$
s. t. $\mathbf{a} = \mathbf{B} \mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^{2} \le \varepsilon$
(21)

let $\hat{\mathbf{a}}$ denotes the solution to (21), we can find the weight vector:

$$\mathbf{w} = \mathbf{G} (\mathbf{G}^H \mathbf{R} \mathbf{G})^{-1} \mathbf{\hat{a}}$$
(22)

In essence, the method proposed by Dr. Stoica removes the GPS multipath signals by "pre-filtering" the received data via the matrix \mathbf{G}^{H} . Although this algorithm is appropriate to GPS vertical array, its main idea can be further extended to the general array to suppress GPS multipath signals.

5. Conclusion

Several methods used to mitigation GPS interference have been discussed in this paper. The conventional spatial techniques can adaptively null the interference, but they are incapability of canceling many narrowband interferences as well as wideband and mutipath ones due to the limited degrees of freedom. STAP can overcome the above problem. However, the computational complexity is a troublesome question. Fortunately, the reduced-dimension technique proposed by Dr. Zoltowski has made a breakthrough in GPS interference mitigation. Different from the above reduced-dimension method, the next work we will do is to develop an adaptive recursive least square (RLS) space-time algorithm combined with cyclostationary properties of GPS signal, which will improve speed of convergence by RLS algorithm. Also the algorithm belongs to blind adaptive algorithm by only using the nature of GPS signal, so it is very robust.

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A Simulation Tool for Space-time Adaptive Processing in GPS

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Abstract—With the wide use of GPS, it becomes more and more important to improve the positioning accuracy. The GPS signal is very weak, and can be easily interfered. Space-time adaptive processing(STAP) can suppress the jamming not only in the temporal domain, but also in the spatial domain. STAP has now become a good candidates for jamming mitigation in GPS.

In order to evaluate the performance of various kinds of STAP algorithms, and to develop the novel STAP algorithms. we need to simulate space-time array GPS data with high fidelity and the software GPS receiver. This paper mainly presents a simulation method for space-time GPS data, as well as the simulation of some typical kinds of jammers. The simulation method for software GPS receiver is also introduced. The data that after the process of STAP is computed by the simulated GPS receiver. The simulation results show that the simulation tool is a good platform for the development of STAP algorithms.

1. Introduction

GPS is a network which is composed of many satellites, it can provide accurate position and time information [1]. The signal received by GPS receiver has the characteristics not only in temporal domain, but also in spatial domain. With the advent of STAP algorithms, the jammings can be suppressed through spatial and temporal characteristics effectively, and the positioning precision can be further improved. To evaluate the effect of the STAP algorithms on GPS jamming mitigation, we set up this simulation platform for Space-Time array GPS data. Besides we also simulate the software GPS receiver to validate the data after being processed by algorithms [2, 3].

In the second section of this paper, the simulation method for Space-Time array data is introduced. In the third section we introduce the common jammings to the GPS and simulate some typical of them. In the forth section we introduce the simulation method of GPS receiver briefly. The simulation results are given in the fifth section and in the sixth section we get the conclusion.

2. The Method for Simulating Space-time Array Data

2.1. Single Channel Temporal GPS Data Generation

The GPS signal is composed of navigation data, PRN code and carrier for modulation. The navigation data is a binary coded file, and is transmitted in frames according to certain format. It contains ephemeris parameters, satellite almanac and so on. We can compute the satellites position with them. These parameters value can be found from related data resources. In this paper, we choose the data from CDDIS (http://cddisa.gsfc.nasa.gov). They are coded in RINEX format. We should find the corresponding parameters from them first. These parameters, multiply by themselves scale factors, are converted to binary bits that fit the navigation data format. We only need to simulate the first three sub-frames [3].

The GPS signal is of two kinds of PRN codes, that is C/A code and P code. The structure of P code is quite complex and secret to civil users. Here we only introduce the simulation of C/A (coarse/acquisition) code. The C/A code is coded in binary format, and has the characteristics of multi address, searching GPS signal, coarse acquisition and anti-jamming. It is generated by two 10-order feedback shift registers, which can generate $C_{10}^2 + 10 = 55$ kinds of different C/A code [3]. The 24 satellites have different C/A codes. The chip shift between the satellites is fixed. According to one satellite's C/A code, we can obtain the others' C/A codes.

After the navigation data is spread by the C/A code, they modulate the carrier centered at 1575.42 MHz by BPSK. Since the data we need is at the intermediate frequency that after down-conversion, we select it as $f_{IF} = 21.25$ MHz. And after band-pass sampling, the output center frequency is $f_0 = 1.25$ MHz. The power of the signal arrived at the receiver is about -155 db. We select the gain of the receiver antenna as 4 db, and the gain of its' amplifier as 31 db. Thus the signal at IF is -120 db. Once assume the position of the receiver, we can get the propagation time of the GPS signal.

2.2. Multiple Channel Space-time GPS Data Generation

Suppose the signal of a GPS Satellite arrive at the receiver antenna with the direction of θ , the expression of it is f(t), refer to Fig. 1. So the signal that reaches the second antenna [4] is $f(t - \tau)$, $\tau = l/c$, $l = d\cos(\theta)$, where τ denotes the arrival time difference between the first antenna and the second antenna, and d denotes the inter-element distance. So the signal received by the second antenna can be expressed as $f(t - d\cos(\theta)/c)$, approximately to $f(t)e^{-j\omega\tau} = f(t)e^{-j2\pi f \times d\cos(\theta)/c}$.



Figure 1: Uniform linear array.

Similarly, the signal of the satellite that arrives at the nth antenna can be expressed as:

$$f(t)e^{-j\omega(n-1)\tau} = f(t)e^{-j2\pi f \times (n-1)d\cos(\theta)/c}$$
(1)

Suppose that every antenna has M time delays, and each time delay is T, so the Space-Time satellite data can be expressed as:

$$F(t,\theta) = \begin{pmatrix} f(t) \\ f(t-T) \\ \cdots \\ f(t-(M-1)T) \end{pmatrix} (1 \quad e^{-j\omega\tau} \quad \cdots \quad e^{-j\omega(N-1)\tau})$$
(2)

As for the arrived signal that contains four GPS satellites, its' multiple channel Space-Time data model can be expressed as:

$$S(t,\theta) = \sum_{i=1}^{4} \begin{pmatrix} f_i(t) \\ f_i(t-T) \\ \dots \\ f_i(t-(M-1)T) \end{pmatrix} (1 \quad e^{-j\omega\tau_i} \quad \dots \quad e^{-j\omega(N-1)\tau_i})$$
(3)

where $f_i(t)$ denotes the *i*th satellite signal received, τ_i denotes the *i*th of the satellite time delay.

3. Jamming Simulation

During the GPS signal propagation, it can be affected by Satellite Clock error, Ionospheric error, multi-path jamming, radio station RF jamming, noise and so on. The typical jamming sources are broadcast TV–UHF channel, air-borne VHF, personnel electronic device, Ultra broadband communication, Multi-path, etc. For the purpose of build Space-Time data platform, we only consider narrowband RF jamming, broadband FM jamming, and receiver random noise.

Take the broadband FM jamming for example, its' model can be expressed as:

$$J(t) = A_0 \cos[\omega_0 t + k_f \int_0^t V_\Omega(t) dt]$$
(4)

The system frequency scope is $\omega_0 - k_f | V_{\Omega} | \le \omega \le \omega_0 + k_f | V_{\Omega} |$. On the basis of typical value, the jamming requires lowest power lever and has the worst affection when the frequency bias is between 400 K–600 K. So we select the biased frequency $\Delta f = 500$ MHz, centered frequency $f_0 = 1.2$ MHz, and the jamming power is 60 db above the signal power, that is -60 db.

As the single channel temporal data extend to multiple channel Space-Time data, we also need to extend the jamming into multiple channel Space-Time form, its' mathematical model can be expressed as:

$$I(t,\theta) = \sum_{i=1}^{m} \begin{pmatrix} J_i(t) \\ J_i(t-T) \\ \vdots \\ J_i(t-(M-1)) \end{pmatrix} A_i(\theta)$$
(5)

where $A_i(\theta)$ denotes the steering vector of the jamming.

The receiver internal noise n(t) obeys Gauss statistical distribution, it can occupy the whole frequency. The Space-Time GPS data with jamming can be expressed as:

$$D(t,\theta) = S(t,\theta) + I(t,\theta) + n(t)$$
(6)

4. The Simulation Method of Software GPS Receiver

In the simulation of software GPS receiver, we take the advantage of the flexibility of software. We mainly introduce the acquisition and tracking modules.

The traditional acquisition is a two-dimensional search process. The computation amount is quite large. Based on the software, we use the circular correlation method, showed as Fig. 2.

Assume the input signal is y_k . We take the local carrier at f_i as $l_{i,k} = exp(j2\pi f_i t_k)$. First the FFT result of N points of $y_k \cdot l_{i,k}$ is multiplied by the conjugate FFT of the N points of local C/A code. Then the IFFT of the product gives the correlation result in the time domain for all the 1023 code phase offsets.



Figure 2: Acquisition based on circular correlation.

Figure 3: The correlation curve.

In the code tracking module [5], we use the numerical relation of correlation values of prompt code, early code and late code with the input IF sampled signal to adjust the input signal and get the finer code phase offsets x instead of traditional DLL. Assume the correlation values are y_p , y_e , y_l respectively. The relationship of them is showed as Fig. 3, where T_c is the code chip width, p is the code offset between prompt code and early/late code. From Fig. 3, we obtain that

$$r = \frac{y_l}{y_e} = \frac{T_c - x - p}{T_c + x - p} \Rightarrow x = \frac{(1 - r)(T_c - p)}{1 - r}$$
(7)

From x, we shift the input signal left or right one sample, and can evaluate the finer code phase error.

Due to the flexibility of software, we use third-order PLL to fit the high dynamic situation. We choose the loop filter as $F(s) = \frac{1 + s\tau_2}{s\tau_1} \cdot \frac{1 + s\tau_4}{s\tau_3}$, So the error transfer function is $H_e(s) = \frac{\tau_1 \tau_3 s^3}{\tau_1 \tau_3 s^3 + K \tau_2 \tau_4 s^2 + K(\tau_2 + \tau_4) s + K}$, where K is the loop gain. Let $a = \frac{K}{\tau_1 \tau_3} = \omega_n^2 \eta$, $b = a(\tau_2 + \tau_4) = \omega_n^2 + 2\zeta\omega_n\eta$, $c = a \cdot \tau_2 \tau_4 = 2\zeta\omega_n + \eta$, and we can get

$$H_e(s) = \frac{s^3}{s^3 + cs^2 + bs + a} = \frac{s^3}{(s+\eta)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$
(8)

From the formula (8), we can choose the ζ , ω_n and η according to the dynamic situation easily.

5. Simulation Results

In this paper, we choose the digital centered frequency as 1.25 MHz. According to Shannon sampling theorem, we choose acquisition frequency as 5 MHz, the power of jamming 60 db above that of GPS signal, that is -60 db. The receiver internal noise 20 db above that of GPS signal, that is -100 db. The Fig. 4 denotes the waveform of GPS data without jamming on one antenna. And the Fig. 5 denotes the waveform of GPS data without jamming on one antenna. Their spectrums that are with and without the broadband jamming are indicated in Fig. 6 and Fig. 7 respectively. In the experiment, we choose the antenna position as (X:-3173088.339 m;Y:-3625066.392 m;Z:4181362.566 m), and the position computed by our software receiver is(X:-3173092.972m; Y:-3625044.536 m; Z: 4181358.520 m). The Fig. 8 denotes the spectrum of GPS data after STAP processing.



Figure 4: Waveform of Space-Time GPS data without jamming.



Figure 6: Spectrum of Space-Time GPS data without jamming.



Figure 5: Waveform of Space-Time GPS data with broadband FM jamming.



Figure 7: Spectrum of Space-Time GPS data with broadband FM jamming.



Figure 8: The spectrum of GPS data after STAP processing.

6. Conclusion

The simulation result shows that the Space-Time array data simulation method is valid and of high fidelity. And the simulation tool has been testified as a good platform for evaluating the STAP algorithms and using them in GPS. It will be of great help to improve the GPS positioning precision further.

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Analysis and a Novel Design of the Beamspace Broadband Adaptive Array

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Abstract—An analysis of the broadband beamspace adaptive array is provided. There are two conditions imposed on the array. First, the individual beams should have a good frequency invariant property. Second, they should be linearly independent. However, these two conditions are not independent and it is shown that there is a trade-off between them. To improve the interference cancellation capability of the array, we may need to sacrifice the frequency invariant property of the beams to some degree for more linearly independent beams. A DFT-modulated design method is also proposed, where the beam directions are uniformly distributed over the spatial domain and the linear independence of the beams is guaranteed inherently. Simulation results verified our analysis and the proposed method.

1. Introduction

Adaptive beamforming has found numerous applications in various areas ranging from sonar and radar to wireless communications [1]. For arrays to accomplish nulling over a wide bandwidth, tapped-delay lines (TDLs)

are employed, resulting in an array with M sensors and TDLs of length J, as shown in Fig. 1. To perform beamforming with high interference rejection and resolution, we need to employ a large number of sensors and long TDLs, which unavoidably increases the computational complexity of its adaptive part and slows down the convergence of the system. To reduce the computational complexity of a broadband adaptive beamformer and increase its convergence speed, Many methods have been proposed, including the time-domain subband adaptive beamformer [2, 3], a combination of subband decomposition in both the temporal and spatial domains [4], and those based on frequency invariant beamforming techniques [5, 6].

As the broadband counterpart of the narrowband beamspace adaptive array, the beamspace broadband adaptive array was proposed in [5], where several fre-



Figure 1: A signal impinges from an angle θ onto a uniformly spaced broadband linear array with Msensors, each followed by a *J*-tap filter.

quency invariant beams (FIBs) are formed pointing to different directions by a fixed beamforming network with two-dimensional (2-D) filters; thereafter the outputs of these beams are combined adaptively by a single weight for each of them. Since both the number of beams and the number of selected beams are small, the total number of adaptive weights is greatly reduced.

In this paper, we will first give an analysis of the broadband beamspace adaptive array to show a trade-off between two conditions imposed explicitly or implicitly and its impact on the performance of the resultant beamformer. It can be proved that the number of independent beams formed is the same as the length N of the prototype filter for the fan filter design. Although we can design as many frequency invariant beams as we want, only N of them are independent and at most we can only null out N - 1 interfering signals. As the array's interference cancellation ability is dependent on both the number of independent beams and the frequency invariant property of those beams, we can sacrifice the frequency invariant property to some degree to design more independent beams. As a result, the array's interference cancellation property will be improved. With the above analysis, we then propose a new design of the frequency invariant beams, where their beam directions are uniformly distributed in the spatial domain and their independence is guaranteed inherently by the special form of the prototype filters, which are derived from another prototype filter by the discrete Fourier transform (DFT) modulation with appropriately imposed zeros. This paper is organised as follows. A brief review of the broadband beamspace adaptive array is provided in Section. An analysis of the trade-off in its design is given in. The design based on the DFT modulation is proposed in Section. Design examples and simulation results are given in Section, and conclusions drawn in Section.

2. Broadband Beamspace Adaptive Array

In a narrowband beamspace adaptive array [7], a total of N beams are formed by a beamforming network, where one is the main beam pointing to the direction of the signal of interest and the remaining N-1 beams are auxiliary beams pointing to the remaining directions. The output power levels of the auxiliary beams are compared to a threshold and those higher than the threshold will be chosen in the following adaptation. In this way the resultant partially adaptive array can maintain an acceptable performance with a lower computational complexity. Extend this idea to the broadband case, we can also design N broadband beams pointing to different directions to form a broadband beamspace adaptive array. To combine the outputs of the beams with one adaptive weight for each of them, their response should be frequency invariant.

In [5], such a broadband beamspace adaptive array was proposed for an equally spaced linear array. With the recent development in the design of frequency invariant beamformers for one-dimensional (1-D), two-dimensional (2-D) and three-dimensional (3-D) arrays [8], we can easily extend the idea of a beamspace adaptive array to the 2-D and 3-D cases. Here we will focus on the case of a linear array and first we give a brief review of the proposed beamspace approach.

Suppose a signal with an angular frequency ω and an angle of arrival θ impinges on the uniformly spaced linear array of Fig. 1, then its output in continuous form can be written as

$$y(t) = e^{j\omega t} \sum_{m=0}^{M-1} \sum_{k=0}^{J-1} w_{m,k} \cdot e^{-jm\omega\Delta\tau} \cdot e^{-jk\omega T_s}$$
(1)

with $\Delta \tau = \frac{d}{c} \sin \theta$, where T_s is the delay between adjacent samples in the TDL, d is the array spacing, and c is the wave propagation speed. Then the array's response can be written as

$$\tilde{R}(\omega,\theta) = \sum_{m=0}^{M-1} \sum_{k=0}^{J-1} w_{m,k} \cdot e^{-jm\omega\Delta\tau} \cdot e^{-jk\omega T_s}.$$
(2)

With the normalised angular frequency $\Omega = \omega T_s$, we obtain the response as a function of Ω and θ

$$R(\Omega,\theta) = \sum_{m=0}^{M-1} \sum_{k=0}^{J-1} w_{m,k} \cdot e^{-jm\mu\Omega\sin\theta} \cdot e^{-jk\Omega} \quad \text{with} \quad \mu = \frac{d}{cT_s}.$$
(3)

With the substitution of $\Omega_1 = \Omega$ and $\Omega_2 = \mu \Omega \sin \theta$ in (3), we obtain a 2-D digital filter response

$$R(\Omega_1, \Omega_2) = \sum_{m=0}^{M-1} \sum_{k=0}^{J-1} w_{m,k} \cdot e^{-jk\Omega_1} \cdot e^{-jm\Omega_2}.$$
 (4)

We see that the spatio-temporal spectrum of the received signal lies on the line $\Omega_2 = \mu \Omega_1 \sin \theta$. Suppose the desired frequency invariant response of the array is $P(\sin \theta)$. By the substitution $\sin \theta = \left(\frac{\Omega_2}{\mu \Omega_1}\right)$, we can obtain the response $R(\Omega_1, \Omega_2)$ with nominal parameters Ω_1 and Ω_2 . Sample the function $R(\Omega_1, \Omega_2)$ at the (Ω_1, Ω_2) plane and then apply an inverse discrete Fourier transform (DFT) to the resultant 2-D data, we will then find the corresponding coefficients $w_{m,k}$. To fit the spatial and temporal dimensions of the array, we may need to truncate the result from the inverse DFT [5,8].

For the desired response $P(\sin \theta)$, it can comes from a 1-D digital filter $H(e^{j\Omega})$ by the substitution $\Omega = \pi \sin \theta$. If $H(e^{j\Omega})$ is a lowpass filter [5], then signals from the directions around $\theta = 0$ will correspond to its passband, and a beam is formed pointing to the direction $\theta = 0$. If we want to steer this beam to the direction $\theta = \theta_0$ with the same low pass filter $H(e^{j\Omega})$, we can vary it into the form $\tilde{H}(e^{j\pi \sin \theta}) = H(e^{j(\Omega - \pi \sin \theta_0)})$ and consider $\tilde{H}(e^{j\pi \sin \theta})$ as the new desired frequency invariant response. As the sampling frequency is in general twice the highest frequency component of the signal and array spacing is half the wavelength of the highest frequency component, we have $d = \frac{1}{2} \cdot c \cdot (2T_s) = cT_s$ and $\mu = 1$. Therefore, without loss of generality, we will only consider the case with $\mu = 1$ in the design and simulations.

Moreover, we also assume the signal of interest comes from the broadside, then the main beam will point to the direction of $\theta = 0$. For the auxiliary beams, their directions are decided in such a way that the main direction of a beam should ideally coincide with nulls (zero re-

sponses) of all other beams, as mentioned in the simulation part of [5].

A single adaptive weight is applied to each of the auxiliary beams by minimizing the variance of the error signal between the main beam and the auxiliary beams. In the adaptation, some of the auxiliary beam outputs are active and some others are simply discarded if their output signals are below some prescribed level. Fig. 2 shows the diagram of such a broadband beamspace adaptive array, where $\mathbf{x}[n]$ is the vector containing the received signals $x_0[n], \ldots, x_{M-1}[n]$ and w_1, \ldots, w_{P-1} are the adaptive weights attached to each of the beam outputs.



Figure 2: A broadband beamspace adaptive array with P frequency invariant beams (FIBs).

3. Analysis of the Broadband Beamspace Adaptive Array

For the beamspace array to work, the frequency invariant beamforming network needs to meet two conditions, which are imposed explicitly or implicitly.

First, the beams formed should have a satisfactory frequency invariant property for the interested frequency band, which is dependent on the required shape $P(\sin \theta)$ of the beam and the temporal/spatial dimension of the corresponding 2-D filter. The more complicated the shape, the more coecients we need in each of the frequency invariant beams, i.e., a larger M and J.

From the discussion of the last section, the desired beam response can be derived from the corresponding prototype filter $H(e^{j\Omega})$. Suppose the length of filter is N. As the shape is decided by the prototype filter, the dimension M and J of the 2-D fan filter (frequency invariant beamformer) should be at least 3 times that of the prototype filter to maintain the shape of the response of the prototype filter, that is, $N \leq \min\{\frac{M}{3}, \frac{J}{3}\}$ [5].

Secondly, the beams formed should not be linearly dependent. Otherwise, some of the beam outputs will be a linear combination of the others, which leads to a waste of resources and also reduces the number of effective beams. As a result, we will not be able to null out the desired number of interfering signals. This second condition is not mentioned explicitly in [5], but it is a necessary condition to fully exploit the potential of the beamspace adaptive array. We will see later that the beam direction arrangement in [5] guarantees the linear independence of the beams.

These two conditions are not independent and there is a close relationship between them. In the following, we will show that the number of independent beams formed N_{ind} cannot exceed the length N of the prototype FIR filter. We prove this by contradiction.

Suppose we can have P > N independent beams formed by some prototype filters with a length N. These beams have a response of $H_p(e^{j\pi \sin \theta})$, $p = 0, 1, \ldots, P-1$. Each of them is derived from the corresponding prototype filter $H_p(e^{j\Omega})$, $p = 0, 1, \ldots, P-1$, with an impulse response of $\mathbf{h}_p = [h_{p,0}, h_{p,1}, \ldots, h_{p,N-1}]^T$, $p = 0, 1, \ldots, P-1$. These prototype filters $H_p(e^{j\Omega})$, $p = 0, 1, \ldots, P-1$ can further be derived from the same lowpass filter as discussed in the last section, or they can simply be some different filters.

Now consider the linear combination of the following form

$$\mathbf{0} = \alpha_0 \mathbf{h}_0 + \alpha_1 \mathbf{h}_1 + \dots + \alpha_{P-1} \mathbf{h}_{P-1},\tag{5}$$

where $\alpha_0, \dots, \alpha_{P-1}$ are scalars to be found for this equation to hold. Taking the transpose of both sides and then multiplying the equation with the vector $[1 e^{j\pi \sin \theta} \dots e^{j(N-1)\pi \sin \theta}]^T$, we arrive at

$$0 = \alpha_0 H_0(e^{j\pi\sin\theta}) + \alpha_1 H_1(e^{j\pi\sin\theta}) + \dots + \alpha_{P-1} H_{P-1}(e^{j\pi\sin\theta}),$$
(6)

where $H_p(e^{j\pi\sin\theta})$, $p = 0, 1, \ldots, P-1$ is exactly the response of those independent beams. Since they are independent, all the scalars $\alpha_0, \cdots, \alpha_{P-1}$ must be zero for (6) to hold, and then for (5) to hold, which means that \mathbf{h}_p , $p = 0, 1, \ldots, P-1$ are independent. However, as P is larger than the length of each vector \mathbf{h}_p , the

rank of the $N \times P$ matrix formed by $H = [H_0, H_1, \ldots, H_{P-1}]$ cannot be larger than N, that is, it is impossible for all of the vectors \mathbf{h}_p to be independent. Thus, we reach a contradiction.

As the maximum rank of **H** is N, we can see from the proof that the maximum number of independent beams formed will be equal to the length N of the prototype FIR filter. Clearly, although we can design as many frequency invariant beams as we want, only N of them are independent and at most we can only null out N-1 interfering signals. As the array's interference cancellation ability is dependent on both the number of independent beams and the frequency invariant property, there is trade-off between these two factors for a fixed M and J. We may choose a prototype filter with $N = \min\{\frac{M}{3}, \frac{J}{3}\}$ for a good frequency invariant property, but when the number of interferences increases and becomes larger than $(\min\{\frac{M}{3}, \frac{J}{3}\} - 1)$, the array will not be able to null out the additional interferences. Therefore we may need to sacrifice the frequency invariant property a little to increase N and design more independent beams. The loss in frequency invariant property can be compensated by the gain in the increasing number of independent beams. As a result, the interference cancellation ability of the array is improved. We will give some results to show this trade-off in our simulations.

The next question is, provided the length of the prototype filter N, how to design N independent frequency invariant beams. We will propose a DFT-modulated method in the next section with the beam directions uniformly distributed in the spatial space and their independence guaranteed inherently.

4. DFT-modulated Design of the Frequency Invariant Beamformers

Before we proceed further, we want to give a sufficient condition with which the P beams formed by P general prototype filters \mathbf{h}_p , $p = 0, 1, \dots, P - 1$ are linearly independent. This condition is stated as follows.

• As long as for the $\hat{p}-th$ frequency response $H_{\hat{p}}(e^{j\Omega})$, $\hat{p}=0,\ldots,P-1$, there exists a point $\Omega = \Omega_{\hat{p}}$, where $H_{\hat{p}}(e^{j\Omega_{\hat{p}}}) \neq 0$ and all the remaining frequency responses $H_{p\neq\hat{p}}(e^{j\Omega_{\hat{p}}}) = 0$, the set of frequency responses $H_{p}(e^{j\Omega})$, $p = 0, 1, \ldots, P-1$, and hence the set of beams formed by them will be linearly independent.

The proof is given in the following. Consider the equation (5) again. Taking the transpose of both sides and then multiplying the equation with the vector $[1 e^{j\Omega} \dots e^{j(N-1)\Omega}]^T$, we arrive at

$$0 = \alpha_0 H_0(e^{j\Omega}) + \alpha_1 H_1(e^{j\Omega}) + \dots + \alpha_{P-1} H_{P-1}(e^{j\Omega}), \tag{7}$$

For $\hat{p} = 0$, put the value $\Omega = \Omega_0$ into the above equation, we have

$$0 = \alpha_0 H_0(e^{j\Omega_0}) + \alpha_1 H_1(e^{j\Omega_0}) + \dots + \alpha_{P-1} H_{P-1}(e^{j\Omega_0}) = \alpha_0 H_0(e^{j\Omega_0}) + 0 + \dots + 0.$$
(8)

As $H_0(e^{j\Omega_0}) \neq 0$, we have $\alpha_0 = 0$. Similarly, we have $\alpha_p = 0$, $p = 0, 1, \ldots, P - 1$. Therefore, for (7) to hold, all the *P* scalars must be zero, that is, both the vectors \mathbf{h}_p and frequency responses $H_p(e^{j\Omega})$ are linearly independent. The proof is complete.

In [5], the main direction of a beam was arranged to coincide with nulls (zero responses) of all other beams. From the above proof, clearly, this arrangement guarantees the independence of the beams. However, in [5], the authors were simply using the existing nulls of the prototype filter, so the direction of the auxiliary beams can not be controlled by the designer and they can point to anywhere depending on the chosen lowpass prototype filter. Here we propose a DFT-modulated method for the design of the independent frequency invariant beamformers, where the beam directions are uniformly distributed in the spatial domain and their independence is guaranteed inherently.

Assume the impulse response of a lowpass filter is h[n], n = 0, 1, ..., N - 1. Based on h[n], we can obtain the response \mathbf{h}_p of the p-th prototype filter for the p-th beam shape design by the following DFT modulation

$$h_{p,n} = h[n]e^{j\frac{2pn\pi}{P}}.$$
(9)

In the frequency domain, this modulation shifts the response of original prototype filter h[n] along the frequency axis by $\frac{2p\pi}{P}$. If the z-transform H(z) of h[n] can be expressed as

$$H(z) = \prod_{p=1}^{P-1} (1 - e^{j\frac{2p\pi}{P}} z^{-1}),$$
(10)

then after modulation, the main direction of the P resultant beams will coincide with the nulls of the other beams, hence these beams will be independent. Note in this case, we have P = N, i.e., the number of independent beams formed will be the length of the prototype filter. For the main directions of these beams, we have

$$\pi \sin \theta = \begin{cases} \frac{2p\pi}{P} & \text{for } \frac{2p\pi}{P} < \pi\\ \frac{2p\pi}{P} - 2\pi & \text{for } \frac{2p\pi}{P} \ge \pi \end{cases} \Rightarrow \sin \theta = \begin{cases} \frac{2p}{P} & \text{for } \frac{2p}{P} < 1\\ \frac{2p}{P} - 2 & \text{for } \frac{2p}{P} \ge 1 \end{cases},$$
(11)

for p = 0, 1, ..., P - 1. They are uniformly distributed in the sin θ domain, where the first beam point to the direction $sin\theta = 0$ will be the main beam and the others will be the auxiliary beams. Fig. 3 gives an example of the desired beam shapes with P = N = 5, where it can

be seen clearly that each of the five beam directions coincides with the nulls of the other beams. Once we obtain the P desired beam responses $H_p(e^{j\pi \sin \theta})$, we can follow the procedures given in [8] to obtain the coefficients of the corresponding beamformers.

One point to note is, in general, the $H_p(e^{j\pi\sin\theta})$ obtained by DFT modulation is of complex value for different θ that is,

$$H_p(e^{j\pi\sin\theta}) = A_p(\theta)e^{jB_p(\theta)} \tag{12}$$

where $A_p(\theta)$ and $B_p(\theta)$ are some real functions. The change of both $A_p(\theta)$ and $B_p(\theta)$ with respect to different θ will lead to a more complicated (Ω_1, Ω_2) pattern for the design, which will require more coefficients in the temporal domain and therefore larger dimension of the array $\Lambda_0 A_1(\theta)$ contains enough inform



Figure 3: The desired beam shapes with P = N = 5 formed by DFT modulations.

larger dimension of the array. As $A_p(\theta)$ contains enough information about the shape of the beam response, we can ignore the phase part $B_p(\theta)$ and our results show that in this way we can significantly improve the frequency invariant property of the beams with the same array dimensions.

5. Simulations

To show the trade-off between the frequency invariant property and the number of linear independent beams, the spatial and temporal dimensions of the frequency invariant beams are fixed as M = 14 and J = 16. According to [5], ideally we should use a prototype filter of length $\lfloor 14/3 \rfloor = 4$ for the design of the 4 FIBs. Fig. 4 shows the pattern of the main beam based on a 4-tap filter over the bandwidth $[0.4\pi; 0.9\pi]$.



Figure 4: The magnitude response of the main beam over the bandwidth of $[0.4\pi; 0.9\pi]$, based on a 4-tap and a 6-tap prototype filter, respectively.

The signal of interest comes from broadside and with a signal to interference ratio (SIR) of -20 dB and signal to noise ratio (SNR) of 20 dB. Five interfering signals come from the angles of 20°, -25°, 45°, -50°, and -80°, respectively. Both the interfering signals and the signal of interest have a bandwidth of $[0.4\pi; 0.9\pi]$. We used a normalised LMS algorithm for adaptation. The learning curve with a stepsize of 0.01 is shown by the dashed line in Fig. 5. As the number of interfering signals are 5, which is larger than 4 - 1 = 3, the number of auxiliary beams, the 4-beam adaptive array can not null out all of the interferences, although all of the beams have a very good frequency invariant response over the interested bandwidth $[0.4\pi; 0.9\pi]$. As a result, the learning curve only reaches a level of 15 dB. In order to improve its performance, we need to sacrifice the frequency



Figure 5: The learning curves for different number of independent beams.

invariant property a little. So, we increased the length N of the prototype filter to 5, and 5 independent beams were obtained. The learning curve of this new system with the same stepsize is shown by the dotted line in Fig. 5. Compared to the 4-beam array, the ensemble mean square residual error has been reduced to about 8 dB. We can further to improve the performance of the system by designing 6 independent beams based on a 6-tap prototype filter (N = 6). The frequency invariant property of the main beam in this case is also shown in Fig. 4, which is clearly not as good as that of N = 4. However, as there are more independent beams formed in this array, a further improvement of more than 10 dB has been achieved, as shown by the solid line in Fig. 5.

6. Conclusions

An analysis of the broadband beamspace adaptive array has been provided and it is shown that in order to improve the interference cancellation capability of the array, we may need to sacrifice the frequency invariant property of the beams to some degree for more linearly independent beams. We also proposed a DFT-modulated design of the frequency invariant beams employed in the broadband beamspace adaptive array, where the beam directions are uniformly distributed over the spatial domain and the linear independence of the beams is guaranteed inherently. Simulation results verified our analysis and the proposed method.

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A Subspace-based Robust Adaptive Capon Beamforming

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Abstract—Adaptive beamforming suffers from performance degradation in the presence of mismatch between the actual and presumed array steering vector of the desired signal. This idea enlightens us, so we propose a subspace approach to adaptive beamforming that is robust to array errors based on minimizing MUSIC output power. The proposed method involves two steps, the first step is to estimate the actual steering vector of the desired signal based on subspace technique, and the second is to obtain optimal weight by utilizing the estimated steering vector. Our method belongs to the class of diagonal loading, but the optimal amount of diagonal loading level can be calculated precisely based on the uncertainty set of the steering vector. To obtain noise subspace needs eigen-decomposition that has a heavy computation load and knows the number of signals *a priori*. In order to overcome this drawback we utilized the POR (Power of R) technique that can obtain noise subspace without eigen-decomposition and the number of signals *a priori*. It is very interesting that *Li Jian's* method is a special case where m = 1, and the proposed subspace approach is the case where $m \to \infty$, so we obtained a uniform framework based on POR technique. This is also an explanation why the performance of the proposed subspace approach excels that of *Li Jian's* method. The excellent performance of our algorithm has been demonstrated via a number of computer simulations.

1. Introduction

Array signal processing has wide applications in radar, communications, sonar, acoustics, seismology, and medicine. One of the important tasks of array processing is beamforming. The standard beamformers include the delay-and-sum approach, which is known to suffer from poor resolution and high sidelobe problems. The Capon beamformer adaptively selects the weight vectors to minimize the array output power subject to the linear constraint that the signal of interest (SOI) does not suffer from any distortion [1]. The Capon beamformer has better resolution and interference rejection capability than the standard beamformer, provided that the array steering vector corresponding to the SOI is accurately known. In practice, the knowledge of the SOI steering vector may be imprecise, the case due to differences between the presumed signal steering vector and the actual signal steering vector. When this happens, the Capon beamforming may suppress the SOI as an interference, which result in array performance drastically reduced, especially array output signal-to-interference-plus-noise ratio (SINR) [4].

In the past three decades many approaches have been proposed to improve the robustness of the Capon beamforming. Additional linear constraints, including point and derivative constraints, have been imposed to improve the robustness of the Capon beamforming [2,3]. However, for every additional linear constraints imposed, the beamformer loses one degree of freedom (DOF) for interference suppression. Moreover, these constraints are not explicitly related to the uncertainty of the array steering vector. Diagonal loading (including its extended versions) has been a popular approach to improve the robustness of the Capon beamformer [4]. However, for most of the diagonal loading methods, determining the diagonal loading remains an open problem. Recently there are some methods been proposed (for examples, [5–7] and reference therein) to this point.

Mismatch between the presumed steering vector of the SOI and the actual one result in drastically reduced array SINR, therefore if we can estimate actual steering vector of the SOI, robustness of the array will be improved. In this paper, from the point of view of the subspace we propose a novel robust Capon beamformer, which involves two steps, the first step is to estimate actual steering vector of SOI, and the second is to calculate optimal weight by Capon method. The rest of this paper is organized as follows. Section 2 contains background material. In section 3, the robust Capon beamformer is developed. Computer simulation results illustrating the performance of the robust Capon beamformer are presented in Section 4. Finally, Section 5 contains the conclusions.

2. Background

2.1. Signal Model

We consider the standard narrowband beamforming model in which a set of M narrowband plane wave signals, impinge on an array of N sensors with half wavelength spacing, where M < N. The $N \times 1$ vector of received signals is given by

$$\mathbf{x}(t_k) = \sum_{m=0}^{M-1} \mathbf{a}(\theta_m) s_m(t_k) + \mathbf{n}(t_k), \ k = 1, 2, \dots, L$$
(1)

where $s_m(t_k), m = 0, \ldots, M - 1; k = 1, 2, \ldots, L$ are the source signals snapshots,

$$\mathbf{a}(\theta_m) = [1, e^{-j\pi\sin\theta_m}, \dots, e^{-j\pi(N-1)\sin\theta_m}]^T$$

is the steering vector in the direction θ_m , and $\mathbf{n}(t_k)$, k = 1, 2, ..., L are the vectors containing additive white noise samples, L is the number of the snapshots. Also, in this paper, the sources and noise are assumed to be statistically uncorrelated.

We assume that one of the signals is the desired signal, say $s_0(t)$, and treat the remaining signals as interferences. Since $s_0(t)$ is uncorrelated with the noise and interferences, the data covariance matrix has the form,

$$\mathbf{R} = \sigma_0^2 \mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0) + \sum_{k=1}^{M-1} \sigma_k^2 \mathbf{a}(\theta_k) \mathbf{a}^H(\theta_k) + \mathbf{R}_n \triangleq \mathbf{R}_s + \mathbf{R}_{i+n}$$
(2)

where $\mathbf{R}_s = \sigma_0^2 \mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0), \sigma_i^2 = E\{|s_i(t_k)|^2\}$ is the power of *i*th signal, and \mathbf{R}_{i+n} is the interference plus noise covariance matrix. In practice, the covariance matrix \mathbf{R} is estimated by

$$\hat{\mathbf{R}} = \frac{1}{L} \sum_{n=1}^{L} \mathbf{x}_n \mathbf{x}_n^H \tag{3}$$

where all received signals have zero means and L samples are independent.

2.2. Capon Beamforming

The Capon beamforming is as follows.

Determine the $N \times 1$ vector \mathbf{w}_0 that is the solution to the following linearly constrained quadratic minimization problem,

$$\min_{\mathbf{w}} \mathbf{w}^{H} \mathbf{R} \mathbf{w} \quad s.t. \mathbf{w}^{H} \bar{\mathbf{a}}(\theta_{0}) = 1$$
(4)

where $\bar{\mathbf{a}}(\theta_0)$ is presumed steering vector of the desired signal.

Appling Lagrange multiplier method results in the following solution,

$$\mathbf{w}_{0} = \frac{\mathbf{R}^{-1} \bar{\mathbf{a}}(\theta_{0})}{\bar{\mathbf{a}}^{H}(\theta_{0}) \mathbf{R}^{-1} \bar{\mathbf{a}}(\theta_{0})}$$
(5)

The array mean output power p_0 is

$$p_0 = \frac{1}{\bar{\mathbf{a}}^H(\theta_0)\mathbf{R}^{-1}\bar{\mathbf{a}}(\theta_0)} \tag{6}$$

The Capon beamformer has better resolution and much better interference rejection capability than the standard beamformer, provided that the presumed array steering vector of the SOI match actual array steering vector precisely. In practice, the exact steering vector of the SOI is unavailable or its measure/estimation is imprecise, therefore, we only use the presumed $\bar{\mathbf{a}}(\theta_0)$ instead of the actual $\mathbf{a}(\theta_0)$ in the Capon beamformer, and the mismatch between the exact steering vector and the presumed one may drastically degrade the performance of the Capon beamformer.

The array output SINR can be written as,

$$SINR = \frac{E[|\mathbf{w}_0^H \mathbf{s}_0(t)|^2]}{\mathbf{w}_0^H \mathbf{R}_{i+n} \mathbf{w}_0} = \frac{\sigma_0^2 |\mathbf{w}_0^H \mathbf{a}(\theta_0)|^2}{\mathbf{w}_0^H \left(\sum_{k=1}^{M-1} \sigma_k^2 \mathbf{a}(\theta_k) \mathbf{a}^H(\theta_k) + \mathbf{R}_n\right) \mathbf{w}_0}$$
(7)

where $\sigma_0^2 = E(|s_0(t)|)$. Inserting (5) into (7) yields,

$$SINR = \sigma_0^2 \frac{\left| \bar{\mathbf{a}}^H(\theta_0) \mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_0) \right|^2}{\bar{\mathbf{a}}^H(\theta_0) \mathbf{R}_{i+n}^{-1} \bar{\mathbf{a}}(\theta_0)}$$
(8)

where $\mathbf{a}(\theta_0)$ is the actual steering vector, then (8) can be rewritten as:

$$SINR = \sigma_0^2 \mathbf{a}^H(\theta_0) \mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_0) \times \frac{\left| \mathbf{a}^H(\theta_0) \mathbf{R}_{i+n}^{-1} \bar{\mathbf{a}}(\theta_0) \right|^2}{(\mathbf{a}^H(\theta_0) \mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_0)) (\bar{\mathbf{a}}^H(\theta_0) \mathbf{R}_{i+n}^{-1} \bar{\mathbf{a}}(\theta_0))}$$
$$= SINR_m \cdot \cos^2(\mathbf{a}(\theta_0), \bar{\mathbf{a}}(\theta_0); \mathbf{R}_{i+n}^{-1})$$
(9)

where $SINR_m = \sigma_0^2 \mathbf{a}^H(\theta_0) \mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_0)$ and $\cos^2(\cdot)$ is defined as,

$$\cos^{2}(\mathbf{a}, \mathbf{b}; \mathbf{Z}) = \frac{\left|\mathbf{a}^{H} \mathbf{Z} \mathbf{b}\right|^{2}}{\left(\mathbf{a}^{H} \mathbf{Z} \mathbf{a}\right) \left(\mathbf{b}^{H} \mathbf{Z} \mathbf{b}\right)}$$
(10)

Clearly, $0 \le \cos^2(\mathbf{a}, \mathbf{b}; \mathbf{Z}) \le 1$. Therefore, array output SINR is reduced due to mismatch between the presumed steering vector of the SOI and its true value.

In recent years, diagonal loading (DL) is a popular approach to improving the robustness of Capon beamformer to the mismatch above. In DL methods, the data covariance $\hat{\mathbf{R}}$ is replaced by $\hat{\mathbf{R}} + \gamma \mathbf{I}$, where γ is positive constant (see reference [4–6] for details). The DL method proposed in [4] is used in Section 4 for comparisons. In the following section, a novel robust beamforming is developed to alleviate the effects of the steering vector mismatch on the SINR performance of Capon beamformer.

3. Robust Capon Beamforming

The robust beamforming problem we will deal with in this paper can be briefly stated as follows: Extend the Capon beamformer so as to improve array output SINR even only an imprecise knowledge of steering vector $\mathbf{a}(\theta_0)$ is available. To simplify the notation, in what follows, we sometimes omit the argument θ of $\mathbf{a}(\theta)$ and $\bar{\mathbf{a}}(\theta)$. We assume that the only knowledge we have about $\mathbf{a}(\theta_0)$ is that it belongs to the following uncertainty [5]

$$[\mathbf{a}(\theta_0) - \bar{\mathbf{a}}]^H \mathbf{C}^{-1} [\mathbf{a}(\theta_0) - \bar{\mathbf{a}}] \le 1$$
(11)

where \mathbf{C} are given positive definite matrix.

As shown above, array performance loses will occur in the presence of mismatch between the presumed and actual steering vectors of the SOI. If we estimate the actual steering vector of the SOI as more precise as we can, then performance of the beamformer will be improved. The proposed robust Capon beamforming is based on this idea. From subspace theory we know that the actual steering vector of desired signal is orthogonal to noise subspace, our approach is based on the optimizing the projection of signal steering vector onto noise subspace. The steering vector is normed as $||\mathbf{a}||^2 = \mathbf{a}^H \mathbf{a} = N$. To derive our robust Capon beamformer, we use following constrained optimization

$$\min_{\mathbf{a}} \mathbf{a}^{H} \mathbf{U}_{n} \mathbf{U}_{n}^{H} \mathbf{a}$$

$$s.t.(\mathbf{a} - \bar{\mathbf{a}})^{H} C^{-1}(\mathbf{a} - \bar{\mathbf{a}}) \leq 1$$

$$||\mathbf{a}||^{2} = N$$
(12)

where $\bar{\mathbf{a}}$ is known to us in advance, but has error (mismatch to the actual steering vector of the SOI). \mathbf{U}_n is the noise subspace, which is obtained by the eigen-decomposition of $\hat{\mathbf{R}}$. To make up the noise subspace, we assume that the number M, of plane waves impinging on the array is known *a priori*. We use this assumption only for derivations and cancel it later. Note that we can improve the estimation accuracy of the actual steering vector of the SOI from (12), and then obtain optimal weight \mathbf{w}_0 by Capon method.

Without loss of generality, we will consider solving (12) for the case in which $\mathbf{C} = \varepsilon \mathbf{I}$, (ε is user parameter), then, (12) becomes

$$\min_{\mathbf{a}} \mathbf{a}^{H} \mathbf{U}_{n} \mathbf{U}_{n}^{H} \mathbf{a}$$

$$s.t. ||\mathbf{a} - \bar{\mathbf{a}}||^{2} \le \varepsilon$$

$$||\mathbf{a}||^{2} = N$$
(13)

We use the Lagrange multiplier methodology again, which is based on the function

$$L(\mathbf{a},\lambda,\mu) = \mathbf{a}^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{a} + \mu(2N - \varepsilon - \bar{\mathbf{a}}^H \mathbf{a} - \mathbf{a}^H \bar{\mathbf{a}}) + \lambda(\mathbf{a}^H \mathbf{a} - N)$$
(14)

where $\mu \geq 0$, $\lambda \geq 0$ are the Lagrange multiplier.

Hence, the unconstrained minimization of (14) for fixed μ , λ , is given by

$$\frac{\delta L(\mathbf{a},\mu,\lambda)}{\delta \mathbf{a}} = 2\mathbf{U}_n \mathbf{U}_n^H \mathbf{a} - 2\mu \bar{\mathbf{a}} + 2\lambda \mathbf{a} = 0$$
(15)

Clearly, the optimal solution of **a** is

$$\hat{\mathbf{a}} = \mu (\mathbf{U}_n \mathbf{U}_n^H + \lambda \mathbf{I})^{-1} \bar{\mathbf{a}}$$
(16)

Inserting $\hat{\mathbf{a}}$ into (14), minimizing $L(\mathbf{a}, \lambda, \mu)$ with respect to μ gives

$$\frac{\delta L(\hat{\mathbf{a}},\mu,\lambda)}{\delta\mu} = 2N - \varepsilon - \bar{\mathbf{a}}^H \hat{\mathbf{a}} - \hat{\mathbf{a}}^H \bar{\mathbf{a}} = 0$$
(17)

Then, we obtain

$$\hat{\mu} = \frac{2N - \varepsilon}{2\bar{\mathbf{a}}^H (\mathbf{U}_n \mathbf{U}_n^H + \lambda \mathbf{I})^{-1} \bar{\mathbf{a}}}$$
(18)

Inserting $\hat{\mu}$ into (14), minimizing Lagrange function with respect to λ yields

$$\frac{\delta L(\hat{\mathbf{a}}, \hat{\mu}, \lambda)}{\delta \lambda} = \hat{\mathbf{a}}^H \hat{\mathbf{a}} - N = 0$$
(19)

and the following equation can be derived,

$$\frac{\bar{\mathbf{a}}^{H}(\mathbf{U}_{n}\mathbf{U}_{n}^{H}+\hat{\lambda}\mathbf{I})^{-2}\bar{\mathbf{a}}}{[\bar{\mathbf{a}}^{H}(\mathbf{U}_{n}\mathbf{U}_{n}^{H}+\hat{\lambda}\mathbf{I})^{-1}\bar{\mathbf{a}}]^{2}} = \frac{N}{(N-\frac{\varepsilon}{2})^{2}}$$
(20)

Then, the solution of $\hat{\lambda}$ can be obtained by some simple manipulations.

Substituting (18) into (16) yields

$$\hat{\mathbf{a}} = (N - \frac{\varepsilon}{2}) \frac{(\mathbf{U}_n \mathbf{U}_n^H + \hat{\lambda} \mathbf{I})^{-1} \bar{\mathbf{a}}}{\bar{\mathbf{a}}^H (\mathbf{U}_n \mathbf{U}_n^H + \hat{\lambda} \mathbf{I})^{-1} \bar{\mathbf{a}}}$$
(21)

To summarize, the proposed robust Capon beamforming consists of following steps. **The algorithm**:

Step 1: Calculate data covariance matrix, i.e.,

$$\hat{\mathbf{R}} = \frac{1}{L} \sum_{n=1}^{L} \mathbf{x}_n \mathbf{x}_n^H$$

Step 2: Compute the eigen-decomposition of $\hat{\mathbf{R}}$ and obtain the noise subspace \mathbf{U}_n .

Step 3: Solve $\hat{\lambda}$ in (20).

Step 4: Use the $\hat{\lambda}$ in Step 3 to calculate

$$\hat{\mathbf{a}} = (N - \frac{\varepsilon}{2}) \frac{(\mathbf{U}_n \mathbf{U}_n^H + \hat{\lambda} \mathbf{I})^{-1} \bar{\mathbf{a}}}{\bar{\mathbf{a}}^H (\mathbf{U}_n \mathbf{U}_n^H + \hat{\lambda} \mathbf{I})^{-1} \bar{\mathbf{a}}}$$
(22)

Step 5: Compute optimal weight by Capon method, i.e.,

$$\mathbf{w}_0 = \alpha \hat{\mathbf{R}}^{-1} \hat{\mathbf{a}}, \ \alpha = \frac{1}{\hat{\mathbf{a}}^H \hat{\mathbf{R}}^{-1} \hat{\mathbf{a}}}$$
(23)

The proposed robust beamforming belongs to the class of diagonal loading, but the optimal amount of diagonal loading level can be precisely calculated based on the uncertainty set of the steering vector. In the Section 4 computer simulation results demonstrate excellent performance of the proposed algorithm.

In order to avoid eigen-decomposition and knowing the number of signals *a priori*, we use the POR approach to obtain noise subspace. In [8], \mathbf{R} is decomposed by EVD as

$$\mathbf{R} = \begin{bmatrix} \mathbf{U}_s \ \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \Lambda_s + \sigma_v^2 \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma_v^2 \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix}$$
(24)

where $\Lambda_s = diag\{\delta_1^2, \ldots, \delta_M^2\}$, \mathbf{U}_s denotes the signal subspace. It approximates the noise subspace of \mathbf{R} based on \mathbf{R}^{-m} (*m* is a positive integer). Accordingly

$$\sigma_v^{2m} \mathbf{R}^{-m} = \mathbf{U}_n \mathbf{U}_n^H + \mathbf{U}_s diag \left\{ \left(\frac{\sigma_v^2}{\delta_i^2 + \sigma_v^2} \right)^m \right\} \mathbf{U}_s^H$$
(25)

Clearly, $(\sigma_v^2/(\delta_i^2 + \sigma_v^2))^m$ is less than 1 and converge to zero for sufficiently large m. Theoretically, $\lim_{m\to\infty} \sigma_v^{2m} \mathbf{R}^{-m} = \mathbf{U}_n \mathbf{U}_n^H$. As result, we modify the criterion (12) and consider the following POR cost function

$$\min_{\mathbf{a}} \mathbf{a}^{H} \hat{\mathbf{R}}^{-m} \mathbf{a}$$

$$s.t. ||\mathbf{a} - \bar{\mathbf{a}}||^{2} \le \varepsilon$$

$$||\mathbf{a}||^{2} = N$$
(26)

By contrast, the (26) avoids estimating that dimension directly. Moreover, as $m \to \infty$, the proposed the POR beamforming method in (26) converges to the subspace one in (12), and it can be shown that the performance of the POR method for finite m will converge to the subspace one through computer simulation. We compared our method with previous one in [6], where m = 1 in the section 4.



Figure 1: Output SINR versus different SNR, pointing errors $\Delta = 3^{\circ}$, for (a) $\varepsilon = 0.7$, for (b) $\varepsilon = 7$.



Figure 2: The Output SINR versus pointing errors for (a) $\varepsilon = 0.7$, for (b) $\varepsilon = 7$.

4. Computer Results

Our main motivation of simulation is to demonstrate the performance in the presence of some errors in the steering vector. In all of the examples considered below, we assume a uniform linear array (ULA) with N = 20 sensors and half-wavelength spacing is used. The sources emitted mutual independent narrowband waveforms. All the results are achieved via 50 Monte Carlo trials.

In the first example, we consider the effect of the pointing error of the SOI on array output SINR. The exact direction of arrival of SOI is θ_0 , of which assumed value is $\theta_0 + \Delta$, i.e., $\bar{\mathbf{a}}(\theta_0) = \mathbf{a}(\theta_0 + \Delta)$. We assume that $\mathbf{a}(\theta_0)$ belongs to the uncertainty set

$$\|\mathbf{a}(\theta_0) - \bar{\mathbf{a}}(\theta_0)\|^2 \le \varepsilon \tag{27}$$

where ε is a user parameter. Let $\varepsilon_0 = ||\mathbf{a}(\theta_0) - \bar{\mathbf{a}}(\theta_0)||^2$. Then, choosing $\varepsilon = \varepsilon_0$. However, since Δ is unknown in practice, the ε we choose may be greater or less than ε_0 . To show that the choice of ε is not a critical issue for our algorithm, we will present simulation results with several values of ε in equation (21). In this example, the directions of the SOI and an interference source are $\theta_0 = 30^\circ$, $\theta_1 = -30^\circ$, respectively. The assumed direction of the SOI is $\theta_0 + \Delta = 33^\circ$, which results exact $\varepsilon_0 = 5.7750$. The interference-to-noise ratio (INR) is 40 dB.

Figure 1 plots array output SINR versus the SNR of the SOI when the number of snapshots is set to be L = 100. It is observed that the proposed algorithm (12) performs better than other two algorithms at all input SNR. Also, since the error in steering vector of SOI is relatively large and cannot be negligible, the standard Capon beamformer and its diagonal loading version suffer from severe performance degradation when SNR increases. However, the proposed beamformer has SINR loss of 5 dB when SNR = 20 dB. The proposed the POR method for different m over various input SNRs is also illustrated in Figure 1. Obviously, the Output SINR for m = 2 and m = 3 all converge to subspace approach (12), the counterpart for m = 1 [6] has the large output SINR loss.

Figure 2 shows the array output SINR curve versus the pointing errors, in which $SNR = 0 \, dB$, $INR = 20 \, dB$, L = 100. In this figure, the excellent performance achieved by the proposed algorithm is observed, which shows the robustness to the pointing errors. It is noted that, similar to other robust approaches, our method will worsen if there is/are strong interference spatially closed to the SOI. The reason is that for a given uncertainty region (11), the solution of **a** in optimization (12) is converge to the strong interference source. Also, it can be seen that the Output SINR of the proposed POR method increases as m increases, with m = 3 has same performance with subspace one (12).

5. Conclusion

In this paper, we discuss the performance degradation due to the presence of steering vector uncertainty of the SOI, such as, direction of arrival estimation error, finite number of snapshots, and array response error, etc. A robust Capon beamformer is developed by utilizing the orthogonality between signal and noise subspace. A more accurate estimate of the actual steering vector of the SOI is obtained via constrained optimization, by which the optimal weight is computed according to Capon beamforming. We have shown that the proposed algorithm belongs to the class of diagonal loading approaches, and the optimal amounts of diagonal loading can be precisely calculated. In order to avoid eigen-decomposition and knowing the number of signals *a priori*, we have proposed a POR-based robust beamforming scheme. It significantly outperforms the method proposed in [6] and converge to the subspace one (12). The excellent performance of our algorithm has been demonstrated via a number of computer simulations.

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Amplitude Estimation of Multichannel Signal in Spatially and Temporally Correlated Noise

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Abstract—This paper examines the problem of complex amplitude estimation of a multichannel signal in the presence of colored noise with unknown spatial and temporal correlation. A number of amplitude estimators are developed, including the optimum maximum likelihood (ML) estimator, which involves nonlinear optimization, and several suboptimal but computationally more efficient estimators based on least-squares (LS) or weighted LS (WLS) estimation. The Cramér-Rao bound (CRB) for the estimation problem is presented. Numerical results are presented to illustrate the performance of these estimators with or without training data.

1. Introduction

Amplitude estimation occurs in numerous signal processing applications. A survey of amplitude estimation techniques for sinusoidal signals with known frequencies in colored noise is found in [1]. While [1] is primarily concerned with *single-channel sinusoidal* signals, we consider amplitude estimation of an *arbitrary multichannel* signal observed in space and time using a sensor array. The observed data is contaminated by *a spatially and temporally correlated* disturbance signal with *unknown* correlation. Among other applications, this problem is encountered in an airborne radar system equipped with multiple antennas (e. g., [2]), where the multichannel signal refers to the space-time steering vector of the antenna array, the amplitude refers to the radar cross section (RCS) of a target, and the disturbance lumps together the thermal noise, radar clutter, and other interferences. Amplitude estimation within such a context would be useful for estimating the spatial and temporal correlation of the disturbance, developing effective target detectors, and finding solutions to several other relevant problems.

To account for its temporal and spatial correlation, our approach is to model the disturbance as a multichannel autoregressive (AR) process. Using extensive real radar data, [2] has shown that multichannel AR models are appropriate and offer efficient representation of the disturbance signal in airborne radars. Our *parametric* approach to the modeling of the disturbance is another major distinction compared to the *non-parametric* approach of [1]. Based on the parametric approach, our problem of interest is to find estimates of the signal amplitude, the AR coefficient matrices, and the spatial covariance matrix of the multichannel signal that drives the AR model. In the sequel, we first examine the optimum ML detector, and show that it involves nonlinear optimization. We then introduce several suboptimal but computationally more efficient LS and WLS amplitude estimators, which can be used to initialize the nonlinear searching involved in the ML estimator. The CRB for the estimation problem is presented as a performance baseline. In our numerical comparison of the different estimators, we focus on the case with *no or very limited training* data, which is of particular interest for applications in non-stationary or dense-target environments (e. g., [3]).

2. Data Model and Problem Statement

The observed noisy multichannel signal $\mathbf{x}_0(n)$ can be written as

$$\mathbf{x}_0(n) = \alpha \mathbf{s}(n) + \mathbf{d}(n), \quad n = 0, 1, \dots, N - 1,$$
 (1)

where all vectors are $J \times 1$ vectors, J is the number of spatial channels, N is the number of temporal observations, $\mathbf{s}(n)$ denotes the signal vector that is assumed known but with unknown complex amplitude α , and $\mathbf{d}(n)$ denotes the disturbance that is correlated in space and time. In addition, there are a set of *disturbance-only training* (i. e., $\alpha = 0$) data $\mathbf{x}_k(n)$, k = 1, 2, ..., K and n = 0, 1, ..., N - 1, available to assist amplitude estimation. In radar systems, training data may be obtained from range cells adjacent to the test cell. However, training is generally limited or may even be unavailable, especially in non-stationary or dense-target environments [3]. We consider amplitude estimation with and without training; in the later case, we have K = 0. Let $\mathbf{x}_k \triangleq [\mathbf{x}_k^T(0), \mathbf{x}_k^T(1), \dots, \mathbf{x}_k^T(N-1)]^T$, and **d** and **s** are formed similarly from $\mathbf{d}(n)$ and $\mathbf{s}(n)$, respectively. It is assumed that the training data $\{\mathbf{x}_k\}_{k=1}^K$ and **d** are independent and identically distributed (i.i.d.) with complex Gaussian distribution $\mathcal{CN}(\mathbf{0}, \mathbf{R})$, where **R** denotes the unknown space-time covariance matrix. A *J*-channel AR process is used to model the disturbance:

$$\mathbf{x}_k(n) - \alpha \mathbf{s}(n) = -\sum_{p=1}^P \mathbf{A}^H(p) \{ \mathbf{x}_k(n-p) - \alpha \mathbf{s}(n-p) \} + \boldsymbol{\varepsilon}_k(n), \quad k = 0, 1, \dots, K,$$
(2)

where $\{\mathbf{A}^{H}(p)\}_{p=1}^{P}$ denote the unknown $J \times J$ AR coefficient matrices and $\varepsilon_{k}(n)$ denotes the driving spatial noise with distribution $\mathcal{CN}(\mathbf{0}, \mathbf{Q})$, where \mathbf{Q} denotes the unknown $J \times J$ spatial covariance matrix. With some notational abuse, we have $\alpha = 0$ (i. e., disturbance-only) for $k \neq 0$ in (2). To focus on the amplitude estimation problem, we assume the model order P is known. In practice when P is unknown, it can be estimated by using a variety of model selection techniques [4].

The problem is to estimate the amplitude α , which is the *signal parameter* of primary interest, as well as *nuisance parameters* $\{\mathbf{A}^{H}(n)\}$ and \mathbf{Q} , from observations $\{\mathbf{x}_{k}(n)\}$.

3. Amplitude Estimators

For compact presentation, let $\mathbf{A}^H \triangleq [\mathbf{A}^H(1), \dots, \mathbf{A}^H(P)] \in \mathbb{C}^{J \times JP}$ which contains all the coefficient matrices involved in the *P*-th order AR model, $\mathbf{y}_k(n) \triangleq [\mathbf{x}_k^T(n-1), \dots, \mathbf{x}_k^T(n-P)]^T$ which contains the regression subvectors formed from the observed signal \mathbf{x}_0 or the *k*-th training signal \mathbf{x}_k , and $\mathbf{t}(n) \triangleq [\mathbf{s}^T(n-1), \dots, \mathbf{s}^T(n-P)]^T$, which contains the regression subvectors formed from the steering vector s. In the following, we first consider the optimal ML estimator, followed by the suboptimal LS and WLS estimators.

3.1. Optimal ML Amplitude Estimator

In Appendix 1, we show that the ML estimator of α is given by

$$\hat{\alpha}_{ML} = \min_{\alpha} \left| \hat{\mathbf{R}}_{xx}(\alpha) - \hat{\mathbf{R}}_{yx}^{H}(\alpha) \hat{\mathbf{R}}_{yy}^{-1}(\alpha) \hat{\mathbf{R}}_{yx}(\alpha) \right|,$$
(3)

where the correlation matrices are given by

$$\hat{\mathbf{R}}_{xx}(\alpha) = \sum_{n=P}^{N-1} [\mathbf{x}_0(n) - \alpha \mathbf{s}(n)] [\mathbf{x}_0(n) - \alpha \mathbf{s}(n)]^H + \sum_{n=P}^{N-1} \sum_{k=1}^K \mathbf{x}_k(n) \mathbf{x}_k^H(n),$$
(4)

$$\hat{\mathbf{R}}_{yy}(\alpha) = \sum_{\substack{n=P\\N-1}}^{N-1} [\mathbf{y}_0(n) - \alpha \mathbf{t}(n)] [\mathbf{y}_0(n) - \alpha \mathbf{t}(n)]^H + \sum_{\substack{n=P\\N-1}}^{N-1} \sum_{\substack{k=1\\K}}^K \mathbf{y}_k(n) \mathbf{y}_k^H(n),$$
(5)

$$\hat{\mathbf{R}}_{yx}(\alpha) = \sum_{n=P}^{N-1} [\mathbf{y}_0(n) - \alpha \mathbf{t}(n)] [\mathbf{x}_0(n) - \alpha \mathbf{s}(n)]^H + \sum_{n=P}^{N-1} \sum_{k=1}^K \mathbf{y}_k(n) \mathbf{x}_k^H(n).$$
(6)

Although statistically optimal, there is no closed-form expression for the above ML estimate. The cost function (3) is a highly nonlinear bivariate function (α is complex-valued). A brute-force exhaustive search over the two-dimensional parameter space is generally impractical. Alternatively, we can resort to Newton-like iterative nonlinear searches, providing an initial estimate of α is available. Next, we discuss suboptimal estimators that can be used for initialization.

3.2. LS Estimator

A linear LS amplitude estimator based on \mathbf{x}_0 only is given by

$$\hat{\alpha}_{\rm LS} = \frac{\mathbf{s}^H \mathbf{x}_0}{\mathbf{s}^H \mathbf{s}},\tag{7}$$

which ignores the coloredness of the disturbance signal. Albeit simple, the LS estimator is useful when training is unavailable. In addition, it can be used in combination with the WLS amplitude estimator presented next for improved estimation accuracy.

3.3. WLS Estimator

Suppose we have some initial estimates of \mathbf{A} and \mathbf{Q} , denoted by $\hat{\mathbf{A}}$ and $\hat{\mathbf{Q}}$, respectively. Then, as shown in Appendix 2, a WLS amplitude estimator is given by

$$\hat{\alpha}_{\text{WLS}} = \frac{\sum_{n=P}^{N-1} \{\mathbf{s}(n) + \sum_{p=1}^{P} \hat{\mathbf{A}}^{H}(p) \mathbf{s}(n-p) \}^{H} \hat{Q}^{-1} \{\mathbf{x}_{0}(n) + \sum_{p=1}^{P} \hat{A}^{H}(p) \mathbf{x}_{0}(n-p) \}}{\sum_{n=P}^{N-1} \{\mathbf{s}(n) + \sum_{p=1}^{P} \hat{\mathbf{A}}^{H}(p) \mathbf{s}(n-p) \}^{H} \hat{Q}^{-1} \{\mathbf{s}(n) + \sum_{p=1}^{P} \hat{A}^{H}(p) \mathbf{s}(n-p) \}}.$$
(8)

To find initial estimates $\hat{\mathbf{A}}$ and $\hat{\mathbf{Q}}$, we consider two cases with and without training. First, if training is available (i.e., $K \geq 1$), an ML estimator based on only the training data can be used to estimate \mathbf{A} and \mathbf{Q} . Following similar steps in Appendix 1, we can show that the *training-only* ML estimates are given by

$$\hat{\mathbf{A}}^{H} = -\hat{\mathbf{R}}^{H}_{yx,t}\hat{\mathbf{R}}^{-1}_{yy,t}, \tag{9}$$

$$\hat{\mathbf{Q}} = \frac{1}{K(N-P)} \Big(\hat{\mathbf{R}}_{xx,t} - \hat{\mathbf{R}}_{yx,t}^{H} \hat{\mathbf{R}}_{yy,t}^{-1} \hat{\mathbf{R}}_{yx,t} \Big),$$
(10)

 $\hat{\mathbf{R}}_{xx,t} = \sum_{n=P}^{N-1} \sum_{k=1}^{K} \mathbf{x}_k(n) \mathbf{x}_k^H(n)$, and $\hat{\mathbf{R}}_{yy,t}$ and $\hat{\mathbf{R}}_{yx,t}$ are correlation matrices formed similarly as in (5) and (6), however, using only the training signals.

On the other hand, if no training data are available (K = 0), we can create artificially one "training signal" by by subtracting. $\hat{\alpha}_{LS} \mathbf{s}(n)$ from the observed signal, i.e.,

$$\bar{\mathbf{x}}_0 \triangleq \mathbf{x}_0 - \hat{\alpha}_{LS} \mathbf{s}$$

where α_{LS} is given by (7). Then, the training-only ML estimator (9) and (10) can be used to estimate **A** and **Q** as if K = 1. Finally, it is noted that once the WLS estimate $\hat{\alpha}_{WLS}$ is obtained, it can be used to update estimates of **A** and **Q**. We can iterate the above procedure a few times.



Figure 1: MSE of the signal amplitude estimate $\hat{\alpha}$ versus the input SINR when J = 4, N = 32, and K = 0.



Figure 3: MSE of the signal amplitude estimate $\hat{\alpha}$ versus the input SINR when J = 4, N = 16, and K = 1.



Figure 2: MSE of the signal amplitude estimate $\hat{\alpha}$ versus the input SINR when J = 4, N = 128, and K = 0.



Figure 4: MSE of the signal amplitude estimate $\hat{\alpha}$ versus the input SINR when J = 4, N = 64, and K = 1.

4. Cramér-Rao Bound

The CRB provides a lower bound on the variance of the parameter estimates obtained by any unbiased estimators, and it can be used to access the accuracy of various amplitude estimation schemes. It can be shown that CRB for the signal amplitude estimation is given by

$$CRB(\alpha) = \left[\sum_{n=P}^{N-1} \left\{ \mathbf{s}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p)\mathbf{s}(n-p) \right\}^{H} \mathbf{Q}^{-1} \left\{ \mathbf{s}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p)\mathbf{s}(n-p) \right\} \right]^{-1}.$$
 (11)

5. Numerical Results

We present numerical results to compare the proposed amplitude estimation schemes. In the following, the SINR is defined as SINR= $|\alpha|^2 \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}$ where \mathbf{R} is the $JN \times JN$ joint space-time covariance matrix of the disturbance \mathbf{d} . For the no training case (K = 0), we consider 1) LS amplitude estimator given by (7); 2) WLS1 amplitude estimator given by (8) with estimates $\hat{\mathbf{A}}$ and $\hat{\mathbf{Q}}$ obtained by using the artificially created training signal; 3) WLS2 estimator which extends WLS1 with another iteration; 4) ML amplitude estimator given by (3). For the case when training is available (K > 0), we consider 1) WLS amplitude estimator given by (8) along with (9) and (10); 2) ML amplitude estimator given by (3). In both cases, the CRB (11) is included.

Figures 1 and 2 depict the mean-squared error (MSEs) of the various amplitude estimates versus the input SINR. We can see that 1) the MSE of WLS1 estimator is slightly larger than the CRB when N = 32, but is close to the CRB when N = 128; 2) as N increases, the MSEs of the WLS1, WLS2, and ML estimators are getting close to the CRB; 3) the MSE of the LS estimator is away from the CRB even at N = 128.

Figures 3 and 4 depict the MSEs of the various amplitude estimates versus the input SINR when very limited training is available (K = 1). It is seen that as N increases, the WLS estimates are close to the ML estimates and the CRB.

6. Conclusion

We have examined the problem of amplitude estimation of a known multichannel signal in the presence of a temporally and spatially correlated disturbance signal. To deal with temporal and spatial coloredness, the disturbance signal is modeled as a multichannel AR process with unknown AR coefficient matrices and spatial covariance matrix. We have derived the ML estimate of the signal amplitude which involves two-dimensional nonlinear searches. We have also introduced several suboptimal LS and WLS estimators that can be utilized to initialize the searching.

Appendix 1: Derivation of ML Estimators

The exact maximization of the joint PDF or likelihood function with respect to the unknown parameters produces a set of highly nonlinear equations that are difficult to solve. For large data records, the likelihood function can be well approximated by a joint conditional PDF (12) conditioned on $\{\mathbf{x}_k(n)\}_{n=0}^{P-1}\}, k = 0, 1, \ldots, K$ [5]. For brevity, the conditional PDF is referred to as the likelihood function henceforth. The loglikelihood function is proportional to (within an additive constant) [6]

$$\sum_{k=1}^{H} \sum_{n=P}^{K} \left[\mathbf{x}_{k}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \mathbf{x}_{k}(n-p) \right]^{H} \mathbf{Q}^{-1} \left[\mathbf{x}_{k}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \mathbf{x}_{k}(n-p) \right]$$

$$- \sum_{n=P}^{N-1} \left[\{ \mathbf{x}_{0}(n) - \alpha \mathbf{s}(n) \} + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \{ \mathbf{x}_{0}(n-p) - \alpha \mathbf{s}(n-p) \} \right]^{H} \mathbf{Q}^{-1}$$

$$\times \left[\{ \mathbf{x}_{0}(n) - \alpha \mathbf{s}(n) \} + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \{ \mathbf{x}_{0}(n-p) - \alpha \mathbf{s}(n-p) \} \right]$$

$$(12)$$

where L = (K + 1)(N - P). Taking the derivative of the likelihood function with respect to **Q** and equating the result to zero produce the ML estimates of **Q** given α and **A**:

$$\hat{\mathbf{Q}}(\alpha, \mathbf{A}) \triangleq \frac{1}{L} \left\{ \sum_{k=1}^{K} \sum_{n=P}^{N-1} \left[\mathbf{x}_{k}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \mathbf{x}_{k}(n-p) \right] \left[\mathbf{x}_{k}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \mathbf{x}_{k}(n-p) \right]^{H} + \sum_{n=P}^{N-1} \left[\{ \mathbf{x}_{0}(n) - \alpha \mathbf{s}(n) \} + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \{ \mathbf{x}_{0}(n-p) - \alpha \mathbf{s}(n-p) \} \right] \times \left[\{ \mathbf{x}_{0}(n) - \alpha \mathbf{s}(n) \} + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \{ \mathbf{x}_{0}(n-p) - \alpha \mathbf{s}(n-p) \} \right]^{H} \right\}.$$
(13)

Substituting the above $\hat{\mathbf{Q}}$ back in the likelihood function, we find that maximizing the loglikelihood reduces to minimizing $|\hat{\mathbf{Q}}(\alpha, \mathbf{A})|$. Therefore, the ML estimates of α and \mathbf{A} can be obtained by minimizing $|\hat{\mathbf{Q}}(\alpha, \mathbf{A})|$ with respect to α and \mathbf{A} . In turn, we can get the ML estimate of \mathbf{Q} by replacing α and \mathbf{A} with their ML estimates in (13). Next, observe that

$$L\hat{\mathbf{Q}}(\alpha, \mathbf{A}) = \hat{\mathbf{R}}_{xx}(\alpha) + \mathbf{A}^{H}\hat{\mathbf{R}}_{yx}(\alpha) + \hat{\mathbf{R}}_{yx}^{H}(\alpha)\mathbf{A} + \mathbf{A}^{H}\hat{\mathbf{R}}_{yy}(\alpha)\mathbf{A}$$

$$= \left(\mathbf{A}^{H} + \hat{\mathbf{R}}_{yx}^{H}(\alpha)\hat{\mathbf{R}}_{yy}^{-1}(\alpha)\right)\hat{\mathbf{R}}_{yy}(\alpha)\left(\mathbf{A}^{H} + \hat{\mathbf{R}}_{yx}^{H}(\alpha)\hat{\mathbf{R}}_{yy}^{-1}(\alpha)\right)^{H} + \hat{\mathbf{R}}_{xx}(\alpha) - \hat{\mathbf{R}}_{yx}^{H}(\alpha)\hat{\mathbf{R}}_{yy}^{-1}(\alpha)\hat{\mathbf{R}}_{yx}(\alpha)$$
(14)

where the correlation matrices are given by (4), (5), and (6). Since $\hat{\mathbf{R}}_{yy}(\alpha)$ is non-negative definite and the remaining terms in (14) do not depend on \mathbf{A} , it follows that $\hat{\mathbf{Q}}(\alpha, \mathbf{A}) \geq \hat{\mathbf{Q}}(\alpha, \mathbf{A})|_{\mathbf{A}=\hat{\mathbf{A}}(\alpha)}$, where

$$\hat{\mathbf{A}}^{H}(\alpha) = -\hat{\mathbf{R}}_{yx}^{H}(\alpha)\hat{\mathbf{R}}_{yy}^{-1}(\alpha).$$
(15)

When $\hat{\mathbf{Q}}(\alpha, \mathbf{A})$ is minimized, the estimate $\hat{\mathbf{A}}^{H}(\alpha)$ of \mathbf{A}^{H} will minimize any non-decreasing function including the determinant of $\hat{\mathbf{Q}}(\alpha, \mathbf{A})$ [7]. Hence, $\hat{\mathbf{A}}^{H}(\alpha)$ is the ML estimate of \mathbf{A}^{H} given α . Replacing \mathbf{A}^{H} in (14) by $\hat{\mathbf{A}}^{H}(\alpha)$ yields the ML amplitude estimator (3).

Appendix 2: Derivation of WLS Estimator

Suppose that \mathbf{Q} and \mathbf{A} are known. Then, taking the derivative of the loglikelihood function (12) and setting the result to zero yield

$$\alpha \sum_{n=P}^{N-1} \left\{ \mathbf{s}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \mathbf{s}(n-p) \right\}^{H} \mathbf{Q}^{-1} \left\{ \mathbf{s}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \mathbf{s}(n-p) \right\} - \sum_{n=P}^{N-1} \left\{ \mathbf{s}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \mathbf{s}(n-p) \right\}^{H} \mathbf{Q}^{-1} \left\{ \mathbf{x}_{0}(n) + \sum_{p=1}^{P} \mathbf{A}^{H}(p) \mathbf{x}_{0}(n-p) \right\} = 0.$$
(16)

By solving (16), we have the ML estimate of α . It is different from the ML estimate (3) which assumes \mathbf{Q} and \mathbf{A} are unknown. In practice, \mathbf{Q} and \mathbf{A} are unknown. If these matrices are replaced by their estimates $\hat{\mathbf{Q}}$ and $\hat{\mathbf{A}}$, the resulting WLS estimator is given by (8).

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A Neural Network Approach for Electromagnetic Diagnostic Applications

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Abstract—In this paper, a neural network approach is applied to a typical electromagnetic diagnostic problem consisting in the prediction of the electromagnetic field absorption inside a dielectric phantom. The approach has been tested by considering, at the input of the neural network, the values of the incident electric field at a fixed number of locations. Also phaseless measures have been taken into account. The outputs are some features describing the electromagnetic absorption, such as the peak amplitude of the induced field, the location of the absorption peak and a measure of the extension of the induced spot inside the phantom.

Preliminary results show that the approach shows a satisfactory accuracy in reconstructing the selected absorption features, and that it is able to estimate these characteristics very quickly.

1. Introduction

The purpose of this paper is to investigate the potentialities of the application of neural networks to an electromagnetic diagnostic problem.

Electromagnetic non-destructive diagnostics is usually referred to the investigation and the reconstruction of not accessible scenarios by the exploitation of measurable data coming from the interactions between electromagnetic waves and the unknown scatterers. Recent years have experienced a relevant interest in developing electromagnetic inverse scattering approaches for NDE/NDT applications (subsurface mapping of underground utilities, detection of mines or unexploded ordnances, archaeological surveys, inspection of industrial products, monitoring of buildings' deterioration and so on). Among other techniques, new approaches based on neural networks have been recently proposed in order to reconstruct the position, the dimension and the dielectric properties of a subsurface object [1–3].

Nonetheless, electromagnetic non-destructive diagnostics also refers to the prediction of the electromagnetic field distribution inside not accessible scenarios. In this case, also, research areas are manifold. In particular, the scientific community is largely interested in developing electromagnetic non-invasive techniques to achieve the prediction of the absorption of the electromagnetic field inside biological bodies exposed to wireless communication systems. A large amount of research efforts have been spent in order to develop accurate solution approaches, mainly based on the solution of the direct scattering problem and thus requiring the exact knowledge of the electromagnetic field distribution inside the dielectric properties of the investigated biological body, which had to be replicated in a reference phantom. Nowadays, some researchers turned to inverse scattering approaches to predict the electromagnetic field distribution inside the human head [4, 5]. The idea which is behind the inverse scattering approach is that it allows, in principle, to recover the electric field and the dielectric properties of the investigated scenario though the exploitation of the measurements of the scattered field in an observation domain external to the investigation area, and the values of the exposure inside the investigation area. This way, it is indeed possible to avoid the numerical modelling of the electromagnetic source, which is often a very difficult task to be achieved. Also, the approach is completely non-destructive and it could be suitable for in-vivo evaluations.

Since in general this kind of approach is characterized by large computational costs, an alternative approach can be taken into account which is based on neural networks. The fundamental difference with respect to the microwave imaging technique already investigated in [4,5] is that the new approach is based on a "learningby-examples" technique able to estimate the function relating the inputs and outputs of the problem provided that a training set of examples is available. It turns out that, as we have already experienced when we applied neural networks to reconstruct buried objects [2,3], the solution of the problem is very fast, thus making it very appealing for applications requiring a fast and accurate monitoring of in-vivo biological tissues.

The input data for the proposed approach are the complex values of the incident field at few locations inside the investigation domain. However, in real application, near-field phase measurements often require the use of sophisticated equipments and they can be quite inaccurate due to a number of different factors. Thus, accordingly to what has been done for the validation of the microwave imaging approach in [4, 5], in the present

paper also the exploitation of amplitude-only information has been considered. This has been done to simulate real conditions measurements but also in order to fully investigate the potentialities of the proposed technique and to underline its limits.

The paper is organized as follows: in section 2 the investigated electromagnetic problem will be described and the neural network approach will be introduced; in section 3, some numerical results, concerning the prediction of the absorption features inside the cross-section of a simplified biological phantom, will be reported.

2. Problem Definition and Neural Network Approach

The neural network is a learning machine whose architecture was inspired by a biological analogy with the human brain. It is well known that a neural network is a very general tool that can be used to solve both classification (pattern recognition) problems and regression ones [6,7]. Indeed, different models of neural networks have been proposed in the literature. However, as stated by the so-called Universal Approximation Theorem [1], [6], a two-layer feed-forward perceptron with a non-constant, bounded and monotone increasing continuous activation function can approximate any non-linear function relating two sets of variables.

Let us describe the problem we have considered. We have modelled a simple dielectric phantom by means of a homogeneous cylindrical biological scatterer Ω of rectangular cross section. The phantom is characterized by known relative permittivity $\varepsilon_r = 47.76$, relative permeability $\mu_r = 1.0$ and conductivity $\sigma = 1.0$ S/m while the external medium is free-space ($\varepsilon = \varepsilon_0, \mu = \mu_0$). These values are the mean values of the dielectric characteristics of human tissues inside a cross-section of a human head at the operating frequency f = 900 MHz [5]. The object is illuminated by an electric current line which radiates a TM polarized wave, in the near field of the phantom.

It is well known that the electromagnetic scattering can be described trough the two well-known integral equations c

$$\bar{E}(\bar{r}) = \bar{E}_{inc}(\bar{r}) + \int_{\Omega} \chi(\bar{r}') \bar{E}(\bar{r}') \cdot \bar{\bar{G}}(\bar{r}/\bar{r}') d\bar{r}' \quad \text{for } \bar{r} \in \Omega$$

$$\tag{1}$$

$$\bar{E}(\bar{r}) - \bar{E}_{inc}(\bar{r}) = \int_{\Omega} \chi(\bar{r}') \bar{E}(\bar{r}') \cdot \bar{\bar{G}}(\bar{r}/\bar{r}') d\bar{r}' \quad \text{for } \bar{r} \notin \Omega$$
⁽²⁾

where \bar{E}_{inc} is the incident electric field, $\bar{\bar{G}}$ is the Green's dyadic and $\chi(\bar{r}) = \varepsilon_r(\bar{r}) - 1 - j \frac{\sigma(\bar{r})}{2\pi f \varepsilon_0}$ is the object function.

Generally speaking, the application of an inverse scattering technique to predict the electric field absorption in the dielectric phantom results in the exploitation of both the incident field measurements in the investigation domain Ω and the scattered field measurements in an observation domain external to Ω . Thus, in [4, 5] a cost functional has been constructed by computing the errors between the measurements and the reconstructed field distribution obtained by the discretization of the integral equations. If we consider Eq. (1), data quantities are thus the measurements of the incident field in the investigation domain, while the unknowns are the values of the electric field inside Ω . It is possible then to define an operator ϕ such that

$$\bar{E}(\bar{r}) = \phi(\bar{E}_{inc}(\bar{r})) \quad \text{for } \bar{r} \in \Omega \quad . \tag{3}$$

Equation (3) states that the inverse problem can be recast into a regression problem in which the function ϕ can be estimated through the knowledge of a finite set of input/output pairs. In this work, the estimation is performed by implementing a single-layer feed-forward perceptron neural network with sigmoid activation [6]. Thanks to the flexibility of the prediction tool, it is indeed possible to simplify the problem. Instead of estimating the field values inside Ω , we can focus on the relevant features of the absorption. To this aim, we have identified three output variables: the absorption peak, the location of the absorption peak and the area of the spot around the peak inside which the ratio between the peak value and the amplitude of the fields remains over 6 dB. Thus, the problem can be rewritten as

to find an approximation of ϕ such that

$$\bar{\Psi} = \phi(\bar{E}_{inc}(\bar{r}_i)) \quad \text{for } i = 1, \dots, N, \bar{r}_i \in \Omega$$
(4)

being $\overline{\Psi}$ the array of the output variables, provided a training set of examples $\{\overline{\Psi}, \overline{E}_{inc}(\overline{r}_i)_{i=1,...,N}\}_j, j = 1,..., M$ (input/output pairs).

The neural network here considered is a two-layer feed-forward perceptron neural network. The training of the network has been achieved through implementation of a back-propagation algorithm [6]. During this phase, the neural network is presented with all the examples (input/output patterns) of a suitably defined training set and an iterative procedure, based on a gradient descent, attemps to minimize the error between the outputs of the network and the actual ones, provided that the initial weights of the network are randomly



Figure 1: Peak position prediction: (a) reconstructed values vs actual values at the output of the neural network exploiting 15 amplitude and phase measurements; (b) absolute error as a function of the number of receivers.

chosen (batch mode of back-propagation learning). For all the computations reported in this paper we cosidered a single training cycle. After the training phase, we have tested the generalization capabilities of the neural network (i. e., we tested the accuracy of the input-output relationship computed by the neural network for input/output patterns never used in training it [6]). Both for the training and testing of the neural network we used synthetic data that have been obtained through the finite element modelling of the problem at hand. The numerical domain has thus been discretized into triangular elements 0.003 m in size. In order to construct a set of input/output patterns for the validation of the approach, we have simulated 1848 different exposure conditions for the training set and 240 for the test set. Thus, 2088 different sets of incident field (input data) distributions have been generated by considering different positions of the electromagnetic source and different amplitudes of the current flowing through the line. For each of them, we also computed the associated features describing the electric field absorption inside the phantom (output variables).

3. Numerical Results

In this section, we are reporting some preliminary numerical results in order to investigate the capabilities of the approach described above. The input data come from the measurements of the electric field exposure while the output data are the identified features describing the electric field absorption inside the dielectric phantom. The measurements of the incident field have been collected at a finite number of points inside the investigation domain.

In the first set of simulations, we tested the capability of the neural network in predicting the electric field absorption for different numbers of observation points. We started from a configuration employing fifteen measurement points equally spaced inside the investigation domain. Input data are represented by the complete information (amplitude and phase) on the incident field at the observation points. Thus, a neural network with 30 neurons in the input layer, 30 neurons in the hidden layer and three nodes in the output layer has been implemented. Results of the neural network processing of the test set patterns are showed in Figs. 1(a), 2(a) and 3(a). The dispersion curves in the graphs represent the reconstructed values of the position of the absorption peak, the amplitude of the absorption peak and the 6 dB area, respectively, as compared to the actual features. In this case, the position of the absorption peak is reconstructed with an average absolute error equal to 0.00065 m and a maximum error of 0.00232 m. It is remarkable to notice that the error doesn't exceed the characteristic dimension of the finite element mesh. It is thus always possible to discriminate between two adjacent positions of the absorption peak, also this variable seems to be correctly evaluated since the average error is 5.0% while the maximum error is 10.8%. The extension of the absorption spot inside the phantom is predicted with an average error equal to 8.4% while the maximum error is 21.8%.

The number of measurement points has then been reduced in order to investigate the influence of the field sampling on the performances of the approach. Results are quite interesting, as the reader can infer from the



Figure 2: Peak amplitude prediction: (a) reconstructed values vs actual values at the output of the neural network exploiting 15 amplitude and phase measurements; (b) relative error as a function of the number of receivers.



Figure 3: 6 dB area prediction: (a) reconstructed values vs actual values at the output of the neural network exploiting 15 amplitude and phase measurements; (b) relative error as a function of the number of receivers.

plots in Figs. 1(b), 2(b) and 3(b). In addition, on the same graphs, we reported the results of the neural network processing of phaseless data as compared to the performances achieved when the complete information on the field measurements is available. It is immediately evident that the performances of the neural network undergo a significant deterioration only when very few measurement points are considered. When we consider both the information on the amplitude and phase of the incident field at the input, performances seem not to worsen if the number of measurement points remains over four. This seems to be a relevant feature in the considered neural network application since it it proved that satisfactory results can be achieved even when the information on the exposure condition are limited to a very restricted number of measurements. When phaseless data are considered, performances are in general worse with respect to consider full information on the field measurements. As an example, let's consider the neural network exploiting fifteen phaseless measurements of the incident field. In this case, we implemented a neural network with 15 nodes in the input layer, 15 neurons in the hidden layer and three nodes in the output layer. The prediction of the absorption peak location is achieved with an average absolute error equal to 0.00192 m; the average error on the estimated absorption peak is around 9.3%; the average error on the predicted 6 dB area is equal to 10.1%. Moreover, as we have already observed when full data were considered, as the number of measurement points reduces, a significant worsening of the performances is seen to apply only when that number reduces to four or less.

The second set of simulations concerned the investigation of the robustness of the proposed approach. To this aim we have considered the performances of the technique when noisy measurement data are taken into account at six measurement points. We have simulated the presence of a Gaussian noise over the test data by adding to the incident field values a complex quantity whose real and imaginary parts are Gaussian variables characterized by zero mean and variance depending on the considered signal-to-noise ratio (SNR). Table 1 reports the average errors achieved in predicting the electric field absorption features for different values of the signal-to-noise ratio. As can be expected, performances deteriorate when considering lower values for the SNR. In particular, it seems that the prediction of the amplitude of the absorption peak and the extension of the spot is significantly worse when SNR<30 dB.

As far as computational costs are concerned, the training phase of the neural network is usually an intensive task often requiring tens of minutes or few hours, depending on the number of input data, to be ended. However, after the training phase, the processing of the entire test set of data requires less than one second.

	no noise	SNR=50dB	SNR=40dB	SNR=30dB	SNR=20dB
absolute error on peak position	0.00074	0.000752	0.000821	0.00110	0.00245
relative error on peak amplitude	0.042	0.044	0.057	0.084	0.331
relative error on 6 dB area	0.092	0.099	0.110	0.116	0.292

Table 1: Mean errors in the prediction for different signal-to-noise ratios.

4. Conclusion

In this paper a neural network approach has been investigated for the prediction of relevant features of the electric field absorption inside an exposed dielectric phantom. The input data are the exposure measurements at few locations inside the investigation domain. Moreover, also phaseless data have been considered in the analysis. Preliminary results confirm that the approach seems to provide satisfactory performances in locating the absorption peak and in predicting its amplitude and extension. The robustness of the approach has been tested by considering the effect of noise on measurement data and good results have been obtained for SNRs greater than or equal to 30 dB. These results are encouraging as they show the potentialities of the technique in predicting the absorption with few measurement data and in real-time. Further investigations must be carried out in order to improve the robustness of the approach.

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2.5D AGILD Electromagnetic Modeling and Inversion

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Abstract—In this paper, we propose a new 2.5D AGILD electromagnetic (EM) modeling and inversion algorithms. We derive 2.5D differential integral equations for EM field on the boundary strip and center strip Ω_s with poles in cylindrical and spherical coordinate system. A 2.5D EM field Garlekin equation is derived on the remainder domain. It supposes that the electrical parameters in the rotational direction are uniform. In the cylindrical coordinate system, the EM field is function of the r, θ , and z. However, the electrical parameters are only depended on ρ and z. Upon substituting the Fourier serious of the EM field into the strip differential integral equation and Galerkin equation, we propose the 2.5D AGILD EM modeling and inversion algorithm and develop its software. The 2.5D AGILD EM modeling and inversion algorithms and software are explained in the ../files/051002183125/paper/Figs.1–6. The AGILD method has the following advantages. (1) It vanishes error on the artificial boundary; (2) It reduces the full matrix and the ill posed for inversion; (3) It resolves coordinate singularities in cylindrical and spherical coordinate system in continuous caster, geophysics, singularities in north and south poles in Earth for EM field and Navier Stokes flow simulation in atmosphere; (4) AGILD has widely applications in geophysics, atmosphere, nano-materials, caster, medical, radio, motor, etc. areas; (5) The applications show that the 2.5D AGILD and GL are fast, accurate, and have reasonable high resolution.

1. Introduction

The Global Integral and Local Differential field modeling and parameter inversion GLID algorithms have been proposed since 1997 by Xie and Li [6–8]. The new AGILD EM modeling and inversion has been published in PIERS 2005 in Hangzhuo and PIERS 2006 [1–5]. In this paper, we propose 2,5D AGILD EM modeling and inversion in cylindrical coordinate.

The AGILD modeling and inversion has widely applications for EM field and parameter in the motor and generator. They are useful for GEOMAIL, VEMP, cross hole imaging, MT, EM sheet, nanometer pipe and ball, blood pipe flow, MRI, medical instrument, microwave, coaxial antenna, and EM stirring, sensor, nondestructive testing etc. in the science and industrial engineering.

The arrangement of this paper is as follows: In the section 1, we have introduced 2.5D AGILD method and arranged our plan. In section 2, We propose the 2.5D EM differential integral equations. The 2.5D AGILD modeling algorithms are presented in the section 3. In section 4, we propose the 2.5D AGILD inversion. The Applications are described in the section 5. Finally, we present discussions and conclusion in the section 6.

2. The New 2.5D Em Differential Integral Equations

2.1. The 2.5D Em Differential Integral Equations

In the 2.5D EM modeling, the EM parameters σ , ε , and μ are independent on θ , and only variable in ρ and z. We choose the continuous component E_{θ} and H_{θ} to be unknown function. Upon substituting $[E, H] = \sum_{m=-\infty}^{\infty} [E_m, H_m](\rho, z)e^{im\theta}$ into the Maxwell equation in cylindrical coordinate, we derive the following 2.5D differential integral equations on the boundary strip and center strip domain containing pole $\rho = 0, E_{b\rho}^{J}, \ldots, H_{bz}^{M}$ are known green tensor in the background medium, they are calculated in rectangle, cylindrical, and spherical coordinate system respectively.

$$\begin{split} E_{\theta}(\rho', z', \theta'; \rho_s, z_s, \theta_s) &- \int_{\partial \Omega -} E_{b\rho}^J H_{\theta} \rho d\rho + E_{bz}^J H_{\theta} \rho dz - H_{b\rho}^J E_{\theta} \rho d\rho - H_{bz}^J E_{\theta} \rho dz \\ &+ \int_{\partial \Omega -} E_{\rho} H_{b\theta}^J \rho d\rho + E_z H_{b\theta}^J \rho dz - H_{\rho} E_{b\theta}^J \rho d\rho - H_z E_{b\theta}^J \rho dz \\ &- \int_{\Omega} (((\sigma_b + i\omega\varepsilon_b) - (\sigma + i\omega\varepsilon))(E_{b\rho}^J E_{\rho} + E_{bz}^J E_z - E_{\theta} E_{b\theta}^J)) \rho d\rho dz \end{split}$$

$$+ \int_{\Omega} ((-i\omega\mu_{b}) - (-i\omega\mu))(H_{b\rho}^{J}H_{\rho} + H_{bz}^{J}H_{z} - H_{\theta}H_{b\theta}^{J})\rho d\rho dz$$

$$= \int_{\Omega} ((J_{\rho})E_{\rho} + (J_{z})E_{z})\rho d\rho dz - \int_{\Omega} ((-i\omega\mu_{b})M_{b\rho}H_{\rho} + (-i\omega\mu_{b})M_{bz}H_{z})\rho d\rho dz$$

$$- \int_{\Omega} ((J_{\rho})E_{b\rho}^{J} + (J_{z})E_{bz}^{J} - J_{\theta}E_{b\theta}^{J})\rho d\rho dz$$

$$+ \int_{\Omega} ((-i\omega\mu)M_{\rho}H_{b\rho}^{J} + (-i\omega\mu)M_{z}H_{bz}^{J} + (i\omega\mu)M_{\theta}H_{b\theta}^{J})\rho d\rho dz \qquad (1)$$

$$(-i\omega\mu_{b})H_{\theta}(\rho',z',\theta';\rho_{s},z_{s},\theta_{s}) + \int_{\partial\Omega_{-}} E_{b\rho}^{M}H_{\theta}\rho d\rho + E_{bz}^{M}H_{\theta}\rho dz - H_{b\rho}^{M}E_{\theta}\rho d\rho - H_{bz}^{M}E_{\theta}\rho dz$$

$$-\int_{\partial\Omega_{-}} E_{\rho}H_{b\theta}^{M}\rho d\rho + E_{z}H_{b\theta}^{M}\rho dz - H_{\rho}E_{b\theta}^{M}\rho d\rho - H_{z}E_{b\theta}^{M}\rho dz$$

$$+\int_{\Omega} (((\sigma_{b}+i\omega\varepsilon_{b})-(\sigma+i\omega\varepsilon))(E_{b\rho}^{M}E_{\rho}+E_{bz}^{M}E_{z}-E_{\theta}E_{b\theta}^{M}))\rho d\rho dz$$

$$-\int_{\Omega} ((-i\omega\mu_{b})-(-i\omega\mu))(H_{b\rho}^{M}H_{\rho}+H_{bz}^{M}H_{z}-H_{\theta}H_{b\theta}^{M})\rho d\rho dz$$

$$=-\int_{\Omega} ((J_{\rho})E_{\rho}+(J_{z})E_{z})\rho d\rho dz + \int_{\Omega} ((-i\omega\mu_{b})M_{b\rho}H_{\rho}+(-i\omega\mu_{b})M_{bz}H_{z})\rho d\rho dz$$

$$+\int_{\Omega} ((J_{\rho})E_{b\rho}^{M}+(J_{z})E_{bz}^{M}-J_{\theta}E_{b\theta}^{M})\rho d\rho dz$$

$$-\int_{\Omega} ((-i\omega\mu)M_{\rho}H_{b\rho}^{M}+(-i\omega\mu)M_{z}H_{bz}^{M}-(-i\omega\mu)M_{\theta}H_{b\theta}^{M})\rho d\rho dz \qquad (2)$$

where the $E\rho$, Ez, $H\rho$, and Hz will be represented by E_{θ} , H_{θ} in the Eqs. (1) and (2).

2.2. The $2.5\,\mathrm{D}$ EM Garlekin Equation in the Cylindrical System

Upon substituting Fourier series $[E, H] = \sum_{m=-\infty}^{\infty} [E_m, H_m](\rho, z)e^{im\theta}$ into the Maxwell equation in cylindrical coordinate, let E_{θ} , H_{θ} denote $E_{\theta m}$, $H_{\theta m}$, we derive the following 2.5 D EM Garlekin equation,

$$\begin{split} &\int_{\Omega} \frac{\rho^2}{(k^2 \rho^2 - m^2)} \left(\left(\frac{1}{\rho} im \frac{\partial H_{\theta}}{\partial z} + (\sigma + i\omega\varepsilon) \frac{1}{\rho} \frac{\partial \rho E_{\theta}}{\partial \rho} \right) \frac{1}{\rho} \frac{\partial \rho \phi}{\partial \rho} - \left(\frac{1}{\rho} im \frac{1}{\rho} \frac{\partial \rho H_{\theta}}{\partial \rho} - (\sigma + i\omega\varepsilon) \frac{\partial E_{\theta}}{\partial z} \right) \frac{\partial \phi}{\partial z} \right) \rho d\rho dz \\ &- \oint_{\partial\Omega} \frac{\rho^2}{(k^2 \rho^2 - m^2)} \left(\left(\frac{1}{\rho} im \frac{\partial H_{\theta}}{\partial z} + (\sigma + i\omega\varepsilon) \frac{1}{\rho} \frac{\partial \rho E_{\theta}}{\partial \rho} \right) \rho \phi dz + \left(\frac{1}{\rho} im \frac{1}{\rho} \frac{\partial \rho H_{\theta}}{\partial \rho} - (\sigma + i\omega\varepsilon) \frac{\partial E_{\theta}}{\partial z} \right) \rho \phi d\rho \right), \\ &= -\int_{\Omega} \frac{\rho^2}{(k^2 \rho^2 - m^2)} \left(\left(\frac{1}{\rho} im (J_z) + (k^2) M_{\rho} \right) \frac{\partial \phi}{\partial z} + \left(\frac{1}{\rho} im J_{\rho} - k^2 M_z \right) \frac{1}{\rho} \frac{\partial \rho \phi}{\partial \rho} \right) \rho d\rho dz \\ &+ \int_{\Omega} ((\sigma + i\omega\varepsilon) (E_{\theta} + J_{\theta}) \rho \phi d\rho dz + \oint_{\partial\Omega} \frac{\rho^2}{(k^2 \rho^2 - m^2)} \left(\left(\frac{1}{\rho} im J_{\rho} - k^2 M_z \right) \rho \phi dz - \left(\frac{1}{\rho} im (J_z) + (k^2) M_{\rho} \right) \rho \phi d\rho \right), (3) \end{split}$$

$$\begin{split} &\int_{\Omega} \frac{\rho^2}{(k^2 \rho^2 - m^2)} \left(\left(\frac{1}{\rho} im \frac{\partial E_{\theta}}{\partial z} + (-i\omega\mu) \frac{1}{\rho} \frac{\partial \rho H_{\theta}}{\partial \rho} \right) \frac{1}{\rho} \frac{\partial (\phi\rho)}{\partial \rho} - \left(\frac{1}{\rho} im \frac{1}{\rho} \frac{\partial \rho E_{\theta}}{\partial \rho} - (-i\omega\mu) \frac{\partial H_{\theta}}{\partial z} \right) \frac{\partial \phi}{\partial z} \right) \rho d\rho dz, \\ &- \oint_{\partial\Omega} \frac{\rho^2}{(k^2 \rho^2 - m^2)} \left(\left(\frac{1}{\rho} im \frac{\partial E_{\theta}}{\partial z} + (-i\omega\mu) \frac{1}{\rho} \frac{\partial \rho H_{\theta}}{\partial \rho} \right) \phi \rho dz + \left(\frac{1}{\rho} im \frac{1}{\rho} \frac{\partial \rho E_{\theta}}{\partial \rho} - (-i\omega\mu) \frac{\partial H_{\theta}}{\partial z} \right) \phi \rho d\rho \right) \\ &= \int_{\Omega} \frac{\rho^2 (-i\omega\mu)}{(k^2 \rho^2 - m^2)} \left(\left(-\frac{1}{\rho} im M_z - J_{\rho} \right) \frac{\partial \phi}{\partial z} - \left(\frac{1}{\rho} im M_{\rho} - J_z \right) \frac{1}{\rho} \frac{\partial (\phi\rho)}{\partial \rho} \right) \rho d\rho dz \\ &+ \int_{\Omega} (-i\omega\mu) (H_{\theta} + M_{\theta}) \phi \rho d\rho dz + \oint_{\partial\Omega} \frac{\rho^2 (-i\omega\mu)}{(k^2 \rho^2 - m^2)} \left(\left(\frac{1}{\rho} im M_{\rho} - J_z \right) \phi \rho dz + \left(-\frac{1}{\rho} im M_z - J_{\rho} \right) \phi \rho d\rho \right), (4) \end{split}$$

where function ϕ is test function. When m = 0, the Eqs. (3) and (4) have singularity in the pole $\rho = 0$. The traditional FEM and FD method have singularity in the pole $\rho = 0$.

3. The New 2.5 D AGILD Modeling

Fortunately, our 2.5D differential integral Eqs. (1) and (2) have no coordinate singularity in the pole $\rho = 0$. We use collocation FEM points to discrete the Eqs. (1) and (2) on the boundary strip and center strip containing the pole point $\rho = 0$. The double layered discrete matrix equation is as follows

$$K_{BB} \begin{bmatrix} E_{\theta B_y} \\ H_{\theta B_y} \end{bmatrix} + K_{BI} \begin{bmatrix} E_{\theta I_y} \\ H_{\theta I_y} \end{bmatrix} = \begin{bmatrix} J_{sB} \\ M_{sB} \end{bmatrix}$$
(5)

where $[E_{\theta B}, H_{\theta B}]$ is the θ component EM field on the external layer of the center strip and boundary strip, $[E_{\theta I}, H_{\theta I}]$ is the θ component EM field on the internal layer of the center boundary strip. $[J_{sB}, J_{sB}]$ is the discrete source term on the boundary and center strip. On the remainder domain which has no pole point, we use Garlekin FEM to discrete the 2.5 D EM Garlekin Eqs. (3) and (4) and get sparse matrix equation. By GILD processes from internal to external, we obtain

$$K_{IB} \begin{bmatrix} E_{\theta B_y} \\ H_{\theta B_y} \end{bmatrix} + K_{II} \begin{bmatrix} E_{\theta I_y} \\ H_{\theta I_y} \end{bmatrix} = \begin{bmatrix} J_{sI} \\ M_{sI} \end{bmatrix}, \tag{6}$$

where $[J_{sI}, M_{sI}]$ is the composed source term in the internal layered. By solving the coupled Eqs. (5) and (6), we obtain $[E_{\theta B}, H_{\theta B}]$ and $[E_{\theta I}, H_{\theta I}]$. Using backward processes, we obtain $[E_{\theta}, H_{\theta}]$ in the whole domain for each m sheet (see Figure).

Because the Garlekin FEM (6) is build on the domain without singularity pole, and the differential integral discrete Eq. (5) has no coordinate singularity, Therefore, Our 2.5D AGILD method resolve the coordinate singularity historical difficulty.

4. The New 2D AGILD Inversion

In the 2.5D AGILD EM modeling, we suppose the EM parameter is invariable in the θ . The EM parameters are variable in the ρ and z of 2D. We present the 2D AGILD inversion here.

4.1. Variance EM Differential Integral Equation

We propose the 2.5D EM variation differential integral equation on the strip for inversion

$$\delta E_{\theta}(\rho', z', \theta'; \rho_{s}, z_{s}, \theta_{s}) - \delta \int_{\partial \Omega_{-}} E_{b\rho}^{J} H_{\theta} \rho d\rho + E_{bz}^{J} H_{\theta} \rho dz - H_{b\rho}^{J} E_{\theta} \rho d\rho - H_{bz}^{J} E_{\theta} \rho dz$$

$$+ \delta \int_{\partial \Omega_{-}} E_{\rho} H_{b\theta}^{J} \rho d\rho + E_{z} H_{b\theta}^{J} \rho dz - H_{\rho} E_{b\theta}^{J} \rho d\rho - H_{z} E_{b\theta}^{J} \rho dz$$

$$= \int_{\Omega} \delta(\sigma + i\omega\varepsilon) (E_{b\rho}^{J} E_{\rho} + E_{bz}^{J} E_{z} - E_{\theta} E_{b\theta}^{J}) \rho d\rho dz$$

$$- \int_{\Omega} \delta(-i\omega\mu) (H_{b\rho}^{J} H_{\rho} + H_{bz}^{J} H_{z} - H_{\theta} H_{b\theta}^{J}) \rho d\rho dz$$

$$= \delta \int_{\Omega} (J_{\rho} E_{\rho} + J_{z} E_{z}) \rho d\rho dz - \delta \int_{\Omega} ((-i\omega\mu_{b}) M_{b\rho} H_{\rho} + (-i\omega\mu_{b}) M_{bz} H_{z}) \rho d\rho dz$$

$$+ \delta \int_{\Omega} ((-i\omega\mu) M_{\rho} H_{b\rho}^{J} + (-i\omega\mu) M_{z} H_{bz}^{J} + (i\omega\mu) M_{\theta} H_{b\theta}^{J}) \rho d\rho dz, \qquad (7)$$

$$\delta E_z = \frac{-2k\delta k\rho^2}{(k^2\rho^2 - m^2)^2} \left(\frac{1}{\rho} im \frac{\partial E_\theta}{\partial z} + (-i\omega\delta\mu) \frac{1}{\rho} \frac{\partial\rho H_\theta}{\partial\rho} + (-i\omega\delta\mu) \frac{1}{\rho} im M_\rho - (-i\omega\delta\mu) (J_z) \right),\tag{8}$$

Similarly, we can derive the variance of δE and δH . Also, we can derive the variance equation of the differential integral Eq. (2), and Gerlekin Eqs. (3) and (4).

4.2. Discrete Variance Em Differential Integral Equation

By using collocation FEM discretization of the variance EM differential integral equations, and regularizing with parameter and linearization, the matrix equation of the (7) relative $[\delta\sigma, \delta\varepsilon, \delta\mu]_{Bs}$ in boundary strip to $[\delta\sigma, \delta\varepsilon, \delta\mu]_{is}$ in inside strip will be

$$\Im_{\mathbf{B}_{s}B_{s}}\begin{bmatrix}\delta\sigma_{B_{s}}\\\delta\varepsilon_{B_{s}}\\\delta\mu_{B_{s}}\end{bmatrix} + \Im_{B_{s}I_{s}}\begin{bmatrix}\delta\sigma_{I_{s}}\\\delta\varepsilon_{I_{s}}\\\delta\mu_{I_{s}}\end{bmatrix} = \begin{bmatrix}\delta E_{\theta}\\\delta H_{\theta}\end{bmatrix}_{D_{s}},$$
(9)

and using EM discretization of the variance Garlekin equation and weaker regularizing linearization, we obtain matrix equation relative $[\delta\sigma, \delta\varepsilon, \delta\mu]_{is}$ in inside strip to $[\delta\sigma, \delta\varepsilon, \delta\mu]_{Bs}$ in boundary strip. By solving the coupled matrix equation and backward processes, we obtain the variance of the EM parameter $[\delta\sigma, \delta\varepsilon, \delta\mu]$ that is used to update the parameter. The 2.5D AGILD EM modeling and inversion are explained by ../files/051002183125/paper/Figs. 1-6.



Fig. 4: AGILD inverse block scheme. Fig. 5: Inside block layer to outside layer. Fig. 6: Outside block layer to inside layer.

5. The Applications

The 2.5D AGILD modeling and inversion have widely applications in GEOMAIL, VEMP, EM sheet, cross hole imaging etc. geophysical exploration, Earthquake exploration, coaxial antenna, motors and generators design, EM stirring in caster, microwave cell phone design, environment, EM in nanometer pipe and ball, senor design, nondestructive testing. AGILDMAIL, AGILDVEMP, AGILDSheet, and ADILDEMS [5] etc software have been developed. The Earth sphere magnetic field is simulating. 2.5D AGILD can be parallelization by using frequency f_i , wave number k_{zj} and angle number θ_m . Many GL [2] and AGILD results are publishing in Journals.

6. Discussions and Conclusion

Many synthetic and field data imaging show that AGILD method is fast and accurate without any boundary error reflection, and AGILD inversion is high resolution. 2.5 AGILD has same merits as AGILD and GILD. Its main merits are (1) to reduce cost to 2D, (2) vanish error boundary reflection, and resolve singularities in cylindrical and spherical coordinate.

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Measured Electromagnetic Pulses Verify Asymptotics and Analysis for Linear, Dispersive Media

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Abstract—Electromagnetic precursors and other pulses in dispersive media have been studied theoretically since 1914. Yet a recent National Research Council study [1, p. 73–75] found few measurements that verify the relevant theories. Verification is useful because, where independent theories and measurements agree, the results are highly reliable. This report describes two laboratory verifications of transient-electromagnetic theory that I completed recently.

1. Introduction

Laboratory measurements of dc-content, electromagnetic pulses verify two groups' asymptotics for the dispersive decay and spread of pulses in a Debye model and the spread in a Lorentz model [2]. The measured peak amplitudes of pulses decay with depth nearly as $x^{-1/2}$ (water) and $x^{-1/3}$ (concrete) in coaxial cables. The products of the measured peaks and full-widths at half-maximum (FWHMs) are nearly constant. This was predicted for water and for all Debye models; for concrete, it was a surprise.

Other, independent measurements verify analysis showing that any pulse decays exponentially in a lossy material if the pulse's spectrum has nonzero separation from $\omega = 0$ (dc) [3–5]. The energy and peak amplitude then decay at least as fast as their respective slowest rates of exponential decay for the material and spectrum at hand. By basic antenna theory, frequencies near dc have infinitesimal efficiency of radiating into the far field [3]. Thus, exponential decay is nearly universal in far-field, lossy objects.



Figure 1: Peaks $\propto x^{-0.450}$ and FWHMs $\propto x^{-0.448}$ for x > 1 cm in water. Peaks $\propto x^{-0.388}$ for x > 8 cm in concrete.

2. Verification of Debye- and Lorentz-model Asymptotics

Petropoulos and I showed that the Dirac $\delta(t)$ response of any Debye model will asymptotically approach the δ response of an advection-diffusion equation as the depth x greatly exceeds an easily computed quantity that we named the time-domain skin depth [6]. As $x \to \infty$, the δ response's peak amplitudes decay as $x^{-1/2}$ and the full widths at half-maximum (FWHMs) spread as $x^{1/2}$ in Debye models. Some of these results, and further results, were obtained independently by Kelbert and Sazonov [7, secs. 2.2 and 2.3] in the same year (1996) as

[6]. In mid-1997, Farr and Frost published relevant experimental data for water and concrete [8]. Because the three groups were unaware of each other's work, the results were profoundly independent.

A theorem on "element convolution" [6] predicted that Farr and Frost's measured pulse would propagate in a Debye model of water with peaks $\propto x^{-1/2}$ and FWHMs $\propto x^{1/2}$, approximately [2]. This is evident in Fig. 1, where peaks×FWHMs are constant to within ±1.4% for water, verifying much of the three groups' work. For concrete, peaks×FWHMs are constant to within ±6.2% but the reason is not known.

Debye models are not good for concrete. But Farr and Frost's best Lorentz model for concrete is a good fit for the real part of the complex permittivity, $\varepsilon(\omega)$. Unfortunately, the same Lorentz-model fit badly understates Im ε , typical of Lorentz-model fits for solids [9, ch. 3, sec. 2.2 and 2.3]. Even in these circumstances, a derivation [2, sec. IV] shows that the pulse's measured FWHM would be well accounted for by asymptotics for the Lorentz model, provided the asymptotics themselves are good.

One basic test of Lorentz-model asymptotics is that, as $x \to \infty$, the peak amplitude of any propagated pulse decays asymptotically as $x^{-1/3}$ if the incident pulse's Fourier transform, $\tilde{f}(\omega)$, is nonzero at $\omega = 0$; but the decay is $x^{-2/3}$ if \tilde{f} has a first-order zero at $\omega = 0$ [6, endnote 30]. This basic test has precedence in Brillouin's $x^{-1/3}$ revision, in 1932, of his incorrect $x^{-1/2}$ Lorentz-model result from 1914; albeit for only one variety of incident pulse [10, p. 105–110, 124–127]. Some more-recently-published Lorentz-model asymptotics [11, paragraph 2 of p. 352] fail the basic test. Kelbert and Sazonov's asymptotics [7, sec. IV] pass the test.

Kelbert and Sazonov's asymptotics accommodate Farr and Frost's Lorentz-model fit and their incident pulse for concrete. The result is Fig. 2, which shows reasonably good agreement of theory and the FWHMs measured beyond 6 cm in concrete.



Figure 2: Measured and theoretical FWHMs agree for x > 6 cm in concrete.

In this manner, one measurement group (Farr and Frost) and two theory groups (Kelbert and Sazonov, and Petropoulos and me) published mutually verifying results within nearly a year of each other, without even knowing of each other. The mutual verifications include both the Debye and Lorentz models. The verification for Lorentz-model asymptotics is partial—valid for only FWHMs—because Lorentz models typically understate the Im ε of solids [9].

3. Verification of Exponential Decay

From 1914 [10] until July 4, 2002 [3], apparently, almost every pulse-decay rate predicted for a dispersion model was of the form $x^{\text{-const}}$, called algebraic decay. Decay rates are sensitive to the degree of spectral concentration near $\omega = 0$ (dc) [6, endnote 30]. What happens if an incident pulse's spectrum is confined to a non-infinite band of frequencies separated by a nonzero amount from dc, as broadcast regulations and practicality may require? The frequency components still would travel as $\exp(ikx)$, where $k(\omega)$ is the complex-valued wave number. Let k_i^{\min} be the smallest positive value that Im k has for the material and spectrum at hand. Intuition
then suggests that the full pulse's peak would decay at least as fast as the slowest rate of exponential decay, $\exp(-k_i^{\min}x)$.

Exponential-decay derivations [12] are suitable for undergraduate textbooks. Let $\overline{\mathcal{E}}(x) = \mathcal{E}(x)/\mathcal{E}(0)$ be the normalized energy, where $\mathcal{E}(x) = \int_{-\infty}^{\infty} |E(x,t)|^2 dt$. Incident pulses f(t) with Fourier transforms $\tilde{f}(\omega)$ propagate linearly as $E(x,t) = \int_{-\infty}^{\infty} e^{i[k(\omega)x+\omega t]} \tilde{f}(\omega) d\omega$. The Parseval equation then implies $\int_{-\infty}^{\infty} |E(x,t)|^2 dt = \int_{-\infty}^{\infty} |e^{ikx}\tilde{f}(\omega)|^2 d\omega$ and algebra shows $\overline{\mathcal{E}}(x) \leq \int_{-\infty}^{\infty} e^{-2k_i^{\min}x} |\tilde{f}(\omega)|^2 d\omega = e^{-2k_i^{\min}x} \int_{-\infty}^{\infty} |\tilde{f}|^2 d\omega = e^{-2k_i^{\min}x} \int_{-\infty}^{\infty} f^2 dt$. The value k_i^{\min} is defined in the previous paragraph.

A similar, 5-step derivation [3,12] proves the exponential decay of normalized peak amplitudes, $\overline{\mathcal{P}}(x) = \mathcal{P}(x)/\mathcal{P}(0)$, where $\mathcal{P}(x) \equiv \max_t |E(x,t)|$. In fact, $\overline{\mathcal{P}}(x) \leq e^{-2k_1^{\min}x} \int_{-\infty}^{\infty} |\tilde{f}| d\omega$. This relation for peaks was verified in 2002 by a 4-parameter, infinite family of numerical examples [3] for the Debye and Lorentz models.

Two years after the exponential decay of $\overline{\mathcal{P}}$ was verified numerically, the exponential decay of $\overline{\mathcal{E}}$ was verified experimentally [4,5]. The experiment is described next.

Choi and Osterberg measured $\overline{\mathcal{E}}(x)$ for a 660–740-nm-wavelength red laser pulse traveling 4.7 m in deionized water [4]. Their explicit motivation was to observe a pulse that decays slower than exponentially.

Choi and Österberg's data analysis yielded a graph [4, Fig. 1] that their concluding paragraph used as evidence of $\approx x^{-1/2}$ decay of $\overline{\mathcal{E}}(x)$. The evidence was weak: The $x^{-1/2}$ claim was based on only the last 4 of 24 data points. The error bars along a *logarithmic* axis were centered on the data and had constant lengths, without explanation for the cause. The dotted curve for $\exp(-2k_i^{\min}x)$ was mislabeled. And the k_i^{\min} value used in [4] represented the experiment's *deionized water* by this uncommon mixture: 25% Sargasso Sea water; $\approx 25\%$ water from Crater Lake National Park, USA; and $\approx 50\%$ doubly-distilled water [5, paragraph 3]. Sea water, especially, is unlike the deionized water used experimentally in [4].

Figure 3 is adapted from [5]. It corrects [4, Fig. 1]. The experiment's deionized water is represented here by the k_i^{\min} value for twice-distilled water at the 660 nm wavelength. The measurements of normalized energy $\overline{\mathcal{E}}(x)$ are marked. Fig. 3 shows that these measurements decay exponentially, as $\overline{\mathcal{E}}(x) \leq \exp(-2k_i^{\min}x)$, for 24 data points whose 0.001–4.7 m span covers 3.7 decades of depth. Please notice how close the measured data are to the theoretically slowest rate of decay (solid curve) for the material and spectrum at hand.



Figure 3: Experimental verification of the exponential decay of $\overline{\mathcal{E}}(x)$, showing each datum in 3.7 decades of distance.

4. Conclusion

Studies of 1D electromagnetic pulses in dispersion models date to 1914 [10]. For the next 88 years, apparently, it was largely unnoticed that many pulses used in that theory could not propagate into the far field because they had dc ($\omega = 0$) content. Such pulses can travel in waveguides with dispersive fill, but few such measurements existed until recently [1, p. 73–75]. Quantitative agreement of measurement and theory has since improved by means mentioned in sec. 2, which regards pulses with dc in the spectrum or at a spectral endpoint.

When a 1D pulse's spectrum is separated from dc by a nonzero amount, the pulse will decay exponentially in lossy materials. The relevant measurements and derivations here in sec. 3 and in [4, 5, 12] seem suitable for the undergraduate curriculum in electromagnetics.

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A New Formulation for Scattering by Impedant 3D Bodies

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Abstract—A new integral equation formulation is introduced for solving, in the frequency domain, the problem of electromagnetic scattering by an impedant (IBC) or perfect electric/magnetic (PEC/PMC) 3D body of arbitrary shape. It is based firstly, on a special application of the equivalence principle [2] where the 0-field exterior domain is filled with another impedant medium and, secondly, on the widely used PMCHW (Poggio, Miller, Chang, Harrington and Wu) formulation which forces field continuity through the scatterer surface [3]. Unlike other IBC formulations such as [4], this one also applies to PEC/PMC. Furthermore, in this last case, it appears to stabilize the numerical scheme in the vicinity of eigen frequencies. We will provide proofs and conditions of the wellposedness of the problem for impedant as well as for PEC/PMC bodies.

1. Introduction

Since the pioneering work of Leontovitch, Impedance Boundary Condition (IBC) has been widely used to simplify electromagnetic scattering problems. It simulates the material properties of a surface by forcing surface electric and magnetic fields to respect: $\mathbf{E}_{tan} = R\mathbf{n} \times \mathbf{H}_{tan}, R \in \mathbb{C}$ [1] where **n** is the unit normal to the surface pointing into the outside of the impedant medium. It is absorbing when Re(R) > 0. Range of validity of IBC for imperfect conductors has been discussed in [1]. Many specific implementations have been surveyed, but only a few general numerical method are available. The last ones are from Lange [5] and Bendali [4]. Beyond the algebric approach, [5] appears to be very similar to the proposed new formulation. It mimics the widely used PMCHW (Poggio, Miller, Chang, Harrington and Wu) method [3] and introduces a specific parameter which behaves like an impedant "complement medium" whose impedance would be equal to the scatterer's one. But, none of [5] and [4] methods extends to perfect electric (PEC) or magnetic (PMC) conductors. The proposed new formulation follows a more physical approach. It is based on a special application of the equivalence principle [2] where the 0-field exterior domain is filled with another impedant medium and on the use of the PMCHW technic. It does not require scatterer and complement domain impedances to be the same and, most of all, it extends to PEC and PMC bodies ($R_s \longrightarrow 0$ or ∞).

This paper describes a way to generalize [5] formulation. Before posing the concerned integral equation system, we briefly remind how the initial problem is decomposed. The well posedness of the formulation is then demonstrated. Finally we give some numerical illustrations which validate this approach and point out its advantages.

2. Subproblem Decomposition

The equivalent principle [2] conduces to decompose any problem into several subproblems, each one being dedicated to a given portion of the original problem. Given a subproblem, we denote "active domain" the piece of problem extracted from the original one. The space surrounding an active domain is named "complement domain". Fields are expected to be null there. 0 field being a Maxwell Equation solution whatever the medium within a source free domain, this allows to choose any medium for the complement domain. This property is often used to transform a subproblem into a free space problem by replacing a scatterer by free space. It is seldom used in other cases. The proposed formulation uses it twice: once, classically, in the first subproblem, by filling the scatterer volume with free space and, another time, in the second subproblem, by filling the complement domain with an impedant medium.

In order to illustrate this approach, let us consider a scatterer in free space lighted by a plane wave. We refer by D_S to the region of space embodying the scatterer. Its surface is denoted Γ . We refer by D_E ("exterior domain") to the rest of the space interesting the problem. Normal vectors will always be supposed to be unit vectors pointing outside the specified domain: n_S and n_E pointing from D_S , respectively D_E , toward D_E , respectively D_S . The initial problem is decomposed into 2 subproblems as follow (Fig. 1):



Figure 1: subproblems decomposition.

PbE: the exterior problem. It includes: an active domain D_E containing free space, a complement domain CD_E filled with free space, a set of surface electric and magnetic fictive currents, respectively \mathbf{J}_E and \mathbf{M}_E on Γ , impressed sources generating a plane wave incident field. It is well known that this construction leads to a standard problem where fictive sources radiate in a free space environment.

PbI: the interior problem. It includes: an active domain D_S containing the impedant scatterer with local impedance R_s , a complement domain CD_S , filled with an impedant medium caracterised by its local impedance R_c , a set of surface electric and magnetic fictive currents, respectively \mathbf{J}_I and \mathbf{M}_I on Γ , no impressed sources. Due to the complement choice, fields must satisfy an impedance bondary condition on both sides of interface Γ . On the scatterer side of Γ (point denoted x_s):

$$\mathbf{E}_{tan}(x_s) = R_s \mathbf{n}_S \times \mathbf{H}_{tan}(x_s) \tag{1}$$

On the complement side of Γ (point denoted x_{cs}):

$$\mathbf{E}_{tan}(x_{cs}) = -R_c \mathbf{n}_S \times \mathbf{H}_{tan}(x_{cs}) \tag{2}$$

3. Integral Equation Formulation

Once all subproblems posed, we evaluate, for each subroblem independantly, the scattered field on the active side of the interface radiated by fictive currents.

PbE radiating operators In a free space environment, fields radiated by surface currents are controlled by the familiar Stratton-Shu and jump relations on the interface. The field observed at point x_e on the D_E side of surface Γ is given by (Refer to [4] for expressions of Z and operators T and K):

$$\begin{aligned} \mathbf{E}_{tan}^{pbE}(x_e) &= \mathbf{E}_{tan}^{inc}(x) + ikZ(T\mathbf{J}_{\mathbf{E}})_{tan} + (K\mathbf{M}_{\mathbf{E}})_{tan} + \frac{1}{2}\mathbf{M}_{\mathbf{E}} \times \mathbf{n}_E \\ \mathbf{H}_{tan}^{pbE}(x_e) &= \mathbf{H}_{tan}^{inc}(x) - (K\mathbf{J}_{\mathbf{E}})_{tan} + ikZ^{-1}(T\mathbf{M}_{\mathbf{E}})_{tan} - \frac{1}{2}\mathbf{J}_{\mathbf{E}} \times \mathbf{n}_E \end{aligned}$$

PbI radiating operators A right combination of the usual boundary conditions [2] that links E and H fields on both side of a current sheet running on Γ

$$\begin{cases} \mathbf{E}_{tan}^{pbI}(x_s) = \mathbf{E}_{tan}^{pbI}(x_{cs}) + \mathbf{M}_{\mathbf{I}} \times \mathbf{n}_{S} \\ \mathbf{H}_{tan}^{pbI}(x_{cs}) = \mathbf{H}_{tan}^{pbI}(x_s) + \mathbf{J}_{\mathbf{I}} \times \mathbf{n}_{S} \end{cases}$$
(3)

and IBC relations (1) and (2) leads to the E and H field expression:

$$\begin{cases} \mathbf{E}_{tan}^{pbI}(x_s) = \frac{R_s}{R_c + R_s} (-R_c \mathbf{J}_{\mathbf{I}} + \mathbf{M}_{\mathbf{I}} \times \mathbf{n}_S) \\ \mathbf{H}_{tan}^{pb1}(x_s) = -\frac{R_c}{R_c + R_s} (\mathbf{J}_{\mathbf{I}} \times \mathbf{n}_S + \frac{\mathbf{M}_I}{R_c}) \end{cases}$$
(4)

When the scatterer medium tends toward PEC $(R_s \rightarrow 0)$, (4) reduces to:

$$\begin{cases} \mathbf{E}_{tan}^{pb1}(x_s) = 0\\ \mathbf{H}_{tan}^{pb1}(x_s) = -\mathbf{J}_{\mathbf{1}} \times \mathbf{n}_S - \frac{\mathbf{M}_1}{R_c} \end{cases}$$

Beyond there simplicity, they appear to be local operators, the numerical implementation of which does not require any long calculation and leads to a sparse matrix.

Connection— According to PMCHW, integral equations are built by forcing equality between surface fields associated to both subproblems:

$$\begin{cases} \mathbf{J}_{\mathbf{I}} = -\mathbf{J}_{\mathbf{E}} \\ \mathbf{M}_{\mathbf{I}} = -\mathbf{M}_{\mathbf{E}} \end{cases} and \begin{cases} \frac{R_s}{R_c + R_s} (-R_c \mathbf{J}_{\mathbf{I}} + \mathbf{M}_{\mathbf{I}} \times \mathbf{n}_S) = \mathbf{E}_{tan}^{inc}(x) + ikZ(T\mathbf{J}_{\mathbf{E}})_{tan} + (K\mathbf{M}_{\mathbf{E}})_{tan} + \frac{1}{2}\mathbf{M}_{\mathbf{E}} \times \mathbf{n}_E \\ -\frac{R_c}{R_c + R_s} (\mathbf{J}_{\mathbf{I}} \times \mathbf{n}_S + \frac{\mathbf{M}_I}{R_c}) = \mathbf{H}_{tan}^{inc}(x) - (K\mathbf{J}_{\mathbf{E}})_{tan} + ikZ^{-1}(T\mathbf{M}_{\mathbf{E}})_{tan} - \frac{1}{2}\mathbf{J}_{\mathbf{E}} \times \mathbf{n}_E \end{cases}$$
(5)

4. Well Posedness

It worth pointing out that the formulation is not a strict application of the equivalence principle. In particular, nowhere it imposes 0 fields outside active domains. This fundamental characteristic must be proven independently. In this intent, we define a new subproblem called "complement problem" PbC. It is built from the union of the complement domains of both subproblems, CD_E and CD_S plus their interface Γ .

According to the way subproblems are built, PbE and PbI solutions restricted to their respective complement domains CD_E and CD_S are solutions of the complement problem PbC:

on
$$CD_S$$
 side
$$\begin{cases} \mathbf{E}_{tan}^{pbC}(x_{cs}) = \mathbf{E}_{tan}^{pbI}(x_{cs}) \\ \mathbf{H}_{tan}^{pbC}(x_{cs}) = \mathbf{H}_{tan}^{pbI}(x_{cs}) \end{cases}$$
 and on CD_E side
$$\begin{cases} \mathbf{E}_{tan}^{pbC}(x_{ce}) = \mathbf{E}_{tan}^{pbE}(x_{ce}) \\ \mathbf{H}_{tan}^{pbC}(x_{ce}) = \mathbf{H}_{tan}^{pbE}(x_{ce}) \end{cases}$$

Furthermore, PMCHW formulation forces equality between surface fields located into PbE and PbI active sides. By applying (3), one can easily prove that PMCHW formulation works as well with fields observed into the complement sides: $\begin{cases} \mathbf{E}_{tan}^{pbI}(x_{cs}) = \mathbf{E}_{tan}^{pbE}(x_{ce}) \\ \mathbf{H}_{tan}^{pbI}(x_{cs}) = \mathbf{H}_{tan}^{pbE}(x_{ce}) \end{cases}$. Consequently, in the Complement problem

 $\begin{cases} \mathbf{E}_{tan}^{pbC}(x_{cs}) = \mathbf{E}_{tan}^{pbC}(x_{ce}) \\ \mathbf{H}_{tan}^{pbC}(x_{cs}) = \mathbf{H}_{tan}^{pbC}(x_{ce}) \end{cases}, \text{ tangential components of field are continuous through } \Gamma \text{ and, finally, PbC appears} \\ \text{to be a source free problem. AS FAR AS IT IS NOT A SINGULAR PROBLEM subject to eigen modes, its unique solution is ZERO. This proves that field solutions are equal to 0 in all complement domains whatever the subproblem.} \end{cases}$

Consequently, PbI and PbE solutions are the same as the ones provided by the equivalence principle, combination of which is known to be the unique solution of the original problem.

Finally, we can conclude that the well posedness condition requires that the problem built on the complement domains union is a non singular problem.

5. Numerical illustrations

Numerical results obtained with a unit sphere meshed with planar triangles (750 edges) confirm the formulation validity and advantages. Equivalent currents and test functions are expanded using RWG elements [6].

• <u>Accuracy</u>: in the case of an IBC sphere ($R_s = 100$), we have compared numerical results obtained from three formulations: the new formulation, CERFACS implementation of Leontovitch problem [4] and Mie series with boundary condition (1) imposed at the sphere surface [2]. The sphere is lighted from the bottom (+z direction) by a x-polarised plane wave which wave number is set to k = 2. Complement medium impedance is $R_c = 2$. Fig. 2 reports the radar cross section (RCS) observed in different direction using the 3 methods. Angle 0 corresponds to the direction of incidence. The 3 resulting curves are in perfect agreement. New formulation and CERFACS RCS results are strictly superimposed. This visual feeling is confirmed by the relative errors values on equivalent currents computed via the 3 methods (see Tab. 1). • Numerical stabilization: the behaviour of one selected RWG current element of a PEC sphere $(R_s = 0)$ has been followed when wave number k varies in the vicinity of the first eigenfrequency of the spherical cavity: $k_e = 2,76$. In this case, we use edge excitation by turning on edge 1 (excitation vector set to $\begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}$). Fig. 3 reports real and imaginary parts of the observed flux as a function of k when EFIE or proposed formulation is used. One can easily notice that the resonance peak, that clearly appears with EFIE, is suppressed by the new method. The proposed formulation is thus no subject to spurious solutions when $R_s \longrightarrow 0$ or ∞ .

degree of freedom	formulations	error
electric	Cerfacs / New formulation	$1,\!6\%$
electric	New formulation/ Mie	1,7%
magnetic	New formulation / Mie	$1,\!6\%$

 Table 1: Equivalent current relative error.



Figure 2: RCS obtained by 3 methods.



Figure 3: Real and imaginary part of current value computed by EFIE and new formulation.

4. Conclusion

The proposed formulation provides practitioners in computational electromagnetism with a general well posed method to deal with all kinds of impedant bodies, from usual IBC medium up to very good and even perfect conductors without any risk of spurious solution. Interior problem local operators are very easy to implement using RWG elements. They generate a negligible extra computation compared to the one needed for the exterior problem. Since magnetic currents must always be taken into account, even for PEC/PMC, the main drawback is the doubling of the number of degrees of freedom compared to [4]. In addition, it worth noting that the well posedness condition which states that the complement problem must be non singular could be extended to all forms of PMCHW formulations.

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Chasmas Including Magnetic Effects

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Abstract—In a plasma one has by definition quasi-neutrality over distances of the order of the Debye length. In some situations one has no quasi-neutrality over many times the Debye length. Such a non-quasi-neutral plasma or charged plasma has been called chasma. We studied previously fairly simple chasmas [1-5] using an integro-differential equation and using the partial differential equations [6], where we obtained the so called 'chasma frequency', playing a role in the steady state and in the stability. Now we extend the latter analysis by considering the Maxwell equations from the start, i.e., including the magnetic terms. According to the geometrical situation (electron and ion velocities perpendicular to the 'electrodes' or not) one may derive expressions for all steady state quantities. Perturbation yields instability except in special cases, as was the case in the non-magnetic situation.

1. Introduction

In a plasma one has by definition quasi-neutrality in volumes which have dimensions larger than the Debye length or at least a few times the Debye length. In some situations one has no quasi-neutrality over many times the Debye length. E. g., in certain discharges or in the multipactor effect [1] (secondary electron resonance discharge) in the cavities of linear accelerators. It is expected that chasma's occur too in certain extended double layers with currents and magnetic fields playing a role in the solar convective zone and solar atmosphere or in double layers in the terrestrial atmosphere. Callebaut and Knuyt [2] investigated theoretically the steady state, using an approach based on a singular integro-differential equation, cf. [3–5] and [7–13] as well.

Here we want to analyze a fairly idealized situation, although not unrealistic in view of the studies cited above, however now using the basic partial differential equations. The study made in [6] did not involve a magnetic field, which was an inconsistency as currents were allowed to flow. Including the Maxwell equations complicates the system very much. For the steady state this is still no big problem. However, the investigation of the (linear) stability is more involved. In fact an extended quasi-nonneutral region requires already a particular configuration and circumstances to allow for an equilibrium, or rather a steady state. Thus it may be expected often to be unstable. Actually, observations in linear accelerators and in separated cavities seemed to indicate sometimes a stable behavior and sometimes an unstable one, although the latter might have been caused in some cases by exhaustion of the power supply.

2. Basic Equations

We have three sets of basic equations.

2.1. Maxwell Equations and Material Equations

With conventional notations we have

 $rot\mathbf{H} = \mathbf{j} + \partial_t \mathbf{D} = e(n_+ \mathbf{v}_+ - n_- \mathbf{v}_-) + \epsilon \partial_t \mathbf{E},\tag{1}$

$$div\mathbf{B} = 0, \quad \rightarrow div\mathbf{H} = 0, \tag{2}$$

$$rot\mathbf{E} = -\partial_t \mathbf{B} = -\mu \partial_t \mathbf{H},\tag{3}$$

$$div\mathbf{D} = \rho_+ + \rho_-, \quad \to \epsilon div\mathbf{E} = e(n_+ - n_-), \tag{4}$$

where we have inserted the material equations: $\mathbf{B} = \mu \mathbf{H}$ and $\mathbf{D} = \epsilon \mathbf{E}$. ϵ is the electric permittivity and equals in vacuum $8.85 \times 10^{-12} \text{ C/Vm}$; μ is the magnetic permeability and usually is very close to its value in vacuum: $\mu = 4\pi \times 10^{-7} \text{ kgm/C}$ (or henry/m), SI units. We assume that ϵ and μ are constant which is a reasonable hypothesis as it will turn out that we shall deal for the steady state with a homogeneous density; however, as some other quantities are varying in space an extension may be to have ϵ in tensorial form. The charge density of the electrons is ρ_- , their number density is n_- , their velocity is \mathbf{v}_- , their charge is -e and their mass is m_- . For the ions, supposed ionized once only and all of the same mass, the same notation applies, but with a +sign replacing everywhere the -sign.

2.2. The Equations of Motion

With an obvious notation we have

$$n_{\pm}m_{\pm}\mathbf{v}_{\pm}.\nabla\mathbf{v}_{\pm} = \pm en_{\pm}\mathbf{E} + \pm\mu en_{\pm}\mathbf{v}_{\pm} \times \mathbf{H}.$$
(5)

These are the equations of motion for respectively ions and electrons in the cold plasma (chasma) approximation that the pressure terms are negligible.

2.3. Conservation of Charges (and Particles)

We express the conservation of charges, as well as the conservation of particles.

$$\partial_t \rho_{\pm} + \nabla .(\rho_{\pm} \mathbf{v}_{\pm}) = \pm P, \tag{6}$$

with P the ion charge density produced per unit time. Clearly there is a background of neutral particles that may get ionized by collisions. P is supposed here to be constant. P is usually the product of the beam density (here supposed to be constant), the density of the neutral gas and the ionization coefficient. In view of charge conservation during an ionization -P corresponds to the electron charge produced per unit time and unit volume. In total we have 27 equations and 26 unknowns. After dropping **B**, **D**, **j**, ρ_+ and ρ_- we still have 16 equations with 15 unknown functions. (The equations for the charge conservation and the equations (1) and (4) are not independent.)

3. Steady State

3.1. General

For the steady state we have $\partial_t = 0$ and thus the basic equations reduce to

$$rot\mathbf{H} = e(n_+\mathbf{v}_+ - n_-\mathbf{v}_-),\tag{7}$$

$$div\mathbf{H} = 0, \tag{8}$$

$$rot\mathbf{E} = 0, \quad \rightarrow \mathbf{E} = -\nabla\varphi, \tag{9}$$

$$\epsilon \Delta \varphi = -e(n_+ - n_-),\tag{10}$$

$$m_{\pm}\mathbf{v}_{\pm}.\nabla\mathbf{v}_{\pm} = \mp e\nabla\varphi + \pm e\mu\mathbf{v}_{\pm} \times \mathbf{H},\tag{11}$$

$$\nabla (n_{\pm} \mathbf{v}_{\pm}) = \pm P/e, \tag{12}$$

where φ is the electric potential. We have omitted here for simplicity the index $_0$ for the steady state quantities. Note that e.g., \mathbf{v}_{+0} and \mathbf{v}_{-0} and φ_0 are still functions of space.

We consider the case of two infinite, plane-parallel plates ('electrodes') or infinite plane-parallel 'potential borders' of a double layer, which are far enough from each other so that we may neglect boundary effects. A homogeneous ionizing beam is creating electrons and ions while passing through the chasma region, either parallel to the plates or perpendicular to them. We choose the x-direction perpendicular to the plates and the y-direction parallel to them, along the beam if this one is parallel to the electrodes. The gradient of the potential is then in the x-direction. We have $\partial_y = 0$ and $\partial_z = 0$. The equations for the magnetic field yield

$$n_{+}v_{+} = n_{-}v_{-}, \tag{13}$$

$$\partial_x H_x = 0, \quad \partial_x H_y = 0, \quad \partial_x H_z = 0. \tag{14}$$

The total current has to vanish, i.e., the electron current compensates the ion current; otherwise there is no steady state. Moreover **H** has to be a constant field. It cannot be created by the chasma as there is no current, hence it is an applied magnetic field. Its orientation does not matter much as the charges moving in one place and deviated to another place replace other charges coming from elsewhere. For simplicity and with an eye on experimental situation we take $\mathbf{H} = \mathbf{H}_0$ either perpendicular or parallel to the electrodes.

Without an applied magnetic field, or when it is parallel to the velocities, the system of equations reduces to the one studied in our previous work [6], however, now with the supplementary relation (13). Note that this relation is rather natural as the same numbers of electrons and ions are generated in the ionization process, however, their densities are inversely proportional to their velocity, which suits equation (13) very well. (The fast back and forth sweeping h.f. beam, as e.g., in the multipactor effect, may be considered approximately as averaging to a zero current.)

3.2. Homogeneous Chasma

We know from the studies using the singular integro-differential equation [2–5] that steady states exist in which both n_+ and n_- are constant. Integration of the Poisson equation (10) yields then:

$$\varphi = -e\frac{(n_+ - n_-)x^2}{2\epsilon},\tag{15}$$

where we have chosen the origin where the gradient of φ vanishes and omitted the arbitrary constant.

3.3. Beam Parallel to the Electrodes

With $\mathbf{v}_{\pm} \times \mathbf{H} = 0$ we recover the analysis given in [6]. As the production is usually proportional to the beam density, multiplied by an ionization frequency ω_{ch} we may write $P = \omega_{ch}\rho_{-0}$. There results

$$\omega_{ch}^2 = \frac{e^2(n_{+0} - n_{-0})}{\epsilon m_-},\tag{16}$$

where we have called ω_{ch} the chasma (angular) frequency in [6]. It is a strange mix of the quantities constituting the electron and ion plasma frequencies. Moreover its meaning is different: this is an entity occurring in the steady state, and as such occurring in the stability analysis too, while the plasma frequency, although constituted by equilibrium quantities, appears in the perturbation analysis only.

4. Stability

The stability analysis of the chasma for the previous case is now much more involved than without current and magnetic field. Again there is mostly instability for some particular situations.

5. Conclusion

The present results using the full set of basic equations complement and confirm the result previously obtained with a singular integro-differential equation and the analysis in which the magnetic effects were neglected [6]. A constant beam and constant ion production leads to a homogeneous ion density and a potential quadratic in the coordinates. We introduced a so-called chasma frequency ω_{ch} which has a similar structure as the electron plasma frequency, but uses the difference in ion and electron number density instead of the equilibrium number density of either the ions or the electrons. Moreover its function is different. There is stability in particular cases, depending on ω_{ch} and the geometry.

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Generation of Solar Magnetic Fields Using a Quadripolar Seed Field

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Abstract—The exact solution for the kinematic dynamo problem in spherical coordinates r, ϑ, φ is given in Ref. 1. The velocity is supposed to be azimuthal and to be an arbitrary function of r and ϑ only. Using a bipolar seed field yielded a qualitative agreement with the sunspot butterfly diagram and the polar faculae butterfly diagram. Here we investigate the case that a quadripolar seed field resides in the Sun (maybe in the whole convective zone or rather only in the tachocline at the bottom of the convective zone). In fact some observations reveal a quadripolar contribution to some surface phenomena. A combination of a bipolar and a quadripolar field yields better agreement for a suitable choice of their amplitudes. The separation between the sunspot region and the polar faculae, although both are generated by the same mechanism, is manifest: the region where the radial variation of the angular frequency of the rotation vanishes.

1. Introduction

Here the dynamo is considered in the ideal magnetohydrodynamic (MHD) approximation: no resistivity and no α -effect. The basic equations are the equation for the evolution of the magnetic field **H** and the conservation of the magnetic flux

$$\partial_t \mathbf{H} = rot(\mathbf{v} \times \mathbf{H}),\tag{1}$$

$$div\mathbf{H} = 0, \tag{2}$$

where \mathbf{v} is the velocity (SI units). The velocity field in the solar case may be assumed as being mainly a rotation around the solar axis (there is quite a good symmetry, sometimes a perfect anti-symmetry, with respect to the equator). Superposed on this main rotation are some turbulent motions, usually of much smaller amplitude. (Sometimes the turbulent speeds may be much higher, but they usually do not last long and alternate their direction.) They are not considered here, although their azimuthal component may be included in the treatment.

2. Exact Solution for Given Azimuthal Velocity

The system of spherical coordinates (r, ϑ, φ) is chosen so that the velocity $\mathbf{v}(r, \vartheta)$ has an azimuthal component only, v_{φ} . As only differential rotation matters (in our approach) for the amplification of the magnetic field this may be the differential velocity with respect to some chosen frame with uniform angular velocity, which is co-moving with some point e.g., on the equator. Hence

$$v_{\omega} = r\omega(r,\vartheta)\sin\vartheta. \tag{3}$$

Here $\omega(r, \vartheta)$ is the angular frequency. Using the requirement that the magnetic field has to be single-valued Callebaut [1] obtained:

$$H_r = \frac{-1}{r^2 \sin \vartheta} \partial_\vartheta \Phi + P_r(r, \vartheta, \omega t - \varphi), \tag{4}$$

$$H_{\vartheta} = \frac{1}{r \sin \vartheta} \partial_r \Phi + P_{\vartheta}(r, \vartheta, \omega t - \varphi), \tag{5}$$

$$H_{\varphi} = \frac{-t}{r} \frac{\partial(\omega, \Phi)}{\partial(r, \vartheta)} + P_{\varphi}(r, \vartheta, \omega t - \varphi), \tag{6}$$

where the Jacobian is introduced. P_r , P_ϑ and P_φ are purely periodic functions of $\omega t - \varphi$, and contain r and ϑ in addition; they are related through the equations of flux conservation and field evolution. As one is essentially interested in growth with time and not in a periodic waxing and waning during one solar rotation, one does not bother much about the periodic terms P, although they may be relevant in connection with e.g., the mean field theory.

 Φ is an arbitrary function of r and ϑ only. It follows from Eqs. (4) and (5) that H_r and H_ϑ do not vary with time except in their periodic terms. The only interesting time dependence (at present) is provided by the *linear* time dependence occurring as a coefficient of the Jacobian in H_{φ} . That the effective growth is linear with time may easily be understood physically. In fact the differential rotation tears the plasma, and the frozen-in magnetic field, differently in the φ direction, thus increasing H_{φ} . As the rotation is supposed steady, the increase is the same at all times, thus linear with time.

3. Quadripolar Seed Field

A contribution to the seed field consists of what is left over after part of the field has escaped at the solar surface. On the other hand there may be a part generated e.g., at the bottom of the convective zone. By lack of data on the seed field we have considered in [1] as an example a bipolar field. We gave there several arguments in favor of this choice: it is a fairly easy field, satisfying Eq. (2) and moreover planets have bipolar fields, although inside their field is far more complicated. Moreover this gave qualitatively good results. However, at the surface of the Sun, where huge currents encircling the Sun flow in filament bands, large-scale unipolar magnetic regions occur which yield a field with latitude bands of alternating signs (Callebaut and Makarov, 1992; Makarov, Callebaut, and Tlatov, 1997; Makarov et al., 2001; Makarov, Tlatov, and Sivaraman, 2003; Makarov, Tlatov, and Callebaut, 2002, 2005). Thus the choice of bipolar magnetic fields may be reasonable as a start, but one may be able to improve the results by using a combination of various types, e.g., a bipolar field together with a multipolar one. Moreover, we may consider the seed field to be in a shell only instead of in the whole region for r > 0.7 R with R the solar radius. Of course, the large and small scale turbulence and granular cells perturb the field continuously and so do the mechanisms loosing magnetic fields to space (sunspots, flares, polar faculae, bright points, coronal holes, ...) or to dissipation. We consider now the following quadripolar field separately (at first) as a tentative choice for the seed field:

$$H_r = r^{-4} H_p \cos \vartheta, \tag{7}$$

$$H_{\vartheta} = r^{-4} H_p \sin \vartheta, \tag{8}$$

in the region r > R/2 or rather r > 0.7R. The starting value for H_{φ} is irrelevant for the growth, however, it has to be independent of φ to make the field divergence vanish. H_p is a constant. It is clear that this initial field may be weak in general: to fix the ideas we may think of H_p/R^{-3} , the field at the equator, as a few gauss. However, in a narrow shell at the bottom layer of the convective zone the field may be thousands of gauss, even several hundred kilogauss. Of course, for the quadripolar field component the value may be much lower than for the bipolar one. (A combination of bipolar, quadripolar and octopolar seed fields will be envisaged later.) The components (7) and (8) may be matched to the expressions (4) and (5), which is an argument in favor for the choice of a bipolar seed field. The main difference with a bipolar field is that the latter has an r^{-3} dependence. We find

$$\Phi = -r^{-2}H_p \sin^2 \vartheta. \tag{9}$$

4. Analytic Expression for the Differential Rotation

In [1] we obtained an approximate analytic expression for the angular frequency of the solar rotation per year: $5.77(\dots)(-2.0,\dots,2.0)(1+0.07,\dots,2.0)$

$$\omega = \omega_{r_0} + \frac{5.77(r - r_0)(\cos^2\vartheta_0 - \cos^2\vartheta)(1 + 0.87\cos^2\vartheta)}{(R - r_0)\cos^2\vartheta_0},\tag{10}$$

Here $\cos \vartheta_0 = 0.6$ is approximately the present value where $\partial_r \omega$ vanishes.

5. Growth

Using Eqs. (6), (9), and (10) we obtain

$$H_{\varphi} = \frac{5.77 \ H_p t \sin \vartheta \cos \vartheta}{r^4 (R - r_0) \cos^2 \vartheta_0} \left[r(\cos^2 \vartheta_0 - \cos^2 \vartheta) (1 + 0.87 \cos^2 \vartheta) + 2(r - r_0) \sin^2 \vartheta (1 - 0.87 \cos^2 \vartheta_0 + 1.74 \cos^2 \vartheta) \right]. \tag{11}$$

 $+2(r-r_0)\sin^2\vartheta(1-0.87\cos^2\vartheta_0+1.74\cos^2\vartheta)$]. (11) The formula is very similar to the one for a bipolar field: now r^{-4} appears as a factor instead of r^{-3} and a factor 2 in the second part of the formula. Proceeding with $r_0 = 0.7R$ (and $r_0 = 0.5R$ too, to see the influence) and $\cos\vartheta_0 = 0.6$ we obtain for H_{φ} and for X, the amplification factor per year, results similar to those obtained for the bipolar field.

1. Again we have very small growth in the region around latitude 37° or $\vartheta = 53^{\circ}$. Here $\cos \vartheta = \cos \vartheta_0$ and $\partial_r \omega = 0$. Again for the latitudes in the vicinity of 37° there may be some growth of the field in one sense near $r = r_0$, while the growth is in the opposite sense near the solar surface r = R, resulting in a small total growth. This latitude band marks the separation between the equatorial region with sunspots and the polar region with polar faculae as $\partial_r \omega$ reverses sign.

- 2. Again there is no growth at the equator and at the poles. Again H_{φ} has opposite signs in both hemispheres. This suggests again that applying a time dependent seed field with period of about 22 years may be very suitable to explain the magnetic cycle (22 years) and not only the cycle of 11 years. However, as pointed out in [1], it is not a simple matter to explain the origin of such a time variation of the seed field.
- 3. An amplification of one order of magnitude is easily possible in some latitude bands. However, the growth rates are somewhat smaller than with a bipolar field. The general result matches qualitatively the sunspot butterfly diagram and the polar faculae butterfly diagram.

Using a combination of a bipolar and a quadripolar field may improve further the results by using appropriately chosen coefficients for both fields. This is a snag as we do not have an independent estimate of the relative ratios. However, from the observations of Makarov [7, 8, and 10] it may be possible to derive such an estimate, at least approximately. Similarly one may add an octopolar field with appropriate small coefficient, again in agreement with the observations of Makarov.

6. Conclusion

Growth rates of more than an order of magnitude during one solar cycle are easily possible in certain latitude bands when using a quadripolar seed field. We obtained, without using yet the α -effect, a qualitative correspondence for two of the main features of the solar activity depending on the latitude: the sunspot and polar faculae activities are explained by the same mechanism, but with some latitude gap between them due to the reverse of sign of $\partial_r \omega$ near latitude 37°. Making the bold (and still difficult to explain) hypothesis that the seed field oscillates with a period of 22 years would even allow to explain the magnetic cycle. Moreover, it turned out that the poleward migration of the circulation is not essential for the generation of the magnetic field.

The use of a quadripolar magnetic field as a seed field for the field generation in the solar dynamo seems plausible as an additional effect to the use of a bipolar field. One may even add a weak octopolar seed field. The appropriate ratios of bipolar, quadripolar and octopolar seed field may possibly be determined from certain observations on the solar surface. The use of a more involved dependence on the latitude than just $\sin^2 \vartheta$ for Φ doesn't seem necessary. The main feature, the separation of the equatorial region with sunspots from the polar region with polar faculae, is mainly due to the fact that $\partial_r \omega$ reverses sign at a certain latitude (presently 37°) and not to the choice of the seed field.

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Higher Order Fourier Analysis for Multiple Species Plasma

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Abstract—Several first order perturbations of moderate amplitude may easily occur together in nature in a small interval of time. Each separately leads to a family of higher order terms which may total a somewhat larger amplitude. However, all the nonlinear terms of all first order perturbations may lead for a certain phase to a very large and even a divergent result. A kind of bunching, concentration of the energy in a small phase interval, occurs. This may act as a trigger and explain sudden outbursts which occur in nature (e. g., in solar flares, CMEs, bright points, prominences, etc.), on Earth (interruption of power generators) and in the laboratory. Inducing several moderate perturbations in a quiet plasma (e. g., a Q-machine) may allow experimental verification of the theoretical convergence limit.

1. Introduction

The nonlinear Fourier method of Callebaut consists in concentrating on the family of higher order terms of a single Fourier term of the linearized analysis [1–3]. Thus we have obtained the higher order terms of plasma perturbations, gravitational ones, etc. In the simplest case of a cold plasma this resulted in obtaining an analytical expression for the higher order terms. This allowed to investigate the convergence of the series, which in this case is e^{-1} of the equilibrium density. For the cases without an analytical expression we developed a numerical-graphical method to obtain the convergence limit. Near this limit the total amplitude of the wave becomes very large. The convergence limit decreases with increasing pressure.

We made an attempt [5] to explain a baffling aspect of some plasmas, e.g., prominences, that remain quiet during weeks, even months, and then suddenly burst out in an explosion with no apparent reason. Now we add a powerful argument to the reasoning in [5]: several first order perturbations of moderate amplitude may easily occur together in nature in a small interval of time. Each separately leads to a family of higher order terms which may total somewhat larger amplitudes. However, all the nonlinear combination of all first order perturbations may lead for a certain phase to a very large and even a non-convergent result.

2. A Basic Result

The nonlinear Fourier analysis by Callebaut was developed in several papers [1–3]. In particular an analytic expression for the cold plasma case was developed [2].Consider a first order perturbation n_1 of the density with amplitude A: $n/n_0 = Ae^{i(\omega t + kx)}$ normalized to the equilibrium density, using conventional notations. The associated family for the density) reads

$$n/n_0 = \sum_{s=1}^{\infty} \frac{s^s}{s!} A^s e^{is(\omega t + kx)}.$$
(1)

Similar expressions are valid for the velocity, the potential and the pressure.

This analytic expression allows to calculate the convergence ratio: $A < e^{-1}$. This means that the amplitude of the first order term has to be less than 37 per cent of the equilibrium density. Otherwise the associated sum, i.e., the full perturbation, diverges for at least one phase.

3. Several First Order Terms

Two remarks are here in order. The first one is that the above analysis is in fact an ordinary Fourier analysis of a specific solution of the system of equations with specific initial conditions: A, ω, k (which are in fact given by the first order term). In that sense the Fourier analysis is mathematically safe, as long as some very general conditions are satisfied of which the main one is that the situation is periodic, which is the case for the cold plasma. Nonlinearity comes in only when considering more than one of such first order terms. In that case interference or mixing occurs between the various families. However, once the series (1) is known, the solution for the nonlinear situation follows immediately from the binomial theorem:

$$n/n_0 = 1 + \sum_{s=1}^{\infty} \frac{s^s}{s!} (Ae^{i(\omega t + kx)} + Be^{i(\mu\omega t + kx)} + Ce^{i(\lambda\omega + kx)})^s.$$
(2)

B and C are first order amplitudes like A. The factors μ and λ are introduced as the various perturbations have various frequencies; for simplicity we kept the same wavenumber k. The extension to any number of first order terms is obvious.

The second remark concerns the initial amplitude required to have a divergent series: for the case of a single first order term this amplitude has to exceed 37 per cent of the equilibrium density to yield an instability. This requires usually an extremely strong perturbation, only possible if an extraordinary (big) bang is applied to the plasma. The limit is lowered when pressure terms are included, but even 5 or 10 per cent of the equilibrium are strong first order perturbations. This was a weak point in our paper (5). In nature or in the laboratory you need practically a neighboring explosion or the application of a (sudden) very large perturbing field. Ordinary shocks may not be sufficient. however, the application is quite different when we consider several perturbations as illustrated in equation (2). Now it is the sum of A + B + C + ... that has to reach 0.37 to have a diverging total sum for some phases. If in nature e.g., perturbation occur with amplitudes around 0.01 then some 37 of such perturbations may blow up the plasma once the phase is reached where the sum becomes divergent. This is not an impossible demand, especially as the perturbations do not have to be originally in phase or have their frequencies in a rational proportion. This may allow quite a delay before the instability sets in, a feature which suits many observations. See below: applications.

We have drawn figures for several values of the parameters A, B, C, μ, λ : A+B+C varied from 0.05 to 0.4, μ and λ took values $2, 5, \sqrt{2}, \sqrt{10}$. We considered various orders: s = 1, 3, 10, 100. Here we put some conclusions

- 1. If the total sum of the amplitudes is smaller than about 0.2 then the difference between order 3, 10, and 100 is very small.
- 2. If the total sum of the amplitudes is greater than 0.2 but less than about 0.3 then the difference between order 10 and 100 is very small.
- 3. If the total sum of the amplitudes is near 0.35, then one sees some difference between order 10 and 100.
- 4. For a total sum of the amplitudes above 0.35 the difference increases more and more. Above 0.37 (or e^{-1}) the series diverges (except for very special cases like the one in the cosine development). Moreover, the total density becomes negative (total density becomes less than zero for some arguments, which is physically impossible); this confirms that the series is not convergent.
- 5. The calculation time (using pentium 4 computer) is reasonable up to order 100. However, the time increases for each wave which is added especially for high orders.
- 6. Starting from several small amplitudes one may reach a solitonlike behavior for a certain phase (either in time or in space) with a quite large amplitude, especially when the sum of the initial amplitudes approaches 37% of the initial density. As in nature several (many) small perturbations may occur more or less together, although they may be generated at different places, this phenomenon may be important.

4. Applications

We just mention a few aspects. We refer to our paper (9), of course now having in mind several perturbations of reasonable amplitudes instead of the very big one required there.

4.1. Solar Flares, Filament Bands, Bright Points and CMEs

E.g., for a solar flare to be initiated waves may come from all sides at all times and it may take a long time (days, weeks) before the limiting value for instability is reached. Moreover, the strip in which the instability is initiated is very narrow (a finite energy is bunched together into a narrow space). From that strip the instability may spread over the whole flux tube (e.g., anomalous resistivity may occur) and thus some time elapses between the ignition and the flash (typically a quarter of an hour)liberating a tremendous amount of energy from a magnetic flux tube.

The same applies to e.g., a so-called "bright point" near the solar surface. Note that several small perturbations may ignite a bright point and that several bright points may ignite a prominence.

4.2. Power Generators on Earth

It is well known that power generators may crack down due to some perturbation which was apparently too small to cause the instability. Cf. March 1989 when the whole state of Quebec, Canada, was a day without electricity due to a solar storm which caused a magnetic perturbation spreading to the Earth. Again those perturbations seem often too small to have such an effect, but adding all the higher order terms and all the various perturbations may yield instability.

4.3. Ball Lightning

The most baffling feature of ball lightning is that it involves one or two orders of magnitude more energy than what may be expected from its light emission; the corresponding stability is equally surprising, sometimes followed by a quiet evanescent phenomenon, sometimes followed by a strong explosion. In [5] we have attempted to explain the huge energy contained in ball lightning by waves of the type of equation (1), rotating (or moving back and forth), but peaked in a narrow phase band. Moreover the cases where an explosion happens may be due to the combination of several perturbations of moderate amplitude as explained above.

4.4. Experimental Suggestion

In some experimental setups like the Q-machine one obtains a quiet plasma during reasonable periods. One may attempt to reach the instability limit by applying a very strong (sudden) external field, as suggested in [2]. However it may be easier to apply several perturbations (e. g., of different frequencies) each requiring a smaller amplitude. Plasmas with heavy negatively charged ions (fullerenes attach electrons) and positive ions may be suitable for this as their frequencies are much lower than for ordinary plasmas. However, a snag is that some electrons do not attach, resulting in a three species plasma. We are developing the appropriate extension of our nonlinear theory for this multiple species plasma.

5. Conclusion

In [5] we argued that a first order term may have a whole family of associated higher order terms which for some phase all combine together to form a powerful wave which may act as a trigger causing instability. The weak point was that the first order term had to be already very strong. Now, we have shown that the combination of several moderate first order terms and their families can yield a very strong (even divergent) perturbation in a narrow phase strip, thus having the possibility to act as a trigger locally or in a neighboring configuration.

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Accuracy of Air Ion Field Measurement

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Abstract—An analysis of the electric state of air shows the presence of various ion sorts. The therapeutic effect of negative high-mobility ions of proper concentration is known. This positive effect was observed in caves that are used for speleotherapy. This article presents the capability of methods for measuring ion concentration and for ion spectral analysis.

1. Introduction

Air ion concentration and composition belong to the frequently monitored parameters of the atmosphere [5]. Their influence on living organisms has been the subject of intensive studies. Earlier research has demonstrated the positive influence of light negative ions and air cleanness on human health. The Department of the Theoretical and Experimental Electrical Engineering of Brno University of Technology and the Institute of Scientific Instruments of the Academy of Sciences of the Czech Republic are involved in the research of ion field in office and living spaces. The objective is to increase the concentration of light air ions in these spaces. Another task is to set up a simulated therapy room, with conditions similar to speleotherapy caves. It sets the requirements for accurate measurement of ion field with good repeatability. The article deals with the design of gerdien tube and peripheral measuring devices. An optimal design is important for eliminating the inaccuracy of ion concentration measurement.

2. Measuring Method

Several methods are currently used to measure air ion fields: the dispersion method, the ionspectrometer method, the Faraday cage method, and the gerdien tube method, whose principle is shown in Fig. 1 d_1 —inner electrode diameter, d_2 —outer electrode diameter, l—length of gerdien tube, M—air flow volume rate, v—air flow velocity, e—elementary charge of electron, \oplus positive air particle (ion), \ominus negative air particle (ion). The gerdien tube consists of two electrodes. There is an electric field between the inner electrode (the collector) and the outer electrode. The field is imposed by voltage source U. Air ions flow from the fan through the gerdien tube. Negative ions in the electric field impact the collector, and the current produced is measured by a pA-meter. The current measured is proportional to air ion concentration.



Figure 1: Principle of gerdien tube method.

The model of the measuring system is shown in Fig. 2. The values measured carry systematical measurement errors. This is due to leakage currents and parasitic capacitances (modeled by I_{LEAK} in Fig. 2) [6]. We have to consider leakage resistances R_{AK} of gerdien tube, leakage resistances and capacitance of the pA-meter input $(R_{EH}, C_{EH}, R_{EL}, C_{EL})$, insulation resistance (R_V) of the collector voltage source. The current measured is further affected by the input resistance of pA-meter and the input resistance of voltage source (R_U, C_U) . To minimize the measurement error, R_{AK} , and R_V should be much larger than R_I , and R_{EH} , and R_{EL} should also be much larger than R_{OUT} . Time constant $R_U C_U$ has to be much larger than the measuring time.



Figure 2: Model of a system for measuring air ion concentration—the gerdien tube method.

3. New Design of Gerdien Tube

The inner and outer electrodes are elliptical in shape. This shape ensures that the flow of air is laminar. Air flow turbulence can distort the accuracy of measurement. The surface of the electrodes is required to be as smooth as possible. These aspects make the design of gerdien tube quite demanding (fine grinding, lapping, etc.,). The new design of gerdien tube is shown in Fig. 3.

Since in the measurement of air ion concentration very small currents are detected, it is necessary to eliminate the influence of ambient electric charge. The influence of magnetic fields has to be minimized too.



Figure 3: New gerdien tube.

4. Weak Current Amplifier

Table 1: Amplifier parameters.

I _{IN}	U_M	U_{out}	Gain	R_G
[pA]	[V]	[V]	[-]	$[\Omega]$
0,1	$1\mathrm{m}$	1	1000	$1\mathrm{M}$
1	10 m	1	100	100 k
10	0,1	1	10	10 k
100	1	10	10	10 k

The current flowing through the gerdien tube consists of ions. Current intensity depends on polarization voltage, on the dimension and parameters of gerdien tube, and on ion concentration. The specific current range for the designed gerdien tube is $10^{-10} \text{ A} - 10^{-13} \text{ A}$.

For the following measurement it is suitable to convert the current to voltage. Because the current is very weak, it is suitable to do this near the gerdien tube. The transimpedance configuration is used for the conversion and amplification in the first stage.

The transimpedance amplifier is realized with an INA 116 opamp. The INA 116 has a very low input bias current $I_{b,max} = 100$ fA. The design of the amplifier is shown in Fig. 5. The first stage has transimpedance $R_T = 10$ GΩ. The second stage is a variable-gain amplifier. The gain is set by resistor R_G . Table 1 shows the values of gain, voltage and current for various gain resistors. The resulting current-to-voltage conversion constant can be set to 0.1-1-10 pA/V.



Figure 4: Design of pA-amplifier.

5. Comparison of Gerdien Tubes

The gerdien tube of new design was compared with two others. Gerdien tube configuration and parameters are shown in Figs. 5–7. Measurement results of tube are shown in Fig. 8.



Figure 5: Gerdien tube [5]. $M = 10,62\,{\rm dm^3},\,v = 4,3\,{\rm ms^{-1}},\,I_{leak}{=}0,4\,{\rm pA} @~150\,{\rm V}$



Figure 6: Gerdien tube [5]. $M=12,14\,{\rm dm}^3,\,v=3,75\,{\rm ms}^{-1},\,I_{leak}=0,3\,{\rm pA}\ @\ 150\,{\rm V}$



Figure 7: New design of gerdien tube.





Figure 8: Results of measurment gerdien tube.

6. Conclusion

The new design of gerdien tube and the optimization of peripheral measuring devices have minimized the systematic error of measurement. The new system allows measuring air ion concentration with a sensitivity > 100 ions/cm³. The ion mobility is in the interval 0.3–100 cm²V⁻¹s⁻¹. The system will be used to measure ion field distribution in living and office spaces.

Very low leakage currents were achieved in the new design of gerdien tube. It allows higher sensitivity measurement. A long-term research task is to create an environment with suitable ion concentration and humidity in living spaces. The ion distribution in the environment will be simulated.

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Numerical Simulation of Nonlinear and Parametric Oscillations in a Semiconductor Resonator Structure

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Abstract—The rigorous mathematical modeling of nonlinear oscillations in microwave strip-slot resonator structure (RS), loaded with a distributed planar Gunn diode, is based on the solution of the threedimensional diffraction boundary problem, formulated rigorously taking into account the full set of Maxwell's equations and the nonlinear equation of transport carriers in a semiconductor. Using our numerical approach to determine the bifurcation points of the solution of nonlinear Maxwell's equations, the transition region from the stationary regime of the nonlinear semiconductor device behavior (i. e., the frequency multiplication and the stable parametric amplification) to the generation (the onset of self-oscillations) caused by the instability process in the distributed Gunn diode, was simulated taking into account constrained geometries of strip-slot RS.

1. Introduction

Microwave technology of monolithic integrated/hybrid circuits (MMIC) is moving up to higher frequencies and higher bandwidths, into the mm wave range, up and above 100 GHz. As the industry turns to MMIC devices, planar geometries have to be used. The development and manufacturing of microwave or mm-wave integrated semiconductor devices depends on the development of computer aided design (CAD) tools, based on the accuracy and the adequacy of mathematical models by solving Maxwell's equations rigorously. The goal of the paper is to investigate the nonlinear physical phenomena and effects in a distributed semiconductor insertion, loaded into a resonator structure (RS), using mathematical modeling at the electrodynamic accuracy levels and computing the bifurcation points of the nonlinear Maxwell's operator for the design of prospective MMIC nonlinear semiconductor devices.

2. The Mathematical Model Using the Decompositional Approach on Nonlinear Autonomous Multimode Blocks

The mathematical simulation of nonlinear oscillations in a microwave strip-slot RS, loaded with a distributed planar Gunn diode (Fig. 1) is based on the solution of three-dimensional diffraction boundary value problems for nonlinear Maxwell's equations, complemented with the equation of transport carriers in semiconductor [1]. The computational algorithm for solving the nonlinear diffraction boundary problem is based on the decompositional approach into nonlinear autonomous multimode blocks [2]. The autonomous block, placed between cross-sections S_3 and S_4 (Fig. 1), is nonlinear and it is included in the software package for the mathematical simulation of linear microwave devices [3]. The procedure of the decomposition and recomposition with linear and nonlinear autonomous blocks is described in [2].

For computing of the scattering matrix of the strip-slot resonator and the tapered section in the RS (Fig. 1) a second decomposition was made. The method of the calculation of the scattering matrix of interfaces between regular strip-slot lines (SSL) was proposed in [4], and the method of determining of the scattering matrix of the tapered section is described in details in [5].

The geometry of the tapered section is approximated by a function on $f(z) = \alpha \cdot z - \beta \cdot \sin \gamma \cdot z$, were α , β , γ are approximation coefficients. The accuracy of the results of mathematical modeling of the tapered section depends on the number of step discontinuities p (p = 10), the number N of eigenwaves taken into account at the interfaces between regular SSL (N = 5) and the number n of the basis functions taken into account at virtual waveguides using the numerical method of multimode autonomous blocks [6] n = 25, providing the high accuracy (better than 0.001%) of computation of propagation constants of eigenwaves of SSL.

The mathematical model of the nonlinear semiconductor RS was created by taking into account higher order nonlinearities by using five combination frequencies M = 5 [1], the number of eigenwaves on regular SSL for each of the combination frequencies is N = 5 at "seaming" of fields at the interfaces between SSL. The mathematical model of the distributed planar Gunn diode accounts for the charging in the semiconductor and the ohmic contacts [1].

3. Accurate Electromagnetic Modeling of Nonlinear and Parametric Effects in the Semiconductor Insertion Loaded into the Resonator Structure

Let a monochromatic electromagnetic wave of frequency ω_1 incident upon the input crosssections S_1 , S_2 of RS loaded with the planar Gunn diode (Fig. 1). The waves are the fundamental and the higher-order modes of SSL having known magnitudes $C^+_{n(\alpha)}(\omega_1)$, where α is the index of cross-sections, n are the indexes of eigenwaves of SSL. It is necessary to determine the magnitudes $C^-_{k(\alpha)}(\omega_m)$ of reflected (in local coordinate system on the cross-sections S_1 , S_2) modes on combination frequencies ω_m , where m are the indices of the combination frequencies.

The results of computing the normalized magnitudes $|C_{1(2)}^{-}(\omega_2)| / |C_{1(1)}^{+}(\omega_1)|$ of the reflected (on the output cross-section S_2) fundamental mode at the second time harmonic (m = 2), with respect to the magnitude of the incident (on the input cross-sections S_1) fundamental mode at first time harmonic (m = 1), depending on the resonator length L for variable magnitudes of $|C_{1(1)}^{+}(\omega_1)|$ are represented in Fig. 2. For comparison, the normalized magnitudes $|C_{1(2)}^{-}(\omega_1)| / |C_{1(1)}^{+}(\omega_1)|$ of the reflected fundamental mode at the first time harmonic (m = 1), with respect to the magnitude of the incident fundamental mode, are also shown in Fig. 2.





Figure 1: Resonator structure (RS) with the nonlinear semiconductor insert: 1—tapered section; 2—distributed planar Gunn diode; 3—strip-slot resonator (SSR).



The results of numerical modeling were obtained for the biasing electric field $E_0/\omega_0 = 1000 \text{ V/mm}$ (at point 4 of the observation of electrostatic field [1]), and for the parameters of the epitaxial film $\varepsilon = 12.5$, $\mu = 1$, $D_F = 200 \text{ cm}^2/c$; $n_0 = 1.5 \cdot 10^{15} \text{ cm}^{-3}$, and the substrate $\varepsilon = 12.5$; $\mu = 1$; the length of planar semiconductor insertion is l = 0.3 mm.

The results of computing of the normalized magnitudes $|C_{1(2)}^{-}(\omega_2)| / |C_{1(1)}^{+}(\omega_1)|$, depending on the value of parameter n_0/f , where f is the frequency, n_0 is the electron concentration (in the active layer of the semi-conductor $n_0 = N_D$, were N_D is the doping density) for variable $|C_{1(1)}^{+}(\omega_1)|$ are represented in Fig. 3(a). For comparison, the normalized magnitudes $|C_{1(2)}^{-}(\omega_1)| / |C_{1(1)}^{+}(\omega_1)|$ are also shown in Fig. 3(b).

It follows from the results of electromagnetic modeling, shown in Figs. 2 and 3, that the nonlinear effect of frequency multiplication in the distributed planar Gunn diode in the RS depends on changes of magnitudes $C_{1(1)}^+(\omega_1)$ of incident wave. If $C_{1(1)}^+(\omega_1)$ increases, the efficiency of frequency multiplication $K_1(\omega_2) =$ $20 \lg \frac{|C_{1(2)}^-(\omega_2)|}{|C_{1(1)}^+(\omega_1)|}$ decreases (as for the amplification coefficient $K_1(\omega_1) = 20 \lg \frac{|C_{1(2)}^-(\omega_1)|}{|C_{1(1)}^+(\omega_1)|}$ at the first time harmonics); because the electromagnetic field is extinguished when the value of the electric field in semiconductor becomes smaller than the Gunn threshold. The nonlinear effect of frequency multiplication is significant for the optimum value of the parameter n_0/f , because charging effects, depending on the ratio of the frequency f and the Gunn effect transit-time frequency, determine the increase of the nonlinearity coefficient of the semiconductor medium even for small values of $C_{1(1)}^+(\omega_1)$.

The results of numerical calculations of parametric amplification coefficient $K_y(\omega_1)$ for small signal case,

depending on the distance d between the RS strips, (in fact, d determines the intensity of the biasing electric field E_0), and taking into account variations of the electron concentration n_0 in the semiconductor, are shown in Fig. 4.



Figure 3: Efficiency of the frequency multiplication (a) and the amplification coefficient (b) depending on the electron concentration n_0 : curve $1 - C^+_{1(1)}(\omega_1) = 16 \text{ V/mm}$; 2 - 24 V/mm; 3 - 32 V/mm; 4 - 40 V/mm; $f_1 = 30 \text{ GHz}$; other parameters are the same as in Fig. 1.

4. Numerical Modeling of the Onset of Self-oscillations by Computing the Bifurcation Points

The numerical method, developed by us in [7], was used to determine the bifurcation points of the nonlinear Maxwell's operator. The generation in the distributed planar Gunn diode loaded into the strip-slot RS, caused by the instability process in the semiconductor with a bulk negative conductivity, was simulated taking into account the constrained geometries. The bifurcation points are those values of the bifurcation parameters, i. e., the frequencies fs, where self-excited oscillations appear [1]. Using the auxiliary computing algorithm [7] the necessary and sufficient conditions for the existence of the bifurcation point [8] in neighborhood of the numerical parameter F can be investigated. If only the sufficient condition, that the eigenvalue of matrix A(z) is an integer [8], is satisfied, in this point the magnitude of self-oscillations is equal to zero for F (see Table 1). If the necessary and sufficient conditions for the existence of the bifurcation point [8] are satisfied, then there is a new solution at the bifurcation point, described by the onset of non-zero magnitude self-oscillations at the bifurcation parameter fs (see Table 1).

d, mm	$n_0 =$	$n_0 = 1.5 \cdot 10^{15}, \text{cm}^{-3}$ $n_0 = 3.5 \cdot 10^{15}, \text{cm}^{-3}$				$n_0 = 5.5 \cdot 10^{15}, \mathrm{cm}^{-3}$		
	F,GHz	$C^{-}_{1(1,2)}(\omega)$ V/mm	F, GHz	fs,GHz	$C^{-}_{1(1,2)}(\omega)$ V/mm	F,GHz	fs,GHz	$C^{-}_{1(1,2)}(\omega) { m V/mm}$
0.00150	68.90	0.00	68.83		0.00	68.76		0.00
0.00200	51.68	0.00	51.63		0.00		51.58	75.39
0.00250	40.84	0.00		40.80	50.37		40.78	277.31
0.00300	33.98	0.00		33.95	79.56		33.91	398.16
0.00343	30.06	0.00		30.03	170.18		30.00	475.24
0.00400	25.81	0.00		25.78	281.31		25.76	495.23
0.00450	22.71	0.00		22.69	398.28		22.66	470.31
0.00500	20.43	0.00		20.41	451.71		20.39	463.31
0.00550	18.69	0.00		18.67	485.75		18.65	451.57

Table 1: The bifurcation parameter fs and parameter F depending on the distance d between RS strips, determining the biasing electric field $E_0(\omega_0)$, and the electron concentration n_0 .

The results of computing the bifurcation points depending on the value of the intensity of biasing electric field $E_0(\omega_0)$, are determined by the distance d between RS strips, and the electron concentration n_0 in the semiconductor are presented in Table 1, where F is a numerical parameter, fs are the bifurcation parameters, i. e., the frequencies of the onset of self-oscillations, d is the distance between RS strips, n_0 is the electron concentration in the semiconductor, $C_{1(1,2)}^-(\omega)$ are the magnitudes of self-excited oscillations on the output cross-sections S_1 , S_2 .

The optimum parameters and the size of the planar Gunn diode for the efficiency of the generation and parametric amplification were determined by taking into account constrained geometries. The optimum value of d =0.00343 mm determines the maximum of parametric amplification coefficient $K_y(\omega_1)$ (Fig. 4), when the frequency of self-excited oscillations in the transit-time mode of the planar Gunn diode (fs = 30 GHz from Table 1) is equal to the frequency of pumping wave $f = 2f_1$ in the degenerate regime of parametric amplification. At this frequency for $n_0 = 5.5 \cdot 10^{15} \text{ cm}^{-3}$ the magnitude of self-excited oscillations ($C_{1(1,2)}^-(\omega) = 475.24 \text{ V/mm}$) is more than for $n_0 = 3.5 \cdot 10^{15} \text{ cm}^{-3}$ ($C_{1(1,2)}^-(\omega) = 170.18 \text{ V/mm}$), that is why increasing the nonlinearity of the "dynamic capaciparametric amplification coefficient $K_y(\omega_1)$ (Fig. 4). It for



Figure 4: Efficiency of parametric amplification, depending on the distance *d* between RS strips (i. e., the biasing electric field $E_0(\omega_0)$) for different electron concentrations n_0 : $1-n_0 = 5.5 \cdot 10^{15} \text{ cm}^{-3}$; $2-3.5 \cdot 10^{15} \text{ cm}^{-3}$; $3-1.5 \cdot 10^{15} \text{ cm}^{-3}$; $f_1 = 15 \text{ GHz}$, $C_{1(1)}^+(\omega_1)=0.01 \text{ V/mm}$.

why increasing the nonlinearity of the "dynamic capacity" of the planar Gunn diode provides an increasing parametric amplification coefficient $K_y(\omega_1)$ (Fig. 4). It follows from the results of computing (Table 1) that for $n_0 = 1.5 \cdot 10^{15} \text{ cm}^{-3}$ there is no self-oscillations in the planar Gunn diode, as this is the stable regime of the steady state domains [9].

5. Conclusion

The accurate electromagnetic modeling of nonlinear and parametric oscillations in microwave stripslot RS loaded with a distributed planar Gunn diode shows how the efficiency of the frequency multiplication and the parametric amplification depends on the magnitude and the electron concentration in the semiconductor. The results of computing the bifurcation points by our numerical method permit to analyze the optimum parameters and the size of the distributed planar Gunn diode in RS for the efficient generation and parametric amplification taking into account constrained geometries. The new results of research into nonlinear interactions (self-oscillations, frequency multiplication, parametric amplification) in distributed planar semiconductor Gunn diodes could be used for future MMIC devices, in particular, new microwave mm- or submm-wave generators, frequency multipliers, parametric amplifiers.

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A Parallel Computer Implementation of Fast Low-rank QR Approximation of the Biot-Savart Law

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Abstract—In this paper we present a low-rank QR method for evaluating the discrete Biot-Savart law on parallel computers. It is assumed that the known current density and the unknown magnetic field are both expressed in a finite element expansion, and we wish to compute the degrees-of-freedom (DOF) in the basis function expansion of the magnetic field. The matrix that maps the current DOF to the field DOF is full, but if the spatial domain is properly partitioned the matrix can be written as a block matrix, with blocks representing distant interactions being low rank and having a compressed QR representation. The matrix partitioning is determined by the number of processors, the rank of each block (i.e., the compression) is determined by the specific geometry and is computed dynamically. In this paper we provide the algorithmic details and present computational results for large-scale computations.

1. Introduction

The computation of magnetic fields from a prescribed electric current is a common problem in magnetic design and analysis. One approach is to form the problem as a Partial Differential Equation (PDE) for the unknown field with the prescribed electric current as the source term. Regardless of the particular PDE formulation, e.g., a magnetic vector potential formulation or a mixed B-H formulation, a large volumetric mesh must be employed, and some boundary condition must be applied on the outer boundary of the mesh. In contrast to the PDE approach, the Biot-Savart law can be employed to directly compute the magnetic field due to the prescribed current [1]. The advantage of the Biot-Savart law approach is that a full volume mesh is not required, and no boundary conditions need be applied. The disadvantage of the Biot-Savart approach is the cost is O(N * M). In this paper we review a fast low-rank QR method for compressing the $M \times N$ Biot-Savart matrix. The approach is similar to low-rank QR methods developed for boundary element electrostatics [2, 3] and for low frequency electric field integral equations [4]. The key difference with our approach is that we are concerned with volumetric current densities and implementation on parallel computers.

2. Formulation

The law of Biot and Savart is given by

$$\vec{B}(x) = \nabla \times \vec{A} = \frac{1}{\mu 4\pi} \int_{\Omega'} \frac{\vec{J}(x') \times (x - x')}{|x - x'|^3} d^3 x'.$$
 (1)

where J(x') is the known current density at the source point x', and B(x) is the desired magnetic flux density at the observation point x. We assume that we have a finite element representation for J over the volume Ω' , and a finite element representation for B over a surface Γ ,

$$\vec{J} = \sum_{i=1}^{N} \xi_i \vec{W}_i^2, \quad \vec{B} = \sum_{j=1}^{M} \beta_j \vec{W}_j^1, \tag{2}$$

where ξ_j and β_j are the *i*th degree-of-freedom (DOF), and \vec{W}_i^2 and \vec{W}_j^1 are vector basis functions. Inserting the basis function expansions (2) into (1) yields the discrete Biot-Savart law

$$\mathbf{M}\bar{\boldsymbol{\beta}} = \mathbf{Z}\bar{\boldsymbol{\xi}},\tag{3}$$

where

$$\mathbf{Z}_{ij} = \int_{\Gamma} \int_{\Omega'} \frac{1}{\mu 4\pi} \frac{\vec{W}_i^2(x') \times (x - x') \cdot \vec{W}_j^1(x)}{|x - x'|^3} d\Omega' d\Gamma, \tag{4}$$

and

$$\mathbf{M}_{ij} = \int_{\Gamma} \vec{W}_i^1(x) \cdot \vec{W}_j^1(x) d\Gamma$$
(5)

and where $\bar{\xi}$ and $\bar{\beta}$ are the arrays of DOF. The matrix **M** is a "mass matrix" due to the fact that the basis functions are not orthogonal. The mass matrix is extremely sparse and the computational cost for forming and solving this matrix is negligible. In many applications the problem of determining the *B*-field can be posed in terms of the magnetic vector potential $A = \nabla \times B$ with

$$\vec{A}(x) = \frac{1}{\mu 4\pi} \int_{\Omega} \frac{\vec{J}(x')}{|x - x'|} d^3 x'.$$
(6)

Using a finite element representation for A yields another version of the discrete Biot-Savart law

$$\vec{J} = \sum_{i=1}^{N} \xi_i \vec{W}_i^2, \quad \vec{A} = \sum_{j=1}^{M} \alpha_j \vec{W}_j^1, \tag{7}$$

$$\mathbf{M}\bar{\alpha} = \mathbf{Y}\bar{\xi},\tag{8}$$

where

$$\mathbf{Y}_{ij} = \int_{\Gamma} \int_{\Omega'} \frac{1}{\mu 4\pi} \frac{\vec{W}_i^2(x') \cdot \vec{W}_j^1(x)}{|x - x'|} d\Omega' d\Gamma.$$
(9)

We will refer to the $M \times N$ matrices **Z** and **Y** as Biot-Savart matrices. The computation of these matrices involves singular and near-singular integrals. The surface integration is performed using standard Gaussian quadrature points for each surface element. The volume integration uses an adaptive integration rule, which varies the order of Gaussian quadrature based on the distance between the source point x' and the observation point x. When the surface element containing x is a face of the volume element containing x', a highly accurate height-based singularity cancellation quadrature rule is used [5]. The matrices (4) and (9) are constructed using 2-form or "face elements" for the basis functions W^2 and 1-form or "edge elements" for the basis functions W^1 , see [6] for details on the construction of the basis functions.

Our primary application for the discrete Biot-Savart law is providing boundary conditions for finite element solution of multi-conductor eddy current problems. In each conductor we solve the time-dependent vector diffusion equation using an edge element based A- ϕ finite element formulation [7]. Clearly the *B*-field in the air surrounding the conductors is critical. The finite element formulation requires that either $\hat{n} \times \vec{A}$ or $\hat{n} \times \vec{B}$ be specified on the conductor boundaries, corresponding to inhomogeneous Dirichlet or Neumann boundary conditions, respectively. Our approach for dealing with the B-field in the air surrounding the conductors is to use the discrete Biot-Savart law (3) or (8) as the boundary condition on each conducting surface.

3. Parallel Implementation

We assume that the volume Ω has been partitioned into K partitions, where K is the number of computational processors, with each partition having an equal number of volume elements. The volume elements are distributed via the partitioning. The surface Γ is also partitioned into K equally sized surface partitions. Note however that the surface elements are not distributed via the surface partitions, each processor can access the entire surface mesh. The Biot-Savart matrix is then decomposed into a $K \times K$ block matrix, with every block $Z^{pq}, p \in \{1:K\}, q \in \{1:K\}$ representing the interaction of surface partition Γ_p with volume Ω_q . The qth processor computes blocks Z^{pq} , p = 1 : K, i.e., a column of blocks. Note that the matrix is decomposed via a partitioning of elements, hence the matrices Z^{pq} are overlapping in DOF space. The specific partitioning algorithm used to partition the elements is not critical, in the examples below we employ a graph-based algorithm [8]. The key point is that if the partitions Γ_p and Ω_q are well-separated then the sub-matrix Z^{pq} will have a low-rank QR decomposition. The procedure for computing the low-rank QR decomposition is described below. We define "well-separated" as follows: the bounding spheres for the element partitions Γ_p and Ω_q are computed, if the bounding spheres do not intersect then the partitions are considered well-separated and a low-rank QR representation of Z^{pq} is computed. We employ a recursive procedure for computing Z^{pq} when partitions Γ_p and Ω_q are not well-separated. This results in a hierarchical representation for Z. If Γ_p and Ω_q are not well separated, Ω_q is divided into eight equally sized sub-partitions, Γ_p is divided into four equally sized

sub-partitions, and the "well-separated test" is applied to the sub-partitions Γ_{pi} and Ω_{qj} , i = 1 : 4, j = 1 : 8. A space-filling curve algorithm is used for creating the sub-partitions. The process is applied recursively, with a low-rank QR representation computed for well-separated sub-partitions. The recursion is halted when a volume sub-partition contains fewer than some number of elements, for application example 512. If at the lowest level of recursion the interaction is not well separated, it is simply represented by a dense matrix.

No parallel communication is required in the construction of the hierarchical Biot-Savart matrix, each processor has the elements that it needs to perform the integrals. Each processor has the same amount of work hence the computation of is load balanced. Note, however, that in the low-rank QR approximation the rank k is computed dynamically, and the rank k depends upon the geometry. Hence the application of the hierarchical Biot-Savart matrix, i. e., the matrix-vector multiplication $\bar{\beta} = \mathbf{Z}\bar{\xi}$, may not be perfectly load balanced. Also note that the application of the hierarchical Biot-Savart matrix does require parallel communication. This communication is as follows: (1) each processor q does a gather operation to get the values of $\bar{\xi}$ that it needs, (2) each processor q loops over the sub-matrices Z^{pq} , p = 1 : K and computes $\bar{\beta}_q = \mathbf{Z}^{pq}\bar{\xi}_q$, (3) each processor participates in a global reduction on $\bar{\beta}_q$.



Figure 1: Hierarchical partitioning of the Biot-Savart matrix. The highest level of partitioning is based on the number of processors, as represented by the left-most matrix. The sub-matrices Z^{pq} representing near interactions are hierarchically decomposed into 8 sub-volumes and 4 sub-surfaces, as illustrated by the rightmost matrix.

4. Low-rank QR Decomposition

When Γ_p and Ω_q are well separated the matrix Z^{pq} will have a low-rank representation

$$Z_{m \times n}^{pq} \approx Q_{m \times k} \times R_{k \times n},\tag{10}$$

where k is the rank. We do not want to form the entire Z^{pq} and then compress it, rather we sample the matrix by picking s rows and columns of Z^{pq} , where s is some predetermined number, e.g., 50. The procedure for picking the sampled rows and columns is ad-hoc, the procedure that we employ is described in [4]. The sampling procedure is solely linear algebra, it does not depend upon the particular Green's function, finite element basis functions, etc. For the ad-hoc sampling procedure to be effective we must have s greater than the expected rank. The algorithm for computing $Q_{m\times k}$ and $R_{k\times n}$ is as follows:

Step 1: Form the sampled column matrix $S_{m \times s}^c$ and the sampled row matrix $S_{s \times n}^r$.

Step 2: Compute the rank-revealing QR decomposition $\tilde{Q}_{m \times s} \tilde{R}_{s \times s} = S_{m \times s}^c$ using LAPACK routines DGEQPF and DORGQR. The rank k is determined by the criteria $\tilde{R}_{kk} < thresh \cdot \tilde{R}_{11}$ where thresh is a threshold value, we then keep only k columns of \tilde{Q} , denote this as $Q_{m \times k}$, and discard \tilde{R} .

Step 3: We form a new matrix $\hat{Q}_{s \times k}$ by taking s rows of $Q_{m \times k}$, the exact same rows as used to construct S^r . **Step 4:** Compute the least-squares solution to $\hat{Q}_{s \times k} R_{k \times n} = S^r_{s \times n}$ using LAPACK routine DGELSS.

At this point we have the desired matrices $Q_{m \times k}$ and $R_{k \times n}$ which approximate $Z_{m \times n}^{pq}$. The quality of the approximation, and the amount of compression (the rank k), are determined by the value of *thresh* used in Step

2 above. Our approach, being based on highly tuned LAPACK routines, is efficient both in terms of FLOPS and memory usage. The complexity is $O(m \cdot s) + O(s \cdot n)$, using a fixed value of s yields a linear complexity in m and n.

5. Examples

In these examples we compute a hierarchical low-rank QR approximation of the matrix defined by Eq.(9). For the first example consider the geometry shown in Fig. 2. This geometry consists of 19000 volume elements and is partitioned for 16 processors. Therefore the Biot-Savart matrix will be a 16×16 block matrix. Each block Z^{pq} has roughly 1200 rows and 4000 columns. Using values of s = 50 and thresh = 0.005 gives the parallel rank map shown in (11). The compression is significant, each 1200×4000 matrix is compressed to $Q_{1200 \times k} + R_{k \times 4000}$ where k is the value shown in (11). Note that the blocks labeled with rank 00 are near interactions and have full rank. These blocks were decomposed further as explained in Section 3 above. For example, the Z^{11} near-interaction matrix will be decomposed into 8 sub-volumes and 4 sub-surfaces, each resulting sub-matrix has roughly 270 rows and 560 columns. The resulting rank map for the Z^{11} sun-matrix is shown in (12). Again the blocks labeled with rank 00 are near-interactions and have full rank. In this specific case the sub-partitions have around 150 volume elements each, so they will not be partitioned further. The total compression was $60 \times$ for this specific example.

The second example is shown in Fig. 3. The geometry consists of three conducting coils, the center coil is driven with an independent current source, and we wish to compute the eddy currents in the coils due to the *B*-field in the surrounding air. The problem consists of 20736 volume elements and was partitioned for 24 parallel processors, therefore the Biot-Savart matrix is a 24×24 block matrix. The parallel rank map for this is too large to show here, but the results were as follow: Each processor had 24 matrices to compute at the highest level, on average 19 of these corresponded to well-separated regions and were compressed with an average rank of 10. The remaining 5 full-rank matrices were further partitioned into $4 \cdot 8 = 32$ sub-matrices, and on average 29 of these corresponded to well-separated regions and were compressed with an average rank of 25. At the lowest level of the hierarchy, the near interactions were represented, on average, by dense matrices of dimension 335×439 , there were a total of $24 \cdot 3 = 72$ of these. The total compression was $109 \times$. This compression represents both the memory savings and the reduction in CPU time required to apply the Biot-Savart interaction.

_															_	
00	14	16	00	12	10	5	8	5	4	12	20	12	7	7	7	
5	6	6	5	8	12	00	21	00	21	6	4	9	11	18	11	
19	00	21	00	00	17	7	12	11	7	00	12	24	12	10	10	
11	00	12	12	00	00	11	00	15	9	17	7	13	14	12	19	
6	13	11	9	16	00	14	00	00	12	6	12	16	14	23	5	
6	6	5	8	12	00	21	00	21	6	4	9	11	18	11	5	
9	7	7	10	34	00	00	00	15	8	5	9	12	18	17	21	
12	00	16	9	9	4	8	5	4	12	00	15	9	6	8	19	(11)
16	00	17	17	11	5	9	6	6	45	00	44	12	8	10	13	(11)
00	12	14	00	00	9	23	12	8	25	9	16	11	11	13	11	
21	21	12	24	11	8	12	12	9	00	12	00	00	12	29	10	
14	12	10	16	19	9	17	12	10	00	9	00	00	15	00	5	
8	5	$\overline{7}$	7	11	20	12	15	00	6	5	8	12	00	18	6	
9	11	8	10	18	9	22	14	16	13	6	12	00	45	00	5	
8	7	6	10	11	15	20	16	00	9	4	11	37	00	00	15	
38	00	16	00	16	11	$\overline{7}$	11	10	$\overline{7}$	00	15	00	14	12	11	
-															-	
				00	11	19	38	00	12	31	21					
				23	16	21	16	00	36	17	27					(19)
				26	00	35	00	14	23	43	33					(12)
				26	13	00	42	41	13	00	00					
				-												

The difference in compression for these two examples, 60x vs 109x, indicates that the amount of compression depends upon the number of elements, greater compression will be a achieved as the computational mesh is refined. If the computational mesh were refined, the well-separated interactions would still have the same rank, hence the cost of the well-separated interactions is O(N). Each near-interaction is recursively decomposed



Figure 2: Computational mesh for a linear induction motor partitioned for 16 parallel processors.



Figure 3: Computational mesh for an inductive coupling application partitioned for 24 parallel processors.

into $4 \cdot 8 = 32$ interactions, most of which are again well-separated and have low-rank. The dominant cost is the nearinteractions which are represented as dense $m \times n$ matrices, where m and n are determined by fixed parameters (e.g., the recursion halting parameter of 512 elements) which are independent of the global dimensions M and N. The number of near interactions is, asymptotically, $O(N \log(N))$, hence the overall method is $O(N \log(N))$.

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Full Wave Analysis of RF Signal Attenuation in a Lossy Rough Surface Cave Using a High Order Time Domain Vector Finite Element Method

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Abstract—We present a computational study of signal propagation and attenuation of a 200 MHz planar loop antenna in a cave environment. The cave is modeled as a cylindrical guiding structure with a lossy wall. The wall is nominally circular with a random roughness. To simulate a broad frequency band, the full wave Maxwell equations are solved directly in the time domain via a high order vector finite element discretization using the massively parallel CEM code EMSolve. The numerical technique is first verified against theoretical results for a planar loop antenna in a smooth lossy cave. The simulation is then performed for a series of random rough surface meshes in order to generate statistical data for the propagation and attenuation properties of the antenna in a cave environment. Results for the mean and variance of the power spectral density of the electric field are presented and discussed.

1. Introduction

The study of electromagnetic wave propagation in caves and tunnels is of great practical interest to antenna engineers due to the increasing demands for reliable wireless communications systems in such environments. Current wireless radio frequency (RF) communication systems were not designed to operate reliably in enclosed environments such as caves and tunnels, and signal quality is severely compromised due to the rough and lossy surfaces of the cave. Today there is limited ability to maintain communications in cave-like structures, tunnels or subways, prohibiting the quick deployment of wireless systems in caves and tunnels. If the propagation properties of the tunnel could be better characterized (dissipation, dispersion, fading, and channel capacity), then a more robust communication system could be designed specifically for operation in such environments, hence full wave EM simulations of propagation in this type of environment are very useful.

Much theoretical work in this field has been done in order to develop a better understanding of the RF propagation channel. Dudley recently studied models for propagation in lossy circular tunnels [1]. He produced expressions for the fields in terms of a Fourier transform over the axial variables, and presented the numerical results for the field intensity both as a function of axial distance and as a function of radial distance. However, this work only involved smooth tunnel walls and not the more realistic situation of rough wall tunnels. In this case, the electromagnetic fields can be modeled as a stochastic process in a cave with random rough walls. Recently, Pao and Casey have investigated the statistical properties of wave propagation in straight, rough-walled tunnels [2,3]. This work assumes a perfect electrical conductor (PEC) boundary at the rough wall/air interface. A more realistic model needs to take into account the lossy nature of the rough walls and the cave material (typically granite or some sort of earth like material with electrical conductivities on the order of $0.1 \, \text{S/m}$).

In this paper we use a high order finite element discretization to solve the full wave Maxwell equations directly in the time domain for the case of a planar loop antenna placed at the mouth of a straight, lossy rough walled tunnel. We chose a time domain simulation in order to efficiently compute the response over a broad frequency band. We begin with a brief description of the numerical method employed for this problem (as implemented in the EMSolve code [4]). We then verify the numerical method against the theoretical results of Dudley [1] for a planar loop antenna in a smooth lossy cave and discuss the limitations of the numerical model. Finally, we proceed to solve the electromagnetic wave equation on a sequence of randomly generated meshes to determine statistical properties for the power spectral density of the electric field.

2. Numerical Formulation

We begin with the second order time dependent wave equation for the electric field in a 3 dimensional domain Ω

$$\epsilon \frac{\partial^2}{\partial t^2} \mathbf{E} = -\nabla \times (\mu^{-1} \nabla \times \mathbf{E}) - \sigma \frac{\partial}{\partial t} \mathbf{E} - \frac{\partial}{\partial t} \mathbf{J} \quad \text{in } \Omega$$
$$\nabla \cdot (\epsilon \mathbf{E}) = 0 \qquad \qquad \text{in } \Omega$$
$$\hat{\mathbf{n}} \times \mathbf{E} = \mathbf{E}_{bc} \qquad \qquad \text{on } \partial \Omega \qquad (1)$$

where $\partial\Omega$ is the two dimensional boundary of the domain, $\hat{\boldsymbol{n}}$ is the outwardly directed unit normal of this boundary and **J** is a free current density source that can be added to drive the problem. The value \mathbf{E}_{bc} represents an arbitrary boundary condition imposed on the electric field intensity while ϵ , μ and σ denote, respectively, the dielectric permittivity, magnetic permeability and electrical conductivity of the materials contained in the domain Ω .

Applying an arbitrary order Galerkin finite element discretization to the wave equation (the details of which can be found in [5, 6]) yields the following semi-discrete system of ordinary differential equations

$$M_{\epsilon} \frac{\partial^2}{\partial t^2} e = -S_{\mu} e - M_{\sigma} \frac{\partial}{\partial t} e - \frac{\partial}{\partial t} j$$
⁽²⁾

where M_{ϵ} , M_{σ} are finite element mass matrices, S_{μ} is a finite element stiffness matrix and e, j are discrete arrays of finite element degrees of freedom. Applying a backward difference approximation for the first order time derivative in (2) and a central difference approximation for the second order time derivative yields the following fully discrete linear system of equations

$$M_{\epsilon}e_{n+1} = (2M_{\epsilon} - \Delta t^2 S_{\mu} - \Delta t M_{\sigma})e_n + (\Delta t M_{\sigma} - M_{\epsilon})e_{n-1} - j'$$
(3)

where Δt is the discrete time step, the integer *n* denotes the current time step and the time derivative of the free current source has been directly incorporated into the new source term *j*'.

For the results shown in section 4 below we employ second-order interpolatory H(curl) basis functions along with custom quadrature rules that yield a diagonal "mass" matrix M_{ϵ} . The details of this discretization are presented in [7]. This method is much more accurate than standard FDTD. Indeed, the numerical dispersion for this method is $O(h^4)$ rather than $O(h^2)$ as it is for FDTD. Compared to higher-order FDTD schemes, this method is better at modeling the jump discontinuity of fields across the air-earth interface.

3. Verification

Before we proceed to simulations of a random rough surface, we begin by verifying our numerical method with known theoretical values for the case of a smooth, lossy tunnel. These results were computed by Dudley according to the procedures described in [1]. We consider the case of an axially symmetric circular current loop of radius b = 0.2 m placed at the mouth of a circular tunnel of radius a = 2.0 m. The current loop is driven by a time harmonic source of frequency f = 200 MHz. The electric field is "measured" along the length of the tunnel at a radial observation point ρ such that $\rho/a = 0.3$. The tunnel has a relative dielectric constant of $\epsilon_r = 5.0$. We consider two cases, a tunnel with an electric conductivity $\sigma_{low} = 0.02$ S/m and $\sigma_{high} = 0.1$ S/m.

For the numerical model, we discretize the tunnel domain in two different ways using both a Cartesian (or "stair-step") approximation to the smooth tunnel wall and a more accurate conforming cylindrical mesh (see Figure 1 and Figure 2). For both cases, a planar loop of current of radius 0.2 m is placed at one end of the tunnel, while a simple absorbing boundary condition (ABC) is placed at the other. The ABC is imperfect for anything other than plane waves at normal incidence, hence we make our tunnel mesh 75 m long and ignore field data from the last 20% of the tunnel mesh. A perfect electric conductor (PEC) boundary condition is applied at the cross-sectional limits of the problem space to fully define the problem. For both the Cartesian and Cylindrical meshes, the outermost PEC boundary is made sufficiently large to prevent spurious reflections. The temporal dependence of the current source is a Gaussian pulsed sine wave centered at 200 MHz with a 20% bandwidth. The simulation is performed using high order p = 2 basis functions to mitigate the effects of numerical dispersion.

In Figure 3 and Figure 4 we compare results for both numerical models (Cartesian and cylindrical meshes) to the theoretical results for both conductivity values. Note that the agreement between the theoretical model and the numerical model using the conforming cylindrical mesh are excellent, indicating that the proposed numerical method is working properly. The discrepancies between the Cartesian results and the theoretical

results are due to the "stair-step" approximation to smooth surfaces, which is known to be problematic since such approximations fail to converge to a true cylindrical surface. While the Cartesian mesh is a bad choice for modeling smooth tunnels, it is sufficient for modeling rough surface tunnels, which we will use in the next section.



Figure 1: Cross section of smooth tunnel Cartesian mesh.



Figure 3: Comparison between theoretical model and two different numerical models at 200 MHz for a smooth cave with conductivity $\sigma_{low} = 0.02 \text{ S/m}$.



Figure 5: Example of randomly generated cave mesh with interior removed (close-up view).



Figure 2: Cross section of smooth tunnel conforming cylindrical mesh.



Figure 4: Comparison between theoretical model and two different numerical models at 200 MHz for a smooth cave with conductivity $\sigma_{high} = 0.1 \text{ S/m}$.



Figure 6: Snapshot of computed electric field magnitude at t = 89.6 ps.

4. Computational Results

We now proceed to apply the same process discussed above to the more complicated case of random rough walled caves. The random rough surface is generated as follows. First, we generate a cylindrical surface of radius 2 m and length 75 m. Next, we add a random perturbation with zero mean and a standard deviation 0.28867 m. Then, we smooth the random surface (low pass filter) to introduce a surface correlation of a given length. Finally, we generate a 3D Cartesian mesh, where the electrical conductivity of each element depends upon whether the element is inside the random surface (air) or outside the random surface (earth). For mesh elements that straddle the random surface, a volume-fraction is used to determine the electrical conductivity with values ranging between $\sigma_{low} = 0.02 \text{ S/m}$ and $\sigma_{high} = 0.1 \text{ S/m}$. To model the dielectric properties of the earth, a constant dielectric permittivity of 5 times the free space permittivity ϵ_0 (a typical value for granite) is used. Each computational mesh consists of 583, 200 hexahedral elements, an example of which is shown in Figure 5. Note that the portion of the mesh representing the air has been removed to illustrate the random rough surface. The various simulation parameters for the random rough surface computations are summarized in Table 1.

Table 1:	Summary	of	$\operatorname{computational}$	$\operatorname{statistics}$	for	10	random	cave	simulations.	

Avg. Cave Radius	$2 m (1.33333 \lambda)$
Cave Length	$75\mathrm{m}(50\lambda)$
Element Size (Δx)	$0.167\mathrm{m}(0.13111\lambda)$
Max Deviation of Surface Roughness	$0.5\mathrm{m}(0.33333\lambda)$
Standard Deviation of Surface Roughness	$0.288675\mathrm{m}(0.19245\lambda)$
Signal Type	Modulated Gaussian pulse, planar loop antenna
Pulse Frequency	$200\mathrm{MHz},20\%$ Bandwidth
Gaussian Width, Delay	4.67e-9s, 2.50e-8s
No. of Trials	10
No. Unknowns per Trial	~ 14 million
No. Parallel CPU's per Trial	192





Figure 7: Mean power for 10 random cave simulations at 5 different frequencies.

Figure 8: Variance of power for 10 random cave simulations at 5 different frequencies.

A total of 10 random caves were simulated. A time history was recorded at each x, for all time steps. This data was used to find the spectrum at every spatial step. For each simulation the mean and variance of the power spectral density (PSD) and phase were extracted over the bandwidth of the signal. Each run was normalized by dividing by the total PSD magnitude at the first x-data point, thereby removing the characteristics of the input signal, but preserving the relative magnitudes vs. polarization. The last 20% of the spatial samples were

removed to avoid reflections from the end of the cave (due to the imperfect nature of the ABC). The results for computed mean power at 5 different frequencies are shown in Figure 7. Note that in general, the scattered field from the rough surface walls fills in nulls which are created by destructive interference in the smooth case (see Figure 3 and Figure 4 for comparison). In addition, note that the lower frequencies are attenuated more rapidly than the higher frequencies as expected. In Figure 8 we plot the computed variance in power across all 10 simulations. In Figure 9 we compare the smooth cave results (conforming cylindrical mesh, σ_{low}) to random rough wall results at 200 MHz.



Figure 9: Comparison between smooth wall and mean rough wall results at 200 MHz.

5. Conclusions

We have applied the high order time domain vector finite element methods described in [5–7] to the case of RF electric field propagation in a lossy rough wall tunnel. This particular calculation has proved difficult to solve using direct theoretical analysis. We have verified our numerical results by direct comparison to a theoretical model for propagation in a smooth lossy cave. We have presented statistical data for the power spectral density of a 200 MHz planar loop antenna and have compared our data in a rough walled cave to one with a smooth surface. Further work will allow direct time domain modeling of more complicated cave structures with bends and forks that are too complex for theoretical techniques.

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A Matlab-Based Virtual Propagation Tool: Surface Wave Mixed-path Calculator

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Abstract—A new Matlab-Based, user-friendly virtual propagation tool (VPT) that can be used for multi-mixed path surface wave path loss calculations has been designed. Any multi-mixed-path surface wave propagation scenario may be specified by the user together with all the necessary input parameters, and path loss vs. range plots may be produced. The effects of multi-mixed paths, electrical parameters of each propagation section, as well as the frequency can be observed and extra path losses can be predicted. The VPT can be used both for design and training purposes.

1. Introduction

In addition to decades of long-range marine communication systems in high frequency (HF) band surface wave high frequency radars (HFSWR) have become a great potential in this frequency region for integrated maritime surveillance systems (IMSS) both as primary and complementary sensors. Countries with wide-coastal regions such as USA, Canada, France, Germany, Italy, Brazil, Turkey, Sri Lanka, China, India, etc., have already deployed or completed the designs of such systems for their economic exclusive zones (EEZ) [1,2]. One major problem in HF communication/radar systems is the prediction of surface wave propagation path loss. The propagation scenarios differ quite a lot from region to region. For example, engineers of the IMSS on the East Coast of Canada need to know maximum monitoring range for a given transmitter power. On the other hand, the problem of Turkey in the West Coast is to find out extra multi-mixed path propagation loss because of the existence of many different scaled islands in the region.

At HF frequencies, ground wave propagation is dominated by the surface wave. As long as the transmitter and receiver are close to surface direct and ground reflected waves cancel each other and only surface wave can propagate. The Earth's surface electrical parameters are important in reaching longer ranges. Sea surface is a good conductor, but ground is a poor conductor at these frequencies. A challenging problem is to predict surface wave path loss variations over mixed paths, such as sea-land or sea-land-sea transitions [3,4]. A sharp decrease occurs in signal strength along sea-land transition and the signal recovers itself beyond the island, known as the Millington (recovery) effect [5].

We have introduced a few propagation packages for the calculation of surface wave propagation effects [6–10], where analytical ray and mode models (i. e., Norton and Wait formulations) are hybridized to extend their ranges of validity, accuracy, rate of convergence, etc., depending on such problem parameters as operational frequencies, source/observer locations and the physical propagation environment. The WAVEPROB packages uses analytical ray and mode methods in hybrid form that can handle propagation through standard atmosphere over smooth spherical Earth and can be best used from a few hundred kHz up to 40–50 MHz [7]. The ray shooting algorithm SNELL_GUI [8] shoots a number of rays through a propagation medium characterized by various piecewise linear vertical refractivity profiles, so the user may visualize various ducting and anti-ducting characteristics depending on the supplied parameters. The packages RAY_GUI and HYBRID_GUI [9] can be used to investigate ray/mode formulation inside a 2D non-penetrable parallel plate waveguide. The user may analyze individual ray/mode contributions and their collective effects as well as hybrid forms. Finally, the multipurpose SSPE_GUI package completes the virtual set, which can be used directly in simulations of short- and long-range radiowave propagation over user-specified, non-smooth Earth's surface through non-homogeneous atmosphere [10].

In this study, we have developed and designed a new Matlab-Based, user-friendly virtual propagation tool (VPT) that can be used for multi-mixed path surface wave path loss calculations. The user may design a propagation scenario by just using the computer mouse, specify all other input parameters, and produce path loss vs. range plots. The effects of multi-mixed paths, electrical parameters of each propagation section, as well as the frequency can be observed and extra path losses can be predicted. The VPT can be used both for design and training purposes.

2. Analytical Formulation Based on Ray-mode Approach

The fundamental analytic models are based on ray and mode techniques and are mostly known as Norton [3] and Wait [4] formulations, respectively. The Norton formulation extracts a ray-optical asymptotic approximation from a wavenumber spectral integral representation. The Wait formulation restructures the spectral integral as a series of normal modes propagating along the earth's surface. They both assume a smooth spherical earth (and/or its earth-flattened approximate equivalent) with various smooth, penetrable ground characteristics, a radially homogeneous atmosphere above, and excitation by a vertical or a horizontal electric dipole on or above the earth's surface.

Norton and Wait formulations parameterize the propagation process in terms of different phenomenological models, their ranges of validity, accuracy, rate of convergence, etc., depending on such problem parameters as operational frequencies, source/observer locations and the physical propagation environment, differ as well, with particular impact on computations. Using the ray-mode approaches separately or in hybrid form, one may deal with smooth-boundary problems [11], such as

- Surface wave path loss or field strength variation with respect to range (especially beyond the horizon and when both transmitter and receiver are on the surface).
- Range and/or height propagation variations in interference regions (i.e., when transmitter and receiver are above the surface and within the line-of-sight (LOS)).
- Surface wave path loss over multi-mixed propagation paths to account for, for example, land-sea or sealand-sea (island) transitions.

It should be noted that ray-mode and their hybridized techniques cannot handle problems, such as propagation over rough surface terrain, and/or through surface and/or elevated ducts formed by inhomogeneous vertical as well as horizontal atmospheric conditions. Although height gain functions in mode theory [11] can be used to account for transmitter/receiver heights, it is difficult to deal with receiver heights in diffraction regions (beyond LOS) because of numerical problems in calculating higher order terms in the series representation of Airy functions.

3. Millington Effect and ITU Curve Fitting Method

Although perfectly reflecting boundary assumption provides in general sufficient approximation at VHF and above (i. e., frequencies higher than 100–200 MHz), the use of impedance boundary condition becomes essential at HF frequencies and below. This is especially required for the simulation of long-range marine communication and/or ocean surveillance systems using HF frequencies. A challenging problem is to predict surface wave path loss variations over mixed paths, such as sea-land or sea-land-sea (island) transitions. A sharp decrease occurs in signal strength along sea-land transition and the signal recovers itself after land-sea transition (beyond the island), known as the Millington (recovery) effect [5].

The path loss of a communication system between any pair of transmitter/receiver is defined as

$$L_p(d) = 10 \log\left(\frac{P_r}{P_t}\right) \tag{1}$$

For a $P_t = 1 \,\text{kW}$ transmitter (i.e., for a short electric dipole with a dipole moment of $M = 5\lambda/2\pi$), the received power at an arc distance d can be determined from the computed field strength E via

$$P_r(d) = \frac{E_r(d)^2}{Z_0} \times \frac{\lambda^2}{4\pi}$$
(2)

The path loss is then obtained from these two equations as

$$L_r(d) = 142.0 + 20\log(f_{MHz}) + 20\log(E_{\mu V/m}) \quad [dB]$$
(3)

where the units of the operating frequency and field strength are MHz and $dB\mu V/m$, respectively.

The Millington method uses a graphical interpolation approach to calculate the mixed path losses. Figure 1 shows a multi-mixed propagation medium including 5-paths. Let's consider a scenario for 2-paths with different surface parameters. The Millington method is based on an interpolation of a direct electric field E_D and an



Figure 1: Multi-mixed-path surface wave propagation scenario for 5 paths.

inverse electric field E_I as:

$$E_D = E_1(d_1) + E_2(d_1 + d_2)E_2(d_1)$$
(4)

$$E_I = E_2(d_2) + E_1(d_1 + d_2)E_1(d_2)$$
(5)

Here, the field values $E_1(d_1)$, $E_2(d_2)$, $E_2(d_2)$, $E_1(d_1 + d_2)$, and $E_2(d_1 + d_2)$ are defined as follows:

$E_1(d_1)$:	Calculated field strength at a distance d_1 over homogeneous Med. I
$E_1(d_2)$:	Calculated field strength at a distance d_2 over homogeneous Med. I
$E_2(d_2):$	Calculated field strength at a distance d_2 over homogeneous Med. II
$E_1(d_1+d_2):$	Calculated field strength at a distance $d_1 + d_2$ over homogeneous Med. I
	(the whole path is assumed as Med. I)
$E_2(d_1+d_2):$	Calculated field strength at a distance $d_1 + d_2$ over homogeneous Med. II
	(the whole path is assumed as Med. II)

Then the total electric field is calculated by taking the average as

$$E(d_1 + d_2) = 0.5(E_D + E_I).$$
(6)

The Millington method can be used for 3-paths in a similar way. If the path lengths are d_1 , d_2 and d_3 , respectively, the direct electric field E_D and the inverse electric field E_I are calculated via

$$E_D = E_1(d_1) + E_2(d_1 + d_2)E_2(d_1) + E_3(d_1 + d_2 + d_3)E_3(d_1 + d_2)$$
(7)

$$E_I = E_3(d_3) + E_2(d_3 + d_2)E_2(d_3) + E_1(d_3 + d_2 + d_1)E_1(d_3 + d_2)$$
(8)

and the total electric field is calculated again by taking the average as

$$E(d_1 + d_2 + d_3) = 0.5(E_D + E_I) \tag{9}$$

The extension to n-path formulas is straightforward.

4. Matlab-Based HF_PATH Package

The front panel of the HF_Path package is designed as shown in Figure 2, and is divided into three sub regions. The left part of the GUI is reserved for the user-supplied parameters. The user specified parameters are explained in Table 1. The operating frequency, range increment, transmitter height and receiver height are supplied first. Then the electrical parameters; the conductivity and the relative permittivity of the sea and land are to be specified next ($\mu = \mu_0$ everywhere and the atmosphere is homogeneous). The parameters of all sea paths (or land paths) are assumed same. Although it is doable, the package doesn't allow the user to specify *N*-path with *N*-different electrical surfaces. Finally the user specifies the number of paths along the range using a popup menu. Once the user determines the number of paths *N*, only *N* editable textboxes become visible to enable the user to specify the lengths of the paths. For example, in Figure 2 the number of paths is 6, so there are 6 visible textboxes. However in Figure 3, the number of paths is 3, so there are only 3 visible textboxes. It is also important to note that the first segment is always sea, and that a sea segment is always followed by a land segment and vice versa. The mid-part of the front panel is reserved for the figures. The upper figure
shows the geometry of the scenario and changes whenever the number of paths is changed by the user via the corresponding popup menu. The lower figure displays both the geometry and plots for Path Loss vs. Range or Field Strength vs Range variations. The sea and land segments are shown in blue and green, respectively, as shown in Figure 2. The lengths of the blue and green filled areas correspond to actual lengths specified by the user.



Figure 2: The front panel of HF_PATH package.

Parameter	Explanation	Default Value
Frequency	Operating Frequency	$5\mathrm{MHz}$
Range Increment	The difference between each observation point	0.5 km
Transmitter Height	Height of the Transmitter in [m]	0 m
Receiver Height	Height of the Transmitter in [m]	0 m
Conductivity of Sea	Conductivity of each sea segment in [S/m]	5 S/m
Conductivity of Land	Conductivity of each segment land in [S/m]	0.01 S/m
Relative Permittivity of Sea	Relative permittivity of each sea segment	70
Relative Permittivity of Land	Relative permittivity of each land segment	15
Number of Paths	Number of sea and land segments between the	3
	transmitter and the receiver $(\min : 1; \max; 6)$	
Length of Path 1	Length of the first segment (sea) in [km]	100 km
Length of Path 2	Length of the second segment (land) in [km]	100 km
Length of Path 3	Length of the first segment (sea) in [km]	100 km
:	:	:
:	:	:

Table 1: User-specified parameters of the HF_Path package.

The control push buttons are located at the upper right part of the panel. Pressing the "Info" button opens the MATLAB Help window that includes explanations on how to use the package. Typing "help HF_Path" at the MATLAB command line also displays the same explanations. The "Close" button terminates the program. The "Clear" button clears the graph. Once the "Plot" button is pressed, the user-specified parameters are written line by line to an input file named "HFMIX.INP", then the program HFMIX.EXE is executed and the outputs are both displayed in the figure and written to files "LMIX.DAT" and "EMIX.DAT". Both files consist of 2-columns of data in text format. The first column belongs to the range values in km and the second column of LMIX/EMIX corresponds to Path Losses/Field Strengths in dB. The check boxes below the Plot button are used to select whether to plot the Path Loss vs. Range or Field Strength vs. Range. Operational parameters may be changed by the user and multi-plots may be displayed by pressing the plot button more than once (as long as the propagation scenario, i.e., the segment lengths, is kept same). The user may clear previous plots by using the "Clear button" before the "Plot button".



Figure 3: Path loss vs. range for a 3-section-path propagation scenario at 0.5 MHz, 5 MHz and 10 MHz.



Figure 4: Path loss vs. range for a 4-section-path propagation scenario at 5 MHz, 15 MHz, and 30 MHz.

5. Matlab-Based HF_PATH Package

To show the power and beauty of the HF_PATH package some examples and typical results are presented in this section. The first case belongs to a propagation scenario consisting 3-paths and is displayed in Figure 3. The lengths of the segments are: $d_1(\text{sea}) = 120 \text{ km}$, $d_2(\text{land}) = 80 \text{ km}$, $d_3(\text{sea}) = 200 \text{ km}$, which makes the total range from the transmitter to the receiver 400 km. The conductivity of sea/land are specified as 5/0.01 S/m. The relative permittivities are 70/15. The height of the transmitter and the receiver are both chosen as 0 m. The calculations are performed for three different operating frequencies; 0.5 MHz, 5 MHz, and 15 MHz. All three Path Loss vs. Range graphs corresponding to these frequency values are displayed in the figure with different colors. The Millington effect is observed at MHz frequencies and above as shown in the figure. The sharp decrease on the sea-land transition region, and signal recovery beyond the land-sea transition are also visible in the figure. It should be noted that, the higher the frequency the higher the path loss at the same distance.

The next example consists of a 4-segment propagation path and results are shown in Figure 4. In this example the segment lengths are same and are equal to 100 km. The range variations of path losses at three different operating frequencies are plotted in the figure.

The third example is shown in Figure 5 for a 5-segment-path. The example corresponds to a propagation scenario with 2 islands with lengths of 32 km and 58 km at radial distances 98 km and 274 km from the source. The Path Loss vs. Range graphs correspond to frequency values of 5 MHz, 15 MHz and 30 MHz.

The last example is another 3-section-path propagation scenario as shown in Figure 6. The plots correspond to different types of lands with conductivity values of $\sigma_{LAND} = 0.001 \text{ S/m}, 0.1 \text{ S/m}, 1 \text{ S/m}.$



Figure 5: Path loss vs. range for a 5-section-path propagation scenario at 5 MHz, 15 MHz, and 30 MHz.



Figure 6: Path loss vs. range for a 3-section-path propagation scenario at $\sigma_{LAND} = 0.001, 0.1, 1 \text{ S/m}.$

6. Conclusions

The new Matlab-Based, user-friendly HF_PATH virtual propagation tool can be used for multi-mixed path surface wave path loss calculations. Any multi-mixed-path surface wave propagation scenario may be specified by the user together. The electrical parameters of the propagation segments and the operating frequency are also user-specified parameters. The effects of multi-mixed paths, electrical parameters of each propagation section, and the frequency on to the range variation of path loss can be simulated easily. The HF_PATH can be used for both design and training purposes.

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Optical Binding in Air

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Abstract—Optical binding between micron-sized oil droplets in air has recently been observed. The experimental setup, consisting in two vertical, counter propagating and diverging laser beams, builds up a three dimensional trap. The cloud of oil droplets, enclosed in a glass cell, progressively fills in the trap where droplets interact one with another. Scattered intensity is observed on a video camera. Interactions involve optical, electrostatic, radiometric and capillary forces. Orders of magnitude are discussed.

Chains up to four droplets have been observed, the most stable structure being the doublet and not the single drop. In air, viscosity being one thousand times smaller than in water, mean free path of a micro-sphere is much bigger. That is why mean residence times in metastable states are of the magnitude of a few seconds and that brownian motion quickly drives the trapped droplets in the very minimum of potential energy: the doublet structure. Two stable states have also been obtained for the doublet. Observation of interference indicates that oil droplets are phase-locked onto each other every $\lambda/2$.

The spraying technique we use, gives droplets smaller than the micron in radius. This is the intermediate case of the Mie range between the small and large wavelength cases. Those new experimental results exhibit the role of the short and long range interactions in optical binding. They are then theoretically discussed both in the ray model and in the Rayleigh approximation, and compared with previous works on optical binding in water. Moreover, in our case, the index contrast is much bigger. It implies stronger scattered intensities, bigger interaction forces with light and therefore, bigger binding forces.

1. Introduction

Since the pioneering work by Ashkin [1] in the early 1970's, optical tweezers have nowadays become a commonly used tool for micromanipulation in water. Optical trapping in air and vacuum remains a difficult task due to Van der Waals forces several orders of magnitude larger than optical forces. In the literature, two possibilities were explored: the use of aerosols[1–4] and mechanical vibration coupled with strongly focused cw laser beams [5, 16, 7, 8]. Afterwards, self-assembled structures of microparticles under strong laser illumination have been demonstrated [9–12]. Optical binding was observed when the particle separation is either othogonal or along the light propagation. When the separation is set orthogonal to the beam propagation and to beam polarization, theory predicts potential minima every l for particles in the Rayleigh range [13]. This l periodicity was experimentally observed for polystyrene spheres in the Mie range in water [10]. The Mie correction to Rayleigh approximation was supposed to modify the interaction strength more than the periodicity. In three dimensional optical traps made with two counter-propagating beams, potential minima appeared to be along the beam axis. Due to the weakly focused beams and to gradient forces, the particles are constrained to remain on the beam axis. Optical interactions then lead to chains where spheres are either stuck or separated by more than a diameter away [12]. For spheres in the Rayleigh range, potential minima every $\lambda/2$ are expected.

Trapping in air imposes a tridimensional trap since the Van der Waals forces are not negligible as is the case in water. However, the larger index ratio gives larger cross sections and the optical forces are consequently stronger than in water.

2. Experiment

Our experiment [15] deals with micron-sized oil-droplets in air obtained with a spray nozzle. According to their falling time, their diameter was estimated to be in the range between $1 \,\mu\text{m}$ and $1.5 \,\mu\text{m}$. They are protected from air convection currents by a glass cell. We use a 30 mW frequency doubled YAG laser at 532 nm. The optical trap consists in two weakly focused (N.A. = 1/15) and counter-propagating laser beams (see Fig. 1). The return beam is focused roughly $300 \mu m$ before the forward beam. The equilibrium position of trapped particles is at half distance of both focusing points, where the intensities of the two beams are equal. The geometry is then similar to those previously studied in water [11, 12] with optical fibers. We chose a vertical geometry in order to oppose gravity with the scattering force rather than with the gradient force which is much weaker for spheres in the Mie range.

3. Results

3.1. Trapped Structures

When the cell is filled with an oil droplet cloud, radiation pressure pushes the droplets inside the trap. By far, the most common structure observed was a doublet. We rarely saw a single droplet. Three and four



Figure 1: Experiment principle. The scattering force is opposed to gravity. The forward beam is retro-reflected on a mirror at the focus of a lens. The downward returning beam is focused $300 \,\mu\text{m}$ above the upward direct beam so as to build a stable equilibrium zone. The laterally scattered intensity is observed on a video camera through a 10x, 0.25NA microscope lens. A cell of glass, not shown, protects from air motion.

droplet chains were also seen for a few seconds before changing into a doublet by escaping or merging processes. Coallescence with outer droplets—tends to increase progressively the size of both droplets in a doublet. The increasing finishes when the cloud of droplet has fallen down. We observed that in a doublet, the larger the droplets, the further apart. We never saw a doublet collapsing or splitting away. The optical binding forces in this case, appears to be much stronger than all other forces.





On a few occasions, we observed a sudden change of the doublet's appearance, mainly regarding the interference of both Airy patterns on the camera. On the first picture (see Fig. 3), images of droplets interfere such as to give a dark fringe between droplets while on the second picture, we see bright dots on the symmetry axis. In the first case, droplets are scattering in phase opposition as in the second case, they are emitting in phase, corresponding to a $\lambda/2$ difference of distance. An estimate of the difference of distance separations of doublets gave values close to $\lambda/2$. This measurement is difficult due to the low resolution power of the imaging microscope objective. This phenomenon was assumed to be a swiching between two stable states of a doublet, which can be understood in the dipolar approximation as explained further.

3.2. Clinging to Fringes

When trapped, particles move quikly due to speckle. Static speckle is introduced with dirtyness of optical components. Dynamic speckle is also introduced by the cloud of droplets crossing the trapping beams. Disturbance caused by the cloud is larger when a droplet cross the beam in the vicinity of the focusing point.

The laser we use has two longitudinal modes. There are coherence beatings every three millimeters. After a 30 cm optical path, we do not know if the return beam is coherent with the upward one. In the case where



Figure 3: Two different equilibrium states of a $2.3 \,\mu$ m doublet. The main difference is the interference pattern. On the first picture, droplets are emitting in phase opposition, we see a dark fringe on the symmetry axis. On the second one, they are emitting in phase, bright dots can be seen on the symmetry axis. It suggests that the difference of separation distance between droplets increased. We estimated the increasing to be of the order of half a wavelength in accordance with the interference pattern.



Figure 4: Position spectrum of the doublet in the trap.

the two counter-propagating beams are coherent, a stable $\lambda/2$ fringe pattern should trap the structure. If the brightness contrast is not large enough, the gradient force is weak and small intensity fluctuations between the two counter propagating beams can unbalance the well's minimum in the longitudinal direction. In this case, the doublet quickly (compared with the frame rate of the video camera) sweeps a sinusoidal potential well. As images of droplets are several pixels wide, their positions can be measured with a subpixel resolution. We performed the Fourier transform of the positions y_p of the doublet in the trap when counter-propagating beams were circularly polarized and weakly coherent (see Fig. 4):

$$\mathcal{F}\left(Y\right)\left(\sigma\right) = \sum_{n} e^{2\pi i \sigma y_{p}}$$

The same numerical calculation for the case of a crossed-polarized beams experiment does not give those peaks. According to imaging power $\lambda/2 \simeq 1.6 pixel$, the main harmonic is then the 0.62 peak. The other peaks are folded back harmonics. We can see that they are numerous and Dirac comb like, which means that the doublet very likely mechanically clings to fringes and the position spectrum we obtain cannot be a light modulation measurement artefact.

3.3. Theoretical Discussion on Binding

In our experiment, the spheres radii are from $0.5 \,\mu\text{m}$ up to $1.2 \,\mu\text{m}$ when several droplets have merged. Those values correspond to ka (a being its radius and k the wave vector) between 12 and 14. For particles in the Mie regime which is the case, numerical calculations have to be performed and multiscattering processes must be taken into account to know the exact optical binding forces [14]. However, in this regime, the particles' behaviour looks like both dipoles and large spheres. In this discussion, we aim at giving a flavour of the physics

of the binding effect. We think this experiment can be approximately understood from the two extreme regimes of the ray model and the Rayleigh range.

When spheres are such that ka < 1 or when kr > 1 (r being the separation distance between spheres), the dipole approximation is sufficient to estimate potential minima. In this approximation, optical interactions between particles are maximal when the separation is orthogonal to polarization. It can be either along or transverse to the beam axis. In agreement with dipolar theory, it was experimentally observed [10] that potential wells for two particles were every λ , the wavelength. When the separation is along the wave vector, a similar calculation predicts potential wells roughly every $\lambda/2$:

$$V = -\left|\frac{\cos\varphi + f(kr)e^{ikr}\cos(kr+\varphi)}{1 - f(kr)^2 e^{2ikr}}\right|^2$$
$$f(kr) = k^3 \alpha \left(\frac{1}{kr} - \frac{1}{(kr)^3} + \frac{i}{(kr)^2}\right)$$

with

 $\alpha = \frac{n^2 - 1}{n^2 + 2}a^3$ being the polarizability of the (identical) dielectric spheres of index *n*, and *r* being the separation between the two spheres. The denominator corresponds to the Mossotti resonance in atom trapping. It can only be zero for resonant particles for which the real part of *f* can be larger than one. This can never happen with dielectric or even metallic particles: the spheres touch before the resonance happens. When kr > 1, we can approximate the previous formula by:

$$V \simeq -\cos\varphi \left(\cos\varphi \left(1 + \frac{k^3\alpha}{kr}\right) + \frac{k^3\alpha}{kr}\cos(2kr + \varphi)\right)$$

which exhibit a $\lambda/2$ periodicity which is consistent with the experimental observation (Fig. 3).

However we could not see jumps between many $\lambda/2$ -separated potential wells like in Fournier's experiment [10]. The two droplets remain at a quite stable distance depending on their size. This comportement looks like that of large spheres.



Figure 5: Principle of binding between two spheres in the ray model. Each droplet acts as a tweezer for the other. As there is no reason why focal plans of spheres be the same, the second sphere defocuses its trapping tweezer and can rebuild another tweezer behind it like in triplet cases or 4-droplet cases.

When the radius of spheres is such that ka > 100, optical forces can be calculated in the ray model approximation with a good degree of accuracy [6, 17]. In this model, binding between two spherical dielectric particles can be understood by comparison with optical tweezers. For a sufficiently focused beam, a dielectric particle can be trapped close to the focal point. In our case, the focusing lens is nothing else than the next droplet. Each droplet builds an optical tweezer for the other. The numerical aperture of a spherical lens can be approximated by $NA \simeq \frac{a}{f} = 2\frac{n-1}{n}$ which only depends on the index of the sphere (not on its radius). We think this model explains the high stability of the doublet structure despite speckle: a single plane seems to be sufficient for particles to be bound. For reaching such stability, spheres need to be close enough to each other. When spheres are much more than a diameter away like in the case of experiments in water [11, 12], spheres cannot be in a bound state: the focusing numerical aperture is not sufficient for the optical tweezer to be stable. In this case, microspheres interact repulsively in a single trap so as to give chains. This comparison with optical tweezers can also explain why structures with three and four droplets are less stable than the doublet case. Indeed, while being trapped by the tweezer, the sphere defocuses the beam (see Fig. 5). However, there is no reason why the optical force be zero when the focal point of the first droplet is the same as the focal point of the second. We can then hope, for a given radius of sphere (even more in the Mie range) a configuration where the second sphere will be trapped at two focal lengths of the focusing point. In this case, neglecting spherical

aberrations, the second sphere will rebuild a trap behind it. This argument explains both the possibility to build 4-droplet chains and why the doublet is much more stable than triplets and quadruplets.

3.4. Orders of Magnitude

We can see on movies than despite all the disturbing sources, the doublet is very stable, even when the laser beam is cut for one second. We present here the main forces involved in this experiment.

The strongest forces are capillary forces. For particles smaller than $1.2 \,\mu\text{m}$ in radius, they can be estimated with Laplace's theorem: $\Delta P = \frac{2\gamma}{a} \simeq 10^5 \,\mathrm{pN.\mu m^{-2}}$ where ΔP is the pressure difference between inside and outside the droplet, γ is the capillary coefficient of the liquid and a the radius of the sphere. This pressure must be compared with the electromagnetic pressure of the order of I/c. In a binding case, field can be enhanced between particles so as to increase the optical force by one order of magnitude. However, if we simply consider the trapping pressure, we obtain in the case of our experiment a pressure equal to $1 \text{ pN}.\mu\text{m}^{-2}$.

Brownian motion could also destruct the phase locking observed between bound droplets. As the interference pattern between images of droplets of a doublet remains despite random forces, we can conclude that the distance never changes more than $\lambda/4$. It means that the mean thermal force over a distance $\lambda/4$ is smaller than $\frac{kT}{\lambda/4} \simeq 10^{-2}$ pN. To be compared with the pressures we calculated in the previous paragraph, we can approximate the radius of spheres to be one micron. Finaly, as droplets are negatively charged when sprayed due to triboelectricity effects with the spray nozzle, electrostatic forces causes droplet repealing. Each droplet carry a few elementary charges and the distance between droplets being roughly $2.5 \,\mu\text{m}$: $F = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2} \simeq 10^{-5} \,\text{pN}.$

Electrostatic forces are then three orders of magnitude smaller than optical forces.

We should add an estimate of heating effects. As oil slightly absorbs light, convection currents may appear inside droplets. This effect has already been discussed in a previous article [15] but a precise idea of the forces involved cannot be given.

4. Conclusion

Our experimental results obtained in air differ appreciably from those previously reported in water. Much of the difference probably results from the higher index contrast. Our results fit both with a Rayleigh range binding process and a semi-classical ray model.

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Towards Efficient Modelling of Optical Micromanipulation of Complex Structures

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Abstract—Computational methods for electromagnetic and light scattering can be used for the calculation of optical forces and torques. Since typical particles that are optically trapped or manipulated are on the order of the wavelength in size, approximate methods such as geometric optics or Rayleigh scattering are inapplicable, and solution or either the Maxwell equations or the vector Helmholtz equation must be resorted to. Traditionally, such solutions were only feasible for the simplest geometries; modern computational power enable the rapid solution of more general — but still simple — geometries such as axisymmetric, homogeneous, and isotropic scatterers. However, optically-driven micromachines necessarily require more complex geometries, and their computational modelling thus remains in the realm of challenging computational problems. We review our progress towards efficient computational modelling of optical tweezers and micromanipulation, including the trapping and manipulation of complex structures such as optical micromachines. In particular, we consider the exploitation of symmetry in the modelling of such devices.

1. Introduction

Optical tweezers have seen deployment in a wide range of applications in biology, soft materials, microassembly, and other fields. As well as being used for the trapping and manipulation of a wide range of natural and artificial objects, optically trapped probes are used to measure forces on the order of piconewtons. Compared with this diverse range of experimental applications, theory and accurate computational modelling of optical tweezers has received much less attention and has remained relatively undeveloped, especially for non-spherical particles and non-Gaussian beams. This is unfortunate, especially when we consider the growing fields of controlled rotation of complex microparticles — prototype optically-driven micromachines — and fully threedimensional manipulation using complex optical fields, where the application of theory and modelling provide insight into the physics, and allow engineering and optimisation.

Since optical forces and torques result from the transfer of momentum and angular momentum from the trapping beam to the particle via scattering, the theory and computational modelling of optical tweezers is, in essence, the theory and computational modelling of the scattering of light or electromagnetic radiation. Since typical particles that are optically trapped or manipulated are on the order of the wavelength in size, approximate methods such as geometric optics or Rayleigh scattering are inapplicable, and solution or either the Maxwell equations or the vector Helmholtz equation must be resorted to. As scattering by particles in this size range is of interest in many fields, a wide variety of analytical and computational methods have been developed. Thus, there is a solid foundation on which to develop computational modelling of optical micromanipulation.

There are, however, complications that prevent simple direct application of typical light-scattering codes. The first, but not necessarily the most important, is that optical tweezers makes use of a highly focussed laser beam, while most existing scattering codes assume plane wave illumination. Perhaps more fundamental is the need for a large number of repeated calculations to characterise an optical trap — even for an axisymmetric (but nonspherical) particle trapped in a circularly polarised Gaussian beam, we already have four degrees of freedom. Clearly, this places strong demands on computational efficiency.

Due to this requirement for repeated calculation of scattering by the same particle, we employ the T-matrix method [1,2]. Below, we outline the employment of the T-matrix method for the calculation of optical forces and torques. While most implementations of the T-matrix method are restricted to simple geometries, this is not a limitation inherent in the method; fundamentally, the T-matrix method is a *description* of the scattering properties of a particle, not a method of calculating the scattering properties. Therefore, in principle, any method of calculating scattering can be used to obtain the T-matrix for a scatterer. We discuss such "hybrid" methods, where a computational method not usually associated with the T-matrix method is used to calculate the T-matrix of a scatterer, and hence the optical force and torque.

A further important consideration is that optical micromachines, while complex, are likely to possess a high degree of symmetry; this can be exploited to reduce computation times by orders of magnitude. We demonstrate the effectiveness of this approach by modelling the optical trapping and rotation of a cube. The two principal symmetries of such shapes — mirror symmetry and discrete rotational symmetry about the normal to the mirror symmetry plane — are exactly the symmetries that typify the ideal optically-driven rotor.

2. *T*-matrix Formalism for Optical Force and Torque

The *T*-matrix method in wave scattering involves writing the relationship between the wave incident upon a scatterer, expanded in terms of a sufficiently complete basis set of functions $\psi_n^{(inc)}$, where *n* is a mode index labelling the functions, each of which is a solution of the Helmholtz equation,

$$U_{\rm inc} = \sum_{n}^{\infty} a_n \psi_n^{\rm (inc)},\tag{1}$$

where a_n are the expansion coefficients for the incident wave, and the scattered wave, also expanded in terms of a basis set $\psi_k^{(\text{scat})}$,

$$U_{\rm scat} = \sum_{k}^{\infty} p_k \psi_k^{\rm (scat)},\tag{2}$$

where p_k are the expansion coefficients for the scattered wave, is written as a simple matrix equation

$$p_k = \sum_{n=1}^{\infty} T_{kn} a_n \tag{3}$$

or, in more concise notation,

$$\mathbf{P} = \mathbf{T}\mathbf{A} \tag{4}$$

where T_{kn} are the elements of the *T*-matrix. The *T*-matrix formalism is a Hilbert basis description of scattering. The *T*-matrix depends only on the properties of the particle — its composition, size, shape, and orientation — and the wavelength, and is otherwise independent of the incident field.

This means that for any particular particle, the T-matrix only needs to be calculated once, and can then be used for repeated calculations. This is the key point that makes this an attractive method for modelling optical tweezers, providing a significant advantage over many other methods of calculating scattering where the entire calculation needs to be repeated.

The natural choice of basis functions when describing scattering by a compact particle is to use vector spherical wavefunctions (VSWFs) [1]. The optical force and torque are given by sums of products of the modal amplitudes [3–5].

Notably, neither how the VSWF expansion of the incident field nor how the T-matrix can be calculated has entered the above description of scattering. A variety of methods exist for the former [6, 5], and the latter task is generally the more challenging computationally.

Most implementations of the T-matrix method use the extended boundary condition method (EBCM), also called the null field method, to calculate the T-matrix. This is so widespread that the T-matrix method and the EBCM are sometimes considered to be inseparable, and the terms are sometimes used interchangeably. However, from the description above, it is clear that the T-matrix formalism is independent of the actual method used to calculate the T-matrix [7, 8].

A number of alternative methods have been used for the calculation of T-matrices. Notably, such "hybrid" methods, for example the discrete dipole approximation (DDA) method used by Mackowski [9] can be used for the calculation of T-matrices for particles of arbitrary shape, internal structure, and electromagnetic properties. Complex internal structure will generally require a discretisation of the internal volume of the particle, rather than a method based on surface discretisation. We are working on both finite-difference frequency-domain (FDFD) and DDA based hybrid T-matrix solvers.

3. Optical Torque and Symmetry

The *T*-matrix elements are strongly dependent on the symmetry of the scatterer [1]. We can deduce the principal features from Floquet's theorem, relating solutions to differential equations to the periodicity of their boundary conditions.

If we have a scatterer with *n*th-order rotational symmetry about the z-axis, an incident mode of azimuthal index m couples to scattered modes with azimuthal indices $m, m \pm n, m \pm 2n, m \pm 3n$ and so on. For scatterers that are mirror-symmetric, upward and downward coupling must be equal, in the sense that, for example, a mirror-symmetric scatterer of 2nd order rotational symmetry (such as a long rod), T-matrix elements coupling from m = 1 modes to m = -1 modes will have the same magnitudes as the elements coupling from m = -1 to m = 1 modes. For chiral scatterers, these T-matrix elements will, in general, be different.

This directly affects the optical torque; the vector spherical wavefunctions are eigenfunctions of the angular momentum operators J^2 and J_z . Essentially, the radial mode index n gives the magnitude of the angular momentum flux, while the azimuthal mode index m gives the z-component of the angular momentum flux. Therefore, the coupling between orders of different m describes the generation of optical torques about the beam axis.

For the case of a rotationally symmetric scatterer, this means that there is no coupling between modes with differing angular momenta about the z-axis [1, 10, 11]. Therefore, it is not possible to exert optical torque on such scatterers except by absorption (or gain) — since the incoming and outgoing angular momenta per photon are the same, the only optical torque can result from a change in the number of photons. In general, the use of absorption for the transfer of optical torque is impractical, due to excessive heating. Therefore, a departure from rotational symmetry is required. This can be either at the macroscopic (the shape of the particle) or microscopic (optical properties of the particle) level.

Birefringent and elongated or flattened particles are simple examples of introducing such asymmetry; notably, such particles were the first to be controllably optically rotated through means other than absorption, for example by Beth in the first measurements of optical torque [12]. Particles with these properties have also been rotated in optical traps [13–16]. As such particles can still be axisymmetric about one axis, rapid calculation of optical forces and torques is still possible [15, 16].

More complex particles have also been fabricated and rotated [17–19], but in these cases, there are few results from computational modelling [20].

As such structures typically possess discrete rotational symmetry, the restrictions on coupling between azimuthal orders can be used to reduce the number of T-matrix elements that need to be calculated. This can greatly reduce the time required. This is also the case for the hybrid methods described above. For a scatterer with *p*th-order discrete rotational symmetry, it is only necessary to perform calculations for a 1/p portion of the entire structure. If, in addition, there is mirror symmetry about the xy plane, the parity of the VSWFs will be preserved. Therefore, an odd-n TE mode will only couple to odd-n TE modes and even-n TM modes. This halves number of non-zero T-matrix elements, and halves the portion of the structure that needs to be modelled.

4. Example: Optical Trapping of a Cube

A simple example illustrating both the relationship between optical torque and symmetry, and the exploitation of particle symmetry for more efficient calculation of optical forces and torques, is the optical trapping of a cube. The cube embodies both of the symmetries — mirror symmetry and discrete rotational symmetry about the normal to the mirror symmetry plane — that typify the ideal optically-driven rotor.

As the cube has 4th-order rotational symmetry, and mirror symmetry with respect to the Cy plane, each incident modes only couples to approximately 1/8 the number of significant scattered modes. Although the column-by-column calculation of the *T*-matrix still requires the same number of least-squared solutions, each of this is of a smaller system of equations, and much faster. For example, the two wavelengths wide cube used in our example below required 30 minutes for the calculation of the *T*-matrix on a 32 bit single-processor 3 GHz microcomputer, as compared with 30 hours for an object of the same size lacking the cube's symmetries. Only one octant of the cube was explicitly included in the calculation.

If Figure 1, we show the optical force and torque exerted on a cube with relative refractive index of 1.19 = 1.59/1.34, and faces 2λ across, where λ is the wavelength in the surrounding medium. Once the *T*-matrix is calculated, to calculate the optical force and torque at a particular position requires less than 1 second (unless the point is far from the beam focus, in which case, up to 10 seconds or so can be needed).

In Figure 1(a), we see that cubic shapes can be stably trapped axially, while 1(b)-(d) show that optical torque can be generated by such structures. The increased efficiency resulting from the use of orbital angular momentum [5] is clear.



Figure 1: Optical force and torque on a dielectric cube. (a) shows the axial force as a function of position along the beam axis, showing that the cube can be trapped. (b)–(d) show the dependence of the optical torque on the beam convergence angle and the polarisation and orbital angular momentum. In (b), the beam is Gaussian (ie LG_{00}), while in (c) and (d), the beams are LG_{01} and LG_{02} respectively. The solid lines are for plane polarised beams, while dotted and dashed lines are for circularly polarised beams with spin parallel to and antiparallel to the orbital angular momentum.

5. Conclusion

The symmetry properties of a scatterer can be used to dramatically speed the calculation of the scattering properties of a particle. If these are expressed in the form of the T-matrix, this enables rapid and efficient calculation of optical forces and torques. Since typical optically-driven microrotors possess discrete rotational symmetry, they are ideal candidates for this method. In addition, mirror symmetry about a plane can also be used to further reduce the computational burden. Finally, "hybrid" T-matrix methods can be used for particles with geometries or internal structure making them unsuitable for traditional methods of calculating T-matrices.

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Analysis of Evanescent Waves Scattering by a Single Particle in Total Internal Reflection Microscopy

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Abstract—Since its invention in the mid of eighties [1] Total Internal Reflection Microscopy (TIRM) has proven to be an effective technique to measure weak interactions between spherical colloidal particles and surfaces with a resolution of a few femtonewton. It is a single particle evanescent light scattering technique. In an experimental setup a laser beam is coupled into a prism and hits the glass-water interface with an angle slightly above the critical angle of total internal reflection. This generates an evanescent field near the interface that decays in the lower refractive index medium (water) with a characteristic penetration depth which depends on the angle of incidence. A colloidal particle that is dispersed in the medium will scatter light from the evanescent wave if it is in the vicinity of the surface. By registering a scattered intensity it is possible to deduce the particlesubstrate distance. Compared to other methods for measure particle wall interactions like the surface force apparatus or the atomic force microscopy where a colloidal particle is attached to the tip, TIRM is the most sensitive technique because thermal fluctuations where limit the other methods in their resolution are exploited to determine the interaction potential. In this way forces in the order of a few femtonewton can be detected. TIRM has proven to be a valuable tool for the precise measurement of weak colloidal interactions as double layer forces, van der Waals forces, magnetic interactions and depletion forces. Review on TIRM can be found for example in [2, 3].

To compare experimental results with results of mathematical modeling an effective light scattering method is needed. For this purpose the Discrete Sources Method (DSM) has been chosen. The DSM is a well-known method for light scattering analysis, which has recently been applied for evanescent wave scattering [4].

1. Discrete Sources Method

For the theoretical modeling the Discrete Sources Method (DSM) has been chosen. The DSM is a well-known method for the analysis of light scattering. It has recently been applied to the evanescent wave scattering [4]. In frame of the DSM the mathematical statement can be presented as follows:

$$\nabla \times \mathbf{H}_{\zeta} = jk\varepsilon_{\zeta}\mathbf{E}_{\zeta}; \qquad \nabla \times \mathbf{E}_{\zeta} = -jk\mu_{\zeta}\mathbf{H}_{\zeta} \quad \text{in } D_{\zeta}, \quad \zeta = 0, 1, i,$$

$$\mathbf{n} \times (\mathbf{E}_{i} - \mathbf{E}_{0}) = 0, \ \mathbf{n} \times (\mathbf{H}_{i} - \mathbf{H}_{0}) = 0, \ \text{at } \partial \mathbf{D}$$

$$\mathbf{e}_{z} \times (\mathbf{E}_{0} - \mathbf{E}_{1}) = 0, \ \mathbf{e}_{z} \times (\mathbf{H}_{0} - \mathbf{H}_{1}) = 0, \ \text{at } \Sigma$$
(1)

and radiation conditions at infinity.

Here, D_0 is an ambient media, D_1 is a glass prism, D_i is an interior particle domain ∂D is a particle boundary, Σ is a prism-air border, **n** is the outward unit normal vector to ∂D , $k = \omega/c$ and $\{\mathbf{E}_{\zeta}, \mathbf{H}_{\zeta}\}$ stands for the total field in the corresponding domain D_{ζ} . We assume that the exciting field $\{\mathbf{E}_1^i, \mathbf{H}_1^i\}$ is a plane wave propagating from D_1 at the angle θ_1 with respect to the z-axis and transmitting at the interface following Snell's law. Note that the total field in D_0 is a superposition of the refracted incident field $\{\mathbf{E}_0^i, \mathbf{H}_0^i\}$ and the scattered $\{\mathbf{E}_0^s, \mathbf{H}_0^s\}$ field. If Im $\varepsilon_{\zeta}, \mu_{\zeta} \leq 0$ (the time dependence for the fields is chosen as $\exp\{j\omega t\}$) and the particle surface is smooth enough $\partial D \subset C^{(1,\alpha)}$, then the above boundary-value problem is uniquely solvable.

We construct an approximate solution to the scattering problem (1) according to the DSM outlines [5]. The amplitudes of discrete sources are determined from the boundary conditions at the particle surface, which can be rewritten as

$$\mathbf{n} \times (\mathbf{E}_i - \mathbf{E}_0^s) = \mathbf{n} \times \mathbf{E}_0^i \qquad \mathbf{n} \times (\mathbf{H}_i - \mathbf{H}_0^s) = \mathbf{n} \times \mathbf{H}_0^i, \quad \text{at } \partial D \qquad (2)$$

To construct the fields of dipoles and multipoles analytically satisfying the transmission conditions at the plane interface Σ we apply the Green tensor for a stratified interface [6].

An approximate solution takes into account an axial symmetry of the particle and the polarization of an external excitation. For P-polarized excitation for fields presentations outside the particle the following electric

and magnetic vector potentials are used

$$\mathbf{A}_{m,n}^{e,0} = \left\{ g_m^e(\eta, z_n) \cos(m+1)\phi; -g_m^e(\eta, z_n) \sin(m+1)\phi; -f_{m+1}(\eta, z_n) \cos(m+1)\phi \right\}, \\
\mathbf{A}_{m,n}^{h,0} = \left\{ g_m^h(\eta, z_n) \sin(m+1)\phi; g_m^h(\eta, z_n) \cos(m+1)\phi; -f_{m+1}(\eta, z_n) \sin(m+1)\phi \right\}, \\
\mathbf{A}_{0,n}^{e,h,0} = \left\{ 0; 0; g_m^{h,e}(\eta, z_n) \right\}.$$
(3)

where $g_m^{e,h}$, f_m Fourier harmonics of the corresponding Green tensor components [4]. For the total field inside the particle we define the following vector potentials:

$$\mathbf{A}_{m,n}^{e,i} = \{ \mathbf{J}_{m}^{i}(\eta, \mathbf{z}_{n}) \cos(m+1)\phi; -\mathbf{J}_{m}^{i}(\eta, \mathbf{z}_{n}) \sin(m+1)\phi; 0 \},
\mathbf{A}_{m,n}^{h,i} = \{ J_{m}^{i}(\eta, \mathbf{z}_{n}) \sin(m+1)\phi; J_{m}^{i}(\eta, \mathbf{z}_{n}) \cos(m+1)\phi; 0 \},
\mathbf{A}_{0,n}^{e,h,i} = \{ 0; 0; J_{0}^{i}(\eta, \mathbf{z}_{n}) \}.$$
(4)

Here $J_m^i(\eta, z_n) = j_m(k_i, R_{\eta z_0}) (\rho/R_{\eta z_0})^m$, $j_m(\cdot)$ is the cylindrical Hankel function, $\{z_n\}_{n=1}^{\infty}$ is a dense set of the points distributed over a segment $\Gamma_z^i \in D_i$ of the axis of symmetry. The approximate solution for the P-polarized excitation can be represented as

$$\begin{pmatrix} \mathbf{E}_{\varsigma}^{n} \\ \mathbf{H}_{\varsigma}^{n} \end{pmatrix} = \sum_{m=0}^{M} \sum_{n=1}^{N_{\varsigma}^{m}} \left\{ p_{mn}^{\varsigma} \mathbf{D}_{1}^{\varsigma} \mathbf{A}_{m,n}^{e,\varsigma} + q_{mn}^{\varsigma} \mathbf{D}_{2}^{\varsigma} \mathbf{A}_{m,n}^{h,\varsigma} \right\} + \sum_{n=1}^{N_{\varsigma}^{0}} r_{n}^{\varsigma} \mathbf{D}_{1}^{\varsigma} \mathbf{A}_{0,n}^{e,\varsigma}.$$
(5)

Where $\mathbf{D}_{1}^{\varsigma} = \left(\frac{j}{k\varepsilon_{\varsigma}\mu_{\varsigma}}\nabla\times\nabla\times, -\frac{1}{\mu_{\varsigma}}\nabla\times\right)^{T}$, $\mathbf{D}_{2}^{\varsigma} = \left(\frac{1}{\varepsilon_{\varsigma}}\nabla\times, \frac{j}{k\varepsilon_{\varsigma}\mu_{\varsigma}}\nabla\times\nabla\times\right)^{T}$. The case of an S-polarized excitation can be considered in a similar way [4].

Now we would like briefly describe the numerical realization of the computational algorithm. As mentioned above representation (5) satisfies all the conditions of the scattering problem (1) except the transmission conditions at the particle surface (2). These conditions are used to determine the unknown amplitudes of discrete sources $\{p_{mn}^{0,i}, q_{mn}^{0,i}, r_n^{0,i}\}$. Since the scattering problem geometry is axially symmetric with respect to the Z-axis and discrete sources are distributed over the axis of symmetry, fulfilling the transmission conditions (2) at surface ∂D can be reduced to a sequential solution of the transmission problems for the Fourier harmonics of the fields. So, instead of matching the fields on the scattering surface, we can match their Fourier harmonics separately thus reducing the approximation problem on the surface to a set of problems enforced at the particle surface generatrix \Im . By solving these problems one can determine the discrete sources amplitudes $\{p_{mn}^{0,i}, q_{mn}^{0,i}, r_n^{0,i}\}$.

For the determination of amplitudes the generalized point-matching technique is used [7]. The DSM is a direct method and hence it allows to solve the scattering problem for the entire set of incident angles θ_1 and for both polarizations (P and S) at the same time. Besides, the numerical scheme provides an opportunity to control the convergence of the approximate solution to the exact one by a posterior error estimation [5].

After the amplitudes of the discrete sources (DS) are determined, the far field pattern $\mathbf{E}_{\infty}(\theta, \varphi)$ of the scattered field, can be calculated. It is determined at the upper part of the unit semi-sphere $\Omega = \{0^{\circ} \leq \theta < 90^{\circ}, 0^{\circ} \leq \phi \leq 360^{\circ}\}$ and is given by

$$\mathbf{E}_{0}^{s}(\mathbf{r})/\mathbf{E}^{0}(0) = \frac{\exp\left\{jk_{0}r\right\}}{r}\mathbf{F}\left(\theta,\varphi\right) + O\left(r^{-2}\right) \quad r \to \infty$$

Using asymptotical estimation of the Weyl-Sommerfeld integrals of the Green's tensor components, the representation of the elements of a far field pattern gets a form of finite linear combinations of elementary functions [4]. This circumstance ensures an economical computer analysis of the scattering characteristics in the wave zone.

One of the most important scattering characteristics is an intensity of scattered light

$$I^{P,S}(\theta_0,\theta,\varphi) = \left| F_{\theta}^{P,S}(\theta_0,\theta,\varphi) \right|^2 + \left| F_{\varphi}^{P,S}(\theta_0,\theta,\varphi) \right|^2$$
(6)

where $F_{\theta,\varphi}^{P,S}(\theta_0,\theta,\varphi)$ are the components of the far field pattern for P (5) and S polarized incident wave, in a spherical coordinate system θ, φ .

In the paper we will examine the objective response function, which presents the intensity scattered into a certain solid angle:

$$\sigma_s^{P,S}(\theta_0) = \int_{\Omega} I^{P,S}(\theta_0, \theta, \varphi) \ d\omega \tag{7}$$

where $\Omega = \{ 0 \le \varphi \le 360^{\circ}; 0; 0 \le \theta \le \theta_{NA} \}, \theta_{NA}$ is an angle, which corresponds to the Numerical Aperture (NA) of the objective lens in accordance with $\theta_{NA} = \arcsin(NA/n_0)$.

The number of matching points where the DS amplitudes are defined increases till the necessary accuracy of the results is achieved. The DS number usually is 2-3 times less than the number of the matching points on the particle generatrix. As a rule the discrete sources are deposited on the axis of symmetry inside the particle. The order of multipoles (M) is a priori defined from the condition that the plane wave approximation should be less then 0.1%. The detailed algorithm of matching point's choice and deposition is described in [6].

2. Results and Discussion

In the paper we would like to present some results of numerical modelling of the TIRM calibration curve. As an example we took a PSL sphere of diameter $D = 1.6 \,\mu\text{m}$ at wavelength of 658nm. In figures the objective response (7) is plotted as a function of particle-prism distance. To show the results we have chosen two incident angles which differ from critical one. In Figure one the results for PP, PS, SP and SS polarization are presented for deviation of the incident angle from critical one by 0.58° , and in figure 2 similar results are presented for deviation of 2.6°. In both figures the intensity for P polarized light has less distortions then for S polarized. It is in good agreement with a multiple reflections theory, as the reflectance for P-polarized light is always lower then for S-polarized one.





Figure 1: The objective response (7) versus particle- Figure 2: The objective response (7) versus particleprism distance for a PSL sphere of diameter D = $1.6\,\mu\mathrm{m}$ for different polarizations and incident angle deviation of 0.58° .

prism distance for a PSL sphere of diameter D = $1.6\,\mu\mathrm{m}$ for different polarizations and incident angle deviation of 2.6° .

In the oral presentation more numerical results and their comparison with experimental data will be shown.

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T-matrix Simulation of Plasmon Resonances of Particles on or Near a Surface

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Abstract—We present the light scattering response of gold and silver particles on or near surfaces consisting of different materials. A comparison is made between a particle near a perfectly conducting surface and near a gold surface. The resulting scattering diagrams are found to be different. Beyond this, an approximation with a mirror particle shows little aggrement with a particle near a metal surface. Furthermore, we compare the spectral response of a combination of gold and silver materials for particles at different heights.

1. Introduction

Surface plasmons of small noble metal spheres can be detected as resonance peaks in the measured light scattering spectra. Transmission dark field microscopy is a technique where only the particles scatter light into the direction of the microscope objective. Such a measuring device can visualize very small particles as colored discs. The surface plasmon resonance frequency from a nonspherical particle or from a particle aggregate is different compared with a single spherical particle. With this effect, measuring techniques which use white light as illumination are capable to differ between aggregated particles and a single particle because of their different color. Even when a bio receptor molecule attached to a gold or silver sphere detects a biomolecular counterpart, the resonance frequency changes.

In the following we first describe the underlying scattering theory. Then we give some simulation examples of particles on or near a surface. We compare these results to some approximations found in the literature. This leads us to statements about the applicability of these approximations.

2. Theory

The scattering geometry is shown in figure 1. The incident field \vec{k}_0 and the particle are in the same medium.



Figure 1: Scattering geometry; the z-axis is perpendicular onto Σ , the boundary surface.

In the T-matrix formalism using the null-filed method, the scattered intensities I_{sca} are calculated from the scattered field coefficients f_{mn} and g_{mn} . These coefficients are related to the T-matrix [1]:

$$\begin{bmatrix} f_{mn} \\ g_{mn} \end{bmatrix} = \mathbf{T} \left(\begin{bmatrix} a_{mn_1} \\ b_{mn_1} \end{bmatrix} + \begin{bmatrix} f_{mn_1}^R \\ g_{mn_1}^r \end{bmatrix} \right)$$
(1)

with the T-matrix $\mathbf{T} = T_{mn,mn_1}$ of the particle, the total incident field coefficients $a_{mn_1} = a_{mn_1}^0 + a_{mn_1}^R$ and $b_{mn_1} = b_{mn_1}^0 + b_{mn_1}^R$ consisting of the direct $(a_{mn_1}^0 \text{ and } b_{mn_1}^0)$ and the reflected $(a_{mn_1}^R \text{ and } b_{mn_1}^R)$ incident fields,

and the coefficients $f_{mn_1}^R$ and $g_{mn_1}^R$ representing the fields scattered on the particle and reflected back from the surface to the particle. The $a_{mn_1}^R$ and $b_{mn_1}^R$ involves the Fresnel reflection coefficients. The scattered reflection coefficients for the interacting fields $f_{mn_1}^R$ and $g_{mn_1}^R$ are related to the scattered fields f_{mn_1} and g_{mn_1} :

$$\begin{bmatrix} f_{mn}^{R} \\ g_{mn}^{R} \end{bmatrix} = \mathbf{A} \begin{bmatrix} f_{mn} \\ g_{mn} \end{bmatrix}.$$
(2)

T is calculated from well known algorithms [2] and **A** can be found using radiating vector spherical wave functions [1]. By combining the matrix equations (2) and (3), the far field intensity can be computed. In the case of illumination from above ($\vec{\mathbf{k}}_0$ shows in the reversed direction) the incident field is calculated in the way described so far. If the incident angle β_0 get bigger than the critical angle $\theta_c = \arcsin(n_1/n_2)$ with $n_1 < n_2$ and n_1 and n_2 are the refractive indices of the medium above and below Σ , respectively, then the incident field from above will be totally reflected on Σ . However, an evanescent wave with typical exponential decrease is traveling into medium n_2 . In this case the Fresnel transmission coefficients used in the T-matrix method are changing [3].

3. Results

We calculate intensities at different scattering angles over the visible spectrum of wavelengths of small particles with diameter d = 80 nm. The intensities will be detector integrated over a range of $\theta_{NA} = 25^{\circ}$ which corresponds to a numerical aperture of $NA = n \sin(\theta_{NA})$ of the objective lens. The particles consist of silver or gold. The wavelength dependent refractive indices are interpolated values from Johnson et al., [4]. The numerical aperture depends on the medium surrounding the particle. We use air, water and immersion oil with an assumed constant refractive index.

In figures 2 and 3 the scattering diagrams of three systems are shown. In all systems the particle is a gold sphere and the scattering medium is air. The incident beam angle with respect to the normal is $\beta_0 = 30^{\circ}$ with an incident wavelength of $\lambda = 570$ nm. For that wavelength, the refractive index of the particle is n = 0.296 + i2.899. We first compare a system consisting of two spheres without an interface ('double-sphere' in the legend) with diameter d = 80 nm and distance z = 4 nm. The idea behind this system can be found in electrostatic theory where a system consisting of two point charges shows an identical electrical field compared to a point charge near a conducting plane. We approximate this second system with a surface having a nearly perfect conducting material ('sphere-perfect-conductor' in the legend of the figures) with a wavelength independant refractive index n = 0.00001 + i80, and the distance between the surface of the sphere to the plane surface is half of the first system (z = 2 nm). This idea is confirmed with figure 2.



Figure 2: Scattering diagrams of a sphere before a perfectly conduction plane and two spheres.

Figure 3: Scattering diagrams of a sphere before a perfectly conduction plane and before a gold surface.

Considering the different coordinate systems one can see that the horizontal-horizontal polarized scattering diagrams are very similar. The vertical-vertical polarized scattering diagram of the second system cannot be a straight line because of the Fresnel reflectance coefficients.

In contrast to the different geometries of the systems used in figure 2, the two systems of figure 3 are geometrically identical. A gold sphere with diameter of d = 80 nm is located near an infinite surface (distance between surface of the sphere to the plane surface z = 2 nm). The only difference is that in the first system the surface is an approximation of a perfectly conducting material used above, while in the second system the plane surface consists of gold ('gold surface' in the legend). The first system with the perfectly conducting surface shows a distinct minimum. This is due to a very small transmission coefficient and a corresponding reflection coefficient of nearly r = 1 [3]. Therefore the particle near to the surface is excited 'ideally' from a plane wave. In the second system this minimum vanishes because the Fresnel transmission coefficient do not vanish and therefore the particle is excited differently.

We state that the scattering response of a particle located near a noble metal surface cannot be well approximated with a system consisting of two identical spheres because of the different Fresnel reflection coefficients.

Now we want to consider measurement problems where an optical device pick up the light spectrum scattered from an object on or near a surface. For example a gold particle within a liquid medium is illuminated from a wave at oblique incidence. For the following examples the bottom (substrate) is an optically thick layer. In practice this means a thickness of a few hundred nanometers of a noble metal [5]. The particle medium is water (n = 1.333). We first show the scattering diagram for a particle with diameter d = 80 nm, $\beta_0 = 30^{\circ}$ and $\lambda = 570$ nm, but for three different heights (figure 4). At a distance of z = 200 nm, distinct minima appear because of multiple reflections between the particle (in the Rayleigh regime) and the surface. For a low distance these multiple reflections vanish and with it the minima.



Figure 4: Comparison of the scattering diagram of three different systems.



Figure 5: Detector integrated scattering diagrams of a particle at z = 2 nm in water.

In the following figures, the intensities are detector integrated values with an aperture angle of $\alpha = 25^{\circ}$. The spectral resolution of the wavelengths is $\Delta \lambda = 5$ nm. We use a spectrum of unpolarized incident waves $(\lambda = 450...700 \text{ nm})$ which irradiates four different scattering systems:

- silver particle in water above a silver surface;
- silver particle in water above a gold surface;
- gold particle in water above a silver surface;
- gold particle in water above a gold surface.

For a distance between substrate surface and particle surface of z = 2 nm (this means a $z_0 = 42 \text{ nm}$ in figure 1, the spectrum of detector integrated scattering intensities are shown in figure 5. When the particle is situated higher at a distance of z = 20 nm above the noble metal surface ($z_0 = 60 \text{ nm}$), the resulting scattering response of the same four systems can be seen in figure 6. A further increase of the height to z = 200 nm above the surface ($z_0 = 240 \text{ nm}$) results in figure 7.

Last of all we want to compare the spectral scattering response for a gold particle near to a gold surface for three different media:

- gold particle in air (n = 1.0) above a gold surface;
- gold particle in water (n = 1.333) above a gold surface.
- gold particle in immersion oil (n = 1.518) above a gold surface.

We assume constant refractive index over all wavelengths of the media (air, water and oil). The spectral detector integrated intensities are shown in figure 8.

The spectral response differs considerably. Especially the both liquid media show different characteristic spectras.



Figure 6: Detector integrated scattering diagrams of a particle at z = 20 nm in water.



Figure 7: Detector integrated scattering diagrams of a particle at z = 200 nm in water.



Figure 8: Spectrum of detector integrated intensities for different media (air, water and immersion oil); the particle position is z = 200nm.

4. Conclusion

We show that approximations like the double sphere system are far away from a qualitative similarity with the system under investigation. This means that simulations of the real circumstances are necessary, particulary if one needs quantitative statements of an observed system.

The results of the spectral simulations suggest that for increasing heights of a particle above the surface there is a shift of the intensity maximum towards lower wavelengths (see figure 5–figure 7). This fact may be used for measuring techniques. So altogehter, we want to emphasize the usage of exact techniques like the T-matrix method used for the simulations shown in this paper as a design tool for experimental investigations (e.g., [6]).

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Magnetic Nanostructure Hysteresis Loop Calculation for Modified Thin Film Multi-layer by Ion Irradiation

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Abstract—The nonlinear dependence of magnetization on direction of the applied magnetic field and history is described by statistical domain behavior using phenomenological adaptive parameters (like: g [1], h [A/m], k [J/m³], and q that are related to anisotropy, saturation field, static hysteresis loss, and pinning site density). The loop simulation data could be used also as parameters for thermal stability equation to calculate the relaxation time of the stored information on any magnetic nano particles (dots) of patterned magnetic media.

1. Introduction

Magnetic nanostructures are subjects of growing interest because of their potential applications in high density magnetic recording media and their original magnetic properties [1]. Multilayer thin films (like Co/Pt) are well known for their high magnetic anisotropy, and the origin of this high magnetic anisotropy has been the subject of interest for many researchers [2]. Demands for the continuous increase in the data storage density bring the challenge to overcome physical limits for currently used magnetic recording media [3]. Patterned magnetic media could be a way of realizing ultra high density storage media. Recently, demonstrations of areal recording density over 60 Gb/in^2 in both longitudinal and perpendicular magnetic recordings have been successfully made [4]. Determining the properties of small magnetic structures is extremely important for the development of data storage devices [5]. Better understanding of the micromagnetic processes in magnetic recording media is essential for developing novel materials for future ultrahigh density recording [6]. Good understanding of the noise mechanism in magnetic recording is required for developing heads and media for future applications [7].

In perpendicular recording, the magnetization pattern corresponding to the bits is provided perpendicular to the plane of the medium. The information is being stored in vertical domains or other structures of uniform magnetization [8]. The magnetic properties of an ultra thin multilayer can be patterned by controlled ion beam irradiation [9]. The basic step in this technique is to control the changes in the magnetic properties induced by the irradiation process.

In magnetic materials two characteristic length scales have to be considered [10]:

- at the atomic level, nearest neighbour exchange interaction is dominating,
- at a mesoscopic level, the domain wall width is the characteristic length dominating the magnetization reversal.

When the physical dimensions of a system become comparable to the interatomic spacing, strong modifications of the intrinsic magnetic properties (ordering temperature, magnetic anisotropy, spontaneous magnetization) are expected.

Micromagnetic modeling of the behavior of a nanostructured film beautifully describes the magnetization process, but requires a high calculational effort and long computation times. Furthermore, it is difficult to predict changes of the macroscopic physical behaviour due to variation of parameters. Phenomenological models, on the other hand, are very useful to simulate the behaviour of the magnetic material under the influence of varying parameters, especially when the parameters are based on physical constants.

2. Experiments

An assembly of ferromagnetic amorphous nanoparticles has been prepared by heavy ions irradiation of paramagnetic YCo₂ thin films [11, 12]. Several irradiation experiments carried out on YCo₂ samples have shown that fluences on the order 10^{12} U ions/cm² causes changes in magnetic properties of the samples [12]. Important changes are reported to take place after the irradiation:

- change of spontaneous magnetization, coercivity and initial susceptibility [12], and
- a distinct change of the anisotropy perpendicular to the film plane [11].

3. Energetic Model

The magnetic behaviour of magnetic moments is mainly described by the well known equations of Schrödinger (exchange interaction) and Landau, Lifshitz, and Gilbert (dynamics of magnetization reversal). Above this fundament is the shell of the physical constants describing spontaneous magnetization, anisotropy, magnetostriction, etc. The energetic model (EM) is designed as an interface between this shell and the macroscopic hysteresis phenomenon, able to predict many magnetic properties due to the relation of the parameters with the physical constants. The EM has been applied for different magnetization processes and materials [13–17].

The hysteresis of the magnetization M depending on the applied field H is described by the following equations, with the spontaneous magnetization M_s , the geometrical demagnetizing factor N_d , and the following phenomenological parameters:

1. g [1] related to anisotropy, reversible processes;

2. h [A/m] related to saturation field H_s , reversible processes;

3. $k \, [J/m^3]$ related to static hysteresis loss, irreversible processes;

4. q [1] related to pinning site density, irreversible processes.

In the cases of large domains, the microscopic constant c_r describes the influence of reversal speed. The sgn(x) function provides the correct four quadrant calculation (with the related magnetization $m = M/M_s$):

$$H = H_d + \operatorname{sgn}(m)H_R + \operatorname{sgn}(m - m_o)H_I.$$
(1)

(2)

The first term of Eq. (1) describes linear material behaviour, using the demagnetizing field $H_d = -N_d M_s m,$

the second term represents non-linear behaviour using the reversible field

$$H_R = h \left[\left((1+m)^{1+m} (1-m)^{1-m} \right)^{g/2} - 1 \right],$$
(3)

including saturation at a field $H_s(M_s)$, and the third term describes hysteresis effects like initial susceptibility χ_0 , remanence M_r , coercivity H_c , static losses, and accomodation, using the irreversible field

$$H_I = \left(\frac{k}{\mu_0 M_s} + c_r H_R\right) \left[1 - \kappa \exp\left(-\frac{q}{\kappa}|m - m_o|\right)\right].$$
(4)

For the initial magnetization, beginning with M = 0, H = 0, we set $m_o = 0$ and $\kappa = 1$. The function κ describes the influence of the total magnetic state at points of magnetization reversal. Therefore, κ (previous value κ_o) depends on the unit magnetization reversals $s = |m - m_o|$ up to this point of field reversal (m_o is the starting value of m at the last field reversal) with the simplification $e^{-q} \ll 1$:

$$\kappa = 2 - \kappa_o \exp\left[-\frac{q}{\kappa_o}|m - m_o|\right].$$
(5)

The calculation always starts with the initial magnetization curve and m is increased stepwise (the stepwidth determines the desired resolution of the calculation), which gives the corresponding field by Eq. (1). At a point of field reversal κ is calculated by Eq. (5) and m_o is set to the actual value of m at this point. Then m is decreased stepwise until the next reversal point, etc.

3.1. Identification

The identification of the EM with measurements or data sheets can be done easily. At given M_s and N_d the parameters are directly calculated from special points of the hysteresis loop. Considering reference conditions, the index 0 is to indicate that the identification is done at a temperature $T = T_0$ without applied mechanical stress σ , using the following equations:

$$k_0 = \mu_0 M_s H_c \tag{6}$$

$$q_0 = \frac{M_s}{H} \frac{1 - N_d \chi_0}{1 - N_d \chi_0} \tag{7}$$

If χ_0 is not available one can also use the total static losses $w_l = \int_{-M_s}^{+\infty} H dM + \int_{+M_s}^{-\infty} H dM$ corresponding to the area of the closed major loop (upper and lower branch of hysteresis)

$$w_l = 4k\left(1 - \frac{2}{q}\right) \tag{8}$$

and we can write the equation for q_0 as

$$q_0 = \frac{8\mu_0 M_s H_c}{4\mu_0 M_s H_c - w_l} \,. \tag{9}$$

These relations allow even an estimation of M_s (at $c_r \approx 0$), using Eqs. (6), (7), and (9) to

$$M_s = \frac{2\chi_0 H_c}{1 - N_d \chi_0} + \frac{w_l}{4\mu_0 H_c} \,. \tag{10}$$

Furthermore, q_0 can also be determined by the reduced remanence m_r of the upper branch of a loop with the measured reduced maximum magnetization m_m . Using f_q as a factor related to m_r ,

$$f_q = \left[(1+m_r)^{1+m_r} (1-M_r)^{1-m_r} \right]^{g_0/2} - 1, \qquad (11)$$

we identify q_0 as

$$q_0 = \frac{2}{m_m - m_r} \ln \frac{2H_c}{H_c - h_0 f_q + N_d M_s m_r}$$
(12)

By using f_g as a factor related to m_g which is the reduced magnetization at $H = H_g$, in the knee of the lower branch of the hysteresis,

$$f_g = \frac{1}{\ln\sqrt{(1+m_g)^{1+m_g}(1-m_g)^{1-m_g}} - \ln 2},$$
(13)

hence g_0 is

$$g_0 = f_g \ln \frac{H_g - H_c - N_d M_s m_g}{H_s - H_c - N_d M_s}$$
(14)

Using f_c as a factor related to m_r and m_m ,

$$f_c = 1 - 2 \exp\left[q_0 \frac{m_r - m_m}{2}\right],\tag{15}$$

the microscopic constant describing the domain (grain) geometry ratio becomes

$$c_r = \frac{f_q \frac{H_s - H_c - N_d M_s}{M_s \exp g_0 \ln 2} - f_c \frac{H_c}{M_s} + N_d m_r}{\left(f_q \frac{H_s - H_c - N_d M_s}{M_s \exp g_0 \ln 2} + f_c \frac{H_c}{M_s}\right) - N_d m_r}$$
(16)

Finally, the identification equation of h_0 is

$$h_0 = \frac{H_s - H_c - N_d M_s}{(c_r + 1)(\exp[g_0 \ln 2] - 1)}$$
(17)

If H_s is not available, one can estimate H_s from the measured maximum field H_m at m_m using the approximation $H_s \gg H_c + N_d M_s$. Using f_h as a factor related to m_g and m_m

$$f_h = \frac{\ln\sqrt{(1+m_m)^{1+m_m}(1-m_m)^{1-m_m}} - \ln 2}{\ln\sqrt{\frac{(1+m_g)^{1+m_g}(1-m_g)^{1-m_g}}{(1+m_m)^{1+m_m}(1-m_m)^{1-m_m}}}}$$
(18)

we find

$$H_{s} = (H_{m} - H_{c} - N_{d}M_{s}m_{m}) \left(\frac{H_{m} - H_{c} - N_{d}M_{s}m_{m}}{H_{g} - H_{c} - N_{d}M_{s}m_{g}}\right)^{J_{h}}.$$
(19)

If N_d of the experimental arrangement is unknown then it can be estimated roughly by the differential susceptibility χ_c at coercivity of a measured hysteresis loop:

$$N_d \approx \frac{1}{\chi_c} \bigg|_{H=H_c} \,. \tag{20}$$

If N_d of the sample is rather large so that the magnetization curve is strongly sheared $(M_r N_d > H_c)$, then it can be necessary to identify g_0 and c_r by the backsheared curve $(N_d = 0)$.

3.2. Calculation

The calculations have been done as following: At a given $N_d = 0.47$, the parameters $g_0 = 5.24$, $h_0 = 2.79 \text{ kA/m}$, $k_0 = 1.10 \text{ kJ/m}^3$, and $q_0 = 8.79$ are identified for the perpendicular hysteresis at $\Phi = 5 \cdot 10^{12} \text{ ions/cm}^2$ with $M_s = 20 \text{ kA/m}$. In the next step we vary only M_s in order to calculate the hysteresis of the other irradiation cases. Using Eqs. (6) and (7), we find the dependencies

$$H_c = \frac{k_0}{\mu_0 M_s} \tag{21}$$

and

$$\chi_0 = \frac{\mu_0 M_s^2}{k_0 q_0 + \mu_0 M_s^2 N_d} \tag{22}$$

which strongly affects the shape of the hysteresis Curve.

4. Conclusions

The rapid development of magnetic recording leads to a large increase of the bit density. Multilayer thin films with a perpendicular magnetic anisotropy devices may play an active role in the development and establishment of future storage technologies. Patterning magnetic media is a potential solution for ultrahigh density magnetic recording [18]. Ion beam modification of magnetic layers may be the possible future of ultra high density magnetic recording media.

After ion irradiation of YCo₂ thin films with different fluence values, the measured magnetizazion curves clearly show a perpendicular aniyotropy [11]. The shape of the hysteresis loops depends strongly on M_s , which is predicted by the EM. It turns out that H_c is inversely proportional to M_s and χ_0 is proportional to M_s^2 , if N_d is neglected. As the EM parameters are also related to anisotropy it will be possible also to calculate the direction dependence of these magnetization curves, which is subject to further work.

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Energetical Model Interpretation of Thermal Stability by Changing Direction of the Magnetization of Nano Magnetic Structure

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Abstract—The nonlinear dependence of magnetization on the direction of the applied magnetic field and history is described by statistical domain behavior using phenomenological adaptive parameters (like: g[1], h[A/m], $k[J/m^3]$, and q that are related to anisotropy, saturation field, static hysteresis loss, and pinning site density). The loop simulation data could be used also as parameters for thermal stability equation to calculate the relaxation time of the stored information on any magnetic nano particles (dots) of patterned magnetic media.

1. Introduction

Magnetic nano particle thermal stability calculation is essential for development of patterned ultra high magnetic storage media. The use of reliable model (like: Energetic Model (EM) in the predication of non linear ferromagnetic materials properties [1], wich may depend also on direction and history of magnetization) is very important. EM simulation of hysteresis opens a very big opportunities to calculate values of parameters which we then use directly for interpretation of the stability condition of stored information on a nano magnetic structure. The main idea behind that is to change the direction of the applied field H and then see the stability conditions on a given nano bit volume. The value of the f_{BS} depends strongly on K_u and the volume of the nano structure which holds the stored magnetic information (a what so-called nano bit or nano dot). Research and development teams in companies implementing nano-technology are gaining more and more importance in the field of sensor systems and material science.

2. Interpretation of Magnization Processes and Result

The EM calculates the magnetic state of ferromagnetic materials by minimizing the total energy density w_T [1] (see Table 1) as the sum of the energy density:

Table 1: Relation between k_u , and relaxation time τ at dot-width $D_w = 22 \text{ nm}$ and T = 10 K, as a result of dependence of magnetization on direction of the applied magnetic field at different Φ with values (0°, 45°, 90°), where f_{BS} condition is satisfied.

magnetization direction Φ	$k_u [J/m^3]$	τ [Years]
90°	602.51	22.3×10^3
45°	1110.67	22.5×10^{15}
0°	1895.46	12.4×10^{35}

$$w_T = w_H + w_M \tag{1}$$

Where:

$$w_H = -\mu_0 \vec{M} \cdot \vec{H} \tag{2}$$

of the applied field \vec{H} and the magnetization \vec{M} and the material energy density

 w_N

$$I = w_d + w_R + w_I \tag{3}$$

The latter term is divided into the energy density of demagnetizing fields w_d and into contributions described by statistical domain behaviour: The reversible energy density w_R and the irreversible energy density w_I . It's very important to verify the components of the demagnetization factor that appears with in the magnetization process of a nano magnetic particles. Magnetization hysteresis loops, which display the magnetic response of a magnetic sample to an external field, have been widely used to characterize the behavior of nanostructured magnetic materials [2]. The effective demagnetization factor (or total-demagnetization factor) is compound of two types (where Nd is a geometric and Ni is the inner demagnetizing factor, e. g., due to the magnetostatic stray Fields within the microstructure of grains or particles) as the following:

$$N_e = N_d + N_i \tag{4}$$

The characteristic features of the hysteresis loop are dependent on the material, the size and shape of the entity, the microstructure, the orientation of the applied magnetic field with respect to the sample, the magnetization history of the sample, and the demagnetization factor.

3. Equations

If \hat{N}_d of the experimental arrangement is unknown then it can be estimated roughly by the differential susceptibility χ_c at coercivity of a measured hysteresis loop [3]:

$$N_d \approx \frac{1}{\chi_c} \Big|_{H=H_c} \tag{5}$$

The relation between N_d , $N_{e,0}$, $N_{e,\pi/2}$, and K_u is given as

$$N_d = N_{e,0} - \frac{K_0}{K_u} (N_{e,\pi/2 - N_{e,0}})$$
(6)

$$K_u = \frac{K_0}{N_{e,0} - N_d} (N_{e,\pi/2 - N_{e,0}})$$
(7)

 K_u which is important for calculation of the bit stability factor (see Table 1) is also related to the N_d that depends on the magnetization. Further the identification of K_0 is given by

$$K_0 = \mu_0 M_s^2 \tag{8}$$

4. Conclusion

Nano-technology is providing a critical bridge between the physical sciences and engineering, on the one hand, and modern molecular biology on the other. Materials scientists, for example, are learning the principles of the nanoscale world by studying the behavior of biomolecules and biomolecular assemblies. Nano-technology will increase its influence in electrical engineering and electrical materials strongly. The need for further development in nano-technology is required. Companies with market-oriented innovation, research and advanced development strategies like EVGroup have had important positions and an excellent reputation in the practical implementation of nano-technology. New and light magnetic devices will be invented to make life in the 21st century more functional and the researchers have to gain more knowledge of quantum effects within nano-meter body size. The energetic model is used to identify hysteresis by changing the the direction of the applied field to nano magnetic particles of irradiated samples. Effects of changing the direction of the magnetization field on thermal stability and relaxation time could be then calculated. Choosing a reliable model (EM) for hysteresis simulation of nano magnetic particles is essential. Demands for the continuous increase in the data storage density bring the challenge to overcome physical limits for currently used magnetic recording media [5–14].

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Innovation Use of Nano Technology in Magnetic Storage Devices and Nano Computers

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Abstract—The fluence of ion irradiation on polycrystalline thin films affects both anisotropy and spontaneous magnetisation M_s . The dependence of coercivity and initial susceptibility on M_s is predicted by a hysteresis model considering the balance of energy with good qualitative agreement.

1. Introduction

Demands for the continuous increase in the data storage density bring the challenge to overcome physical limits for currently used magnetic recording media [1,2]. Ferromagnetic nano-particles of different polycrystalline thin films have been formed by heavy or light ion irradiation [2-4]. Although this modification technique may be a way to produce nano-magnetic particles, there are some critical size limits of nanomagnetic structures like the superparamagnetic limit (SPML) which faces magnetic nanotechnology. The magnetic properties of thin films are strongly influenced by their structure [5]. Small changes in the way a thin film is produced often give rise to large changes in some of the magnetic properties of the thin film [6]. This is best understood by observing how the microstructure of the film changes with processing and then correlating the microstructure directly with the properties of the thin film [6]. The behaviour of magnetic nanoparticles has fascinated materials scientists for decades [7]. Magnetic nanostructures have become a centre of great interest in the scientific community and in industry as the core technologies behind magnetic recording devices [8]. And the magnetic properties of an ultra thin multilayer can be patterned by controlled ion beam irradiation [4]. There are fundamental limits due to the atomic nature of matter which may impose ultimate physical bounds to nanofabrication and miniaturization [9]. Over the past several decades, amorphous and more recently nano-crystalline materials have been investigated for applications in magnetic devices [10]. The benefit found in the nanocrystalline alloys stem from their chemical and structural variations on a nano-scale which are important for optimizing magnetic recording devices [10].

2. Irradiation Process and Results

Several irradiation experiments carried out on the Co/Pt multi-layers samples (A1, A2, and A3 see Table 1) cause changes in the magnetic properties of the thin films [1,4]. High aspect ratio silica masks on Co/Pt multi-layers were obtained by e-beam lithography and reactive ion etching with feature sizes down to 30 nm for lines and 20 nm for dots [3]. He+ ion irradiation of the magnetic layers through these masks was used to pattern the magnetic properties [3] (with fluences between 2.10^{14} and 2.10^{16} ions/cm² [4]). After mask removal, high resolution and high density, planar magnetic nano-structures were obtained [3]. The results of the irradiation show perfectly square hysteresis loops at room temperature, the coercive field decreasing progressively to zero [4]. The high perpendicular anisotropy of Pt/Co multi-layers originates from the interfaces between the layers [11]. Other experiments carried out on YCo₂ samples (thin polycrystalline film targets of polycrystalline with thicknesses of approximately $1 \mu m$ [[12]) have shown that fluences in range of 10^{12} U ions/cm² cause changes in magnetic properties of the samples [1]. The result of these experiments were changes of the anisotropy perpendicular to the film plane [1], and change of spontaneous magnetisation, coercivity and initial susceptibility [13].

2.1. Equations and Calculation

The energetic model (EM) [14] is used to calculate the dependence of the shape of the hysteresis loop on M_s . The parameters of the model are calculated directly from measurements of special points of the hysteresis loop. The identification of the parameters is done at reference conditions (index 0) at a temperature $T = T_0$ without any applied mechanical stress σ , and at given M_s and N_e (N_e is the effective demagnetizing factor which is the sum of the geometric demagnetizing factor N_d and the inner demagnetizing factor N_i). The following equations show how to determine the parameter of the model from spontaneous magnetization M_s , coercivity H_c , and from the effective demagnetization factor N_e .

$$k_0 = \frac{\mu_0 M_s H_c}{1 - 2 \exp(-q_0)} \tag{1}$$

k in J/m³ related to static hysteresis loss (irreversible processes), and

$$q_0 = \frac{\mu_0 M_s^2}{k_0} \frac{1 - N_e \chi_0}{\chi_0} \tag{2}$$

q (dimensionless) is related to pinning site density (irreversible processes). Using Eqs. (1) and (2) with the approximation $\exp[-q_0] \ll 1$, we find the dependencies

$$H_c = \frac{k_0}{\mu_0 M_s} \tag{3}$$

and

$$\chi_0 = \frac{\mu_0 M_s^2}{k_0 q_0 + \mu_0 M_s^2 N_e} \tag{4}$$

which strongly affects the shape of the hysteresis. Figure 1 shows the initial magnetization curves calculated and measured major hysteresis loops in dependence of the measured value of M_s . No other change of the parameter values has been made. Table 1 shows the result of the evaluation of the equations above. Figure 2 shows the initial magnetization curves calculated with the parameters depending on M_s due to irradiation, compared to measurements.



Figure 1: Calculated hysteresis loops for thin Films after different ion irradiation fluences perpendicular to the film plane and measured points. The H-values are related to the maximum field $H_m = 160 \text{ kA/m}$. The Mvalues are related to the respective saturation values of $M_s = 20 \text{ kA/m}$, $M_s = 40 \text{ kA/m}$, and $M_s = 60 \text{ kA/m}$ 1. Only these values have been changed to calculate the different major hysteresis loops.

3. Anisotropy Energy after Irradiation

The anisotropy energy k_u is essential for evaluation of the thermal stability condition on a given bit. For the three irradiated samples (A1, A2, A3), k_u was calculated (Eq. 5).

$$k_u = \frac{H_k \mu_0 M_s}{2} \tag{5}$$



Figure 2: Calculated initial hysteresis curves loops for thin films after different ion irradiation fluences perpendicular (Tab. 1) to the film plane and measured points. The H-values are related to the maximum field $H_m = 160 \text{ kA/m}$. The calculation has been done by varying the value of M_s , only.

Table 1: Macroscopic hysteresis features depending on M_s [14].

Φ	$[ions/cm^2]$	A1, 10^{12}	A2, $5 \cdot 10^{12}$	A3, $2 \cdot 10^{13}$
M_s	[kA/m]	20	40	60
χ_0	[1]	0.157	0.199	0.536
H_c	[kA/m]	43.6	21.8	14.5

4. Nano Bits Stability Factor and Its Relaxation Time Calculation

Assuming a factor f_{BS} a simple abbreviation for "bit stability factor", which represents the information stability of stored data on a given nano-bit, where: $f_{BS} = k_u V_{nano}/k_B T$, V_{nano} is the nano magnetic structure volume, $k_u V_{nano}$ is the energy barrier (ΔE), and $k_B = 1.38 \times 10^{-23}$ J/deg Boltzmann constant. The magnetic stored information on a nano dot is then stable: if only the condition ($f_{BS} > 40$) is satisfied [16]. The relaxation time (time duration of stored information) or switching time of stored information τ can be obtained from the Arrhenius relation as:

$$\tau = \frac{1}{f_0} \exp(f_{BS}) \tag{6}$$

Where f_0 is the thermal attempt frequency [17], which is usually assumed to be $10^9 \ s^{-1}$. The irradiation of the samples (A1, A2, A3) with different fluences as shown in Table 1, caused changes of the calculated values of k_u for each sample. Hence different values of f_{BS} and τ were calculated (see Table 2).

Table 2: Relation between k_u , and relaxation time τ at dot-width $D_w = 22 \text{ nm}$ and T=10 K, as a result of irradiation, where f_{BS} condition is satisfied.

samples	k_u [J/m ³]	τ [Years]
A_1	592.51	20.3×10^2
A ₂	1017.67	$23.6{ imes}10^{16}$
A ₃	1975.46	11.8×10^{38}

5. Conclusion

Magnetic nano-structures are subject of growing interest because of their potential applications in high density magnetic recording media and their original magnetic properties [11, 14–16]. The rapid development of magnetic recording leads to a large increase of the bit density. Multilayer thin films with perpendicular magnetic anisotropy devices may play an active role in the development and establishment of future storage technologies. Patterning magnetic media is a potential solution for ultrahigh density magnetic recording [3]. The shape of hysteresis loops depends strongly on M_s , where H_c is inversely proportional to M_s , and χ_0 is proportional to M_s^2 . Thermal stability is one of the serious issues for developing high density recording, and thus much effort has been made to overcome this issue [18]. The idea to use a regular array of physically isolated grains/dots promises an improvement in thermal stability of the recorded bits [19]. The anisotropy energy k_u is essential for evaluation of the thermal stability condition on a given bit, because k_u value is used in calculation of bit stability factor f_{BS} . A given nano bit is then thermaly stable: if only the condition $(f_{BS} > 40)$ is satisfied.

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Application of Finite Network Theory to the Transient Process of Electromagnetic Forming

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Abstract—Electromagnetic forming typically consists of a coil in front of a workpiece. The discharging of a large capacitor connected to the coil induces eddy currents in the workpiece. The discharging process is simulated by the finite network theory (FNM) for non-magnetic materials. A comparison with the commercial FE program ANSYS is made to verify FNM. Since FNM excludes air from calculation the evaluation results in a relatively small system of differential equations which allows a much faster calculation of the transient compared to ANSYS.

1. Introduction

In recent times efforts are made in workpiece forming using magnetic repulsion forces. The so-called electromagnetic forming (EMF) is usually composed of a coil connected to a large capacitor and a workpiece in front of the coil. The discharging of the capacitor-coil network induces large eddy currents in the workpiece resulting in a perpendicular repulsion force. The aim of the authors is the calculation of an example problem with non-magnetic materials using the finite network theory ([1–4]). Roughly speaking, the finite network theory (FNM) uses inductive coupled resistor networks to create the system of differential equations of current loops whereas most of common FE programs use the magnetic vector potential and the electric scalar potential on finite element nodes as unknowns. FNM results in a relatively small system of differential equations compared to commercial FE programs such as ANSYS. In all calculations the authors used ANSYS as reference FE program to proof the calculations made by FNM. It can be shown that the calculation of the transient of the EMF process is much faster using FNM than the calculation in ANSYS:

- 1. FNM holds a lower number of degrees of freedom compared to ANSYS.
- 2. In the case of low-resistivity conductors (coil and workpiece) the transient process in FNM can be calculated by standard methods to solve the related system of differential equations.
- 3. Once the inverse of the impedance matrix is calculated in FNM only back substitution is necessary in every time step.



Figure 1: Geometry of example problem and external circuit.

Our example consists of an rectangular single-loop coil made of copper in front of an aluminum plate (see Fig. 1). The external network consists of an external inductance, an external resistor and a capacitor connected to the rectangular coil. The external network provides an additional single loop electric network to the problem called main loop.

2. Simulation of the Transient by the Use of FNM

FNM discretises the conductive volumes of the coil and the aluminum workpiece into rectangular resistor elements i and j with cross sections q_i and q_j and an the lengths l_i and l_j . The self and mutual inductances of resistor elements can be calculated by equation (1):

$$L_{i,j} = \frac{\mu_0}{4\pi} \frac{1}{q_i q_j} \int_{q_i} \int_{l_j} \int_{l_j} \int_{l_j} \frac{\mathrm{d}q_i \mathrm{d}q_j \mathrm{d}l_i \mathrm{d}l_j}{|\vec{r}_i - \vec{r}_j|} \tag{1}$$

Since the network equations are formulated in terms of the mesh current method the self inductances $M_{\xi,\xi}$ and mutual inductances $M_{\xi,\eta}$, $\xi \neq \eta$ of (2) result from a summation of inductances $L_{i,j}$ of (1) over closed current loops C_{ξ} and C_{η} :

$$M_{\xi,\eta} = \sum_{i \, (C_{\xi})} \sum_{j \, (C_{\eta})} L_{i,j} \cdot \vec{e}_i \cdot \vec{e}_j \tag{2}$$

The inductances $L_{i,j}$ vanish for perpendicular resistor elements. Since our method uses loop inductances the orientations $\vec{e_i}$ of resistor elements are treated in (2) for simplicity. The $\vec{e_i}$ are parallel to the current flow of a single resistor and include the orientation of a resistor with respect to the loop orientation by sign. The summation $\sum_{i (C_{\xi})}$ of (2) indicates the summation of all elements *i* contained in the closed loop C_{ξ} . The $R_{\xi,\xi}$ of matrix **R** in (3) are closed loop resistances. In the case $\xi \neq \eta$ the value $R_{\xi,\eta}$ is the resistance of common branches of the loops C_{ξ} and C_{η} with respect to the orientation. $R_{\xi,\eta} = 0, \xi \neq \eta$ is fulfilled if two loops don't share common branches. FNM results in a 1st order system of differential equations with the matrices **M** and **R**:

$$\mathbf{M} = \begin{bmatrix} M_{1,1} & \cdots & M_{1,n} \\ \vdots & & \vdots \\ M_{k+1,1} & \cdots & M_{n,n} \\ M_{n,1} & \cdots & \tilde{M}_{n,n} \end{bmatrix}$$
$$\mathbf{R} = \begin{bmatrix} R_{1,1} & \cdots & R_{1,n} \\ \vdots & & \vdots \\ R_{n-1,1} & \cdots & R_{n-1,n} \\ R_{n,1} & \cdots & \tilde{R}_{n,n} \end{bmatrix}$$
$$\tilde{R}_{n,n} = R_{n,n} + R_{ext}$$
$$\tilde{L}_{n,n} = L_{n,n} + L_{ext}$$
$$(3)$$

The external circuit of Fig. 1 enforces the correction of its loop (number n) by the external impedance L_{ext} and the serial resistor R_{ext} . The 1st order system of differential equations is:

$$\mathbf{i}^{(m)}(t) = \left(\mathbf{i}_{1}^{(m)}(t), \dots, \mathbf{i}_{n}^{(m)}(t)\right)^{T}$$
$$\mathbf{u}(t) = \left(0, \dots, u_{C}(t)\right)^{T}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}^{(m)}(t) = \mathbf{M}^{-1} \cdot \left[\mathbf{u}(t) - \mathbf{R} \cdot \mathbf{i}^{(m)}(t)\right]$$
$$\frac{\mathrm{d}}{\mathrm{d}t}u_{C}(t) = -\frac{1}{C_{ext}} \cdot \mathbf{i}_{n}^{(m)}(t)$$
(4)

For the solution of (4) ANSYS offers the implicit Euler method [6]. To make our transient calculation comparable with ANSYS we choose an approximate implementation of the implicit Euler method:

$$\mathbf{y}(t) = \left(\mathbf{1}_{1}^{(m)}(t), \dots, \mathbf{1}_{n}^{(m)}(t), u_{C}(t) \right)^{T}$$

$$\frac{d}{dt} \mathbf{y}(t) = \mathbf{A} \cdot \mathbf{y} \tag{5}$$

$$\mathbf{y}_{\nu+1}^{(0)} = \mathbf{y}_{\nu} + \Delta t \cdot \mathbf{A} \cdot \mathbf{y}_{\nu} \tag{6}$$

$$\mathbf{y}_{\nu+1} = \mathbf{y}_{\nu} + \Delta t \cdot \mathbf{A} \cdot \mathbf{y}_{\nu+1}^{(0)} \tag{7}$$

During step $\nu \to \nu + 1$ of the transient an estimation of the new solution vector $\mathbf{y}_{\nu+1}^{(0)}$ is calculated by the explicit Euler method [5], see equation (6) and algorithm 1. By the use of $\mathbf{y}_{\nu+1}^{(0)}$ a second application of the explicit Euler method in (7) results in an approximation of the implicit Euler method.

Equations (6) and (7) are calculated by the Cholesky decomposition of $\mathbf{M} = \mathbf{B}_L \cdot \mathbf{B}_L^T$ of the inductance matrix \mathbf{M} instead of the use of the system matrix \mathbf{A} of the general form of a linear system of differential equations (6). The algorithm of equation (6) is:

Algorithm 1 Single step of explicit Euler method
1: Back substitution for vector \mathbf{x}
$\mathbf{B}_L \cdot \mathbf{x} = \mathbf{u}_ u - \mathbf{R} \cdot \mathbf{i}_ u^{(m)}$
2: Back substitution for vector y
$\mathbf{B}_{L}^{T}\cdot\mathbf{z}=\mathbf{x}$
3: $\mathbf{i}_{\nu+1}^{(m)} = \mathbf{i}_{\nu}^{(m)} + \Delta t \cdot \mathbf{z}$
4: $u_{C,\nu+1} = u_{C,\nu} - \Delta t \cdot \frac{1}{C_{ext}} \cdot i_{n,\nu}^{(m)}(t)$

3. Numerical Results

The discharging of the capacitor of the example problem was calculated by FNM and ANSYS. In both cases the same discretisation of the aluminum plate $(20 \times 20 \times 3 \text{ elements})$ and of the coil $(3 \times 4 \text{ elements})$ for height and width of cross section, 3 mm discretisation for length) was used. Fig. 2 shows the discretisation of FNM including the main loop containing the external circuit and a cutout of the finite element discretisation of ANSYS. In ANSYS we used SOLID97 and INFIN111 elements (see Tab. 1).



Figure 2: Discretisation of ANSYS (a) and finite network theory (b).

Table 1: ANSYS discretisation and degrees of freedom (DOF).

Area	Type	Number of FE
Coil	SOLID97,4,0	636
Plate	SOLID97,1,0	1200
Air	SOLID97,0,0	9584
Infinite area	INFIN111,1	130
DOF		16834

Since FNM excludes air from calculation this method results in a system of differential equations with significantly smaller degrees of freedom of about 1/6 compared to ANSYS (see Tab. 2). Once the Cholesky

decomposition of the inductance matrix is calculated, the application of the Euler method in FNM for the evaluation of the transient process is straight forward. Our ANSYS version provides for transient calculations of electromagnetic-circuit coupled fields only a direct standard solver [6]. In connection with the higher number of degrees of freedom the total computing time in ANSYS is about 80 times the computing time of FNM for an 83-point transient (see Tab. 3). A more appropriate solver of the transient calculation could improve the total computing time in ANSYS. Despite this fact efforts of the finite network method (FNM) still remain since FNM results in a system of differential equations with a much lower number of degrees of freedom.

Table 2: FNM discretisation and degrees of freedom (DOF).

Area	DOF
Coil	907
Plate	1881
Mesh of external circuit	1
Capacitor of external circuit	1
DOF	2790

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Method	Processor	Time
FNM	Mesh generation	127.7 s
	Matrix generation	$213.0~\mathrm{s}$
	Cholesky	
	decomposition	$47.7 \mathrm{\ s}$
	Transient	$129.1 {\rm \ s}$
	Total time	8 min 38 s
ANSYS	Model generation	negligible
	Transient	$11~\mathrm{h}~38~\mathrm{min}$
	Total time	11 h 38 min



Figure 3: Comparison of the discharging of the external circuit calculated by ANSYS and FNM.

The discharging current of the capacitor is mainly determined by the external current. We calculated the transient with a time step of 1 μ s. Care must be taken on the solution algorithm of linear system of differential equations. There may be deviations of about 10 percent in the peak values of the transient comparing the explicit and implicit Euler method. Fig. 3 depicts that our approximate implementation (6), (7) is in very good agreement with the exact realisation in ANSYS. Furthermore, FNM results in a right evaluation of eddy currents. Fig. 4 shows the total eddy current density on the lower side of the aluminum workpiece facing the copper coil. Smaller deviations are expected: In ANSYS the faces of the single-loop coil are connected to the


Figure 4: Total eddy current density in $A \cdot m^{-2}$ at $t = 20 \,\mu s$ calculated by ANSYS (a) and FNM (b). Both figures show the eddy current density on the lower side of the aluminum plate facing the rectangular copper coil. Both models use a discretisation of 3 in height of the workpiece with rectangular elements.

external circuit whereas in FNM the coil is attached only by single mesh to the external circuit.

4. Conclusion

The transient calculation of electromagnetic forming can be done by the use of commercial finite element codes or by of the finite network method (FNM). Since the finite network method discretises conducting areas into inductively coupled resistor networks and excludes air from calculation our method results in a system of 1st order differential equations with a significantly smaller number of unknowns. That's why FNM is the preferred method to simulate inductance phenomena of conductors with relatively small volumes embedded in air.

We confirmed our hypothesis by a comparison of ANSYS and FNM. The efforts in computing time of FNM reinforces when a transient simulation is necessary since every time step of the transient requires the solution of an associated linear equation system. A comparison of the absolute value of the eddy currents was made using the same discretisation of the source of the magnetic field and the eddy current domain in both methods (ANSYS and FNM). The good agreement of the distribution of total eddy currents of FNM and the well-known finite element program ANSYS confirms the applicability of FNM to eddy current problems of non-magnetic materials.

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Exact Expressions of the Orbit-curvature and Curvature-radius of the Toroidal/Helical Orbits

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Abstract—Closed-from, exact expressions of the Toroidal/Helical orbit-curvature and curvature-radius, have been obtained by first computing the first and second derivatives of the x, y and z components of the orbit position-vector \mathbf{r} as function of the wrapping-angle $\boldsymbol{\theta}$, and by then substituting those derivatives in the general expression of the curvature of a parametric space-curve in three dimensions. Such closed-form exact expressions open the possibility of computing the value of the continuously-evolving dipole magnetic-field \boldsymbol{B} , required to guide an electron-beam of known given energy \boldsymbol{E} , expressed in Mev , along the given orbit. Additionally, the maximum and minimum values of the dipole magnetic-field were given by performing preliminary numerical computations with n = 9 and c = 0.2.

1. Introduction

High power microwave (HPM) sources are almost always designed as vacuum-electronic devices, and are characterized by the capability of generating output powers in the range of Megawatt to Gigawatt, by using beam-voltages of hundreds of kilovolts, and beam-currents of tens of ampere. HPM sources operate in either of three broadly-defined modes: a) Short Pulse, at pulse lengths of $0.1-10\,\mu$ s, b) Long Pulse, at pulse lengths of $0.1-10\,\mu$ s, and c) Continuous Wave (CW). The design of high power microwave (HPM) sources has been gradually evolving during at least the past thirty years, primarily stimulated by applications to high energy charged-particle accelerators, and to directed-energy weapons. A number of classic review-papers document that evolution (see [1, 2]). Currently, high energy charged-particle accelerators use almost exclusively high-power klystron amplifiers [3], that attain peak-powers of hundreds of Megawatt in short-pulse operation, tens of Megawatt in long-pulse operation, and about a Megawatt in CW mode. Such amplifiers provide, while converting DC to microwaves, power-efficiency of 50%-60%, and power-gain of 40 dB-60 dB. The electron beam of klystron amplifiers is sharply bunched by *velocity modulation*, followed by a *drift-space* where the accelerated faster electron catch-up with the decelerated slower electron. The so attained sharp bunching generates, upon the continuous-current electron-beam, the required microwave-frequency component, that is the essential source of the generated high-power microwave output.

Quite recently, a number of multi-beam klystrons (MBK) have been developed experimentally, and at least three different MBK models are already commercially available (see [4–8]). Multi-beam klystrons operate at reduced electron-gun voltage, and higher total beam-current than the single-beam designs, thus preventing occasional destructive gun-diode discharges, and increasing the power-efficiency up to $\sim 75\%$. The powerefficiency (measured as the ratio of output microwave power to input DC power) is however still limited, even in MBK, as it is obviously impossible to extract *all* the microwave energy from a sharply-bunched electron-beam, without having the high space-charge-density of the slowing beam force it into uncontrollable defocusing. All high-power klystron amplifiers include therefore a device known as *the beam dump*, which is a high-volume expansion of the klystron vacuum-enclosure, located beyond the microwave-power extraction-structure, where the not-quite completely energy-depleted electron-beam is collected, while converting (*wasting !*) its residual energy to heat *and* X-rays.

2. Toroidal/Helical Orbits

A new, innovative design of High Power Microwave (HPM) Electron-Beam Amplifier was presented by the Author at the PIERS 2004 Symposium, in Pisa, Italy [14]. That new design has the capability of attaining multimegawatt output power levels, even in long-pulse, high duty cycle or even continuous wave (CW) operation, with very high efficiency, very high spectral-purity, and very low levels of phase and amplitude noise. The new design was initially conceived as a combination of a multi-beam klystron (MBK), with an Electron Storage Ring (ESR). Very high power-efficiency may be attained by having a sharply-bunched High-Current, Relativistic Electron Beam (HIREB) circulate around a *closed*, *re-entrant* multi-turn orbit, within a strong-focusing, alternatinggradient (AG) magnetic field, generated by an azimuth-periodic lattice of beam-guiding magnetic-dipoles, and magnetic quadrupole lenses. High beam currents may then be attained because, by using a multi-turn *helical* electron-beam orbit, running on the outer surface of a virtual torus-surface, the beam current and the space-charge density in each of the individual orbit-turns can be much lower than in a single-turn orbit. That orbit configuration was initially conceived as a way of increasing the power-efficiency of high-power klystron amplifiers, by eliminating *the beam dump*, and by introducing a mechanism of beam-energy recovery, similar to that of Energy-Recovery Linacs (ERL).

It was soon seen however that the use of such closed multi-turn electron-orbit would essentially reduce the foot-print of the newly-conceived device by a factor in the order of the square of the integer number n of turns, while keeping the total orbit length unchanged, relative to that of a single-turn Electron Storage Ring (ESR).

It was also seen that, by keeping the electron-beam energy always in a relativistic range (such as for instance from 50 Mev to 100 Mev), much higher single-turn beam-current, and much higher total stored beam-energy (expressed in Joule) could be attained under a strong-focusing, alternating-gradient (AGS) magnetic field, while at the same time any partial extraction of microwave-energy from the bunched circulating beam would not appreciably change the electron orbital-frequency. Indeed, the total stored beam-energy (expressed in Joule) is obviously stored in the relativistic $(\gamma - 1) m_0$ mass-increase of the electrons, multiplied by the square of the constant speed of light. Then, by keeping the electron-energy (expressed in Mev) in a relativistic range, very large amounts of microwave energy (expressed in Joule) could be extracted from the circulating sharply-bunched beam, while hardly changing the electron relativistic velocity-factor β ($\beta = \sqrt{(\gamma^2 - 1)/\gamma^2}$, while $\Delta E = \Delta \gamma m_0 c^2$). As a consequence, such partial microwave-power extraction would hardly change the electron orbit-frequency, provided the beam-energy (expressed in Mev) is kept within a relativistic range, where β is a very slow function of γ . In the light of these considerations, the new HPM amplifier design, that was initially conceived as a combination of the multi-beam klystron (MBK) with an Electron-Storage Ring (ESR), actually appears to perform the function of an Energy-Storage Ring (while still being nevertheless an "ESR"). Quite obviously, in any closed-orbit electron-device, the local orbit curvature is a parameter of fundamental significance, as it determines the magnetic-field flux-density required in the beam-steering dipole-magnets, and in the beamfocusing quadrupole lenses, as function of the electron-beam energy (expressed in Mev), and also determines the orbit-frequency of the electron-bunches, by determining the orbit curvature-radius. A closed-form, exact expression of the orbit-curvature, has now been obtained by first computing the first and second derivatives of the x, y and z components of the orbit position-vector r as function of the wrapping-angle θ , and by then substituting those derivatives in the general expression of the curvature of a parametric space-curve in three dimensions [13].

3. Orbit Equations

The selected *toroidal/helical* orbit-configuration was defined as a parametric space-curve in three dimensional space (\mathbb{R}^3), with its Cartesian coordinates being functions of the azimuth-angle φ (measured around the torus-axis), and of the *wrapping-angle* θ (measured around the torus circular cross-section), with the implied condition that the ratio of the two angle periods be rational, such that the orbit closes on itself after an integer number of turns \mathbf{n} (for $0 \leq \varphi \leq 2\mathbf{n}\pi$). The parametric equations of that orbit are expressed by:

$$\hat{r}(\varphi,\theta) = x(\varphi,\theta) \cdot \hat{i} + y(\varphi,\theta) \cdot \hat{j} + z(\varphi,\theta) \cdot \hat{k}$$
(1)

where φ is the azimuth angle around the torus-axis, and θ is the helical "wrapping angle" around the torus circular cross-section. The three Cartesian components x, y and z of the position-vector r, and the linear relation between the angles φ , and θ are given by:

$$x = (R + r \cos \theta) \cos \varphi \tag{2}$$

$$y = (R + r \cos \theta) \sin \varphi \tag{3}$$

$$z = r \sin \theta \tag{4}$$

$$\theta = \frac{n-1}{n} \varphi \tag{5}$$

A 3D display of the selected Toroidal/Helical orbit configuration is shown in Figure 1, including a 3D display of the locus of the moving curvature center (*the "evolute"* !). A 2D display of the corresponding X–Y plane projections is shown in Figure 2.

4. Curvature

Closed-form expressions have now been obtained for the multi-turn electron-orbit curvature, and for the orbit curvature-radius, as function of the azimuth-angle φ , and wrapping-angle θ . The general expression of the curvature of a space-curve in 3D is [11]:



Figure 1: 3D display of the toroidal/helical orbit and of its moving curvature center.



orbit and of its moving curvature center.



Figure 2: X-Yplane projection of the toroidal/helical Figure 3: Normalized curvature radius of the toroidal/helical orbit for n = 9 and c = 0.2.

$$\kappa(\varphi) = \frac{\vec{r'}(\varphi) \times \vec{r''}(\varphi)}{\left\| \vec{r'}(\varphi) \right\|^3} \quad \text{where}$$
(6)

$$\vec{r'}(\varphi) = \frac{dx}{d\varphi} \cdot \vec{i} + \frac{dy}{d\varphi} \cdot \vec{j} + \frac{dz}{d\varphi} \cdot \vec{k}$$
 and (7)

$$\vec{r''}(\varphi) = \frac{d^2x}{d\varphi^2} \cdot \vec{i} + \frac{d^2y}{d\varphi^2} \cdot \vec{j} + \frac{d^2z}{d\varphi^2} \cdot \vec{k}$$
(8)

are the first and second derivatives of the position-vector $\hat{r}(\varphi, \theta) = x(\varphi, \theta) \cdot \hat{i} + y(\varphi, \theta) \cdot \hat{j} + z(\varphi, \theta) \cdot \hat{k}$ The vector-product in the numerator of the general expression (6) is given by:

$$\vec{r}'(\varphi) \times \vec{r}''(\varphi) = \begin{bmatrix} i & j & k \\ \frac{dx}{d\varphi} & \frac{dy}{d\varphi} & \frac{dz}{d\varphi} \\ \frac{d^2x}{d\varphi^2} & \frac{d^2y}{d\varphi^2} & \frac{d^2z}{d\varphi^2} \end{bmatrix}$$
(9)

and it expands to:

$$\vec{r}'(\varphi) \times \vec{r}''(\varphi) = \begin{pmatrix} \frac{dy}{d\varphi} & \frac{d^2z}{d\varphi^2} & -\frac{dz}{d\varphi} & \frac{d^2y}{d\varphi^2} \end{pmatrix} \cdot \vec{i} \\ - \begin{pmatrix} \frac{dx}{d\varphi} & \frac{d^2z}{d\varphi^2} & -\frac{dz}{d\varphi} & \frac{d^2x}{d\varphi^2} \end{pmatrix} \cdot \vec{j} \\ + \begin{pmatrix} \frac{dx}{d\varphi} & \frac{d^2y}{d\varphi^2} & -\frac{dy}{d\varphi} & \frac{d^2x}{d\varphi^2} \end{pmatrix} \cdot \vec{k}$$
(10)

Further, the cube of the position-vector norm in the denominator of (6) is given by:

$$\left\|\vec{r'}\left(\varphi\right)\right\|^{3} = \left[\sqrt{\left(\frac{dx}{d\varphi}\right)^{2} + \left(\frac{dy}{d\varphi}\right)^{2} + \left(\frac{dz}{d\varphi}\right)^{2}}\right]^{3} \tag{11}$$

The actual expression of the Toroidal/Helical orbit-curvature is then obtained by substituting the first and second derivatives of the Cartesian coordinates \boldsymbol{x} , \boldsymbol{y} , and \boldsymbol{z} in the expressions (7), (8), and (11), and simplified to obtain:

$$\kappa(\varphi) = \frac{\sqrt{\frac{1}{8n^2} \left[k_0 + k_1 \cos\left(\frac{n-1}{n}\varphi\right) + k_2 \cos\left(2\frac{n-1}{n}\varphi\right) + k_3 \cos\left(3\frac{n-1}{n}\varphi\right) + k_4 \cos\left(4\frac{n-1}{n}\varphi\right)\right]}{\left[\sqrt{\left(1 + c\cos\theta\right)^2 + c^2\left(\frac{n-1}{n}\right)^2}\right]^3}$$
(12)

where the five K_i coefficients are functions of the torus aspect-ratio c, and of the number of turns n:

$$k_{0} = \frac{1}{8n^{6}} \left\{ 8n^{6} + 4c^{2}n^{2} \left\{ 1 + 2(n-1)n\left[2 + 3(n-1)n\right] \right\} + c^{4}(n-1)^{2} \left\{ 8 + n \left\{ n\left[57 + 10n(2n-5)\right] \right\} \right\} \right\} (13)$$

$$k_{1} = \frac{1}{2n^{4}} c \left\{ 3c^{2} \left(n-1\right)^{2} \left(1+2 \left(n-1\right)n\right) + 2n^{2} \left[1+n \left(3n-2\right)\right] \right\}$$
(14)

$$k_{2} = \frac{1}{2n^{4}}c^{2} \left\{ \left\{ 2n \left[2 + n \left(2n - 3 \right) \right] - 1 \right\} + (n - 1)^{2} \left[2 + n \left(3n - 4 \right) \right] \right\}$$
(15)

$$k_3 = \frac{1}{2n^4} c^3 \left(n-1\right)^2 \left[1+2n\left(n-1\right)\right] \tag{16}$$

$$k_4 = \frac{1}{8n^4} c^4 \left(n-1\right)^2 \left(2n-1\right) \tag{17}$$

while the electron-orbit *curvature-radius* ρ is quite obviously expressed by $\rho(\theta) = 1/\kappa(\theta)$ and is a periodic function of the *wrapping-angle* θ , in the $0 \le \theta \le (n-1)2\pi$ drange (Figure 3), with period $0 \le \theta \le 2\pi$.

Figure 1 shows a 3D display of a Toroidal/Helical orbit, with n = 9 and c = 0.2, and includes the locus of its moving curvature-center (the orbit "evolute" !), while Figure 2 shows the corresponding projection on the X–Y plane. Finally, Figure 3 shows one period of the orbit curvature-radius for $0 \le \theta \le 2\pi$, normalized to R = 1. The closed-form exact expressions of the Toroidal/Helical orbit-curvature, and curvature-radius provide the possibility of computing the value of the continuously-evolving dipole magnetic-field B, required to guide an electron-beam of known given energy E, expressed in Mev, along the given orbit. Preliminary numerical computations with n = 9 and c = 0.2 have shown the maximum and minimum values of the dipole magneticfield to be $B_{MAX}=4274.66$ Gauss, and respectively $B_{MIN}=2578.4$ Gauss, for an electron-energy E=150 Mev, and $B_{MAX}=1434.489$ Gauss, and respectively $B_{MIN}=865.258$ Gauss, for an electron-energy E=50 Mev. The space-orientation of such dipole field would, however, necessarily need to be also continuously evolving, following the continuous evolution of the Toroidal/Helical orbit torsion. The general expression of the orbit-torsion is given by [11]:

$$\tau\left(\varphi\right) = \frac{\vec{r'}\left(\varphi\right) \times \vec{r''}\left(\varphi\right) \cdot \vec{r''}\left(\varphi\right)}{\left\|\vec{r'}\left(\varphi\right) \times \vec{r''}\left(\varphi\right)\right\|^{2}}$$

The corresponding closed-form, exact expression for Toroida/Helical orbits will be given in a following report.

5. Conclusions

We proposed closed-from, exact expressions of the Toroidal/Helical orbit-curvature and curvature-radius by first computing the first and second derivatives of the x, y and z components of the orbit position-vector r as function of the wrapping-angle θ , and by then substituting those derivatives in the general expression of the curvature of a parametric space-curve in three dimensions. The closed-form exact expressions provide of the possibility of computing the value of the continuously-evolving dipole magnetic-field B, required to guide an electron-beam of known given energy E, expressed in Mev, along the given orbit. Also, preliminary numerical computations were performed.

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Dual-band/Broadband Circular Polarizers Designed with Cascaded Dielectric Septum Loadings

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Abstract—A simple method is presented in this paper for realizing a dielectric septum-loaded type circular polarizer with either dual-band or a single broadband response. Two dielectric septum sections with different dielectric constants and lengths that introduce various phase delay to the x- and y-polarizations of the electric field in the same frequency range are cascaded orthogonally to obtain a dual-band response. A single wideband response can also be achieved if two dielectric septum sections are cascaded in parallel instead. Simulations in Ansoft HFSS show that flatter phase response and wider bandwidth can be obtained by the proposed polarizer comparing to single section ones. Moreover, a dual-band response can only be achieved with a two-section design. Taking advantage of dielectric septum-loaded type circular polarizers, the fabrication error or inaccurate dielectric constants can easily be compensated by adjusting the lengths of dielectric septum sections.

1. Introduction

Circular polarizers have been widely studied and discussed because of the important roles they play in communication systems. Groove- or iris-type circular polarizers [1-2] are robust but require precise fabrication processes. Metal septum-type circular polarizers [3] are easy and effective to design and modify but suffer from large signal reflection. A circular polarizer designed with a dielectric septum loading is proposed [4] with simple design procedure and easy compensation of fabrication error while keeping signal reflection level in an acceptable range. This paper gives a further study on dielectric septum-loaded type circular polarizers by extending its concept to two dielectric septum sections. Two orthogonally cascaded dielectric septum sections lead to a dual-band polarizer while a single wideband design can be achieved with two septum sections cascade in parallel.



Figure 1: Three dimensional view of the circular polarizer with (a) two orthogonally cascaded septum sections and (b) two septum sections cascaded in parallel.

2. Theory

Figure 1 shows the geometry of the proposed circular polarizers, in which two dielectric septum loadings cascaded either orthogonally (Figure 1(a)) or in parallel (Figure 1(b)) are inserted in the middle of the waveguide. Slots on the waveguide wall are needed for precisely locating the dielectric septum. An incident wave E_0 oriented at 45° relative to the dielectric septum can be decomposed into two equal orthogonal projections as shown in Figure 2(a). These two components will propagate through the septum regions with different propagation

constants. The electric field component which is in parallel with the septum is strongly perturbs. As a result the effective dielectric constant for this component is greater and vice versa. If the relative dielectric constants and the lengths of these two septum regions are allowed to be different, various phase differences between the two field components will be introduced by the two septum regions in the same frequency range, as shown in Figure 2(b). If these parameters are chosen properly, a dual-band circular polarizer as well as a single broadband circular polarizer can be achieved.



Figure 2: Field components and propagation constants. (a) Cross-sectional view of the circular polarizer and (b) Propagation constant for various dielectric constant.

Polarizer prototype		Dual-band	Broadband
Operation band (GHz)		$11.7 12.7, \ 19.7 20.2$	11.7 - 20.2
Septum orientation		Orthogonal	Parallel
Waveguide radius (mm), a		8.7	8.7
Slot dimensions (mm),	s	2.2	2.2
	t	1.57	1.57
Septum lengths (mm),	l_1	207.7	34.9
	l_2	125.7	23.7
Septum materials,	ε_{r1}	2.94	2.2
	ε_{r12}	3.48	2.94

Table 1: Specifications and design parameters of the circular polarizers.

3. Design of Dual-band/broadband Circular Polarizers

Table 1 shows the specifications and the design parameters of the circular polarizers. Waveguide radius is firstly determined by the strategy proposed in [4] to obtain a flatter phase response in the desired frequency ranges. For physical strength and precise location of the dielectric septum, the slot on the waveguide wall can also be determined. Once the cross-sectional dimension of the waveguide is determined, propagation constants with various dielectric septums inserted are then calculated.

To design a dual-band circular polarizer, the prototype shown in Figure 1(a) is utilized. By properly choosing the length and relative dielectric constants of the two septum sections, a 90° and a 270° phase differences can be obtained at the center frequency of the lower and upper operation bands, respectively.

On the other hand, if the second prototype (Figure 1(b)) is used, a single broadband circular polarizer can be designed by properly placing the maximum variation point of phase difference in the desired frequency range. Figure 3 shows the simulation results by Ansoft HFSS for the frequency response of phase difference of the designed dual-band and broadband circular polarizers.



Figure 3: Phase difference of the polarizers with (a) dual-band and (b) a single broadband responses.

4. Conclusion

Circular polarizers with two cascaded dielectric septum loadings for dual-band and broadband applications are proposed. Simulations results by Ansoft HFSS shows that for the broadband design not only the flatter phase response near the center frequency but also a broader bandwidth are obtained comparing to single section ones [4]. A dual-band design that can never be achieved with single dielectric septum section is also accomplished in this paper. These circular polarizers are currently under fabrication and the measurement results will be presented later in the conference.

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High-accuracy Approximation to the Integrated Length of Toroidal/Helical Orbits

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Abstract—The new innovative concept of High Power Microwave (HPM) Amplifier recently introduced, combines Multi-Beam Klystron (MBK) and Electron Storage-Ring (ESR) technologies, by using closed, multi-turn Toroidal/Helical electron-orbits. The selected toroidal/helical orbit-configuration was defined as a parametric space-curve in three dimensional space R^3 . We get the parametric equations of that orbit. A high-accuracy approximation has now been obtained for the total length of the Toroidal/Helical orbit, that attains much faster numerical computation. Higher accuracy could be attained by using a higher-order expansion of the orbit-length rate-of-increase.

1. Introduction

High power microwave (HPM) sources are almost always designed as vacuum-electronic devices, and are characterized by the capability of generating output powers in the range of Megawatt to Gigawatt, by using beam-voltages of hundreds of kilovolts, and beam-currents of tens of ampere. HPM sources operate in either of three broadly-defined modes: a) Short Pulse, at pulse lengths of $0.1-10\,\mu$ s, b) Long Pulse, at pulse lengths of $0.1-10\,\mu$ s, and c) Continuous Wave (CW). The design of high power microwave (HPM) sources has been gradually evolving during at least the past thirty years, primarily stimulated by applications to high energy charged-particle accelerators, and to directed-energy weapons. A number of classic review-papers document that evolution (see [1, 2]). Currently, high energy charged-particle accelerators use almost exclusively high-power klystron amplifiers [3], that attain peak-powers of hundreds of Megawatt in short-pulse operation, tens of Megawatt in long-pulse operation, and about a Megawatt in CW mode. Such amplifiers provide, while converting DC to microwaves, power-efficiency of 50%-60%, and power-gain of 40 dB-60 dB. The electron beam of klystron amplifiers is sharply bunched by *velocity modulation*, followed by a *drift-space* where the accelerated faster electron catch-up with the decelerated slower electron. The so attained sharp bunching generates, upon the continuous-current electron-beam, the required microwave-frequency component, that is the essential source of the generated high-power microwave output.

Quite recently, a number of multi-beam klystrons (MBK) have been developed experimentally, and at least three different MBK models are already commercially available (see [4–8]). Multi-beam klystrons operate at reduced electron-gun voltage, and higher total beam-current than the single-beam designs, thus preventing occasional destructive gun-diode discharges, and increasing the power-efficiency up to $\sim 75\%$. The powerefficiency (measured as the ratio of output microwave power to input DC power) is however still limited, even in MBK, as it is obviously impossible to extract *all* the microwave energy from a sharply-bunched electron-beam, without having the high space-charge-density of the slowing beam force it into uncontrollable defocusing. All high-power klystron amplifiers include therefore a device known as *the beam dump*, which is a high-volume expansion of the klystron vacuum-enclosure, located beyond the microwave-power extraction-structure, where the not-quite completely energy-depleted electron-beam is collected, while converting (*wasting !*) its residual energy to heat *and* X-rays.

2. Toroidal/Helical Orbits

A new, innovative design of High Power Microwave (HPM) Electron-Beam Amplifier was presented by the Author at the PIERS 2004 Symposium, in Pisa, Italy [14]. That new design has the capability of attaining multimegawatt output power levels, even in long-pulse, high duty cycle or even continuous wave (CW) operation, with very high efficiency, very high spectral-purity, and very low levels of phase and amplitude noise. The new design was initially conceived as a combination of a multi-beam klystron (MBK), with an Electron Storage Ring (ESR). Very high power-efficiency may be attained by having a sharply-bunched High-Current, Relativistic Electron Beam (HIREB) circulate around a *closed*, *re-entrant* multi-turn orbit, within a strong-focusing, alternatinggradient (AG) magnetic field, generated by an azimuth-periodic lattice of beam-guiding magnetic-dipoles, and magnetic quadrupole lenses. High beam currents may then be attained because, by using a multi-turn *helical* electron-beam orbit, running on the outer surface of a virtual torus-surface, the beam current and the spacecharge density in each of the individual orbit-turns can be much lower than in a single-turn orbit. That orbit configuration was initially conceived as a way of increasing the power-efficiency of high-power klystron amplifiers, by eliminating *the beam dump*, and by introducing a mechanism of beam-energy recovery, similar to that of Energy-Recovery Linacs (ERL).

It was soon seen however that the use of such closed multi-turn electron-orbit would essentially reduce the foot-print of the newly-conceived device by a factor in the order of the square of the integer number n of turns, while keeping the total orbit length unchanged, relative to that of a single-turn Electron Storage Ring (ESR).

It was also seen that, by keeping the electron-beam energy always in a relativistic range (such as for instance from 50 Mev to 100 Mev), much higher single-turn beam-current, and much higher total stored beam-energy (expressed in Joule) could be attained under a strong-focusing, alternating-gradient (AGS) magnetic field, while at the same time any partial extraction of microwave-energy from the bunched circulating beam would not appreciably change the electron orbital-frequency. Indeed, the total stored beam-energy (expressed in Joule) is obviously stored in the relativistic $(\gamma - 1) m_0$ mass-increase of the electrons, multiplied by the square of the constant speed of light. Then, by keeping the electron-energy (expressed in Mev) in a relativistic range, very large amounts of microwave energy (expressed in Joule) could be extracted from the circulating sharplybunched beam, while hardly changing the electron relativistic velocity-factor β ($\beta = \sqrt{(\gamma^2 - 1)/\gamma^2}$, while $\Delta E = \Delta \gamma m_0 c^2$). As a consequence, such partial microwave-power extraction would hardly change the electron orbit-frequency, provided the beam-energy (expressed in Mev) is kept within a relativistic range, where β is a very slow function of γ . In the light of these considerations, the new HPM amplifier design, that was initially conceived as a combination of the multi-beam klystron (MBK) with an Electron-Storage Ring (ESR), actually appears to perform the function of an Energy-Storage Ring (while still being nevertheless an "ESR"). Quite obviously, in any closed-orbit electron-device, the total orbit length is a parameter of fundamental significance, as it determines both the total electric-charge, and the total beam-energy (expressed in Joule) stored in the orbit, and also determines the orbit-frequency of the electron-bunches. The closed-form exact expression of the orbit-length that was reported in [14], had been obtained by symbolically integrating the rate-of-increase of the orbit-length (also known as "the speed"!) as function of the wrapping-angle θ , by using Mathematica. That expression was however characterized by an extreme degree of complexity, even after being simplified (using the Mathematica "FullSimplify" command) from an original a 20-page-long print-out to a single-page expression. The computation-time required by the symbolic integration was only in the order of a minute, even on a modest 166 MHz PC (Dell XPS P166c), but the full simplification of the 20-page-long print-out required four full days, for a total in the order of 96 hours. Even on a modern Workstation, with dual 2.4 GHz Xeon processors (HP xw8000), that simplification requires at least in the order of six hours.

3. Orbit Equations

The selected *toroidal/helical* orbit-configuration was defined as a parametric space-curve in three dimensional space \mathbb{R}^3 , with its Cartesian coordinates being functions of the azimuth-angle φ (measured around the torus-axis), and of the *wrapping-angle* θ (measured around the torus circular cross-section), with the implied condition that the ratio of the two angle periods be rational, such that the orbit closes on itself after an integer number of turns n (for $0 \leq \varphi \leq 2n\pi$). The parametric equations of that orbit are expressed by:

$$\hat{r}(\varphi,\theta) = x(\varphi,\theta) \cdot \hat{i} + y(\varphi,\theta) \cdot \hat{j} + z(\varphi,\theta) \cdot \hat{k}$$
(1)

where φ is the azimuth angle around the torus-axis, and θ is the helical "wrapping angle" around the torus circular cross-section. The three Cartesian components x, y and z of the position-vector r, and the linear relation between the angles φ , and θ are given by:

$$x = (R + r \cos \theta) \cos \varphi \tag{2}$$

$$y = (R + r \cos \theta) \sin \varphi \tag{3}$$

$$z = r \sin \theta \tag{4}$$

$$\theta = \frac{n-1}{n} \varphi \tag{5}$$

The closed-form, exact expression given in the original paper [14] for the multi-turn electron-orbit length, as function of the azimuth-angle φ around the torus-axis, and of the *wrapping-angle* θ , shows a rather daunting degree of complexity, by including all three Elliptic Integrals: a) of the first kind **E**, b) of the second kind **F**, and c) of the third kind **I**.

4. Orbit-Length Approximations

A high-accuracy approximation has now been obtained for the total length of the Toroidal/Helical orbit, that attains much faster numerical computation. That approximation was obtained by expanding the orbit-length rate-of-increase $ds/d\varphi$ ("the speed" !) [13] in powers of the torus aspect-ratio c = r / R, and by integrating that expansion term-by-term. The 6^{th} -order power-expansion of the orbit-length rate-of-increase obtained is expressed by:

$$\frac{ds}{d\varphi} = R \sqrt{\left(1 + c \cos \theta\right)^2 + c^2 \left(\frac{n-1}{n}\right)^2} \\ \cong w_0 + w_1 c + w_2 c^2 + w_3 c^3 + w_4 c^4 + w_5 c^5 + w_6 c^6$$
(6)

(7)

where the seven \boldsymbol{w}_i expansion-coefficients are given by:

$$w_0 = R \tag{7}$$
$$w_1 = R \cos \theta \tag{8}$$

$$\frac{1}{1 - n \cos \theta} \left(\frac{n}{1 - 1} \right)^2 \tag{6}$$

$$w_2 = \frac{1}{2} R \left(\frac{1}{n} \right) \tag{9}$$

$$w_3 = -\frac{1}{2} R \left(\frac{n-1}{n}\right)^2 \cos\theta \tag{10}$$

$$w_4 = \frac{R}{8n^2} \left(\frac{n-1}{n}\right)^2 \left[4n^2 \cos^2 \theta - (n-1)^2\right]$$
(11)

$$w_5 = -\frac{R}{8n^2} \left(\frac{n-1}{n}\right)^2 \cos\theta \left[3 (2n-1) - n^2 (3-4\cos^2\theta)\right]$$
(12)

$$w_{6} = \frac{R}{16} \left(\frac{n-1}{n}\right)^{2} \left\{ \left[\left(\frac{n-1}{n}\right)^{2} - 6\cos^{2}\theta \right]^{2} - 28\cos^{4}\theta \right\}$$
(13)

where the wrapping-angle θ is related to φ through the linear, rational relation (5): $\theta = [(n-1)/n] \varphi$.

Preliminary numerical computations, using n = 9 and c = 0.2, have shown the residual error of the 6^{th} -order expansion of the orbit-length rate-of-increase given in (6) to have a residual error of $-4 \cdot 10^{-6}$ to $+3 \cdot 10^{-6}$ across the $0 \le \theta \le 2\pi$ range, that is consistently periodic across the whole $0 \le \theta \le (n - 1) 2\pi$ range (Figure 1).

The orbit-length approximate expression, resulting from a term-by-term integration of the 6^{th} -order expansion (6), includes five terms, and is expressed by:

$$s(\varphi) = h_1 \theta + h_2 \sin \theta + h_3 \sin 2\theta + h_4 \sin 3\theta + h_5 \sin 4\theta \tag{14}$$

where the five h_i coefficients are functions of the torus aspect-ratio c = r / R, and of the number of orbitturns n:

$$h_{1} = R \left\{ 1 + \frac{1}{2} \left(\frac{n-1}{n} \right)^{2} c^{2} + \frac{1}{4} \left(\frac{n-1}{n} \right)^{2} \left[1 - \frac{1}{2} \left(\frac{n-1}{n} \right)^{2} \right] c^{4} - \frac{1}{16} \left(\frac{n-1}{n} \right)^{2} \left[2 \left(\frac{n-1}{n} \right)^{4} - 3 \frac{(2n-1)^{2}}{n^{4}} \right] c^{6} \right\} (15)$$

$$h_2 = R \left[1 - \frac{1}{2} \left(\frac{n-1}{n} \right)^2 c^2 - \frac{3}{8n^2} \left(\frac{n-1}{n} \right)^2 (2n-1) c^4 \right] c$$
(16)

$$h_3 = R \frac{1}{16 n^2} \left(\frac{n-1}{n} \right)^2 \left[\left(2 - c^2 \right) n^2 + 3 c^2 \left(2 n - 1 \right) \right] c^4$$
(17)

$$h_4 = -R\frac{1}{24} \left(\frac{n-1}{n}\right)^2 c^5 \tag{18}$$

$$h_5 = R \frac{1}{64} \left(\frac{n-1}{n}\right)^2 c^6 \tag{19}$$

The graphic displays of the integrated rate-of-increase 6^{th} -order power expansion shown in Figure 2 have been computed for Toroidal/Helical orbits with n = 9 turns, and aspect-ratio c = r / R from 0.1 to 0.4, in steps of 0.1.

A preliminary numerical comparison of the approximate orbit-length expression given in Equation (14), computed using n = 9 and c = 0.2, has shown the residual error of the approximation to be in the order of

 \pm 3 · 10⁻⁶, with a single oscillation period for 0 \leq $\theta \leq 2\pi$ (Figure 3). The residual error appears to be a periodic function of the wrapping-angle θ , through the whole interval $0 \leq \theta \leq (n-1)2\pi$. While the orbit-length exact-expression given in [14] shows, for these parameter-values, a periodic discontinuity jump of -6.3836527 at θ -values that are odd-multiples of π , the approximation given in Equation (14) is completely continuous, and monotonic across the whole $0 \leq \theta \leq (n-1)2\pi$ range, and its computational speed is quite conveniently substantially higher, thus providing the possibility of determining the electron orbital-period, around either a single-turn or an nturn Toroidal/Helical orbit (Figure 3). Quite obviously, higher accuracy could be attained by using a higher-order expansion of the orbit-length rate-ofincrease.



Figure 2: Integrated 6^{th} -order expansion of the toroidal/helical orbit-length rate-of-increase.



Figure 1: Residual error of the 6th-order expansion of the orbit-length rate-of-increase.



Figure 3: Residual error of the 6^{th} -order power-expansion integral $s(\varphi)$.

5. Energy Storage

A tentative baseline design of an HPM amplifier as described in [14], has been generated, attempting to match the 1.3 GHz TESLA-Klystron specifications [15]. The virtual torus-surface radii computed are $\mathbf{R} = 1444.99$ mm, and respectively $\mathbf{r} = 288.998$ mm, while the torus median-circle circumference is $\mathbf{L}_c = 9079.15$ mm.

The total 9-turn orbit-length computed $L_t = 83.019$ m shows the use of a *n*-turn Toroidal/Helical orbit to lead to a very compact "device" having a surface foot-print n^2 times (= 81!) less than a conventional, circular-orbit, electron-storage-ring "tunnel-installation". Further, it appears feasible to have a total of 120 electron-bunch, in 40 sets of three bunch each, nominally spaced by an azimuth increment $\Delta \varphi = 9^{\circ}$ around the torus median-circle circumference, so that the bunch-set cyclotron-frequency is only $f_c = 1.3 \text{ GHz}/40 = 0.0325 \text{ GHz} \equiv 32.5 \text{ MHz}$, corresponding to an electron cyclotron period $t_c = 30.7692$ nanosec. The three electron-bunch in each set are then spaced by a nominal wrapping-angle increment $\Delta \theta = 120^{\circ}$. Also, the orbit-parameters R, r, L_c , and L_t would hardly change if the electron-energy is always kept sufficiently high, such as from 50 Mev to 150 Mev. Further, it appears also feasible to run a total average beam-current of 9 orbit-turn × 1.11 kA each = 10 kA total, attaining a circulating electron-beam power of 500–1500 Gw, and a total beam kinetic-energy content between $E_1 = 500 \ t_c = 15,384.615$ Joule at 50 Mev, and $E_2 = 1500 \ t_c = 46,153.385$ Joule at 150 Mev. Extracting a partial energy $\Delta \mathbf{E} = E_2 - E_1 = 30,769$ Joule, by switching the circulating electron-beam from an accelerating-structure to a microwave-power extraction-structure, would be sufficient to generate a 10 Mw peak-power, 3 msec long microwave pulse, thus exceeding the required TESLA-Collider RF System specification [15] by a factor two in pulse-length. Re-acceleration of the electron-beam could be performed during the pulse-to-pulse 98.5 ms spacing, of the specified maximum 10 Hz pulse-repetition rate. The re-acceleration could be performed at the third sub-harmonic of the required 1.3 GHz output-frequency, by placing the re-acceleration structure along a single orbit-turn, where the azimuth bunch-spacing is $\Delta \varphi = 27^{\circ}$.

6. Conclusion

We give an overview on the design of high power microwave (HPM) sources. As a new, innovative design of High Power Microwave (HPM) Electron-Beam Amplifier was presented not long before, a high-accuracy approximation has now been obtained for the total length of the Toroidal/Helical orbit, that attains much faster numerical computation. Higher accuracy could be attained by using a higher-order expansion of the orbit-length rate-of-increase. A tentative baseline design of an HPM amplifier has been generated, attempting to match the 1.3 GHz TESLA-Klystron specifications.

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Low Cost 60 GHz Gb/s Radio Development

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Abstract—The recent advances of CMOS and SiGe process technologies have now made the design of lowcost highly integrated millimeter-wave radios possible in Silicon. In combination with an optimum organic Liquid Crystal Polymer packaging approach, this represents a unique opportunity to develop Gb/s radio that could address the increasing demand in term of data rate throughput of the emerging broadband wireless communication systems. In this paper we discuss the circuit and module challenges that will enable a successful deployment of 60 GHz gigabits wireless systems.

1. Introduction

The demand for ultra-high data rate wireless communication systems is increasing daily with the emergence of a multitude of multimedia applications. In particular, the needs become urgent for ultrahigh speed personal area networking and point-to-point or point-to-multipoint data link. This demand has since pushed the development of technologies and systems operating at the millimeter-wave frequencies, and overcome the current limitations of alternative solution such as 802.11n and UWB. This trend has also been reinforced by the exponentially growth of the emerging automotive collision avoidance radar applications. Indeed, the availability of several GHz band-width unlicensed ISM bands in the 60 GHz spectrum represents a great opportunity for ultra-high speed short-range wireless communications [1]. Since the mid–90's, many examples of MMIC chip-set have been reported for 60 GHz radio applications using GaAs FET and InP pHEMT technologies [2]. Despite their commercial availability and their outstanding performances, these technologies struggle to enter the market because of their prohibitive cost and their limited capability to integrated advanced base-band processing. In addition, the combination, of a low cost highly producible module technology, featuring low loss and embedded function such as antenna, is required to enable a high volume commercial use of the 60 GHz systems.



Figure 1: Photo of SiGe 60 GHz integrated MMIC.



Figure 2: Photo of fabricated LCP substrate.

2. Device Technology

In this paper, we will present and discuss the advances of CMOS and SiGe technology has advanced to enable a complete chipset for 60 GHz applications [4–6]. The front-end architecture using a sub-harmonic approach will be detailed and analyzed and example of circuits such as millimeter-waves LNA, mixer and VCO will be presented. An example of integrated front-end chip that has been developed for this application is shown in Figure 1. It includes a 30 GHz cross-coupled VCO oscillating at a center frequency of 30.1 GHz and exhibiting 2.3 GHz tuning range. The maximum output power after buffer is around -11.7 dBm at 29.54GHz. The subharmonic APDP mixer has a measured minimum down-conversion loss of 8.3 dB with a greater than 4 GHz of single-sided 3-dB baseband bandwidth with a 5.5 dBm local oscillator signal at 30.5 GHz. An input 1dB compression point of -7dBm has been recorded. This is the first report of a 60 GHz sub-harmonic mixer on SiGe processes that can be applicable to multi-gigabit wireless personal area network application. A single stage cascode LNA has been measured to have about 15dB of gain. A 2 stages cascode LNA is under development to be combined with the others building blocks.

3. Module Technology

At last, the packaging of the 60 GHz radio represents a major challenge. The Liquid Crystal Polymer has emerged as a promising low-cost alternative for millimeter-wave module implementation [7]. It combines uniquely outstanding microwave performances, low cost and large area processing capability. It appears as a platform of choice for the packaging of the future 60 GHz gigabit radio. We will present the recent advances in developing mmW functions on LCP substrate such as filter and antennas as shown in Figure 2. The optimum combination and co-design of these technologies (Figure 1) is the key for the successful deployment of ultrahigh speed, high capacity, 60 GHz WLAN access for very dense urban network and hot spot coverage. Many other commercial applications will directly benefit from this advance. This includes high data rate Wireless Multimedia Access, compact Wireless Gigabit Ethernet and Wireless FireWire/IEEE–1394 link that can be ultimately combined with a fiber or cable backhaul network.



Figure 3: Concept view of a 60 GHz Gb/s radio module.

4. Conclusion

We discussed in this paper the circuit and module challenges for the next generation gigabits radio operating at 60 GHz. We highlighted the technology issues and choices based on application, system architecture, circuit and packaging considerations. The recent advances of CMOS and SiGe process technologies have now made the design of low-cost, highly integrated millimeter-wave radios possible in Silicon. In combination with an optimum packaging approach, such as a Liquid Crystal Polymer platform, these advances could have a major impact on the cost and the performances of the future high speed systems and lead a to a successful deployment of the 60 GHz gigabit wireless radio.

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Nonlinear Inversion of Multi-frequency Microwave Fresnel Data Using the Multiplicative Regularized Contrast Source Inversion

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Abstract—This paper presents the results of profile inversion of multi-frequency electromagnetic scattered field data, measured by the Institute Fresnel, Marseille, France, from cylindrical objects, both for TM and TE illuminations. The reconstructions are obtained by applying the Multiplicative Regularized Contrast Source Inversion (MR-CSI) method. The results show that the MR-CSI method successfully performs 'blind' inversion of a wide class of scattered field data. Further, we also show that by inverting both TM and TE data simultaneously, a more accurate reconstructed image can be obtained.

1. Introduction

We discuss the performance of the Contrast Source Inversion (CSI) method [1,2], enhanced with a Multiplicative Regularization technique (MR-CSI) [3]. The MR-CSI method has been applied to invert the first set of data measured by the Institut Fresnel, Marseille, France [4]. The inversion results obtained using the MR-CSI method from these first Fresnel data sets were presented in [5]. Following these experiments, the MR-CSI method has been improved by the introduction of the so-called weighted L_2 -norm regularizer, see [6]. The inversion results of the first Fresnel data set using the MR-CSI method with weighted L_2 -norm regularizer can be found in [7].

With this version of the MR-CSI method we demonstrate the reconstructions from the second set of data measured by the Institut Fresnel. We carry out a 'blind' inversion of these data sets without explicitly taking into consideration any *a priori* information regarding the type of objects (either dielectric or metallic) to be reconstructed. In all cases we reconstruct both the permittivity and the conductivity of the unknown objects. The only *a priori* information which is used in the inversion is the positivity constraint on both permittivity and conductivity. The inversion results show that the MR-CSI method seems to handle the experimental field data very well. Furthermore we will show that by inverting both TM and TE data simultaneously we are able to arrive at more accurate reconstructed images.

2. Methodology

The Institute Fresnel experimental setup consists of a transmitting and a receiving antennas, both of which are double-ridged horn antennas. The antennas are moved on a circular rail around the object(s). The objects are elongated in the direction perpendicular to the plane in which the antennas are rotated (i. e., the plane of measurement), that a two-dimensional (2D) model is appropriate. In the plane of illumination, we choose a 2D rectangular test domain D containing the object(s). The transmitting antenna illuminates the objects from different locations distributed equidistantly around the object. We use the subscript j to denote the measured frequency and the subscript s to denote the dependence on the transmitter position. The receiving antenna measures the total field and the incident field from a number of different locations distributed equidistantly around the object. The scattered field, which is needed in the inversion, can then be found by subtracting the incident field from the total field.

The experimental data are collected at a number of frequencies with time factor $\exp(-i\omega_j t)$ where $i^2 = -1$, ω_j is the radial frequency and t is time. We introduce the vectors p and q as the spatial positions in 2D. We use the Maxwell model for the constitutive parameters of the object. Hence the contrast function for each frequency is defined as follows:

$$\chi_j(\boldsymbol{q}) = \frac{\varepsilon(\boldsymbol{q}) - \varepsilon_0}{\varepsilon_0} + i \frac{\sigma(\boldsymbol{q})}{\omega_j \varepsilon_0},\tag{1}$$

where ε and σ denote the permittivity and conductivity, which are frequency independent. The symbol ε_0 denotes the permittivity in vacuum. Since ε and σ are frequency independent, it is obvious that in the inversion we need only to invert for one value of the contrast function. Let χ_1 be the contrast function value at the angular frequency ω_1 , then the other values of the contrast as a function of frequency can be calculated through:

$$\chi_j(\boldsymbol{q}) = \operatorname{Re}[\chi_1(\boldsymbol{q})] + i \frac{\omega_1}{\omega_j} \operatorname{Im}[\chi_1(\boldsymbol{q})].$$
(2)

Since all the objects lie inside a test domain D, the contrast function is therefore non-zero inside D and zero elsewhere.

In the TM-case where the non-zero component of the electric field is the only one parallel to the cylindrical objects, we deal with a scalar wave field problem. The domain integral representation for the scattered field as a function of the total field u_s, j and the contrast χ_j is given by

$$u_{s,j}^{sct}(\boldsymbol{p}) = K_j^{TM}[\chi_j u_{s,j}] = k_{0,j}^2 \int_D g_j(\boldsymbol{p}, \boldsymbol{q}) \chi_j(\boldsymbol{q}) u_{s,j}(\boldsymbol{q}) \mathrm{dv}(\boldsymbol{q}), \quad \boldsymbol{p} \in S,$$
(3)

where $k_{0,j} = \omega_j \sqrt{\varepsilon_0 \mu_0}$ is the wave number in free-space and S is the data domain where the transmitter and receiver are located. The scalar homogeneous Green function is given by

$$g_j(\boldsymbol{p}, \boldsymbol{q}) = \frac{i}{4} H_0^{(1)}(k_{0,j} |\boldsymbol{p} - \boldsymbol{q}|),$$
(4)

where $H_0^{(1)}$ denotes the first kind Hankel function of zero order.

In the TE-case, the field quantities are two-components vectors representing the electric field components in the transversal plane of the cylindrical objects. The domain integral representation for the scattered field vector as a function of the total field $u_{s,j}$ and the contrast χ_j is given by

$$\boldsymbol{u}_{s,j}^{sct}(\boldsymbol{p}) = K_j^{TE}[\chi_j \boldsymbol{u}_{s,j}] = (k_{0,j}^2 + \nabla \nabla \cdot) \int_D g_j(\boldsymbol{p}, \boldsymbol{q}) \chi_j(\boldsymbol{q}) \boldsymbol{u}_{s,j}(\boldsymbol{q}) \mathrm{dv}(\boldsymbol{q}), \quad \boldsymbol{p} \in S,$$
(5)

where ∇ is the spatial differentiation operator with respect to p.

The TM and TE total field, and the contrast inside the test domain D satisfy the following integral equation:

$$u_{s,j}^{inc}(\mathbf{p}) = u_{s,j}(\mathbf{p}) - K_j^{TM}[\chi_{j,n}u_{s,j,n}], \quad \boldsymbol{u}_{s,j}^{inc}(\mathbf{p}) = \boldsymbol{u}_{s,j}(\mathbf{p}) - K_j^{TE}[\chi_{j,n}\boldsymbol{u}_{s,j,n}], \quad \boldsymbol{p} \in D$$
(6)

where the operators $K_j^{TM}[\chi_j u_{s,j}]$ and $K_j^{TE}[\chi_j u_{s,j}]$ are defined in (3) and (5), for the TM-case and TE-case respectively. Eqs. (3), (5) and (6) are the basic equations for developing any inversion algorithm based on the integral equation formulation. The goal of solving the inverse scattering problem can be formulated as follows: Solve (3) or (5) to obtain the contrast χ_1 on D from the knowledge of the scattered field $u_{s,j}^{sct}$ on S and the incident field $u_{s,j}^{inc}$ on D subject to the necessary condition that the total field $u_{s,j}$ on D and the contrast χ_1 on D satisfy the integral equation in (6).

We consider the inverse scattering problem as an optimization problem where, in each iteration n, we update the contrast sources $w_{s,j,n} = \chi_j u_{s,j,n}$ and the contrast $\chi_{j,n}$ alternatingly, by minimization of the cost function. For the TM inversion the cost function is given by

$$F_{n}(\chi_{1,n}, w_{s,j,n}) = \left[\frac{\sum_{s,j} \|u_{s,j}^{sct} - K_{j}^{TM}[w_{s,j,n}]\|_{S}^{2}}{\sum_{s,j} \|u_{s,j}^{sct}\|_{S}^{2}} + \frac{\sum_{s,j} \|w_{s,j,n} - \chi_{j,n}u_{s,j,n}\|_{D}^{2}}{\sum_{s,j} \|\chi_{j,n-1}u_{s,j}^{inc}\|_{D}^{2}}\right] \int_{D} \frac{|\chi_{1,n}(\boldsymbol{p})|^{2} + \delta_{n}^{2}}{|\chi_{1,n-1}(\boldsymbol{p})|^{2} + \delta_{n}^{2}} \mathrm{dv}(\boldsymbol{p}), \quad (7)$$

where

$$u_{s,j,n} = u_{s,j}^{inc} + K_j^{TM}[w_{s,j,n}], \quad \delta_n^2 = \frac{1}{\Delta^2} \frac{\sum_{s,j} \|w_{s,j,n-1} - \chi_{j,n-1} u_{s,j,n-1}\|_D^2}{\sum_{s,j} \|\chi_{j,n-1} u_{s,j}^{inc}\|_D^2}$$
(8)

and $\|\cdot\|_S^2$ and $\|\cdot\|_D^2$ denote the L_2 -norm on the data domain S and the object domain D, respectively. The symbol Δ denotes the mesh size of the discretization grid. In this CSI method, we use the back-propagation step to arrive at initial estimates for the contrast sources and the contrast. After the initial step, in each iteration the contrast sources and the contrast are updated alternatingly each by using one conjugate gradient step. The optimization process may be terminated if one of the following stopping conditions is satisfied:



Figure 1: The configuration used to obtain the data sets FoamMetExtTM.exp and FoamMetExtTE.exp.

- The difference between the normalized data error F_n at two successive iterates, *n*-th and (n-1)-th, is within a prescribed error quantity (it set to be 10^{-5}).
- The total number of iterations exceeds a prescribed maximum $N_{max}=512$.

The *a priori* information that the permittivity and the conductivity are positive are implemented by enforcing the negative value to zero after each iteration. This simple procedure is employed in all of the inversion runs. Details of this so-called MR-CSI method for multi-frequency problem can be found in [5]. However the procedure to update the contrast function is replaced by the improved version in [7]. For the TE inversion the cost function in (7) is replaced by

$$F_{n}(\chi_{1,n}, \boldsymbol{w}_{s,j,n}) = \left[\frac{\sum_{s,j} \|\boldsymbol{u}_{s,j}^{sct} - K_{j}^{TE}[\boldsymbol{w}_{s,j,n}]\|_{S}^{2}}{\sum_{s,j} \|\boldsymbol{u}_{s,j}^{sct}\|_{S}^{2}} + \frac{\sum_{s,j} \|\boldsymbol{w}_{s,j,n} - \chi_{j,n}\boldsymbol{u}_{s,j,n}\|_{D}^{2}}{\sum_{s,j} \|\chi_{j,n-1}\boldsymbol{u}_{s,j}^{inc}\|_{D}^{2}}\right] \int_{D} \frac{|\chi_{1,n}(\boldsymbol{p})|^{2} + \delta_{n}^{2}}{|\chi_{1,n-1}(\boldsymbol{p})|^{2} + \delta_{n}^{2}} \operatorname{dv}(\boldsymbol{p}), \quad (9)$$

where

$$u_{s,j,n} = u_{s,j}^{inc} + K_j^{TE}[w_{s,j,n}].$$
(10)

Further, for joint TM and TE data inversion, the cost function to be minimized is given by

$$F_{n}(\chi_{1,n}, w_{s,j,n}, \boldsymbol{w}_{s,j,n}) = \left[\frac{\sum_{s,j} \|\boldsymbol{u}_{s,j}^{sct} - K_{j}^{TM}[w_{s,j,n}]\|_{S}^{2}}{\sum_{s,j} \|\boldsymbol{u}_{s,j}^{sct}\|_{S}^{2}} + \frac{\sum_{s,j} \|\boldsymbol{w}_{s,j,n} - \chi_{j,n}\boldsymbol{u}_{s,j,n}\|_{D}^{2}}{\sum_{s,j} \|\boldsymbol{u}_{s,j}^{sct} - K_{j}^{TE}[\boldsymbol{w}_{s,j,n}]\|_{S}^{2}} + \frac{\sum_{s,j} \|\boldsymbol{w}_{s,j,n} - \chi_{j,n}\boldsymbol{u}_{s,j,n}\|_{D}^{2}}{\sum_{s,j} \|\boldsymbol{u}_{s,j}^{sct}\|_{S}^{2}} + \frac{\sum_{s,j} \|\boldsymbol{w}_{s,j,n} - \chi_{j,n}\boldsymbol{u}_{s,j,n}\|_{D}^{2}}{\sum_{s,j} \|\boldsymbol{u}_{s,j}^{sct}\|_{S}^{2}} + \frac{\sum_{s,j} \|\boldsymbol{w}_{s,j,n} - \chi_{j,n}\boldsymbol{u}_{s,j,n}\|_{D}^{2}}{\sum_{s,j} \|\boldsymbol{\chi}_{j,n-1}\boldsymbol{u}_{s,j}^{inc}\|_{D}^{2}} \right] \\ \int_{D} \frac{|\chi_{1,n}(\boldsymbol{p})|^{2} + \delta_{n}^{2}}{|\chi_{1,n-1}(\boldsymbol{p})|^{2} + \delta_{n}^{2}} \mathrm{dv}(\boldsymbol{p}).$$
(11)

3. Numerical Results

In this proceeding paper we only show the inversion results of the data sets FoamMetExtTM.exp and FoamMetExtTE.exp. The inversion results of other data sets will be presented during the conference. These data sets FoamMetExtTM.exp and FoamMetExtTE.exp are obtained by measuring a configuration as shown in Fig. 1. This configuration consists of one circular dielectric cylinder with a relative permittivity value of $\varepsilon_r = 1.45$ with a diameter of 80 mm and one metallic cylinder with a diameter of 28.5 mm. In the experiment, there are 18 transmitters distributed uniformly on a circle with a radius of 1.67 m from the center of the experimental setup. For each transmitter the data are measured using 241 receivers located on a circle with a radius of 1.67 m. The data are collected at 17 frequencies in the range of 2–18 GHz. In the experimental setup the fields are generated and received by horn antennas. However as we previously argued, the problem is predominantly 2D. Hence



Figure 2: Reconstruction of the configuration with two disjoint cylinders, a dielectric one and a metallic one, for TM data polarization (data set: FoamMetExtTM.exp) (a) and TE data polarization (data set: FoamMetExtTE.exp) (b); and for joint inversion of both TM and TE data polarizations (c).

both receivers and transmitters are approximated as line receivers and line transmitters. Therefore, we carry out the calibration procedure outlined in [5].

In the inversion we take a test domain D of 16.775 cm by 16.775 cm. The test domain D is discretized into 122 by 122 rectangular subdomains. The side length of each subdomain is 0.1375 cm. The wavelength at 18 GHz is 1.67 cm, hence the width and height of the test domain D is 10 time the wavelength in vacuum. The data for different frequencies are inverted simultaneously. However, in the figures we plot the complex contrast function χ_1 only. This is the complex contrast at the lowest frequency.

The reconstructed images from the TM and TE data sets are shown in Figs. 2(a) and (b). The left plots give the distribution of the real part of the reconstructed contrast function and while the right plots give the distribution of the imaginary part of the reconstructed contrast function. The inversion results from TM data set (see Fig. 2(a)) show that the metallic cylinder is retrieved with real and imaginary parts having the same order of magnitude. These inversion results also show that there is an ambiguity in the inversion. In principle, when carrying out the inversion of a perfectly conducting cylinder one can only reconstruct uniquely the boundary of the object. Inside the metal object the contrast sources are invisible, with the consequence that any contrast inside the object may be arbitrarily arrived at. The small circular object with a large permittivity

value appearing in the image of $\operatorname{Re}(\chi_1)$ is obviously an artefact of the inversion algorithm. However since the reconstructed circular object in $\operatorname{Re}(\chi_1)$ lies completely inside the circular cylinder in $\operatorname{Im}(\chi_1)$, one can conclude that we are dealing with a metallic object. On the other hand, the imaginary parts of the contrast of the TE inversion do not exhibit any significant features (see Fig. 2(b)). However the shape of the large dielectric cylinder is better reconstructed using the TE inversion than the one using the TM inversion.

Next we invert both the TM and TE data simultaneously. The results of this joint inversion are given in Fig. 2(c). By inverting both TM and TE data simultaneously we obtain an improved reconstructed image of the large dielectric cylinder. Furthermore the small artefact in the image of $\text{Re}(\chi_1)$ is obviously lied completely inside the circular cylinder in $\text{Im}(\chi_1)$. Hence, we can conclude that by inverting both the TM and TE data simultaneously we can obtain more accurate reconstructed images than by inverting the TM and TE data sets separately.

4. Conclusions

In view of the present results and our crude approximation of the transmitting and receiving antennas, the Multiplicative Regularized Contrast Source Inversion method seems to be very robust and is capable of 'blindly' handling a wide class of inverse scattering problems. Finally we note that by inverting both TM- and TE-data simultaneously, we can obtain more accurate reconstructed images.

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New Stochastic AGLID EM Modeling and Inversion

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Abstract—The AGILD modeling and inversion have been published in PIERS2005 in Hangzhou. The new 2.5D AGILD and new GL electromagnetic (EM) modeling and inversion are publishing in PIERS2006 in Cambridge of USA. We have proposed the stochastic SGILD modeling and inversion in nano-physics and geophysics using magnetic field equations in 1999 and 2002. In this paper, we present a new stochastic AGILD (SAGILD) EM modeling and inversion and software. We propose a new EM integral equation on the strip domain. Based on the strip integral equation, we propose new stochastic EM field strip integral equations for mean, covariance, and second order mean EM field and parameters. These stochastic EM moments strip integral equations are employed on the boundary strip domain. In the cylindrical and spherical coordinate system, the strip domain includes the boundary and pole strip involving pole $\rho = 0$ or r = 0, North Pole $\theta = 0$ and South Pole $\theta = \pi$. In the remainder internal domain, we use stochastic moments Galerkin equations. We coupled these equations to construct 3D and 2.5D SAGILD modeling algorithms. By decomposing the variance of the parameter mean, $\delta < \sigma + i\omega\varepsilon >= \delta < \sigma + i\omega\varepsilon >_0 + \delta < \sigma + i\omega\varepsilon >_2, \ \delta < \mu >= \delta < \mu >_0 + \delta < \mu >_2$, we derive the new stochastic EM parameters increment moment strip integral equation on the strip sub domain for isotropic or anisotropic random materials. We used the equation pair of EM parameter stochastic integral equations on the strip sub domain and EM parameter stochastic Galerkin equation in remainder sub domain to construct the SAGILD inversion. The SAGILD modeling and inversion have widely applications in the Earth, Sun, and Luna space EM field exploration, nanometer material and material sciences, geophysical and Earthquake exploration, MT, MAIL, VEMP, weather radar imaging, medical MRI and X-ray imaging, and environmental engineering, EM stirring in steel and metal continuous casting, seismic, and finances.

1. Introduction

We have proposed the stochastic SGILD EM modeling and inversion in geophysics and nano-physics using magnetic field equations in 1999 [1] and 2002 [2]. A new GL method and its advantages for resolving historical difficulties will be published in PIERS2006 in Cambridge of USA [3]. The AGILD modeling and inversion has been published in PIERS2005 in Hangzhou [4], and the 2.5D AGILD modeling and inversion will be published in PIERS2006 in Cambridge USA [5]. In this paper, we propose a new SAGILD EM modeling and inversion. First, we propose a new EM integral equation on the boundary strip for rectangle coordinate or boundary pole strip domain for cylindrical and spherical coordinate. Next, we propose new stochastic EM field integral equations for mean, covariance field and parameters, and second order mean field and parameter. These stochastic EM moment strip integral equations are employed on the strip domain. In the cylindrical and spherical coordinate system, the strip domain includes the boundary and pole strip domain with poles $\rho = 0$ or r = 0, North Pole $\theta = 0$ and South Pole $\theta = \pi$. In the remainder internal domain, we use stochastic moments EM Galerkin equations. We couple these equations to construct the 3D/2.5D SAGILD modeling. By decomposing the variance of the mean of the parameters, $\delta < \mu > = \delta < \mu >_0 + \delta < \mu >_2$, $\delta < \sigma + i\omega\varepsilon > = \delta < \sigma + i\omega\varepsilon >_0 + \delta < \sigma + i\omega\varepsilon >_2$. We derive the stochastic EM parameters moment strip integral equations on the strip sub domain for isotropic or anisotropic materials. These new parameter integral equations are described in our internal report in detail [6]. We use the EM parameter stochastic integral equations on the strip sub domain and EM parameter stochastic Galerkin equation in remainder sub domain to construct the SAGILD inversion. In the cylindrical and spherical coordinate, the strip domain contains the poles for resolving the coordinate singularity difficulty.

The SAGILD modeling and inversion have widely applications in the Earth, Sun, and Luna space EM field exploration, nanometer material and material sciences, geophysical and Earthquake exploration, MT, MAIL, VEMP, weather radar imaging, medical MRI and X-ray imaging, and environmental engineering, and EMS stirring in metal casting, random flows, and finances. The new SAGILD modeling and inversion have advantages over existing random method. The SAGILD methods have AGILD's merits and improved field and parameter moments and its confidences.

We arrange contents in this paper as follows. The introduction has been described in the section 1. In section 2, we propose a new EM strip integral equation. The new stochastic EM moment strip integral equation is presented in the section 3. The stochastic EM Garlekin equation is presented in the section 4. In section

5, we propose the stochastic EM modeling. The stochastic EM inversion is presented in the section 6. The applications are described in the section 7. In section 8, we describe conclusions.

2. The New EM Strip Integral Equation

We have proposed EM integral equations in the paper [3], in which, our EM integral equations are

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int_{\Omega} \begin{bmatrix} E_b^J(r', r) & H_b^J(r', r) \\ E_b^M(r', r) & H_b^M(r', r) \end{bmatrix} [D] \begin{bmatrix} E(r') \\ H(r') \end{bmatrix} dr',$$
(1)

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int_{\Omega} \begin{bmatrix} E^J(r', r) & H^J(r', r) \\ E^M(r', r) & H^M(r', r) \end{bmatrix} [D] \begin{bmatrix} E_b(r') \\ H_b(r') \end{bmatrix} dr'.$$
(2)

That is available for the rectangle, cylindrical, and spherical coordinate systems. In our paper [5], we proposed the 2.5D EM differential integral equation in the cylindrical coordinate system. In this section, we propose the new EM strip integral equations on the strip domain in rectangle or strip pole domain in the cylindrical and spherical coordinate for resolving coordinate singularity.

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int_{\Omega_s} \begin{bmatrix} E_b^J(r',r) & H_b^J(r',r) \\ E_b^M(r',r) & H_b^M(r',r) \end{bmatrix} [D] \begin{bmatrix} E(r') \\ H(r') \end{bmatrix} dr' - \int_{\partial\Omega_{s-}} \begin{bmatrix} H_b^J(r',r) & E_b^J(r',r) \\ H_b^M(r',r) & E_b^M(r',r) \end{bmatrix} \times \begin{bmatrix} E(r') \\ H(r') \end{bmatrix} d\vec{S}r', (3)$$

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int_{\Omega_s} \begin{bmatrix} E^J(r',r) & H^J(r',r) \\ E^M(r',r) & H^M(r',r) \end{bmatrix} [D] \begin{bmatrix} E_b(r') \\ H_b(r') \end{bmatrix} dr' - \int_{\partial\Omega_{s-}} \begin{bmatrix} H^J(r',r) & E^J(r',r) \\ H^M(r',r) & E^M(r',r) \end{bmatrix} \times \begin{bmatrix} E_b(r') \\ H_b(r') \end{bmatrix} d\vec{S}r'. (4)$$

The EM strip integral equations are dual equation each other. The equations (3) and (4) are available for the rectangle, cylindrical, and spherical coordinate systems. In the rectangle coordinate r = (x, y, z), $E = (E_x, E_y, E_z)$, dr = dxdydz, in the cylindrical coordinate $E = (E_\rho, E_\theta, E_z)$, $dr = \rho d\rho d\theta dz$, in spherical coordinate, $dr = r^2 \sin \theta dr d\theta d\phi$, $dS = r^2 d\theta d\phi r + r \sin \theta d\phi dr\theta + r dr d\theta \phi$, $E = (E_r, E_\theta, E_\phi)$. For the isotropic materials, the material matrix [D] is the 6×6 diagonal matrix with variance of the conductivity and permittivity, $(\sigma - \sigma_b) + i\omega(\varepsilon - \varepsilon_b)$, and the magnetic permeability, $\mu - \mu_b$. For anisotropic materials, the matrix [D] is a full matrix with variance of the anisotropic materials. Obviously, the EM strip integral equations have no any coordinate singularity.

3. The Stochastic EM Field Moment Strip Integral Equations

Upon substituting the decomposition of the EM field,

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_0^+ \begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_1^+ \begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_2^-,$$
(5)

and the decomposition of the material matrix

$$[D] = \langle [D] \rangle + [D]_s, \tag{6}$$

into the equation (3), we propose the stochastic EM moment strip integral equations,

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_{0} = \begin{bmatrix} E_{b}(r_{s}) \\ H_{b}(r) \end{bmatrix} + \int_{\Omega} \begin{bmatrix} E_{b}^{J}(r',r) & H_{b}^{J}(r',r) \\ E_{b}^{M}(r',r) & H_{b}^{M}(r',r) \end{bmatrix} \langle [D] \rangle \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{0} dr' - \int_{\partial\Omega-} \begin{bmatrix} H_{b}^{J}(r',r) & E_{b}^{J}(r',r) \\ H_{b}^{M}(r',r) & E_{b}^{M}(r',r) \end{bmatrix} \times \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{0} d\vec{S},$$
(7)

and the following three stochastic EM field moment strip integral equations. Let AGILDMI to be the stochastic EM field moment integral operator

$$AGILDMI\left(\begin{bmatrix}C_{\sigma E} \ C_{\mu E}\\C_{\sigma H} \ C_{\mu H}\end{bmatrix}, \begin{bmatrix}C_{\sigma \sigma} \ C_{\mu \sigma}\\C_{\sigma \mu} \ C_{\mu \mu}\end{bmatrix}, \begin{bmatrix}E_{0}\\H_{0}\end{bmatrix}\right) = \begin{bmatrix}C_{\sigma E} \ C_{\mu E}\\C_{\sigma H} \ C_{\mu H}\end{bmatrix}(r) - \int_{\Omega} \begin{bmatrix}E_{b}^{J}(r', r) \ H_{b}^{J}(r', r)\\E_{b}^{M}(r', r) \ H_{b}^{M}(r', r)\end{bmatrix} \langle [D]\rangle \begin{bmatrix}C_{\sigma E} \ C_{\mu E}\\C_{\sigma H} \ C_{\mu H}\end{bmatrix}(r') dr' - \int_{\Omega} \begin{bmatrix}E_{b}^{J}(r', r) \ H_{b}^{J}(r', r)\\E_{b}^{M}(r', r) \ H_{b}^{M}(r', r)\end{bmatrix} \langle [D]\rangle \begin{bmatrix}C_{\sigma E} \ C_{\mu E}\\C_{\sigma H} \ C_{\mu H}\end{bmatrix}(r') dr' + \int_{\partial\Omega_{-}} \begin{bmatrix}H_{b}^{J}(r', r) \ E_{b}^{J}(r', r)\\H_{b}^{M}(r', r)\end{bmatrix} \times \left(\begin{bmatrix}C_{\sigma E} \ C_{\mu E}\\H_{0} \ C_{\sigma \mu} \ H_{0} \ C_{\mu \mu}\end{bmatrix}\right)(r') d\vec{S},$$

$$(8)$$

$$4GILDMI\left(\begin{bmatrix} C_{\sigma E} \ C_{\mu E} \\ C_{\sigma H} \ C_{\mu H} \end{bmatrix}, \begin{bmatrix} C_{\sigma \sigma} \ C_{\mu \sigma} \\ C_{\sigma \mu} \ C_{\mu \mu} \end{bmatrix}, \begin{bmatrix} E_0 \\ H_0 \end{bmatrix}\right) = 0, \tag{9}$$

$$AGILDMI\left(\begin{bmatrix} C_{EE}C_{EH}\\ C_{EH}C_{HH} \end{bmatrix}, \begin{bmatrix} C_{\sigma E}C_{\mu E}\\ C_{\sigma H}C_{\mu H} \end{bmatrix}, \begin{bmatrix} E_{0}\\ H_{0} \end{bmatrix}\right) = 0,$$
(10)

$$\left\langle \begin{bmatrix} E \\ H \end{bmatrix}_{2} \right\rangle = \int_{\Omega} \begin{bmatrix} E_{b}^{J}(r',r) & H_{b}^{J}(r',r) \\ E_{b}^{M}(r',r) & H_{b}^{M}(r',r) \end{bmatrix} \left\langle [D] \right\rangle \left\langle \begin{bmatrix} E(r') \\ H(r') \end{bmatrix} \right\rangle_{2} dr - \int_{\partial\Omega-} \begin{bmatrix} H_{b}^{J}(r',r) & E_{b}^{J}(r',r) \\ H_{b}^{M}(r',r) & E_{b}^{M}(r',r) \end{bmatrix} \times \left\langle \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{2} \right\rangle d\vec{S}(r')$$

$$+ \int_{\Omega} \begin{bmatrix} E_{b}^{J}(r',r) & H_{b}^{J}(r',r) \\ E_{b}^{M}(r',r) & H_{b}^{M}(r',r) \end{bmatrix} \begin{bmatrix} \hat{C}_{\sigma E} \\ \hat{C}_{\mu H} \end{bmatrix} dr - \int_{\partial\Omega-} \begin{bmatrix} H_{b}^{J}(r',r) & E_{b}^{J}(r',r) \\ H_{b}^{M}(r',r) & E_{b}^{M}(r',r) \end{bmatrix} \times \begin{bmatrix} \hat{C}_{\sigma E} \\ \hat{C}_{\mu H} \end{bmatrix} d\vec{S}(r'),$$
(11)

4. The Stochastic EM Field Garlekin Equation

We propose the EM Garlekin equation in the rectangle, cylindrical, and spherical coordinate.

$$\begin{split} \oint_{\partial\Omega} [E \ H]_{0} \times \phi I d\vec{S} &- \int_{\Omega} [E \ H]_{0} \nabla \times \phi I d\Omega = \int_{\Omega} [E \ H]_{0} \begin{bmatrix} 0 & \langle \sigma \rangle \\ -i\omega \langle \mu \rangle & 0 \end{bmatrix} \phi I d\Omega + \int_{\Omega} [J_{s} \ M_{s}] \begin{bmatrix} 0 & 1 \\ -i\omega \langle \mu \rangle & 0 \end{bmatrix} \phi I d\Omega, \quad (12) \\ AGILDMG \left(\begin{bmatrix} C_{\sigma E} \ C_{\mu E} \\ C_{\sigma H} \ C_{\mu H} \end{bmatrix}, \begin{bmatrix} C_{\sigma \sigma} \ C_{\mu \sigma} \\ C_{\sigma \mu} \ C_{\mu \mu} \end{bmatrix}, \begin{bmatrix} E_{0} \\ H_{0} \end{bmatrix} \right) = \oint_{\partial\Omega} \begin{bmatrix} C_{\sigma E} \ C_{\mu E} \\ C_{\sigma H} \ C_{\mu H} \end{bmatrix} \times \phi I d\vec{S} - \int_{\Omega} \begin{bmatrix} C_{\sigma E} \ C_{\mu E} \\ C_{\sigma H} \ C_{\mu H} \end{bmatrix} \nabla \times \phi I d\Omega \\ - \int_{\Omega} \begin{bmatrix} 0 & \langle \sigma \rangle \\ -i\omega \langle \mu \rangle & 0 \end{bmatrix} \begin{bmatrix} C_{\sigma E} \ C_{\mu E} \\ C_{\sigma H} \ C_{\mu H} \end{bmatrix} \phi I d\Omega - \int_{\Omega} \begin{bmatrix} E_{0} C_{\sigma \sigma} \ E_{0} C_{\mu \sigma} \\ H_{0} C_{\sigma \mu} \ H_{0} C_{\mu \mu} \end{bmatrix} \phi I d\Omega, \quad (13) \end{split}$$

$$AGILDMG\left(\begin{bmatrix} C_{\sigma E} & C_{\mu E} \\ C_{\sigma H} & C_{\mu H} \end{bmatrix}, \begin{bmatrix} C_{\sigma \sigma} & C_{\mu \sigma} \\ C_{\sigma \mu} & C_{\mu \mu} \end{bmatrix}, \begin{bmatrix} E_0 \\ H_0 \end{bmatrix}\right) = 0, \tag{14}$$

$$AGILDMG\left(\begin{bmatrix} C_{EE} C_{EH} \\ C_{EH} C_{HH} \end{bmatrix}, \begin{bmatrix} C_{\sigma E} C_{\mu E} \\ C_{\sigma H} C_{\mu H} \end{bmatrix}, \begin{bmatrix} E_0 \\ H_0 \end{bmatrix}\right) = 0,$$
(15)

$$\oint_{\partial\Omega} \langle [E \ H]_2 \rangle \times \phi I d\vec{S} - \int_{\Omega} \langle [E \ H]_2 \rangle \nabla \times \phi I d\Omega = \int_{\Omega} \langle [E \ H]_2 \rangle \begin{bmatrix} 0 & \langle \sigma \rangle \\ -i\omega \langle \mu \rangle & 0 \end{bmatrix} \phi I d\Omega + \int_{\Omega} \begin{bmatrix} \hat{C}_{\sigma E} \ \hat{C}_{\mu H} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -i\omega \langle \mu \rangle & 0 \end{bmatrix} \phi I d\Omega, (16)$$

The above stochastic EM moment strip integral equations are available for isotropic materials. For anisotropic materials, the stochastic EM moment strip integral equations can be derived similarly. The $[C_{\sigma E}]$ is covariance moment matrix, $C_{\sigma E} = \langle E(r)\sigma(\Re) \rangle$, \langle , \rangle is assemble mean, other covariance terms are defined similarly. $\hat{C}_{IJ} = \langle I(r)J(\Re) \rangle|_{r=\Re}I = \sigma$, J = E, H, $[C_{IJ}] = [\langle I(r), J(\Re) \rangle]$, I, $J = \sigma$, μ .

5. The New Stochastic SAGILD EM Modeling

5.1 The AGILD Pair of the Strip Integral Equation and the Garlekin Equation

In our AGILD modeling [4], we couple the strip integral equation or differential integral equation in the strip domain and the Garlekin equation in the remainder internal domain for solving EM field. We call the strip integral equation and Garlekin equation to be AGILDM pair. In the section 4, the stochastic EM moment strip integral equations (7–11) and EM moment Garlekin equation (12–16) are used to form AGILD pair AGILDM{7,12}; AGILDM{9,14}, AGILDM{10,15}, AGILDM{11,16}.

5.2 The New SAGILD EM Modeling

We propose the SAGILD EM modeling as following five steps,

- (M.1) use AGILD modeling to solve the pair equations $AGILDM{7,12}$ for $\langle [E(r), H(r)] \rangle_0$;
- (M.2) use AGILD to solve the pair equations AGILDM $\{9,14\}$ for $[C_{IJ}(r))$, $I = \sigma, \mu, J = E, H$;
- (M.3) use AGILD to solve the pair equations AGILDM{10,15} for $[C_{IJ}(r))]$, I = E, H, J = E, H;
- (M.4) use AGILD to solve the pair equations AGILDM{11,16} for $\langle [E(r), H(r)] \rangle_2$;
- (M.5) to update $\langle [E(r), H(r)] \rangle_{=} \langle [E(r), H(r)] \rangle_{0} + \langle [E(r), H(r)] \rangle_{2}$.

6. The New Stochastic SAGILD EM Inversion

6.1 The EM Parameter Variation Moment Strip Integral Equations

$$\delta \begin{bmatrix} E \\ H \end{bmatrix}_{0} (r_{d}) = \int_{\Omega} \begin{bmatrix} E_{b}^{J}(r',r) & H_{b}^{J}(r',r) \\ E_{b}^{M}(r',r) & H_{b}^{M}(r',r) \end{bmatrix} \delta \langle [D]_{0} \rangle \begin{bmatrix} E \\ H \end{bmatrix}_{0} (r')dr' - \int_{\partial\Omega-} \begin{bmatrix} H_{b}^{J}(r',r) & E_{b}^{J}(r',r) \\ H_{b}^{M}(r',r) & E_{b}^{M}(r',r) \end{bmatrix} \times \delta \begin{bmatrix} E \\ H \end{bmatrix}_{0} (r')d\vec{S}, \quad (17)$$

$$AGILDII\left(\delta\begin{bmatrix} C_{\sigma E} & C_{\mu E} \\ C_{\sigma H} & C_{\mu H} \end{bmatrix}, \delta\begin{bmatrix} C_{EE} & C_{EH} \\ C_{EH} & C_{HH} \end{bmatrix}, \delta[D_0]\right) = \delta\begin{bmatrix} C_{EE} & C_{EH} \\ C_{EH} & C_{HH} \end{bmatrix} (r_d) - \int_{\Omega} \begin{bmatrix} E_b^J(r', r) & H_b^J(r', r) \\ E_b^M(r', r) & H_b^M(r', r) \end{bmatrix} \delta\begin{bmatrix} C_{\sigma E} & C_{\mu E} \\ C_{\sigma H} & C_{\mu H} \end{bmatrix} \begin{bmatrix} E \\ H \end{bmatrix}_0 dr'$$

$$+ \int_{\partial\Omega-} \begin{bmatrix} H_b^J(r',r) & E_b^J(r',r) \\ H_b^M(r',r) & E_b^M(r',r) \end{bmatrix} \times \delta \begin{bmatrix} C_{EE} & C_{EH} \\ C_{EH} & C_{HH} \end{bmatrix} d\vec{S},$$
(18)

$$AGILDII\left(\delta \begin{bmatrix} C_{\sigma E} & C_{\mu E} \\ C_{\sigma H} & C_{\mu H} \end{bmatrix}, \delta \begin{bmatrix} C_{EE} & C_{EH} \\ C_{EH} & C_{HH} \end{bmatrix}_{d}, \delta[D_{0}]\right) = 0,$$
(19)

$$AGILDII\left(\delta\begin{bmatrix} C_{\sigma\sigma} & C_{\mu\sigma} \\ C_{\sigma\mu} & C_{\mu\mu} \end{bmatrix}, \ \delta\begin{bmatrix} C_{\sigma E} & C_{\mu E} \\ C_{\sigma H} & C_{\mu H} \end{bmatrix}_d, \delta[D_0]\right) = 0,$$
(20)

$$\delta \left\langle \begin{bmatrix} E \\ H \end{bmatrix}_{2} \right\rangle (r_{d}) = \int_{\Omega} \begin{bmatrix} E_{b}^{J}(r',r) & H_{b}^{J}(r',r) \\ E_{b}^{M}(r',r) & H_{b}^{M}(r',r) \end{bmatrix} \delta \left\langle [D]_{2} \right\rangle \begin{bmatrix} E \\ H \end{bmatrix}_{0} (r') dr' - \int_{\partial\Omega-} \begin{bmatrix} H_{b}^{J}(r',r) & E_{b}^{J}(r',r) \\ H_{b}^{M}(r',r) & E_{b}^{M}(r',r) \end{bmatrix} \times \delta \left(\begin{bmatrix} E \\ H \end{bmatrix}_{0} + \left\langle \begin{bmatrix} E \\ H \end{bmatrix}_{2} \right\rangle d\vec{S} + \int_{\Omega} \begin{bmatrix} E_{b}^{J}(r',r) & H_{b}^{J}(r',r) \\ E_{b}^{M}(r',r) & H_{b}^{M}(r',r) \end{bmatrix} \delta \left\langle [D]_{0} \right\rangle \left\langle \begin{bmatrix} E \\ H \end{bmatrix}_{2} \right\rangle (r') dr' + \int_{\Omega} \begin{bmatrix} E_{b}^{J}(r',r) & H_{b}^{J}(r',r) \\ E_{b}^{M}(r',r) & H_{b}^{M}(r',r) \end{bmatrix} \delta \left[\hat{C}_{\sigma E} \\ \hat{C}_{\mu H} \end{bmatrix} (r') dr'.$$

$$(21)$$

6.2 The EM Parameter Variation Moment Garlekin Equations

$$\oint_{\partial\Omega} \delta[E \ H]_0 \times \phi I d\vec{S} - \int_{\Omega} \delta[E \ H]_0 \nabla \times \phi I d\Omega = \int_{\Omega} [E \ H]_0 \delta[D]_0 \phi I d\Omega + \int_{\Omega} [J_s \ M_s] \delta[D]_0 \phi I d\Omega, \tag{22}$$

$$AGILDGI\left(\delta\begin{bmatrix} C_{\sigma E} & C_{\mu E} \\ C_{\sigma H} & C_{\mu H} \end{bmatrix}, \delta\begin{bmatrix} C_{EE} & C_{EH} \\ C_{EH} & C_{HH} \end{bmatrix}_{d}, \delta[D_{0}]\right) = 0,$$
(23)

$$AGILDGI\left(\delta \begin{bmatrix} C_{\sigma\sigma} & C_{\mu\sigma} \\ C_{\sigma\mu} & C_{\mu\mu} \end{bmatrix}, \delta \begin{bmatrix} C_{\sigma E} & C_{\mu E} \\ C_{\sigma H} & C_{\mu H} \end{bmatrix}_{d}, \delta[D_{0}]\right) = 0.$$

$$(24)$$

$$AGILDGII\left(\delta\langle [D]_2\rangle, \left\langle\delta[E, H]_2\right\rangle_d, \delta\hat{C}_{DE}, \delta[D_0]\right) = 0.$$
(25)

6.3 The SAGILD EM Parameter Inversion

We propose the SAGILD EM inversion as the following six steps,

- (I.1) use AGILD inversion to solve the pair equations AGILDI{17,22} for $\langle \delta[D]_0 \rangle$;
- (I.2) use AGILD inversion to solve the pair equations AGILDI{19,23} for $\langle \delta[C_{DE}(r)] \rangle$;
- (I.3) use AGILD inversion to solve the pair equations AGILDI{20,24} for $\langle \delta[C_{DD}(r)] \rangle$;
- (I.4) use AGILD inversion to solve the pair equations AGILDI{21,25} for $\langle \delta[D]_2 \rangle$;
- (I.5) to update $\langle \delta[D] \rangle = \langle \delta[D]_0 \rangle + \langle \delta[D]_2 \rangle;$
- (I.6) to do iteration (I.1)–(I.5) with regularizing to find [D] such that $P([D])/P(E,H)(r_d)) = max$.

7. Applications

Because there are random noises in the field data and parameters in the experiment and industrial measurements, it is necessary to study stochastic EM field modeling and parameter inversion. Our SAGILD modeling and inversion have AGILD's significant merits and improved field and parameter moments and its confidences interval. Our SAGILD methods have widely applications in the Earth, Sun, and Luna space EM field exploration, nanometer and material sciences, geophysical and Earthquake exploration, MT, MAIL, VEMP, weather imaging, medical MRI and X-ray imaging, and environmental engineering, EM stirring in casting. SAGILD is used for finances, movie field, game field, seismic wave, acoustic wave, random flow filed, QEM particle wave in nano-physics and nano-biophysics and photosynthesis in anisotropic media.

8. Conclusions

The all integral equations and SAGILD methods in this paper are new original works. Field and synthetic random data tests show that the SAGILD method is high resolution, stable, and reasonable accurate moment modeling and inversion algorithms. It can be used to obtain the improved EM field and parameter mean with the second mean term, covariance, and standard deviations. SAGILD is very useful for estimating uncertainty and confidence interval for field and parameters. Our SAGILD software are effective tools for EM field and parameters, finances, movie field, game field, seismic, acoustic, random flow, QEM particle wave in nano-physics and biophysics and photosynthesis. Our SAGILG MCMC stochastic method and software are developing.

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Geometric Optics and Electromagnetic Models for Cylindrical Obstacles

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Abstract—A software prediction tool called EPICS (Enhanced Propagation for Indoor Communications Systems) was developed at the ESAT-TELEMIC division of the K. U. Leuven in two versions: a Geometric Optics (GO) version and a Physical Optics (PO) version. However, like many other three-dimensional package, this can only determine the signal in an environment that can be decomposed into (ir)regular hexahedral obstacles (with 6 sides like rectangular blocks, cubes, etc.) or (complex) combinations of them. Although most of the real life environment can be approximated by these hexahedral obstacles, this might lead to some artefacts like periodic radar cross section variations, the need for multiple diffractions to calculate the signal behind a cylindrical obstacle, or reflections that are ignored (e.g., because the approximated side plane is positioned so that a reflection on that plane can not reach the receiver) is existing. To calculate the signal more accurately for those cases, we need to implement curved obstacles into EPICS. In a first step to achieve this goal, the introduction of cylindrical obstacles is investigated.

In this paper, the general strategy is discussed. The first step is to determine the different intermediate (i.e., penetration, reflection and diffraction) points on the ray between transmitter and receiver. Efficient computational routines have been written and tested for this purpose, mostly solving the problem first in two dimensions (projected in a plane perpendicular to the axis of the cylinder) and then transforming this solution to the three-dimensional problem. Once these intermediate points have been found, one can start with the computation of the electromagnetic field.

In the case of a penetration, the intermediate point(s) can be found very easily (crossing point(s) of a line and a circle) and the electromagnetic computations don't differ from the computations with hexahedral obstacles. For the reflection by a non perfectly conducting surface, the plane wave Fresnel reflection coefficients can be used. Also the finite thickness of the cylindrical walls can be taken into account, using internal (multiple) reflections, if the losses are high or the reflection coefficient of the wall is not to large.

For the diffractions, the two-dimensional geometric problem that needs to be solved to find the diffraction points is the determination of the tangent line to a circle (both from transmitter and receiver). Note that both can have two tangent lines, and one might have to match the two corresponding diffraction points. In this case, the electromagnetic computations for the vertical (i. e., field component parallel with the axis of the cylinder) and horizontal polarisation are done separately. An important issue in these computations is the convergence of the series used for the calculation of the field.

The reflection points on a cylindrical wall can not be found as easily as in the previous two cases. In general, an iterative process is required. This implies that the search for a good starting value is an important issue. Therefore some efficient computer programs were written to find firstly a good starting value of the Newton-Raphson iteration. As for the electromagnetic computations, one has to take into account that the caustics are transformed after the reflections and thus another amplitude factor has to be taken into account.

Although the described routines are not (yet) a part of the EPICS software, new routines based on Geometric Optics (GO) have been written and tested (in matlab) to predict penetration, reflection and diffraction of electromagnetic fields around cylindrical obstacles. This will be used to compute the effects of a curved airport terminal on an Instrument Landing System (ILS).

1. Introduction

Most of the real life environment can be approximated by hexahedral obstacles, or combinations of different hexahedral obstacles. Of course this leads to some artefacts like periodic radar cross section variations, the need for multiple diffractions to calculate the signal behind a cylindrical obstacle, or reflections that are ignored, because the approximated side plane is positioned so that a reflection on that plane can not reach the receiver (see Figure 1). To calculate the signal more accurately for those cases, we need to implement cylindrical obstacles into the EPICS program [1].



Figure 1: Examples of combinations of hexahedral obstacles to more complex obstacles house (left) and conical tower (right).

For each phenomenon, i. e., penetration, diffraction and reflection we briefly discuss the routines to find the intermediate (penetration, diffraction and/or reflection) points [2]. In most cases, this implies that we first solve a two-dimensional problem which can be easily transformed to the three-dimensional solution. The main part of this paper, however, will be devoted to the electromagnetic computations of the field around these cylindrical obstacles.

1. Penetrations and Attenuation

In EPICS the "direct" field between 2 (intermediate) points is calculated in free space. However, this path might be obstructed by an obstacle. Therefore, each wall/obstacle obstructing this path introduces some attenuation of the signal strength. In general we have 3 possibilities: no penetration (e.g., the line transmitter-receiver is parallel to the axis of the cylinder but the distance between the two lines is bigger than the radius), one penetration (if either the transmitter or the receiver is inside the cylinder, while the other is outside, or in the tangent case) or two penetrations (general case).

1.1. How to Find the Penetration Points?

The routine to find the penetration points is rather easy: first we determine the crossing points of the line transmitter-receiver (or between 2 intermediate points) with the top and bottom plane of the cylinder. If these points are between the transmitter and receiver, and if the distance of these points to the centre of the top/bottom plane respectively is smaller than the radius of the cylinder, these are valid penetration points. The last step is to investigate the cylindrical wall. Therefore, we need to calculate the crossing points of the line between the projected locations of the transmitter and receiver and a circle. Figure 2 shows the side and top view of some examples (the transmitter is denoted by a \diamond , the receiver by a \circ and the penetration point(s) by an *).



Figure 2: Examples of penetration: both through the side walls (left) and one penetration through a side wall combined with a penetration through the reference/bottom plane (right).

1.2. The GO Penetrated Field

Classical Geometrical Optics (GO) states that the high-frequency electromagnetic field propagates along ray paths, which satisfy the principle of Fermat, which states that the propagation of waves associated with these high frequency fields can be reduced to the study of wave paths along which the travel time is minimal. For perpendicular polarisation, the incident field lies in the plane perpendicular to the plane of incidence (soft boundary conditions). Hard boundary conditions require the incident field to be parallel with the plane of incidence. For the reflection by a non-perfectly electromagnetic conducting surface the plane-wave Fresnel reflection coefficients can be used:

$$\Gamma_{\perp} = \frac{\epsilon' \cos \theta - \sqrt{\epsilon} - \sin^2 \theta}{\epsilon' \cos \theta + \sqrt{\epsilon} - \sin^2 \theta}$$

$$\Gamma_{\parallel} = \frac{\cos \theta - \sqrt{\epsilon' - \sin^2 \theta}}{\cos \theta + \sqrt{\epsilon' - \sin^2 \theta}}$$
(1)

where θ is the angle between the incidence ray and the normal of the penetrated plane, ϵ the permittivity and σ the conductivity of the wall. Also the finite thickness of the wall under investigation can be taken into account if the dimensions are small with respect to the distance between transmitter and receiver. In those cases, a plane wave model based on successive reflections within the slab leads to much better results (Figure 3). Only when the losses are small and is not close to 1, edge effects have to be taken into account. However, for practical cases of concrete and thick walls the losses are sufficiently high.



Figure 3: Multiple reflections within a slab.

If we suppose that walls can be approximated by a single slab of dielectric material we can easily see from (Figure 3) that the penetrated field is given by (2), where Γ is the appropriate reflection coefficient. Using this equation, the generalised transmission coefficient can be derived (3).

$$\vec{E}^{t} = \vec{E}^{i} \sum_{n=1}^{\infty} (1+\Gamma) \left(-\Gamma\right)^{2n-2} (1-\Gamma) e^{-2(n-1)s\alpha} e^{-2j(n-1)s\beta} e^{j(n-1)k_{0}d\sin\theta}$$
(2)

$$\tau_g = \frac{(1-\Gamma^2) e^{-s\alpha} e^{-js\beta}}{(1-\Gamma^2) e^{-2s\alpha} e^{-j2s\beta} e^{jk_0 d\sin\theta}}$$
(3)

where k_0 denotes the free space phase constant, while α and β are the plane wave attenuation and phase constant of a lossy medium [3], given by (4). As for the case of the generalised reflection coefficient, the penetration coefficient for given material parameters may depend to a great extent on the frequency and thickness used. Inversely, when thickness and frequency are known penetration measurements can be used to estimate the material parameters of different structures [4].

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}} \sqrt{\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}} \sqrt{\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1}$$
(4)

Figure 4 shows 2 examples of respectively a "perpendicular" incidence, where the line transmitter-receiver is perpendicular to the axis of the cylinder and a "non-perpendicular" incidence. In this last case an extra parameter m can be specified (note that the line transmitter-receiver is still crossing the axis of the cylinder). For these examples we used a wall with a thickness l of 0.1 m, a relative permittivity 2.5 (ϵ_r) and a conductivity of 0.036 (σ). The used frequency was 2.45 GHz.



Figure 4: Examples of penetration through a cylinder: perpendicular (left) and non-perpendicular (right) case.

Note that when m gets very high the losses through the faces are also bigger. For smaller incidence angles, resonance can occur in the wall, so that the losses are not directly proportional with s (see also Figure 3).

2. Diffractions

Again we can then solve the geometrical problem (see Figure 5). The determination of the diffraction points in a two-dimensional environment is rather easy: we draw the lines tangent to the circle from both the transmitter and the receiver (see top views). The last step is to determine which of the two points of the transmitter side corresponds with which point at the receiver side (smooth transmission between the air medium and the cylinder surface). Note that we only take diffractions around the cylinder into account. Thus, if one or both of the two diffraction points of one ray turns out to be above the "top" plane or below the "bottom" plane (reference plane), this ray is not taken into account (e. g., Figure 5).



Figure 5: Examples of diffraction: both diffractions are valid (left) and the righter diffraction is ignored (right).

2.1. Vertical Polarisation

We have considered a plane wave incident upon a perfectly conducting cylinder (Figure 6). The incident wave is linearly polarised with electric vector $\vec{E^i}$ parallel to the axis of the cylinder. The incident \vec{k} -vector is perpendicular to the axis of the cylinder. In terms of cylindrical coordinates, we have

$$\vec{E}^i = \vec{i_z} E_0 e^{jkx} = \vec{i_z} E_0 e^{-jk\rho\cos\theta_0} \tag{5}$$

In this analysis we follow the procedure described by Kong [5].



Figure 6: Scattering by a conducting cylinder.

To match the boundary conditions at $\rho = a$, we transform the plane wave solution into a superposition of cylindrical waves satisfying the Helmholtz wave equation in cylindrical coordinates:

$$e^{-jk\rho\cos\theta_0} = \sum_{m=-\infty}^{\infty} a_m J_m(k\rho) e^{jm\phi}$$
(6)

The constant a_m can be determined by using orthogonality relations for $e^{jm\phi}$. We multiply both sides by $e^{-jn\phi}$ and integrate over ϕ from 0 to 2π . In view of the integral representation for the Bessel function,

$$J_n(k\rho) = \frac{1}{2\pi} \int_0^{2\pi} e^{-jk\rho\cos\theta_0 - jn\phi + jn\pi/2} d\phi$$
(7)

we obtain $a_m = e^{-jn\pi/2}$ and

$$e^{-jk\rho\cos\theta_0} = \sum_{m=-\infty}^{\infty} J_m(k\rho)e^{jm\phi-jm\pi/2}$$
(8)

This expression is referred to as the wave transformation, which represents a plane wave in terms of cylindrical waves.

The scattered wave can also be expressed as a superposition of the cylindrical functions satisfying the Helmholtz wave equation. Expecting outgoing waves, we write the solution in terms of Hankel functions of the first kind. The sum of the incident wave and the scattered wave satisfies the boundary condition of a vanishing tangential electric field at $\rho = a$. We find the total solution to be

$$\vec{E} = \vec{i_z} E_0 \sum_{n=-\infty}^{\infty} \left[J_n(k\rho) - \frac{J_n(ka)}{H_n^{(1)}(ka)} H_n^{(1)}(k\rho) \right] e^{jn\phi - jn\pi/2}$$
(9)

The first summation term represents the incident wave; the second summation term, the scattered wave. Note that for $\rho = a$, the field from (9) becomes zero. In the far-field zone, where $k\rho \gg 1$, we can make use of the asymptotic formula for $H_n^{(1)}(k\rho)$ and find that the scattered wave takes the form of the first expression of (10) for small radii a, which can be expanded with respect to ka.

$$\vec{E}_{s} \approx \vec{i_{z}} E_{0} \sum_{n=-\infty}^{\infty} \sqrt{\frac{2}{\pi k \rho}} \frac{J_{n}(ka)}{H_{n}^{(1)}(ka)} e^{jk\rho + jn(\phi - \pi) - j\pi/4}$$
$$\vec{E}_{s} = \vec{i_{z}} j E_{0} \sqrt{\frac{2}{\pi k \rho}} \left[\frac{1}{\ln(ka)} + (ka)^{2} \cos \phi - \frac{(ka)^{4}}{8} \cos 2\phi + \dots \right] e^{jk\rho - j\pi/4}$$
(10)

This series converges rapidly when the radius of the cylinder is small compared with the wavelength, $ka \ll 1$. The first term is angle-independent and signifies that the scattered wave caused by an infinitely thin wire is isotropic.

2.2. Horizontal Polarisation

We have also generalised the procedure and implemented the diffraction by a conducting cylinder for horizontal polarisation. In this case, the electrical field can be expressed like this (see Figure 6):

$$\vec{E}^i = i_y E_0 e^{-jk\rho\cos\phi} \tag{11}$$

The scattered wave takes the following form:

$$\vec{E}_s = \vec{i_{\rho}} \sum_{n=-\infty}^{\infty} a_n H_n^{(1)}(k\rho) e^{jn(n-\pi/2)} + \vec{i_{\phi}} \sum_{n=-\infty}^{\infty} b_n H_n^{(1)}(k\rho) e^{jn(n-\pi/2)}$$
(12)

Once again, we have to require that the ϕ -component of the total field (incident and scattered field) vanishes for $\rho = a$.

The ϕ -component of the incident field (11) can be written as:

$$\vec{i}_{\phi} = -\vec{i}_{x}\sin\phi + \vec{i}_{y}\cos\phi$$
$$\vec{E}_{\phi}^{i} = -\vec{E}_{0}e^{-jk\rho\cos\phi}\cos\phi$$
(13)

By differentiating Eq. (8) with respect to ρ we obtain:

$$-jke^{jk\rho\cos\phi} = k\sum_{n=-\infty}^{\infty} J'_n(k\rho)e^{-jn(n-\pi/2)}$$
(14)

where the derivative of the Bessel function can be found from [6]:

$$J'_{n}(z) = \frac{J_{n-1}(z) - J_{n+1}(z)}{2}$$
(15)

$$J_0'(z) = -J_1^2(z)$$
(16)

When considering only the ϕ -component of the scattered field (12), we find (17). Indeed, the ϕ -component vanishes in the far field. This expression can be simplified as we have done above for the vertical polarisation.

$$\vec{E} = \vec{i_{\phi}} E_0 \sum_{n=-\infty}^{\infty} \left[J_n'(k\rho) - \frac{J_n'(ka)}{H_n^{(1)}(ka)} H_n^{(1)}(k\rho) \right] e^{jn(\phi - \pi/2)}$$
(17)

In Figure 7 both the vertical and horizontal component are shown for 2 examples. Note that the horizontal component gets stronger as the radius of the cylinder increases.



Figure 7: Examples of diffracted fields around a cylinder.

3. Reflections

3.1. Iterative Process Required to Find the Reflection Points

Whereas for the previous phenomena, the determination of the intermediate points was rather easy, this requires some more attention in the case of a reflection. Of course, one can determine some easy cases as well, e.g., reflections on top/bottom plane, symmetrical cases, etc. The general case for the determination of the reflection point(s), is somewhat more complicated. To find the solutions of the two-dimensional problem we have to solve a fourth degree equation iteratively [2]. This equation is derived by drawing a tangent line on the circle through a chosen reflection point on the circle to determine the mirror images of the transmitter (see Figure 8).

From those points, one can compute the points on the line transmitter-receiver (λ_2 and λ_2) where the signal will be reflected to (i.e., the crossing points between this line and the lines from the mirror image of the



Figure 8: Sketch of general case.

X-axis



Figure 9: Examples of reflection on a cylinder: without (left) and with (right) reflections on the top and bottom plane.

transmitter and the reflection points under investigation, determined by λ_1). The goal is to determine λ_1 so that the vector determined by λ_2 , λ_{2b} respectively, is equal to the projection of the receiver. This implies that λ_2 and λ_{2b} should be equal to 1, leading to Eq. (18).

$$\lambda_{2,2b} = \lambda_1 \left[\frac{\pm 2R_c \sqrt{a' + b'\lambda_1 + c'\lambda_1^2} - 2a' - b'\lambda_1}{\pm R_c \sqrt{a' + b'\lambda_1 + c'\lambda_1^2} - a' + c'\lambda_1^2} \right]$$

$$\lambda_{2,2b} = 1? \Leftrightarrow A_4 \lambda_1^4 + A_3 \lambda_1^3 + A_2 \lambda_1^2 + A_1 \lambda_1 + A_0 = 0$$
(18)

where a' is the quadratic norm of the projected transmitter ($\lambda_1 = 0$), b' twice the scalar product between this vector and the vector between projected transmitter and receiver, c' the quadratic norm of this last vector and Rc the radius of the cylinder.

Unfortunately, we don't always have the possibility to solve a linear equation of the fourth order. Therefore, we will solve this problem iteratively by using the Newton-Raphson method. One can see that equation (18) has 4 possible singularities (nominator equal to zero), and that they are difficult to calculate (start value of Newton-Raphson has to be on the right side of these singularities). Therefore we will search a solution for the

inverse function $(1/\lambda_2 = 1)$. The last step will be again the transformation of the two-dimensional solution to the three-dimensional solution (excluding reflection points on the cylindrical wall that lie above the top plane or below the bottom plane).

3.2. Reflected Field Computations

For the implementation of the computation of the reflected field, one has to keep in mind that after the reflection, the location of the caustics, both for parallel and perpendicular to the axis of the cylinder, might have been changed as is shown in Figure 10.



Figure 10: Reflection against a curved surface (parallel case).

Taking a cross-section along one of the radii of curvature, and expressing the arc on the circle as a function of the viewing angles, one can obtain:

$$a\Delta\alpha\cos\theta_0 = l\Delta\gamma_1 = \rho\Delta\gamma_2 \tag{19}$$

where $\Delta \gamma_1 = \Delta \theta_0 - \Delta \alpha$ and $\Delta \gamma_2 = \Delta \theta_0 + \Delta \alpha$. Eliminating $\Delta \alpha / \Delta \theta_0$ this leads to

$$\frac{1}{\rho_i} = \frac{1}{l} + \frac{2}{R_i \cos \theta_0}$$

$$\frac{1}{R_1} = \frac{\cos^2 \alpha}{a}$$

$$\frac{1}{R_2} = \frac{\sin^2 \alpha}{a}$$
(20)

where R_i represents the radius of curvature (parallel and perpendicular to the axis). Indeed, it can be shown in analysis that the radius of curvature of a function y(x) is given by:

$$R_i = \frac{y''}{\sqrt{(1+y'^2)^3}}$$
(21)

In general the cut of a cylinder is an ellipse which can be expressed by $(x/a')^2 + (y/b')^2 = 1$, where a' = aand $b' = a/\cos \alpha$, bearing in mind that α is the angle between the axis of the cylinder and the cut. Using (21) at the expression of the ellipse, one obtain the formulas of (20). Note that for the parallel case R_2 will become infinite. This implies that the distance to the new caustics can be computed:



Figure 11: A bunch of rays with a different radius of curvature.

$$\frac{1}{\rho_1} = \frac{1}{l} + \frac{2\cos^2\alpha}{a\cos\theta_0}$$
$$\frac{1}{\rho_2} = \frac{1}{l} + \frac{2\sin^2\alpha}{a}$$
(22)

Keeping in mind that the total distance after reflection is given by $d_i = \rho_i + s$, this implies that the field attenuation after reflection can be computed using:

$$|E| = |E_0| \sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}}$$
(23)

where $|E_0|$ is the field at reflection point \vec{M} . This attenuation has to be multiplied by the reflection coefficients which can be determined from the slab-approximation of the wall (see Figure 3).

$$\vec{E}^{r} = \vec{E}^{i} \left[\Gamma + \sum_{n=1}^{\infty} \left(1 + \Gamma \right) \left(-\Gamma \right)^{2n-1} \left(1 - \Gamma \right) e^{-2ns\alpha} e^{-2jns\beta} e^{jnk_{0}d\sin\theta} \right]$$
(24)

$$\Gamma_g = \Gamma \left[1 - \frac{\left(1 - \Gamma^2\right) e^{-2s\alpha} e^{-j2s\beta} e^{jk_0 d\sin\theta}}{\left(1 - \Gamma^2\right) e^{-2s\alpha} e^{-j2s\beta} e^{jk_0 d\sin\theta}} \right]$$
(25)

3.3. Case Study: Brussels Airport Terminal

At Brussels airport, a few years ago a new terminal was build. This A-terminal has a curved shape, to reduce the influence on the Instrument Landing System (ILS) of the neighbouring runway. This ILS systems allows blind landings, and thus has to be very reliable. Using a curved shape, the effect of this new terminal was reduced radically. Figure 12 shows the effect of a rectangular building (left) and a curved building (right) on the differnce pattern of the ILS system (zero along the runway). Note that the building was approximated by a cylinder with a horizontal axis, which comes close to the current shape of this A-terminal. One can clearly see that in the zone where reflections can occur (between 3720 and 5200 m along the x-axis), the effect of the cylindrically shaped building is much smaller.



Figure 12: Comparison between rectangular shaped (left) and curved shaped (right) A-terminal for the difference-pattern of the ILS system.

4. Conclusion

In this paper we investigated the influence of a cylindrical obstacle on the electromagnetic signal. Though it is not presented as a part of the EPICS software yet, new routines based on Geometric Optics (GO) have been written and tested to predict penetration, reflection and diffraction of electromagnetic fields around cylindrical obstacles as a step in a future implementation in EPICS.

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3D and 2.5D AGLID EMS Stirring Modeling in the Cylindrical Coordinate System

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Abstract—We have proposed the new GL and AGILD modeling and inversion in the PIERS 2005 in Hangzhou. In this paper, we propose 2.5D AGILD modeling algorithm for electromagnetic (EM) stirring, motor and generator design. In the cylindrical coordinate system, the EM field is vector function of r, θ , and z. The electrical conductivity is only depended on radial coordinate r and vertical coordinate z. Upon substituting the Fourier serious of the magnetic field into the strip differential integral equation on boundary strip with pole $\rho = 0$ and Galerkin equation in the internal sub domain, we construct 2.5D AGILD EM stirring modeling in cylindrical coordinate system for the steel and metal continuous casting. There are serious difficulties in the EM stirring modeling by using FEM method and FD method. First, there is u/ρ^2 term in the Maxwell magnetic field differential equation in the cylindrical coordinate system, the pole $\rho = 0$ is strong coordinate singularity. The coordinate singularity is difficult in the EM stirring modeling by using FEM and FD method. Our 2.5 AGILDEMS modeling method resolved this difficulty. There is no any coordinate singularity in our 2.5D EM differential integral equation. Second, because the conductivity in air is zero but it is 10^5 in steel, what is a suitable boundary condition on $\rho = 0$ for current, electric field, and magnetic field that is another difficulty when FEM method and FD method to be used. Our AGILDEMS overcome this difficulty. Based on our 2.5D AGILDEMS algorithm, we developed the 2.5D AGILDEMS modeling software. Many applications show that the 2.5D AGILDEMS software is a powerful tool for design of the EM stirring and real time control monitor in the continuous casting. The AGILD K- ε flow modeling and software are developing and joining with our AGILD EMS modeling for continuous casting. GL EMS and AGILD EMS modeling can be used for micro, nano motor, generator and geophysics and materials.

1. Introduction

In the steel and metal continuous caster, the electromagnetic (EM) stirring (EMS) is an established technique and important approach for improving steel quality. Many EMS with variable style have been working in the steel and metal continuous caster industrial in the world. To exactly calculate the EM field and determine the bloom/billet's size and properties in EMS are an important and difficult task. Because the conductivity in the air environmental is zero but 50,000 1/ohm in steel. The sharp high contrast is difficult in inversion. The EM field artificial boundary condition for infinite domain is inaccurate and complicated. The coordinate singularity is another difficulty in FEM for EMS modeling in the cylindrical coordinate system. The existing EM FEM method and software are not accurate to calculate EM field in EMS. The EMS properties inversion for steel material and conductivity is necessary to develop. We have proposed the new GL and AGILD modeling and inversion in the PIERS 2005 in Hangzhou [1,2]. We propose the GL method and its advantages for resolving the historical difficulties [3] and the stochastic AGILD EM modeling and inversion in Piers 2006 in Cambridge [4]. In this paper, we propose the 2.5D AGILD EMS stirring modeling using our magnetic field differential integral equation and magnetic field Garlekin equation. Our AGILD EMS modeling is an important tool for EMS design and EMS real time processes monitoring in the continuous caster. Also EMS modeling and inversion are useful for variable motor and generator design, environment, geophysics, coaxial antenna, etc. sciences and engineering.

The description order in this paper is as follows. In the section 2, we derive the 3D and 2.5D magnetic field strip differential integral equations in the cylindrical coordinate system. The 3D and 2.5D magnetic field strip Garlekin equations are derived in the section 3. In the section 4, we present the 3D and 2.5D EMS modeling. The applications of the EMS modeling is described in the section 5. In the section 6, we describe conclusions.

2. The 3D and 2.5D Magnetic Field Strip Differential Integral Equations

We derive the 3D and 2.5D magnetic field differential integral equations in the strip domain in the cylindrical coordinate system in this section. We call the equations to be the strip magnetic field differential integral equations.

2.1. The 3D Magnetic Field Strip Differential Integral Equation

Upon substituting the field and coordinate transformation between the rectangle and cylindrical coordinate system, we derive the 3D magnetic field strip differential integral equation in the cylindrical coordinate system as follows

$$FH3(H,H_{b\rho},H_{b}^{M_{\rho}},E_{b}^{M_{\rho}}) = H_{b\rho} + \int_{\Omega} \frac{((\sigma+i\omega\varepsilon)-(\sigma_{b}+i\omega\varepsilon_{b}))}{(\sigma+i\omega\varepsilon)} \left(E_{b\rho}^{M_{\rho}} \left(\frac{1}{\rho'}\frac{\partial H_{z}}{\partial \theta} - \frac{\partial H_{\theta}}{\partial z}\right) + E_{b\theta}^{M_{\rho}} \left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho'}\right) + E_{bz}^{M_{\rho}} \left(\frac{1}{\rho'}\frac{\partial}{\partial \rho'}(\rho'H)_{\theta} - \frac{1}{\rho'}\frac{\partial H_{\rho}}{\partial \theta}\right) (r')d\rho'd\rho'd\theta dz + \int_{\partial\Omega-} \frac{1}{(\sigma_{b}+i\omega\varepsilon_{b})} E_{b}^{M_{\rho}} \times H \cdot d\vec{S} - \int_{\partial\Omega-} \frac{1}{(\sigma+i\omega\varepsilon)} \left(\left(H_{b\theta}^{M_{\rho}} \left(\frac{1}{\rho'}\frac{\partial}{\partial \rho'}\rho'H_{\theta} - \frac{1}{\rho'}\frac{\partial H_{\rho}}{\partial \theta}\right) - H_{bz}^{M_{\rho}} \left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho'}\right) \rho dz + \left(H_{b\rho}^{M_{\rho}} \left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho'}\right) - H_{b\theta}^{M_{\rho}} \left(\frac{1}{\rho'}\frac{\partial H_{z}}{\partial \theta} - \frac{\partial H_{\theta}}{\partial z}\right) \right) \rho'd\rho',$$

$$(1)$$

$$H_{\rho}(r) = FH3(H, H_{b\rho}, H_{b}^{-r}, E_{b}^{-r}),$$

$$H_{\theta}(r) = FH3(H, H_{b\theta}, H_{b}^{M_{\theta}}, E_{b}^{M_{\theta}}),$$

$$H_{z}(r) = FH3(H, H_{bz}, H_{b}^{M_{z}}, E_{b}^{M_{z}}),$$

$$(2)$$

where E is the electric field, H is the magnetic field, E_b^m and H_b^m is Green function exciting by the magnetic dipole source, $E_b^m(r', r)$ has weak and integrative singular at r = r', the r locates in the outside boundary of the strip or in the subsurface with $\rho' = 0$, the r' locates in $\partial\Omega$ -, the internal boundary of the strip, therefore, the 3D strip magnetic field differential integral equation has no coordinate singular at pole $\rho' = 0$. It has integrative weak singular kernel.

2.2. The 2.5D Magnetic Field Differential Integral Equation

Substituting the EM field Fourier series, $H(\rho, \theta, z) = \sum_{m=-\omega}^{\infty} H_m(\rho, z) e^{im\theta}$, into the 3D strip magnetic field differential integral equation (2), we derive the 2.5D equations in the cylindrical coordinate system

$$FH25(H, H_{b\rho}, H_{b}^{M\rho}, E_{b}^{M\rho}) = H_{b\rho} + \int_{\Omega} \frac{((\sigma + i\omega\varepsilon) - (\sigma_{b} + i\omega\varepsilon_{b}))}{(\sigma + i\omega\varepsilon)} \left(E_{b\rho}^{M\rho} \left(\frac{1}{\rho'} imH_{z} - \frac{\partial H_{\theta}}{\partial z} \right) + E_{b\theta}^{M\rho} \left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho'} \right) + E_{bz}^{M\rho} \left(\frac{1}{\rho'} \frac{\partial}{\partial \rho'} \rho H_{\theta} - \frac{1}{\rho'} imH_{\rho} \right) (r')\rho' d\rho' d\theta' dz'$$

$$+ \int_{\partial\Omega} \frac{1}{(\sigma_{b} + i\omega\varepsilon_{b})} E_{b}^{M\rho} \times H \cdot d\vec{S}$$

$$(3)$$

$$- \int_{\partial\Omega} \frac{1}{(\sigma + i\omega\varepsilon)} \left(\left(H_{b\theta}^{M\rho} \left(\frac{1}{\rho'} \frac{\partial}{\partial \rho'} \rho' H_{\theta} - \frac{1}{\rho'} imH_{\rho} \right) - H_{bz}^{M\rho} \left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho'} \right) \right) \rho dz + \left(H_{b\rho}^{M\rho} \left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho'} \right) - H_{b\theta}^{M\rho} \left(\frac{1}{\rho'} imH_{z} - \frac{\partial H_{\theta}}{\partial z} \right) \right) \rho' d\rho',$$

$$H_{\rho} = FH25(H, H_{b\rho}, H_{b}^{M\rho} E_{b}^{M\rho}),$$

$$H_{\theta} = FH25(H, H_{b\rho}, H_{b}^{M\rho} E_{b}^{M\rho}),$$

$$H_{z} = FH25(H, H_{bz}, H_{b}^{Mz} E_{b}^{Mz}),$$

3. The 3D and 2.5D Magnetic Filed Garlekin Equation

We derive the 3D and 2.5D magnetic field Garlekin equation in the cylindrical coordinate system.

3.1. The 3D Magnetic Field Garlekin Equation

Substituting field and coordinate transformation from rectangle to cylinder into the magnetic field Galerkin equation [2], we derive the 3D magnetic field Garlekin equation in the cylindrical coordinate system as follows

$$\int_{\Omega} \frac{1}{\sigma + i\omega\varepsilon} \left(\left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho} \right) \frac{\partial \phi}{\partial z} - \left(\frac{\partial}{\partial \rho} \rho H_{\theta} - \frac{\partial H_{\rho}}{\partial \theta} \right) \frac{1}{\rho^{2}} \frac{\partial \phi}{\partial \theta} \right) \rho d\rho d\theta dz + i\omega \int_{\Omega} \mu H_{\rho} \phi \rho d\rho d\theta dz = -i\omega \int_{\Omega} \mu M_{\rho} \phi \rho d\rho d\theta dz, \\
\int_{\Omega} \frac{1}{\sigma + i\omega\varepsilon} \left(-\left(\frac{1}{\rho} \frac{\partial H_{z}}{\partial \theta} - \frac{\partial H_{\theta}}{\partial z} \right) \frac{\partial \phi}{\partial z} + \left(\frac{\partial}{\partial \rho} \rho H_{\theta} - \frac{\partial H_{\rho}}{\partial \theta} \right) \frac{1}{\rho^{2}} \frac{\partial \rho \phi}{\partial \rho} \right) \rho d\rho d\theta dz + i\omega \int_{\Omega} \mu H_{\theta} \phi \rho d\rho d\theta dz = -i\omega \int_{\Omega} \mu M_{\theta} \phi \rho d\rho d\theta dz, \quad (5) \\
\int_{\Omega} \frac{1}{\sigma + i\omega\varepsilon} \left(\left(\frac{1}{\rho} \frac{\partial H_{z}}{\partial \theta} - \frac{\partial H_{\theta}}{\partial z} \right) \frac{1}{\rho} \frac{\partial \phi}{\partial \theta} - \frac{\partial \phi}{\partial \rho} \left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho} \right) \right) \rho d\rho d\theta dz + i\omega \int_{\Omega} \mu H_{z} \phi \rho d\rho d\theta dz = -i\omega \int_{\Omega} \mu M_{z} \phi \rho d\rho d\theta dz, \quad (5)$$

3.2. The 2.5D Magnetic Field Garlekin Equation

Upon substituting the Fourier series, $H(\rho, \theta, z) = \sum_{m=-\omega}^{\infty} H_m(\rho, z) e^{im\theta}$ into the 3D Garlekin equation (5), we derive the 2.5D magnetic field Garlekin equation in the cylindrical coordinate system as follows

$$\int_{\Omega} \frac{1}{\sigma + i\omega\varepsilon} \left(\left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho} \right) \frac{\partial \phi}{\partial z} - \left(\frac{\partial}{\partial \rho} \rho H_{\theta} - imH_{\rho} \right) \frac{1}{\rho^{2}} im\phi \right) \rho d\rho dz + i\omega \int_{\Omega} \mu H_{\rho} \phi \rho d\rho dz = -i\omega \int_{\Omega} \mu M_{\rho} \phi \rho d\rho dz,$$

$$\int_{\Omega} \frac{1}{\sigma + i\omega\varepsilon} \left(-\left(\frac{1}{\rho} imH_{z} - \frac{\partial H_{\theta}}{\partial z} \right) \frac{\partial \phi}{\partial z} + \left(\frac{\partial}{\partial \rho} \rho H_{\theta} - imH_{\rho} \right) \frac{1}{\rho^{2}} \frac{\partial \rho \phi}{\partial \rho} \right) \rho d\rho dz + i\omega \int_{\Omega} \mu H_{\theta} \phi \rho d\rho dz = -i\omega \int_{\Omega} \mu M_{\theta} \phi \rho d\rho dz, \quad (6)$$

$$\int_{\Omega} \frac{1}{\sigma + i\omega\varepsilon} \left(\left(\frac{1}{\rho} imH_{z} - \frac{\partial H_{\theta}}{\partial z} \right) \frac{1}{\rho} im\phi - \frac{\partial \phi}{\partial \rho} \left(\frac{\partial H_{\rho}}{\partial z} - \frac{\partial H_{z}}{\partial \rho} \right) \right) \rho d\rho dz + i\omega \int_{\Omega} \mu H_{z} \phi \rho d\rho dz = -i\omega \int_{\Omega} \mu M_{z} \phi \rho d\rho dz,$$



Figure 1: Rotation magnetic field H_{θ} in time =0 s.



Figure 2: Rotation magnetic field H_{θ} in time =0.1 s.

4. 3D and 2.5 D EMS Modeling

4.1. 3D EMS Modeling

We use collocation FEM the 3D strip magnetic field differential integral equation (2) in the boundary strip domain including pole point $\rho = 0$, and the 3D magnetic field Galerkin equation (5) in the reminder internal domain without pole $\rho = 0$ to construct 3D AGILD EMS magnetic field modeling for EM field in the Stirring and motor etc industrial engineering and sciences.

4.2. 2.5D EMS Modeling

We use collocation FEM the 2.5D strip magnetic field differential integral equation (4) in the boundary strip domain including pole point $\rho = 0$, and use the 2.5D magnetic field Galerkin equation (6) in the reminder internal domain without pole $\rho = 0$ to construct 2.5D AGILD EMS magnetic field modeling for EM field in the stirring and motor etc industrial engineering and sciences.

5. The Applications of the AGILD EMS Modeling

Our 3D and 2.5D AGILD and GL EMS modeling has been used to calculate the EM field for several EM stirring with variable style. Some asynchronous EMS stirring is designed as follows: its outer radius is 500 mm, the internal radius is 350 mm, and it is divided 6 sectors. The electric current has inverse direction for any adjoining two sectors. The input electric current density intensity is 1 A/mm^2 . The frequency is 4 Hz. Before installation of the stirring without steel flow, the factor did measure the magnetic field intensity. By using digit magnetic GAUSS meter, the measurement value of the magnetic field intensity at center of the stirring is 1500 Gauss. By using our 2.5D AGILD EMS modeling simulation, the evaluated magnetic field intensity is 1513.28 Gauss at center of the stirring. The rotational EM field is very accurate and very stable. The AGILD EMS

rotation magnetic field in caster $H\rho(\rho, \theta, zc, t)$ at the 0.0~0.25 second are plotted in the Figures 1 and 4. They show that by using the GL EMS and AGILD EMS modeling, the rotational magnetic field's frequency is exactly 4 Hz. The GL EMS [3, 5] and 2.5D AGILD EMS magnetic field H_{ρ} , H_{θ} intensity are plotted in Figures 5 and 6, the red curve is the GL magnetic field and blue curve is AGILD magnetic field, the two curves are close matched. GL EMS and AGILD EMS modeling can be used for micro, nano motor, generator and group holes geophysics and materials etc. We are developing GL and AGILD K- ε model steel flow driving by the EMS Lorentz force and join it with AGILD EMS modeling to work for the steel and metal continuous casters.



Figure 3: Rotation magnetic field H_{θ} in time =0.2 s.



Figure 4: Rotation magnetic field H_{θ} in time =0.25 s.



Figure 5: The magnetic field H_{ρ} intensity, The red line is GL magnetic field, The blue line is AGILD magnetic field.



Figure 6: The magnetic field H_{θ} intensity, The red line is GL magnetic field, The blue line is AGILD magnetic field.

6. Conclusions

Many EM field in the stirring and motor simulations show that the 3D and 2.5D AGILD, and GL EMS modeling are accurate and fast and stable. The AGILD EMS has merits over existing FEM, FD, and Born approximation. The 3D and 2.5D AGILD and GL EMS modeling will be new tools for widely applications in the sciences and engineering.

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Reduction of FDTD Simulation Time with Modal Methods

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Abstract—In order to simulate electromagnetic phenomena at high frequencies, full wave solvers such as the FDTD method must be used. An alternative to the conventional FDTD method is to compute the zero state response with convolution. Convolution results in an increased computation time with every time step. By performing eigenmodal decomposition of the inputs, a constant time for the convolution can be achieved. We show how the solution can be constructed analytically in terms of the eigenvalues and the eigenvectors of the state transition matrix.

1. Introduction

FDTD is an evolutionary scheme that solves Maxwell's equations in the time domain [1,2]. The evolution continues until steady state or stability in the output is achieved. Schemes of this type are often used when the analytical solution to an electromagnetic problem is prohibitive. Problems to be solved with FDTD are abundant in simulations of aircraft radar cross section at high frequency, microwave ICs, optical pulse propagation, antennas, bioelectromagnetic systems, bodies of revolution, etc. [1]. Situations where it is important to model on-chip interconnect include various microwave circuits such as amplifiers and optoelectronic circuits fabricated in CMOS technology. Reference [3] discusses the design of on-chip waveguides at optical frequencies and reference [4] discusses microwave frequencies. Such real-life problems often require grids with very large numbers of points, due to fine features of the simulated objects and high excitation frequencies. The end result of the fine grids is unreasonable simulation time. With the method proposed in this paper it may be possible to reduce this simulation time to a more acceptable level.

The starting point for the FDTD solution can be the initial conditions, such as an excitation signal. If the solution grid is partitioned into sub grids (i.e., for distributed computation) each containing N field variables, then the starting point is either the initial conditions or the inputs from the adjacent sub grids. We use the $N \times 1$ vector $\mathbf{Q}(n)$ to denote the state of every electric and magnetic field variable in the sub grid. The $N \times N$ state transition matrix $\mathbf{A}(n-i)$ is used to obtain the state at time n from the state at time i. We also define the $N \times 1$ input vector $\mathbf{X}(n)$, to represent the inputs to the sub grid at time n. The manipulation of these matrices in order to get the output of the sub grid, also called a module, was discussed in [5] and only the basic results are given here.

If the inputs are combined in $\mathbf{X}(n)$, an $I \times 1$ vector, then $\mathbf{Y}(n)$, the $O \times 1$ output vector of the module is given as:

$$\mathbf{Y}(n) = [\mathbf{C}\mathbf{A}(n)\mathbf{B}] * \mathbf{X}(n) \tag{1}$$

where the * symbol represents convolution, and the term in brackets is the impulse response $\mathbf{h}(n)$ of the FDTD module. From the results in [5] and from Equation (1) we can observe that the computing time grows with every time step, due to the properties of convolution. Therefore this method is useful only at early stages in the simulation when the number of inputs is small, and the convolution workload does not exceed the time to simulate the module with the standard FDTD.

A strategy to overcome this limitation for the TLM method was discussed in [6]. It involves writing each entry in the location (i, o) of the impulse response matrix as a sum of the eigenvalues of the state transition matrix as follows:

$$h(i,o,n) = \sum_{p=1}^{P} b_{iop} \lambda_p^n = \sum_{p=1}^{P} b_{iop} \left| \lambda_p \right|^n e^{j\omega_p n\Delta t}$$

$$\tag{2}$$

This can be interpreted as the sum of P matrices, each modulated by a different eigenvalue λ_p . Instead of requiring the storage of the entire history of the inputs, this method requires storage of P matrices, where P is some fraction of N, as will be described later in this paper. This method takes a constant amount of time for every time step of the algorithm, with the number of multiplications given by *IOP*. In this paper we propose an alternative method that involves decomposing the input vector into a sum of eigenvectors. With the proposed method, the number of multiplications is reduced to OP.

2. Proposed Method

A technique to express the state $\mathbf{Q}(n)$ as a superposition of eigenvectors and to solve for the zero-input response was discussed in [7]. We propose the extension of that work to the zero-state response. In [7] the initial state is written as:

$$\mathbf{Q}(0) = a_0 \mathbf{y}_0 + a_1 \mathbf{y}_1 + \ldots + a_N \mathbf{y}_N \tag{3}$$

where \mathbf{y}_k are the eigenvectors of the state transition matrix. Using Equation (3), the evolution in time can be expressed as:

$$\mathbf{Q}(n) = (\lambda_0)^n a_0 \mathbf{y}_0 + (\lambda_1)^n a_1 \mathbf{y}_1 + \ldots + (\lambda_N)^n a_N \mathbf{y}_N$$
(4)

Our modification involves expressing the inputs as follows:

$$\mathbf{X}(0) = a_{00}\mathbf{y}_0 + a_{01}\mathbf{y}_1 + \dots + a_{0N}\mathbf{y}_N$$
$$\mathbf{X}(1) = a_{10}\mathbf{y}_0 + a_{11}\mathbf{y}_1 + \dots + a_{1N}\mathbf{y}_N$$
$$\dots$$
$$(5)$$
$$\mathbf{X}(T) = a_{T0}\mathbf{y}_0 + a_{T1}\mathbf{y}_1 + \dots + a_{TN}\mathbf{y}_N$$

With the inputs expressed as in Equation (5), the convolution will involve keeping only a sum for each column as shown below:

$$\mathbf{Y}(1) = \mathbf{h}(1)\mathbf{X}(0) = \lambda_0 a_{00}\mathbf{y}_0 + \lambda_1 a_{01}\mathbf{y}_1 + \ldots + \lambda_N a_{0N}\mathbf{y}_N = s_{10}\mathbf{y}_0 + s_{11}\mathbf{y}_1 + \ldots + s_{1N}\mathbf{y}_N$$

$$\mathbf{Y}(2) = \mathbf{h}(2) \mathbf{X}(0) + \mathbf{h}(1)\mathbf{X}(1) = \lambda_0 s_{10}\mathbf{y}_0 + \lambda_1 s_{11}\mathbf{y}_1 + \ldots + \lambda_N s_{1N}\mathbf{y}_N + \lambda_0 a_{10}\mathbf{y}_0 + \lambda_1 a_{11}\mathbf{y}_1 + \ldots + \lambda_N a_{1N}\mathbf{y}_N = s_{20}\mathbf{y}_0 + s_{21}\mathbf{y}_1 + \ldots + s_{2N}\mathbf{y}_N$$
(6)

From Equation (6) it is clear that a running sum of each column is kept and that convolution involves the multiplication of each column by its eigenvalue. In general the number of multiplications will depend on the number of entries in the y_k vectors and N, the total number of points in the module. Assuming that P out of N eigenvectors are kept for the solution and the remaining ones are discarded, that the size of \mathbf{Y} is $O \times 1$, thus the number of multiplications per time step is reduced to OP.

The complex eigenvalues have a non-zero characteristic frequency obtained by finding the phase angle of the eigenvalue and indicated by $\Omega_i = 2\pi f_i$. The corresponding frequency domain frequency is given by $\omega_i = \Omega_I / \Delta t$ [7]. By properly selecting p, eigenmodes that satisfy the criteria $\Omega_i > 2\pi p$ can be eliminated since it is known that the discretization mechanism of the numerical simulation does not properly propagate these higher frequencies [9]. Adhering to the constraint that only wavelengths that are greater than 10 times the length of a side of a cell can be propagated allows p to be set at 1/10. After the elimination, P indicates the number of remaining eigenmodes.

Hence, the storage of the complex eigenvectors will take up the equivalent of $2OPk_1$ bytes, where k_1 is the number of bytes per double. As can be seen from Equation (6), during every time step two multiplications must be performed for every complex double that is stored. Also, the solution of Equation (5) requires $4OPk_1$ multiplications because the coefficients will in general be complex. Therefore, neglecting additions, every cycle will take roughly $8OPk_2$ milliseconds, where k_2 is the time per multiplication.

3. Results

A module with one interface was analyzed. The dimensions of this module were $1 \times 20 \times 2$ cells. Because the field was assumed zero on the boundary, the module contained only 175 points that participated in the calculation. This resulted in 175 eigenvalues, 116 of which consisted of 58 complex conjugate pairs while the remaining ones were either zero or unity and could be discarded. By setting p = 0.1, all but one of the complex conjugate pairs were discarded.

The module was attached to the terminating face a parallel plate waveguide structure that was simulated with the conventional FDTD and with the algorithm presented in this paper. At the excitation face a constant plane wave source of 10 GHz. was introduced. The dimensions of the waveguide without the module were $58 \times 20 \times 2$ cells, which translates to the dimensions of $2 \times 0.0229 \times 0.002$ wavelengths at 10 GHz. The electric field at various points along the length of the waveguide was obtained for the first 10,000 iterations. The results

were always virtually identical between the conventional FDTD and our methods. In Figure 1 the electric field variation with time in cell (29, 10, 1) is shown. Figure 2 demonstrates that the simulation results match the predictions from electromagnetic theory.





Figure 1: Comparison of results of conventional and proposed methods. The point is located in the middle of the waveguide.

Figure 2: The electric field in the transverse direction. The results correctly indicate the presence of the TM_0 mode.

In order to demonstrate the case when the results were not identical, a module with dimensions of $3 \times 20 \times 2$ cells was utilized. This translated to 525 points. In order to get accurate results from the larger module, p had to be increased to 0.14 in this case. This caused the final system to end up with 16 complex conjugate pairs. The electric field variation in the cell adjacent to the excitation face of the waveguide is displayed in Figure 3. The comparison with the situation where the module produces zero output proves the functionality of the module. Figure 4 shows the small difference between the output of the module at its interface and the electric field produced by the conventional FDTD method at the same point. This difference is barely noticeable in the beginning of the simulation and increases as the simulation progresses in time.



Figure 3: The electric field (Ez) near the waveguide entrance. The overall effect of the module on the simulation can be observed.



Figure 4: The electric field at the module interface.

4. Conclusion

In this paper we discussed the full-wave simulation of interconnect that is found on high frequency integrated circuits. To speed up the simulation, we developed a recursive algorithm for convolution. This recursive

algorithm is based on the modal decomposition approach to the impulse response of the finite-difference timedomain numerical simulation. Its advantages over an earlier approach [5] is that the storage of the history of the impulse responses (IOT) is no longer required. The only storage required is that of the eigenvectors (2OP), eigenvalues (2P), and coefficients (2P). Another improvement over [5] is that the storage of the inputs (IT) is replaced by the much smaller storage of the coefficients. In regards to the approach published in [6], the storage requirement is improved from $\sim (IOP)$ to $\sim (OP)$ and the number of multiplications per time step is improved in the same manner.

The methods discussed in this paper for interconnect can be extended to a majority of other electromagnetic simulation scenarios such as antennas and radar cross section simulation. An important application is the use of the FDTD method to simulate the propagation of electromagnetic waves in semiconductor devices. This is done by coupling the electron transport equations with Maxwell's equations [10].

Future work will involve the investigation into the techniques, such as change of basis, with which multiple modules can be combined together to reduce the overall simulation time. As was seen in the results, the relationship between the N and P varies with the size of the module as well as the choice of p. More insight into this relationship will be required in order to be able to optimize the module for speed or accuracy requirements.

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Approximate Decomposition for the Solution of Boundary Value Problems for Elliptic Systems Arising in Mathematical Models of Layered Structures

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Abstract—We present an alternative approach to the solution of boundary value problems (BVPs) for elliptic systems arising in mathematical models of layered structures. The main idea of the method is to consider auxiliary problems for differential operators separated componentwise and to reduce them to a sequence of iterative problems such that each can be solved (explicitly) by the Fourier method. The solution sequence is then constructed with the help of a contracting transfer operator evaluated explicitly. This method facilitates both analytic and numerical solutions and can be generalized to more complicated mixed BVPs for semilinear partial differential operators.

1. Introduction

The processes which take place in layered structures may be described in terms of boundary value problems (BVPs) for elliptic systems [1, 2], among them are the Laplace, Helmholtz, and Lamè equations, equipped with appropriate boundary conditions of mixed type, including boundary-value contact problems (BVCPs) formulated and investigated in [3].

The simplest examples of BVPs with boundary conditions of mixed type in electromagnetics and acoustics [1, 2] arise when the Dirichlet (or Neumann) conditions are stated on one part of the boundary and the Neumann (Dirichlet) condition on its complement. Such problem are formulated, e.g., in mathematical models of the wave propagation in transmission lines [1]. A decomposition for the solution to the BVPs for the equation systems can be applied when the differential operator can be separated while the boundary value (trace) operators are mixed componentwise on the boundary. In Section 3 we present an example of such a separation (decomposition).

In this work we present an approach for analytical and numerical solution of BVPs in thin layers based on approximate decomposition. The main idea of this method is to simplify the general BVP and to reduce it to a chain of auxiliary problems and then to a sequence of iterative problems such that each of them can be solved (explicitly) by the Fourier method.

2. Formulation

We present the method for the case of a BVCP [3] for the system of Lamè equations in a thin layer (band) equipped with mixed boundary conditions. To this end, consider an elastic band $S = \{-\infty < x_1 < +\infty, 0 < x_2 < h\}$ with Poisson's ratio ν situated on the stiff base $x_2 \equiv 0$. The boundary lines $x_2 = h$ and $x_2 \equiv 0$ are denoted, respectively, by \mathcal{K}_1 and \mathcal{K}_2 (Fig. 1); $\omega = \bigcup_{m=1}^N \omega_m$, where $\omega_m = [a_m, b_m]$, is a set of disjoint segments; and $\omega^* = \mathcal{K}_1 \setminus \omega$. Distribution of shearing strains on line \mathcal{K}_1 , displacements on ω , and elongations on ω^* are given. We denote by u_j and \mathcal{F}_j , (j = 1, 2) the displacements and respectively projections of the body forces in directions x_j . The determination of u_j reduces to a mixed BVP [3] for the Lamè equations in S

$$\Delta u_j + k_0 \frac{\partial}{\partial x_j} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \right) = \mathcal{F}_j, \qquad k_0 = \frac{1}{1 - 2\nu}, \qquad j = 1, 2$$
(1)

with the boundary conditions

$$u_{2} = 0, \qquad \frac{\partial u_{1}}{\partial x_{2}} + \frac{\partial u_{2}}{\partial x_{1}} = 0 \qquad \text{on } \mathcal{K}_{2},$$
$$\frac{\partial u_{2}}{\partial x_{1}} + \frac{\partial u_{1}}{\partial x_{2}} = f_{1}(x_{1}) \qquad \text{on } \mathcal{K}_{1},$$
$$u_{2} = f_{2}(x_{1}) \qquad \text{on } \omega,$$
$$(2)$$

$$(k_0 - 1)\frac{\partial u_1}{\partial x_1} + (k_0 + 1)\frac{\partial u_2}{\partial x_2} = f_3(x_1) \qquad \text{on } \omega^*$$

and the conditions at infinity

$$\Phi_s(u_1, u_2) = \int_S \Pi_S \, ds < \infty,
\Pi_s = (k_0 - 1) \left(\sum_{j=1}^2 \frac{\partial u_j}{\partial x_j}\right)^2 + 2 \sum_{j=1}^2 \left(\frac{\partial u_j}{\partial x_j}\right)^2 + \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)^2,$$
(3)

BVP (1)-(3) has the unique classical solution if the boundary functions are sufficiently smooth. Namely, the following statement is valid (see [3]):

If the functions $\mathcal{F}_1 \in L_p(S)$, $\mathcal{F}_2 \in L_p(S)$, $f_1 \in L_p(\mathcal{K}_1)$, $f_3 \in L_p(\omega^*)$, p > 1 $(f \in L_p(\Omega)$ if $|f|^p$ is integrable over Ω) and function $f_2 \in C^q(\mathcal{K}_1)$, $q \ge 3$, is a smooth (q-times continuously differentiable) compactly-supported function with $supp f_2 \in \omega$ then problem (1)–(3) is uniquely solvable if and only if

$$\int_{\mathcal{K}_1} f_1 \, dx_1 + \int_S \mathcal{F}_1 \, dS = 0$$

and the solutions $u_j \in C^2(\Pi_{ah}) \cap C(\overline{\Pi}_{ah})$ in every rectangle $\Pi = \Pi_{ah} = \{(x_1, x_2) : 0 < x_1 < a, 0 < x_2 < h\}$.

3. Approximate Decomposition

Consider a simplified version of the problem (1)–(3) which will be called *problem* A: body forces $\mathcal{F}_1, \mathcal{F}_2 \equiv 0$; shearing stresses $f_1 \equiv 0$ on \mathcal{K}_2 ; and normal stresses $f_3 \equiv 0$ on ω^* . Consider this problem in a long rectangle Π_{ah} bounded by the curve $\Gamma = \hat{\mathcal{K}}_1 \bigcup \hat{\mathcal{K}}_2 \bigcup \mathcal{H}_1 \bigcup \mathcal{H}_2$, where $\hat{\mathcal{K}}_i = \mathcal{K}_i \bigcap \{0 < x_1 < a\}, (i = 1, 2); \ \hat{\omega}^* = \omega^* \bigcap \{0 < x_1 < a\}; \ \mathcal{H}_1 = \{x = (x_1, x_2): x_1 = 0, 0 < x_2 < h\}, \ \mathcal{H}_2 = \{x = (x_1, x_2): x_1 = a, 0 < x_2 < h\}; \ \text{and} u = (u_1, u_2) \text{ denotes the vector of displacements. Introduce the trace operators <math>L^{(1)}$ and $L^{(2)}$ specifying the boundary conditions on $\hat{\omega}, \hat{\omega}^*$ and Γ :

$$L^{(1)}\boldsymbol{u} = \begin{pmatrix} l_{11}^{(1)} & 0\\ 0 & l_{22}^{(1)} \end{pmatrix} \boldsymbol{u},$$

$$l_{11}^{(1)}u_1 = \frac{\partial u_1}{\partial \nu} (\boldsymbol{x} \in \Gamma), \qquad l_{22}^{(1)}u_2 = u_2 \qquad (\boldsymbol{x} \in \omega \cup \widehat{\mathcal{K}}_2 \cup \mathcal{H}_1 \cup \mathcal{H}_2)$$
(4)

is the operator of the Neumann–Dirichlet boundary conditions, and

$$L^{(2)}\boldsymbol{u} = \begin{pmatrix} l_{11}^{(2)} & l_{12}^{(2)} \\ l_{21}^{(2)} & l_{22}^{(2)} \end{pmatrix} \boldsymbol{u},$$

$$l_{11}^{(2)}u_1 = 0, \qquad l_{12}^{(2)}u_2 = \frac{\partial u_2}{\partial \tau} \qquad (\boldsymbol{x} \in \Gamma),$$

$$l_{21}^{(2)}u_1 = \alpha u_{1,1} \qquad l_{22}^{(2)}u_2 = u_{2,2} \qquad (\boldsymbol{x} \in \hat{\omega}^*),$$

(5)

where

$$\frac{\partial}{\partial \tau} = \begin{cases} \frac{\partial}{\partial x_1}, & \boldsymbol{x} \in \mathcal{K}_1 \bigcup \mathcal{K}_2 \\ \frac{\partial}{\partial x_2}, & \boldsymbol{x} \in \mathcal{H}_1 \bigcup \mathcal{H}_2 \end{cases}, \qquad \frac{\partial}{\partial \nu} = \begin{cases} (-1)^i \frac{\partial}{\partial x_2}, & \boldsymbol{x} \in \mathcal{K}_i \\ (-1)^i \frac{\partial}{\partial x_1}, & \boldsymbol{x} \in \mathcal{H}_i \end{cases}, \qquad \alpha = \frac{k_0 + 1}{k_0 - 1}. \tag{6}$$

The operator $L\boldsymbol{u} = L^{(1)}\boldsymbol{u} + L^{(2)}\boldsymbol{u}$ specifies the boundary conditions of problem A in the form $L\boldsymbol{u} = \boldsymbol{f}$, with $\boldsymbol{f} = (0, \hat{f}_2(\boldsymbol{x}))$ and

$$\widehat{f}_2(\boldsymbol{x}) = \begin{cases} f_2(x_1), & \boldsymbol{x} = (h, x_1) \in \omega, \\ 0, & \boldsymbol{x} \in \Gamma \backslash \omega, \end{cases}$$
(7)

being a differentiable function on Γ with a compact support supp $f_2 \subseteq \omega$. Introduce matrix differential operators of the system in problem (1)–(3) and problem A and rewrite the latter as

$$\mathcal{D}\boldsymbol{u}=0, \qquad L\boldsymbol{u}=\boldsymbol{f}, \tag{8}$$

where

$$\mathcal{D} = \Delta + k_0 A, \qquad \Delta = \begin{pmatrix} \Delta_1 & 0\\ 0 & \Delta_2 \end{pmatrix},$$

$$\Delta_1 u_1 = (k_0 + 1) \frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2}, \qquad \Delta_2 u_2 = \frac{\partial^2 u_2}{\partial x_1^2} + (k_0 + 1) \frac{\partial^2 u_2}{\partial x_2^2}, \qquad (9)$$

$$A = \begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix} \frac{\partial^2}{\partial x_1 \partial x_2}, \qquad \mathbf{f} = (0, \hat{f}_2(\mathbf{x})).$$

Assuming that displacements u_2 are absent on ω^* write problem A in the form

$$\mathcal{D}\boldsymbol{u} = 0, \qquad \widehat{L}\boldsymbol{u} = \boldsymbol{f}, \qquad \widehat{L} = \widehat{L}^{(1)} + \widehat{L}^{(2)}, \qquad (10)$$

where $\widehat{L}^{(1)} = \|\widehat{l}_{ii}^{(1)}\|_{i=1,2}$ is defined as in (4) with the only difference that $\widehat{l}_{22}^{(1)}u_2 = \frac{1}{2}u_2$, $\boldsymbol{x} \in \Gamma$, and $\widehat{L}^{(2)}$ has two nontrivial components: $\widehat{l}_{21}^{(2)}$ defined in (5) and $\widehat{l}_{22}^{(2)}u_2 = \frac{1}{2}u_2$, $\boldsymbol{x} \in \Gamma$.

Define the sequence $\{u_n\}$ of vector-functions according to

$$\Delta \boldsymbol{u}_{0} = 0, \qquad \qquad \widehat{L}^{(1)}\boldsymbol{u}_{0} = \boldsymbol{f}_{0} = \left(-\frac{\partial f_{2}}{\partial x_{1}}, \widehat{f}_{2}(x_{1})\right), \qquad x_{1} \in \omega,$$

$$\Delta \boldsymbol{u}_{n+1} = -k_{0}A\boldsymbol{u}_{n}, \qquad \qquad \widehat{L}^{(1)}\boldsymbol{u}_{n+1} = -\widehat{L}^{(2)}\boldsymbol{u}_{n}, \qquad n = 0, 1, 2, \dots$$

$$(11)$$

The limiting function (if exists) $\boldsymbol{u} = \lim_{n\to\infty} \boldsymbol{u}_n$ (where the limit is determined with respect to an appropriate norm) satisfies (8). In order to prove the existence consider BVP (11) for $\boldsymbol{u}_{n+1} = (u_1^{(n+1)}, u_2^{(n+1)})$. Componentwise, (11) consists of two inhomogeneous BVPs for Poisson equation in the rectangle. The solution to each problem can therefore be obtained as a sum of the corresponding volume and surface (line) potentials. In the vector-operator form the relationship between two intermediate problems (11) can be represented as

$$\boldsymbol{u}_{n+1} = \boldsymbol{K}\boldsymbol{u}_n,\tag{12}$$

where K is a volume-surface integral operator defined in term of the potentials.

Applying the Schauder *a priori* estimates of the solution to BVPs for elliptic PDEs [4, 5], using the explicit form of u_{n+1} and properties of logarithmic and Green's potentials [6, 7], one can show that

$$\|\boldsymbol{u}_{n+1}\|_{C^{2}(\Pi)} \leqslant M_{n} \big(\|\boldsymbol{u}_{n}\|_{C^{2}(\Pi)} + \|f_{2}\|_{C^{2}(\omega)}\big), \qquad n = 1, 2, \dots,$$
(13)

where constant M_n depends on the diameter of Π_{ah} and $M_n \to 0$ if diam $\Pi_{ah} \to 0$. Thus, operator K (12) is a contraction in the space $C^2(\Pi) \cap C(\overline{\Pi})$ of two-component vector-functions if the diameter of set ω , parameter h, and the norm of boundary function f_2 are sufficiently small. This implies the existence of the unique solution $u \in C^2(\Pi) \cap C(\overline{\Pi})$ to problem A.

This approximate decomposition can be applied to the solution of BVPs of the type (1), (2) for semilinear systems with the differential operators $\mathcal{D}\boldsymbol{u} = \Delta \boldsymbol{u} + \mathcal{F}(\boldsymbol{u}, \boldsymbol{u}_{x_1}, \boldsymbol{u}_{x_2}, \boldsymbol{u}_{x_1x_2})$, where \mathcal{F} is nonlinear with respect to \boldsymbol{u} and \boldsymbol{u}_{x_i} . Constructing the iterations similar to (11) or (12) and showing or assuming that the corresponding transfer operator \boldsymbol{K} is contraction, we obtain a recursive procedure (12) to determine displacements \boldsymbol{u} .

4. Solution by the Fourier Method

One can obtain explicit solution to every intermediate BVP (11) in the form of Fourier series

$$u_{2}^{(n+1)} = \sum_{m=1}^{\infty} \sin \frac{\pi m}{a} x_{1} \left(d_{m} \sinh \frac{\pi m \sqrt{k_{0}+1}}{a} x_{2} + e_{m} \sinh \frac{\pi m}{a \sqrt{k_{0}+1}} x_{2} \right),$$

$$u_{1}^{(n+1)} = \sum_{m=1}^{\infty} \cos \frac{\pi m}{a} x_{1} \left(g_{m} \cosh \frac{\pi m}{a \sqrt{k_{0}+1}} x_{2} + q_{m} \cosh \frac{\pi m \sqrt{k_{0}+1}}{a} x_{2} \right),$$
(14)

where

$$a_{m} = -\frac{\sinh \frac{\pi mh}{a\sqrt{k_{0}+1}}}{\sinh \frac{\pi mh\sqrt{k_{0}+1}}{a}} b_{m}, \qquad b_{m} = \frac{f_{m}}{\sinh \frac{\pi mh}{a\sqrt{k_{0}+1}}},$$
(15)

 $f_m = \frac{2}{a} \int_0^a f_2(x_1) \sin \frac{\pi m}{a} dx_1$ are Fourier coefficients for the function f_2 from boundary condition (2) and

$$d_m = \frac{\sqrt{k_0 + 1}}{k_0 + 2} b_m, \qquad e_m = \left(1 + \frac{\sqrt{k_0 + 1}}{k_0 + 2}\right) b_m,$$

$$g_m = \frac{\sqrt{k_0 + 1}}{k_0 + 2} b_m, \qquad q_m = \frac{k_0 + 3}{(k_0 + 2)\sqrt{k_0 + 1}} a_m b_m$$
(16)

are the Fourier coefficients obtained for (11) on the previous stage n.

Series (14) converge absolutely and uniformly in every rectangle $\Pi_{ah}^{\delta} = \{0 \leq x_1 \leq a, \delta \leq x_2 \leq h\}$ with $0 < \delta < h$ and admit term-wise differentiation arbitrary number of times. The rate of convergence is exponential.

In view of the explicit solution (14) it is reasonable to specify a boundary function $f_2(x_1)$ in problem A and (7) as a smooth compactly-supported function $f_2 \in C^p(R)$, $p \ge 3$, with $supp f_2 \in \omega$. One can consider, for example, the case when $f_2(x_1)$ is the so-called hat function of order p (a product of a polynomial in even powers of argument that vanishes at the endpoints of ω and a Gaussian exponent) for which the Fourier coefficients can be calculated explicitly. Such hat functions possess the properties of B–splines; therefore, one can approximate or interpolate a smooth function on the line R with a finite support ω by a finite linear combination of hat functions and apply the approximate decomposition with rapidly converging series solutions to BVPs with virtually arbitrary boundary functions.



517

5. Numerical

Let us present some qualitative results of numerical-analytical solution to problem A (a simplified version of (1)–(3) considered in a long rectangle) obtained using approximate decomposition (first iteration); the profiles of boundary displacements are taken as hat functions presented in Fig. 2. Fig.s 3 and 4 show u_1 and u_2 calculated in the case of a/h = 10 and two disjoint segments $\omega = \bigcup_{i=1}^{2} [x_{S_i} - p_i, x_{S_i} + p_i]$.

Values of displacement u_1 in Fig. 3 are zero at $x_{S_{1,2}}$ because these points shift only in x_2 -direction; values in the support intervals $(x_{S_1} - p_1, x_{S_1})$ and $(x_{S_2} - p_2, x_{S_2})$ are negative because these points shift in the opposite direction; values in the intervals $(x_{S_1}, x_{S_1} + p_1)$ and $(x_{S_2}, x_{S_2} + p_2)$ are positive because these points also shift in the x_2 -direction and take maximum and minimum at the respective points. Function u_2 in Fig. 4 takes only positive values in the intervals $(x_{S_1} - p_1, x_{S_1} + p_1)$ and $(x_{S_2} - p_2, x_{S_2} + p_2)$, maximum and minimum are at the points x_{S_1} and x_{S_2} respectively.

6. Conclusion

We have developed a method of approximate analytical-numerical solution to BVPs for elliptic system in parallel-plane layers based on decomposition of boundary value conditions. An advantage of the method is the possibility of explicit determination and fast computation and visualization of all components at every point of the layer. The method can be extended to wide families of BVPs using spline-type approximations based on hat functions.

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Some Elliptic Traveling Wave Solutions to the Novikov-Veselov Equation

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Abstract—An approach is proposed to obtain some exact explicit solutions in terms of elliptic functions to the Novikov-Veselov equation (NVE[V(x, y, t)] = 0). An expansion ansatz $V \to \psi = \sum_{j=0}^{2} a_j f^j$ is used to reduce the NVE to the ordinary differential equation $(f')^2 = R(f)$, where R(f) is a fourth degree polynomial in f. The well-known solutions of $(f')^2 = R(f)$ lead to periodic and solitary wave like solutions V. Subject to certain conditions containing the parameters of the NVE and of the ansatz $V \to \psi$ the periodic solutions V can be used as start solutions to apply the (linear) superposition principle proposed by Khare and Sukhatme.

1. Introduction

The Novikov-Veselov (NV) equation [1] as a "natural" two-dimensional generalization of the celebrated Korteweg-de Vries (KV) equation [2] has relevance in nonlinear physics (in particular in inverse scattering theory) [3,4] and mathematics (cf. e. g., [5,6]).

As regards to physics, Tagami [3] derived solitary solutions of the NV equation by means of the Hirota method. Cheng [4] investigated the NV equation associated with the spectral problem $(\partial_x \partial_y + u)\psi = 0$ in the plane and presented solutions by applying the inverse scattering transform. With regards to mathematics, Taimanov [5] investigated applications of the (modified) NV equation to differential geometry of surfaces. Ferapontov [6] used the (stationary) NV equation to describe a certain class of surfaces in projective differential geometry (the so-called isothermally asymptotic surfaces).—Apart from these applications solutions of the NV equation are interesting in and of themselves.

In the following we derive some solutions of the NV equation by combining a symmetry reduction method [7,8] and the Khare-Sukhatme superposition principle [9–12].

2. Elliptic Solutions

2.1. General Considerations

Following Novikov and Veselov [1] we consider the system

$$V_t = \partial^3 V + \overline{\partial}^3 V + 3\partial(uV) + 3\overline{\partial}(\overline{u}V), \tag{1}$$

$$\overline{\partial}u = \partial V, \tag{2}$$

where $\partial = \frac{1}{2} (\partial_x - i\partial_y), \overline{\partial} = \frac{1}{2} (\partial_x + i\partial_y)$ are the Cauchy-Riemann operators in \mathbb{R}^2 . System (1), (2) is equivalent to

$$V_t = \frac{1}{4} \left(V_{xxx} - 3V_{xyy} \right) + 3V(u_{1x} + u_{2y}) + 3(u_1V_x + u_2V_y), \tag{3}$$

$$V_x = u_{1x} - u_{2y}, \ -V_y = u_{1y} + u_{2x} \tag{4}$$

with $u(x, y, t) = u_1(x, y, t) + iu_2(x, y, t)$, where u is defined up to an arbitrary holomorphic function $\varphi = \varphi_1 + i\varphi_2$ so that $\varphi_{1x} = \varphi_{2y}$, $\varphi_{1y} = -\varphi_{2x}$. (4) imply

$$u_1 = -2\partial_x^{-1}\partial_y \widetilde{D}V + V + \varphi_1, \quad u_2 = -2\widetilde{D}V + \varphi_2. \tag{5}$$

The operator $\widetilde{D} := (\partial_x^{-1} \partial_y + \partial_y^{-1} \partial_x)^{-1}$ is well-defined [13, (6)], so that u_1, u_2 can be inserted into (3). Traveling wave solutions

$$V(x, y, t) = \psi(z), \ z = x + ky - vt \tag{6}$$

imply $\partial_x^{-1} = k \partial_y^{-1}$ and thus lead to $\varphi \equiv \text{const.} = C_0 + iC_1$. Hence, (3) can be written as

$$-v\psi_z = \frac{1-3k^2}{4}\psi_{zzz} + 6\frac{1-3k^2}{k^2+1}\psi\psi_z + 3\psi_z(C_0+C_1k).$$
(7)

Following an approach outlined previously [7, 8, 14] it seems useful to find elliptic (traveling wave) solutions of the form (p = 2 follows from balancing the linear term of highest order with the nonlinear term in (7))

$$\psi(z) = \sum_{j=0}^{p=2} a_j f(z)^j$$
(8)

with [15]

$$\left(\frac{df(z)}{dz}\right)^2 = \alpha f^4 + 4\beta f^3 + 6\gamma f^2 + 4\delta f + \epsilon \equiv R(f).$$
(9)

The coefficients $a_0, a_1, a_2, \alpha, \beta, \gamma, \delta, \epsilon$ are assumed to be real but otherwise either arbitrary or interrelated.

Inserting (8) into (7) and using (9) we obtain a system of algebraic equations that can be reduced to yield the nontrivial solutions

$$\alpha = 0, \quad \beta = -\frac{2a_1}{1+k^2}, \quad \gamma = -\frac{4a_0}{1+k^2} + \frac{2F}{3(3k^2-1)}, \quad \delta, \quad \epsilon \text{ arbitrary},$$

subject to $a_2 = 0, \ 3k^2 - 1 \neq 0,$ (10)

$$\alpha = -\frac{2a_2}{1+k^2}, \ \beta = -\frac{a_1}{1+k^2}, \ \gamma = \frac{F}{6(3k^2-1)} - \frac{a_1^2 + 4a_0a_2}{4a_2(1+k^2)},$$

$$\delta = \frac{1}{8a_2^2} \left(\frac{a_1^3 - 12a_0a_1a_2}{1+k^2} + \frac{2a_1a_2F}{3k^2-1} \right), \ \epsilon \text{ arbitrary},$$

abject to $a_2 \neq 0, \ 3k^2 - 1 \neq 0$
(11)

with $F = v + 3C_0 + 3kC_1$.

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Thus, the coefficients of the polynomial R(f) are (partly) determined leading to solutions f(z) of (9). As is well known [15, pp. 4–16], [16, p. 454] f(z) can be expressed in terms of Weierstrass' elliptic function $\wp(z; g_2, g_3)$ according to

$$f(z) = f_0 + \frac{R'(f_0)}{4\left[\wp(z;g_2,g_3) - \frac{1}{24}R''(f_0)\right]},$$
(12)

where the primes denote differentiation with respect to f and f_0 is a simple root of R(f).

The invariants g_2, g_3 of $\wp(z; g_2, g_3)$ and the discriminant $\Delta = g_2^3 - 27g_3^2$ are related to the coefficients of R(f) [17, p. 44]. They are suitable to classify the behaviour of f(z) and to discriminate between periodic and solitary wave like solutions [8].

Solitary wave like solutions are determined by (cf. (12) and Ref. [18, pp. 651–652])

$$f(z) = f_0 + \frac{R'(f_0)}{4\left[e_1 - \frac{R''(f_0)}{24} + 3e_1 \operatorname{csch}^2(\sqrt{3e_1}z)\right]}, \quad \Delta = 0, \quad g_3 < 0, \tag{13}$$

where $e_1 = \frac{1}{2} \sqrt[3]{|g_3|}$ in (13).

In general, f(z) (according to (12)) is neither real nor bounded. Conditions for real and bounded solutions f(z) can be obtained by considering the phase diagram of R(f) [19, p. 15]. They are denoted as "phase diagram conditions" (PDC) in the following. An example of a phase diagram analysis is given in [14].

2.2. Periodic Solutions

At first the coefficients according to (10) are considered. For simplicity we assume $\epsilon = 0$, so that $f_0 = 0$ is a simple root of (9). The solution (12) can be evaluated to yield

$$V(x,y,t) = a_0 + a_1 \frac{3(1+k^2)(1-3k^2)\delta}{(1+k^2)F + 6a_0(1-3k^2) + 3(1+k^2)(1-3k^2)\wp(x+ky-vt;g_2,g_3)}$$
(14)

with g_2 , g_3 according to (10) and [8].

Evaluating (12) with coefficients according to (11) (with $\epsilon = 0$ for simplicity) in the same manner we obtain periodic solutions depending on a_0 , a_1 and a_2 .

2.3. Solitary Wave like Solutions

To find the subset of solitary wave like solutions of the NV equation according to (10), (13) the discriminant

 $\Delta \text{ must vanish. This is given if } \delta = 0 \text{ or } \delta = -\frac{(6a_0(1-3k^2)+(1+k^2)F)^2}{8a_1(1-3k^2)^2(1+k^2)}.$ For $g_3 < 0$ we obtain solitary wave like solutions and here the PDC is fulfilled automatically for $g_3 < 0$. If $\delta = 0$, $\epsilon = 0$, $f_0 = \frac{6a_0(1-3k^2)+(1+k^2)F}{2a_1(3k^2-1)}$, we obtain (cf. (8), (13))

$$V(x,y,t) = a_0 + \frac{6a_0(1-3k^2) + (1+k^2)F}{2(3k^2-1)}\operatorname{sech}^2\left[\sqrt{-\frac{6a_0}{1+k^2} + \frac{F}{3k^2-1}}(x+ky-vt)\right].$$
(15)

If
$$\delta = -\frac{(6a_0(1-3k^2)+(1+k^2)F)^2}{8a_1(1-3k^2)^2(1+k^2)}$$
, $\epsilon = 0, f_0 = 0$, (8) reads

$$V(x,y,t) = a_0 + \frac{6a_0(1-3k^2) + (1+k^2)F}{4(3k^2-1)} \tanh^2 \left[\sqrt{\frac{F}{2(1-3k^2)} + \frac{3a_0}{1+k^2}}(x+ky-vt)\right].$$
 (16)

Subject to (10) (15), (16) represent general physical traveling solitary wave solutions of the NV equation for $\epsilon = 0$. While periodic solutions depend on a_0 and a_1 , solitary solutions only depend on a_0 .

Solitary wave like solutions according to (11) can be obtained by an analogous procedure.

3. Superposition Solutions

Khare and Sukhatme proposed a superposition principle for nonlinear wave and evolution equations (NL-WEEs) [9]. They have shown that suitable linear combinations of periodic traveling-wave solutions expressed by Jacobian elliptic functions lead to new solutions of the nonlinear equation in question. Combining the approach above with this superposition principle we have evaluated the following start solutions for superposition [20]

$$f(z) = \begin{cases} -\frac{3\gamma + \sqrt{9\gamma^2 - 16\beta\delta}}{4\beta} \mathrm{dn}^2 \left(\frac{1}{2} \sqrt{3\gamma + \sqrt{9\gamma^2 - 16\beta\delta}} z, \frac{2\sqrt{9\gamma^2 - 16\beta\delta}}{3\gamma + \sqrt{9\gamma^2 - 16\beta\delta}} \right), & \beta\delta > 0, \gamma > 0, \\ \frac{4\delta}{-3\gamma + \sqrt{9\gamma^2 - 16\beta\delta}} \mathrm{sn}^2 \left(\frac{1}{2} \sqrt{-3\gamma + \sqrt{9\gamma^2 - 16\beta\delta}} z, \frac{3\gamma + \sqrt{9\gamma^2 - 16\beta\delta}}{3\gamma - \sqrt{9\gamma^2 - 16\beta\delta}} \right), & \beta\delta > 0, \gamma < 0, \\ -\frac{3\gamma + \sqrt{9\gamma^2 - 16\beta\delta}}{4\beta} \mathrm{cn}^2 \left(\frac{(9\gamma^2 - 16\beta\delta)^{\frac{1}{4}}}{\sqrt{2}} z, \frac{3\gamma + \sqrt{9\gamma^2 - 16\beta\delta}}{2\sqrt{9\gamma^2 - 16\beta\delta}} \right), & \beta\delta < 0. \end{cases}$$
(17)

In (10) we choose $\epsilon = 0$ for simplicity and thus, we obtain start solutions for superposition according to (17). As an example we consider solutions of the form dn^2 for p = 3, further results for cn^2 , sn^2 and according to (11) can be obtained in the same manner.

According to (8), (10) the start solution for superposition reads

$$V(x, y, t) = a_0 + a_1 A \operatorname{dn}^2(\mu(x + ky - vt), m),$$
(18)

with A, μ , m according to (17), so that the superposition ansatz can be written as

$$\widetilde{V}(x,y,t) = a_0 + a_1 A \sum_{i=1}^3 \mathrm{dn}^2 \left[\mu(x+ky-v_3 t) + \frac{2(i-1)K(m)}{3}, m \right].$$
(19)

Inserting $\widetilde{V}(x, y, t)$ (denoting $d_i = dn\left(\mu(x + ky - v_3 t) + \frac{2(i-1)K(m)}{3}, m\right)$) into (7) $(v \to v_3)$ and using well known relations for c_i^2 and s_i^2 [22, p. 16] leads to

$$6Aa_{1}\mu m(1-3k^{2})\left(\mu^{2}-\frac{2Aa_{1}}{1+k^{2}}\right)\sum_{i=1}^{3}c_{i}d_{i}^{3}s_{i}-\frac{12A^{2}a_{1}^{2}m\mu(1-3k^{2})}{1+k^{2}}\sum_{i=1}^{3}d_{i}^{2}\sum_{j\neq i}^{3}c_{j}d_{j}s_{j}$$

$$-2Aa_{1}\mu m\left(\frac{6a_{0}(1-3k^{2})}{1+k^{2}}+3(C_{0}+C_{1}k)+(2-m)(1-3k^{2})\mu^{2}+v_{3}\right)\sum_{i=1}^{3}c_{i}d_{i}s_{i}=0.$$
(20)

Remarkably, $\mu^2 - \frac{2Aa_1}{1+k^2}$ vanishes automatically [20, (13)]. By use of [23], (21) reads

$$-2Aa_{1}\mu m \left(\frac{6a_{0}(1-3k^{2})}{1+k^{2}}+3(C_{0}+C_{1}k)+(2-m)(1-3k^{2})\mu^{2}+v_{3}\right)\sum_{i=1}^{3}c_{i}d_{i}s_{i}$$
$$-2Aa_{1}\mu m \left(-\frac{12Aa_{1}(1-3k^{2})(m-1+q^{2})}{(1+k^{2})(1-q^{2})}\right)\sum_{i=1}^{3}c_{i}d_{i}s_{i}=0.$$
(21)

Thus, the speed v_3 in the superposition solution (19) is given by

$$v_3 = \frac{6a_0(3k^2 - 1)}{1 + k^2} - 3(C_0 + C_1k) + (2 - m)(3k^2 - 1)\mu^2 + \frac{12Aa_1(3k^2 - 1)(m - 1 + q^2)}{(1 + k^2)(q^2 - 1)}.$$
 (22)

The start solution V and the superposition solution \tilde{V} are shown in Fig. 1.



Figure 1: V and \tilde{V} (cf. (18), (19)) for c = -1, k = 1, $a_0 = -1$, $a_1 = -1$, $C_0 = 1$, $C_1 = 1$, $\delta = 4$ (therefore: $v_3 = -8.66008$).

4. Conclusion

For the NV equation we have shown that a rather broad set of traveling wave solutions according to (6), (8) and subject to the nonlinear ordinary differential equation (9) can be obtained. Periodic and solitary wave solutions can be presented in compact form in terms of Weierstrass' elliptic function and its limiting cases ($\Delta = 0$, $g_3 \leq 0$), respectively. The phase diagram conditions (PDC) yield constraints for real and bounded solutions. Finally, it is shown that application of the Khare-Sukhatme superposition principle yields new periodic (real, bounded) solutions of the NV equation.

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Effects of the Resonant Scattering of Intensive Fields by Weakly Nonlinear Dielectric Layer

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Abstract—The transverse inhomogeneous, isotropic, nonmagnetic, linearly polarized, weakly nonlinear (a Kerrlike dielectric nonlinearity) dielectric layer is considered. The results of a numerical analysis of the diffraction problem of a plane wave on the weakly nonlinear object with positive and negative value of the susceptibility are shown. The effects: non-uniform shift of resonant frequency of the diffraction characteristics of a weakly nonlinear dielectric layer; itself the channeling of a field; increase of the angle of the transparency of the nonlinear layer when growth of intensity of the field (at positive value of the susceptibility); de-channeling of a field (at negative value of the susceptibility) are found out.

1. The Nonlinear Problem

Let the time dependence be $\exp(-i\omega t)$ and $\vec{E}(\vec{r})$, $\vec{H}(r)$ complex amplitudes of an electromagnetic field. We consider a nonmagnetic, isotropic, transverse inhomogeneous $\varepsilon^{(L)}(z) = 1 + 4\pi \chi_{xx}^{(1)}(z)$, linearly polarized $\vec{E} = (E_x, 0, 0)$, $\vec{H} = (0, H_y, H_z)$ (*E*-polarized) and Kerr-like weakly nonlinearity $P_x^{(NL)} = \frac{3}{4} \chi_{xxxx}^{(3)} |E_x|^2 E_x$, $\max_{|z| \le 2\pi\delta} \left(|\alpha| \cdot |E_x|^2 \right) \ll \max_{|z| \le 2\pi\delta} |\varepsilon^{(L)}(z)|$ (where $\vec{P}^{(NL)} = \left(P_x^{(NL)}, 0, 0 \right)$ — vector of polarization, $\alpha = 3\pi \chi_{xxxx}^{(3)}$, $\chi_{xxx}^{(1)}$ and $\chi_{xxxx}^{(3)}$ is the components of susceptibility tensor) dielectric layer (Fig. 1), [1, 2].



Figure 1: Weakly nonlinear dielectric layer: $\max_{|z| \le 2\pi\delta} \left(\left| \alpha \right| \cdot \left| E_x \right|^2 \right) << \max_{|z| \le 2\pi\delta} \left| \varepsilon^{(L)}(z) \right|.$

The complete diffraction field $E_x(y,z) = E_x^{inc}(y,z) + E_x^{scat}(y,z)$ of a plane wave $E_x^{inc}(y,z) = a^{inc} \exp\left[i(\phi y - \Gamma \cdot (z - 2\pi\delta))\right]$, $z > 2\pi\delta$ on the nonlinear dielectric layer (Fig. 1) satisfies such conditions of the problem:

$$\nabla^2 \cdot \vec{E} + \frac{\omega^2}{c^2} \cdot \varepsilon^{(L)}(z) \cdot \vec{E} + \frac{4\pi\omega^2}{c^2} \cdot \vec{P}^{(NL)} \equiv \left(\nabla^2 + \kappa^2 \cdot \varepsilon \left(z, \alpha \cdot |E_x|^2\right)\right) \cdot E_x\left(y, z\right) = 0, \tag{1}$$

the generalized boundary conditions:

$$E_{tg} \text{ and } H_{tg} \text{ are continuous at discontinuities } \varepsilon \left(z, \alpha \cdot |E_x|^2 \right);$$

$$E_x \left(y, z \right) = U \left(z \right) \cdot \exp \left(i \phi y \right), \text{ the condition of spatial quasihomogeneity along } y;$$
(2)

the condition of the radiation for scattered field:

$$E_x^{scat}(y,z) = \left\{ \begin{array}{c} a^{scat} \\ b^{scat} \end{array} \right\} \cdot e^{i \ (\phi \ y \pm \Gamma \cdot (z \mp 2 \ \pi \delta))}, \ z \ < \\ z \ < \\ \end{array} \right\} \pm 2 \ \pi \delta \tag{3}$$

Here:
$$\varepsilon \left(z, \alpha \cdot |E_x|^2 \right) = \begin{cases} 1, & |z| > 2\pi\nabla^2 \\ \varepsilon^{(L)}(z) + \alpha \cdot |E_x|^2, & |z| \le 2\pi\delta \end{cases}; \nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}; \alpha = 3\pi\chi^{(3)}_{xxxx}; \Gamma = \left(\kappa^2 - \phi^2\right)^{1/2}; \phi \equiv \kappa \cdot \sin(\varphi); |\varphi| < \pi/2 \text{ (see Fig. 1); } \kappa = \omega/c \equiv 2\pi/\lambda; c = \left(\varepsilon_0 \mu_0\right)^{-1/2}, \varepsilon_0, \mu_0 \text{ and } \lambda \text{ length of the wave are the }$$

parameters of environment. In this case the required solution of the problem (1)-(3) has the form:

$$E_x(y,z) = U(z) \cdot e^{i\phi y} = \begin{cases} a^{inc} \cdot e^{i(\phi y - \Gamma \cdot (z - 2\pi\delta))} + a^{scat} \cdot e^{i(\phi y + \Gamma \cdot (z - 2\pi\delta))}, & z > 2\pi\delta, \\ U^{scat}(z) \cdot e^{i\phi y}, & |z| \le 2\pi\delta, \\ b^{scat} \cdot e^{i(\phi y - \Gamma \cdot (z + 2\pi\delta))}, & z < -2\pi\delta. \end{cases}$$
(4)

Here $U(-2\pi\delta) = b^{scat}$, $U(2\pi\delta) = a^{inc} + a^{scat}$.

The nonlinear problem (1)–(3) is reduced to finding the solutions $U(z) \in L_2([-2\pi\delta, 2\pi\delta])$ (see (4)) of the inhomogeneous nonlinear integrated equation of the second kind [3, 4]:

$$U(z) + \frac{i\kappa^2}{2\Gamma} \int_{-2\pi\delta}^{2\pi\delta} \exp\left(i\Gamma \cdot |z - z_0|\right) \left[1 - \left(\varepsilon^{(L)}(z_0) + \alpha |U(z_0)|^2\right)\right] U(z_0) \, dz_0 = U^{inc}(z) \,, \qquad |z| \le 2\pi\delta, \quad (5)$$

where $U^{inc}(z) = a^{inc} \exp\left[-i\Gamma \cdot (z - 2\pi\delta)\right]$.

The integrated equation (5) with application of the quadrature method and use (4) is reduced to system of the nonlinear equations of the second kind [4].

2. Susceptibility and Effects Resonant Scattering of the Intensive Fields

2.1. Intensity and Resonant Frequency

The effect of non-uniform shift of resonant frequency of the diffraction characteristics of nonlinear dielectric layer is found out at increase of intensity of inciting field [4, 5] (see Fig. 2(a), at positive value of the susceptibility $\alpha = 0.01$, and also Fig. 2(b), at negative value of the susceptibility $\alpha = -0.01$). Growth of intensity of



Figure 2: Parameters of structure: $\delta = 0.5$; $\varphi = 45^{\circ}$; $\kappa = 0.375$; $\varepsilon^{(L)} = 16$. (a)|I| = |inca| = 11.4; $\alpha = 0.01$, (b)|I| = |inca| = 22.4; $\alpha = -0.01$.

the inciting field $|I| = |a^{inc}|$ results in change of the share of the reflected wave $\eta(R(\alpha)) = |R(\alpha)|^2 / |I|^2$ (here $|R(\alpha)| \equiv |a^{scat}(\alpha)|, |T(\alpha)| \equiv |b^{scat}(\alpha)|, |I|^2 = |T(\alpha)|^2 + |R(\alpha)|^2$): reduction of value of resonant frequency with increase and reduction of a steepness of the diffraction characteristics before and after resonant frequency (Fig. 2(a), at $\alpha > 0$); increase of value of resonant frequency with reduction and increase of a steepness of the diffraction characteristics before and after resonant frequency (Fig. 2(b), at $\alpha < 0$).

2.2. Intensity and Angle

The effects: itself the channeling of a field — increase of the angle of the transparency of the nonlinear layer ($\alpha \neq 0$) when growth of intensity of the field (Fig. 3(a), at positive value of the susceptibility, $\alpha > 0$); de-channeling of a field (Fig. 3(b), at negative value of the susceptibility, $\alpha < 0$) are found out, [4,5].



Figure 3: Parameters of structure: $\delta = 0.5$; $\kappa = 0.375$; $\varepsilon^{(L)} = 16$; for linear layer with $\alpha \equiv 0$ and for nonlinear layer: a with $\alpha = 0.01$; b with $\alpha = -0.01$.

The increase of the angle of a transparency with growth of intensity at positive value of the susceptibility $\alpha = 0.01$ is easy for tracking on Fig. 3(a): $|a^{inc}| = 8$, $\varphi \approx 46^{\circ}$ and $|a^{inc}| = 11.4$, $\varphi \approx 85^{\circ}$.

Weak nonlinearity of a dielectric layer $\varepsilon(z, \alpha \cdot |E_x|^2) \equiv \varepsilon(z, \alpha \cdot |U|^2)$,

$$\max_{|z| \le 2\pi\delta} \left(\left| \alpha \right| \cdot \left| E_x \right|^2 \right) << \max_{|z| \le 2\pi\delta} \left| \varepsilon^{(L)}(z) \right|,\tag{6}$$

i.e., the small nonlinear additive $\alpha |U(z)|^2$ to a linear part $\varepsilon^{(L)}(z)$ of the dielectric permeability, caused by intensity $|U^{inc}|$ of a field of excitation of nonlinear object, results in essential changes diffraction characteristics. Exceeding some critical threshold of intensity the statement (6) loses force, computing process is broken. For example, diffraction characteristics reach critical values with growth of intensity of field, see lines for $\alpha > 0$ on Fig. 3(a): point of a transparency $\varphi = \varphi^*(|a^{inc}|)$, where $\eta(R)|_{\varphi=\varphi^*(|a^{inc}|)} = 0$ and $\eta(T)|_{\varphi=\varphi^*(|a^{inc}|)} = 1$, here $\varphi^*(|a^{inc}|)$ defined from: $\frac{d\eta(R)}{d\varphi}|_{\varphi=\varphi^*(|a^{inc}|)} = \frac{d\eta(T)}{d\varphi}|_{\varphi=\varphi^*(|a^{inc}|)} = 0$, weakly nonlinear layer aspires to limiting value $\varphi^*(|a^{inc}|) \to 90^\circ$ at $|a^{inc}| \to \max\{|a^{inc}|\} = 11.5$. The analysis of results for $\alpha < 0$ on Fig. 3(b) shows, that

limiting critical values $\eta(R)|_{\varphi=\varphi^*(|a^{inc}|)\equiv 0^\circ} \to 0.5$ and $\eta(T)|_{\varphi=\varphi^*(|a^{inc}|)\equiv 0^\circ} \to 0.5$ at $|a^{inc}| \to \max\{|a^{inc}|\}=22.4$ lay on curves of translucent $\eta(R) = \eta(T) = 0.5$ weakly nonlinear structure. It allows to estimate numerically size of required intensity of a field of excitation

$$\max_{|z| \le 2\pi\delta} \left(\left| \alpha \right| \cdot \left| U\left(z\right) \right|^2 \right) \le \max_{|z| \le 2\pi\delta} \left(\left| \alpha \right| \cdot \left| U^{inc}\left(z\right) \right|^2 \right) < C \cdot \max_{|z| \le 2\pi\delta} \left| \varepsilon^{(L)}(z) \right|$$
(7)

to make an estimation weakly sizes C, at which (6) does not lose force with growth of intensity of a field of excitation of a nonlinear layer.

For example, see Fig. 3(a), (where: $\varepsilon^{(L)}(z) = 16$, $\alpha = 0.01$), convergence of iterative process is broken when $|U^{inc}| > 11.5$. From (7) it is received: C = 0.083. Hence, weak nonlinearity proves at intensity not surpassing $|U^{inc}| = 11.5$ and variations of small nonlinearity layer: $\max_{|z| \le 2\pi\delta} \left(|\alpha| \cdot |U(z)|^2 \right) < 1.328$.

These effects (see sections 2.1 and 2.2) are connected to resonant properties of a nonlinear dielectric layer and caused by increase at positive value of the susceptibility or reduction at negative value of the susceptibility of a variation of dielectric permeability of a layer (its nonlinear components) when increase of intensity of a field of excitation of researched nonlinear object.

3. Conclusion

The principal fields where the results of our numerical analysis are applicable are as follows: the investigation of wave self-influence processes; the analysis of amplitude-phase dispersion of eigen oscillation-wave fields in the nonlinear objects, see [6]; extending the description of evolutionary processes near to critical points of the amplitude-phase dispersion of nonlinear structure; new tools for energy selecting, transmitting, and remembering devices; etc.

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Multipath Reduction of GPS Measures through Heuristic Techniques of Compensation

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Abstract—The Global Positioning System, also known with the acronym GPS, is today widely used in civil and military applications, as for correct object positioning, as in other fields (ionospheric inferences, soil mapping and characterization, and so on). A limitation in the accuracy retrievable by differential GPS measures is due to multipath error which arises when GPS signal is reflected by surfaces around the antenna. In particular, many GPS receivers are projected with firmware implemented by means of classical mathematic algorithms, which can minimize multipath errors. This paper describes project criteria and experimental results obtained by a multipath rejection system based on Radial Basis Function Neural Networks, compared with measures retrieved by a commercial Differential GPS receiver.

1. Introduction

One of the most relevant source of error in GPS differential measures is due to reflection of signal coming from satellite by surfaces around GPS antenna. This phenomenon introduces a distortion in error phase measurements, which in scientific literature is normally defined as multipath error [1]. In order to reduce its influence, many solutions have been studied, working at level of elaboration of received GPS signal at ground. In order to deject multipath effect, further methods are to position GPS antennas where reflections of signal coming from GPS constellation are minimized, or to give extended ground plane to GPS antennas. In many cases, however, these solutions are practically difficult to apply: an example is the installation of a GPS antenna on a satellite in order to calculate its correct position in real-time. Nowadays, the most part of commercialized Differential GPS (DGPS) receivers are able to grant performances of multipath rejection in the order of 75%, with obtaining times averagely near of 10 s. In scientific environment there is the requirement to obtain an expert and quick inferential model, that can guarantee multipath rejection quality at least comparable to classical models, but with better flexibility and robustness qualities, smaller computational complexity and lower calculus times during regression phase, for real-time applications. Therefore, a Radial Basis Function Neural Network (RBFNN) is used to calculate multipath biases, comparing retrieved simulation results with measurements obtained by a commercial DGPS receiver. Two quantities have been considered to evaluate our RBFNN model: gaps between RBFNN simulations and DGPS measurements, and elapsed times for retrieving RBFNN results and DGPS measurements. In detail, the paper is structured according to the following schema: section 2 describes the fundamental of DGPS, with a brief explanation of multipath effect; in section 3 RBFNN are drawn up; the case study and the campaigns of measurements are respectively pointed out in section 4 and 5; section 6 is used to show RBFNN simulation results and, finally, some conclusions are illustrated in section 7.

2. Differential GPS: the Multipath Minimization Problem

The NavSTAR GPS (Navigation Satellite Timing And Ranging Global Positioning System) system was originally borne in USA for military purposes; it allows the three-dimensional positioning of objects (also moving) to be identified by means of information coming from a geostationary satellite system by using distance measuring spatial intersections (ground receiver - orbit satellite). Mainly two kinds of GPS measures can be used [2]: the pseudo-range and phase measures. GPS satellites transmit on two frequencies, L1 (1575.42 MHz) and L2 (1227.6 MHz), of which the C/A code is modulated only on L1 while the P code is modulated on both frequencies [3]. Mathematically, the pseudorange observable is formulated as follows [4]:

$$p = \rho + d\rho + c(dt - dT) + d_{ion} + d_{trop} + \varepsilon_p \tag{1}$$

where p is the measured pseudorange ([m]), ρ is the geometric range ([m]), $d\rho$ is the orbital error ([m]), c is the speed of light in ([m/s]), dt is the satellite clock error ([s]), dT is the receiver clock error ([s]), d_{ion} is the ionospheric error ([m]), d_{trop} is the tropospheric error ([m]), ε_p is the receiver code noise plus multipath ([m]). In the above equation, there are four unknowns, the three components of the user position and the receiver clock error. Thus, assuming that all other errors are either removed by modelling or are zero mean, at least four pseudoranges must be observed in order to obtain a solution. The carrier phase observable is very similar to the pseudorange observable:

$$\Phi = \rho + d\rho + c(dt - dT) + \lambda N - d_{ion} + d_{trop} + \varepsilon_{\Phi}$$
⁽²⁾

where Φ is the observed integrated carrier phase ([m]), λ is the wavelength ([m]), N is the integer ambiguity (integer number of wavelengths between the satellite and the receiver), ε_{Φ} is the receiver carrier phase noise plus multipath ([m]). In an attempt to reduce the errors in positioning results using equations (1) and (2), the difference between observations from two different receivers but the same satellite are considered. Therefore, the satellite clock error, is completely eliminated while the atmospheric and orbital errors are significantly reduced. This method is commonly referred to as DGPS. By further differencing the observables between satellites (see Fig. 1), the receiver clock error term is also eliminated and the double difference equations (represented by $\Delta \nabla$) become:

$$\Delta \nabla p = \Delta \nabla \rho + \Delta \nabla d\rho + \Delta \nabla d_{ion} + \Delta \nabla d_{trop} + \Delta \nabla \varepsilon_p \tag{3}$$

$$\Delta \nabla \Phi = \Delta \nabla \rho + \Delta \nabla d\rho + \lambda \Delta \nabla N - \Delta \nabla d_{ion} + \Delta \nabla d_{trop} + \Delta \nabla \varepsilon_{\Phi} \tag{4}$$

The main advantage of the double difference observation is that the receiver clock errors are eliminated, and ionospheric, tropospheric, and orbital errors are reduced.



Figure 1: Graphic schematization for double differences calculation

2.1. Multipath errors

Multipath is defined as "the phenomena whereby a signal arrives at a receiver via multiple paths" [5]. It can be caused by almost any reflective surface near the antenna (Fig. 2). For short baselines (i.e. <10 km), multipath is usually the largest error source. Under severe multipath conditions, errors can reach 1 wavelength (i.e. 1 chip length) for code observations or 1/4 of a wavelength for phase observations. Recently, improvement has been made in receiver design to reduce the effect of multipath in code measurements, with percentages equal to 75% depending on the multipath delay.



Figure 2: Graphical description of multipath phenomenon

3. Radial Basis Function Neural Networks

Adaptive Neural Networks are very good tools for non-linear approximation [6]. Neural networks are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems. As in nature, the network function is determined largely by the connections between elements. Commonly neural networks are adjusted, or trained, so that a particular input leads to a specific target output. It is possible to train a neural network to perform a particular function by adjusting the values of the connections (weights) between elements. Typically many such input/target pairs are used, in this supervised learning, to train a network. Therefore, Adaptive Neural Networks are basically adaptive systems that "learn" to correctly execute a defined job (complex, non linear and multi-variable relations) by using some examples [7]. RBFNNs have capabilities to solve function approximation problems [8]. RBFNNs consist of three layers of nodes: more than input and output layers, RBFNNs have a hidden layer, where Radial Basis Functions are applied on the input data. A schematic representation of RBFNN is described in Fig. 3:



Figure 3: RBFNN schema

The ||dist|| box in this figure accepts the input vector p and the input weight matrix IW^{1,1}, and produces a vector having S¹ elements. The elements are the distances between the input vector and vectors $_i$ IW^{1,1} formed from the rows of the input weight matrix. We can understand how this network behaves by following an input vector p through the network to the output a². If we present an input vector to such a network, each neuron in the radial basis layer will output a value according to how close the input vector is to each neuron's weight vector. Thus, radial basis neurons with weight vectors quite different from the input vector p have outputs near zero. These small outputs have only a negligible effect on the linear output neurons. In contrast, a radial basis neuron with a weight vector close to the input vector p produces a value near 1. If a neuron has an output of 1 its output weights in the second layer pass their values to the linear neurons in the second layer. In fact, if only one radial basis neuron had an output of 1, and all others had outputs of 0's (or very close to 0), the output of the linear layer would be the active neuron's output weights. This would, however, be an extreme case. Typically several neurons are always firing, to varying degrees.

4. The Case Study

Faculty of Engineering of University "Mediterranea" of Reggio Calabria (Italy) activated a stable network to monitor relative movements of Sicilian and Calabrian shores inner the area of Stretto di Messina by means GPS techniques. Since vertexes of a main network cannot make movements in the order of the measure uncertainty $(10^{-6} \div 10^{-7})$, it has been necessary to implement a control subnet for each main vertex. One of these subnets is composed by 4 vertexes (Fig. 4); they are located in a surely stable area with an optimal visibility of satellites. It has been chosen short bases in order to avoid tropospheric and ionospheric errors; but it has been retrieved multipath errors in base 2-4, due to reflecting surfaces near one of the vertexes.

Therefore, it has been exploited a RBFNN-based system in order to minimize multipath errors, evaluating multipath as a function of other GPS quantities. RBFNNs are able to approximate the trend of a function after a training procedure, which is carried out by a collection of data examples. Therefore, it is necessary to make a measurement campaign in order to implement a dataset of training (DBTrain) and another to test the behavior of RBFNN (DBTest).



Figure 4: Control subnet implemented to guarantee stability of main network

5. Data Collection Campaign and RBF Simulation Results

The data collection campaign has been carried out inner the area described on section 4, by means of a L1/L2 GPS antenna without multipath rejection, in two different days (acquisition times about equal to 24 hours). Multipath error has subsequently been calculated by means of equation (5), representing a simplified schematization of phase differential measure:

$$b \bullet s = m\lambda + \Delta\varphi + \Delta\varphi_{ric} + \Delta\varphi_{bias} + \Delta\varphi_{mp} \tag{5}$$

where b is baseline direction; s is the arrival direction of GPS signal; $\Delta \varphi$ is the fractional part of phase differential measure (observable); $\Delta \varphi_{ric}$ is the receiver noise (white noise with standard deviation approximately equal to 1-2 mm.); $\Delta \varphi_{bias}$ is the bias noise, considered as a constant (autocalibration procedure); $\Delta \varphi_{mp}$ is the multipath error (highly correlated coloured noise). It is possible to obtain the $\Delta \varphi_{mp}$ value expressed in degree for each base, by a comparison with the opportune "nominal reference" and after approximations of bias and noise in equation (5). Moreover, it is necessary to verify that approximations influence the measure value without changing it, if the measure is compared by means of different antennas. This value is calculated by exploiting the position of the two receivers and the satellite providing GPS signal; the satellite position is defined by azimuth and elevation and is retrievable by ephemeredes of GPS signal. By using a classical GPS antenna without multipath rejection, a set of measurements has been carried out in different places of Reggio Calabria, Italy. So, a set of values has been collected for each of the following quantities (see eq. 5): b, $\Delta \varphi$, $\Delta \varphi_{ric}$, $\Delta \varphi_{bias}$; let us define, for each measurement, these quantities as an input pattern. Subsequently, by using equations (5), (4)and (2) the $\Delta \varphi_{mp}$ values (output patterns) has been calculated, and each input pattern has been linked to its proper output pattern, making a so called data pattern. So, a set of 1500 data patterns has been collected in order to implement the database used for training phase of RBFNN. The trained RBFNN retrieved by Neural Network Matlab Toolbox has been subsequently tested inner the base 2-4 (Fig. 4). Specifically, it has been considered mean, standard deviation and Round Mean Square (RMS) of errors introduced by $\Delta \varphi_{mp}$. They have been obtained after the elaboration of raw data during the 24 hours of acquisition using all visible satellites. Retrieved results demonstrate a reduction of multipath effect in average equal to 45%, in standard deviation of 20% and in RMS of 25%. Moreover, performances of our approach have been tested measuring the slope distance of base 2-4. Measure retrieved by RBFNN-based system has been compared with ones retrieved by a LEICA SR530 receiver.

Table 1 shows the measurements retrieved by LEICA SR530 and our RBFNN-based system for multipath rejection, compared with actual slope distance measured by classical techniques. Performances of our RBFNN-based system are also evaluated by a comparison of elapsed times to obtain 2-4 slope distance with the used commercial GPS receiver (Table 2).

Table 1: Slope distance of base 2-4

	Actual	LEICA SR530	RBFNN-based System
Slope distance 2-4 (meters)	50.000 ± 0.0001	50.12 ± 0.0001	50.004 ± 0.0001

Table 2: Elapsed times to obtain 2-4 slope distance

LEICA SR530	RBFNN-based System
10 seconds	1.2 second

6. Conclusions and Future Works

In this article, the problem of multipath in GPS measures has been analyzed. This is a very significant problem, above all when application field needs a great accuracy in GPS observation. By means of a classical GPS antenna, measurement campaigns has been carried out, and an heuristic system based on RBFNN has been considered in order to reject multipath effects. Performances of our RBFNN-based system for multipath rejection has been evaluated during the acquisition period and compared with measurements retrieved by commercial receiver LEICA SR530. Retrieved results show the following advantages of RBF approach:

- during the 24 hours of acquisition, the RBFNN-based system allows a reduction of multipath effect averagely equal to 30%;
- concerning the calculation of slope distance for base 2-4, a reduction of measure error about equal to 0.234% is obtained exploiting our RBFNN approach against the LEICA receiver;
- RBFNN-based system shows lower elaboration times than LEICA receiver; it is due to the lower computational complexity of the RBFNN-based system

Therefore, the usage of RBFNNs to reduce multipath errors show very interesting results, even if, considering the approximations in (5) for bias and noise, they would have to be verified according to the application environment. Moreover, a further development will be the hardware deployment on FPGA of trained RBFNN, in order to build a Special Purpose Chip (SPC). The aim is to integrate the SPC into a GPS receiver without multipath minimization capabilities, in order to compare its performances with ones obtained by a multipath rejection antenna.

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Optimized Satellite System-like Data Fitting on a Spherical Shell

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Abstract—The measurement of irradiance on a spherical shell is a common in different fields that goes from geological, biological and many others. Accuracy depends of a judicious use of the sampling, and the last is often defined by technical limitations due to the available infrastructure in each field. Such is the case of the so called planetary array, described as a set of satellite like trajectories on the spherical shell. Both, the data acquisition and the ideal data base, in this case spherical harmonics, may be optimized. The measured points are distributed over maximal circles obtained from the equator, and each other are related by the corresponding rotations. Each circle has 2L+1 equidistant points (the first one and the last one coincides), and the field on the sphere is adjusted by the determination of the harmonic coefficients, as an optimization problem, with Nc equation systems of 2L+1 simultaneous equations for $(L+1)^2$ variables.

1. Introduction

We had previously analyzed the problem of the irradiance measurement under circular and spherical geometries, considering a uniform sensor detection system [1]. Now, we are trying to measure a field F, on the surface of the Earth using a spherical detection system. The detectors are localized on trajectories, which are obtained by rotations over the equator, as it is shown in Fig. 1, but the arrangement of the detectors is not uniform distributed on the trajectories as in our earlier spherical experiments.



Figure 1: One of the detectors of the proposed spherical detection system.

The analysis of the new system requires the mathematics development that is described as follows:

At first, we consider an observable $F(\theta_n; \phi_n)$ over a determined number of points in the sphere. The points of measurement are distributed over N_c maximal circles, which are obtained since rotations from the equator C^0 (defined by $\theta = \frac{1}{2}\pi$ and $0 \le \phi < 2\pi$):

$$C^{(\alpha_j,\beta_j,\gamma_j)} := R(\alpha_j,\beta_j,\gamma_j) : C^0$$
⁽¹⁾

by Euler angles $(\alpha_j, \beta_j, \gamma_j)$, $j = 1, 2, ..., N_c$. The normal lines to these circles are (β_j, γ_j) on the sphere, and its phases are α_j respect to the Greenwich meridian.

Over each circle there are distributed an odd number 2L + 1 of equidistant points, where we measured the value of $F(\theta_n; \phi_n)$. In this way, we can calculate its 2L + 1 Fourier coefficients G_m through the FFT [2]. For

the equator case C^0 , we calculate:

$$\widetilde{F}_{m}^{0} := \frac{1}{\sqrt{2l+1}} \sum_{n=-L}^{L} F(\frac{1}{2}\pi, \phi_{n}) e^{i(m\phi_{n})}, \quad \text{where} \quad \phi_{n} := \frac{2\pi n}{2L+1},$$
(2)

with $n, m \in \{-L, -L+1, ..., L\}$ module 2L + 1 in the symmetrical interval f cycling. The Fourier synthesis is given by:

$$F(\frac{1}{2}\pi,\phi_n) = \frac{1}{\sqrt{2L+1}} \sum_{m=-L}^{L} \widetilde{F}_m^0 e^{-im\phi_n}.$$
 (3)

On the maximal circles $C^{(\alpha_j,\beta_j,\gamma_j)}$ we will have the measurements and calculations of the 2L+1 corresponding coefficients, $\tilde{F}_m^{(\alpha_j,\beta_j,\gamma_j)}$, $m \mid_{-L}^L$, $j \mid_{1}^{N_c}$.

The F field over the sphere under measurement has a development in spherical harmonics truncated at L value, given by:

$$F(\theta,\phi) = \sum_{\ell=0}^{L} \sum_{m=-\ell}^{\ell} F_{\ell,m} Y_{\ell,m} .$$
(4)

The number of elements in the series is $1 + 3 + 5 + \cdots + (2L + 1) = (L + 1)^2$. The question is: How we can calculate the $(L + 1)^2$ coefficients of spherical harmonics $F_{\ell,m}$ in terms of the $N_c(2L + 1)$ coefficients $\widetilde{F}_m^{(\alpha_j,\beta_j,\gamma_j)}$ obtained over the circles?

In the following section we present the development over the equator, in section 3 we present the rotation over any maximal circle, and in section 4 we compare the obtained results.

2. Development over the Equator

The spherical harmonics are given by:

$$Y_{\ell,m}(\theta,\phi) = (-1)^m \sqrt{(\ell+2)(\ell+m)!(\ell-m)!} \frac{e^{im\phi}}{\sqrt{2\pi}} \times \sum_k \frac{(-1)^k (\sin\theta)^{2k+m}}{2^{k+m}(k+m)!k!} \frac{(\cos\theta)^{\ell-2k-m}}{(\ell-2k-m)!} \,. \tag{5}$$

The factorials implies that the addition is over all integers k among max(-m, 0) and $\frac{1}{2}(\ell - |m|)$, the number of elements in the addition is given by $\frac{1}{2}(\ell - |m|) + 1$ (where |x| is the integer part of x). Over the equator $\theta = \frac{1}{2}\pi$, the factor $(\cos \theta)^{\ell-2k-m}$ is different of zero, only when its power is zero, $id est \ k = \frac{1}{2}(\ell - m)$ with $\ell - m$ even. Then:

$$Y_{\ell,m}(\frac{1}{2}\pi,\phi) = y_{\ell,m}\frac{e^{im\phi}}{\sqrt{2\pi}}, \quad y_{\ell,m} = y_{\ell,-m},$$
(6)

$$Y_{\ell,m} := \begin{cases} (-1)^{(\ell+m)/2} \frac{\sqrt{(\ell+\frac{1}{2})}(\ell+m)!(\ell-m)!}{2^{\ell}(\frac{1}{2}[\ell+m])!((\frac{1}{2}[\ell-m])!)} & \ell \pm m \text{ even} \\ 0, & \ell \pm m \text{ odd} \end{cases}$$
(7)

Over the equator C^0 , the field $F(\theta, \phi)$ development in a different of zero harmonics series (4) is:

$$F(\frac{1}{2}\pi,\phi) = \sum_{\ell=0}^{L} \sum_{m=-\ell}^{L} F_{\ell,m} Y_{\ell,m}(\frac{1}{2}\pi,\phi), \quad \left(\begin{array}{c} \text{the harmonics of zero} \\ \text{value in } C^{0} \text{ are absent} \end{array} \right)$$
$$= \sum_{m=-\ell}^{L} \frac{e^{im\phi}}{\sqrt{2\pi}} \sum_{\ell=|m|\atop \ell=m \text{ even}}^{L} F_{\ell,m} y_{\ell,m}, \quad \left(\begin{array}{c} \text{interchanging} \\ \text{additions } \ell \text{ and } m \end{array} \right)$$
$$= \frac{1}{\sqrt{2L+1}} \sum_{m=-\ell}^{L} e^{im\phi} \widetilde{F}_{m}^{0}, \quad \left(\begin{array}{c} \text{in agreement with } (3) \\ \text{considering } \phi_{n} = 2\pi n/(2L+1). \end{array} \right)$$
(8)

Comparing the last two terms, related with the coefficients introducing the 2L+1 measured/calculated data:

$$G_m^0 := \sqrt{\frac{2\pi}{2L+1}} \, \widetilde{F}_m^0 = \sum_{\substack{\ell = |m| \\ \ell - m \, \text{even}}}^L F_{\ell,m} \, y_{\ell,m}, \quad m \mid_{-L}^L.$$
(9)

Here we have the key relationships among the 2L + 1 coefficients $\{G_m^0\}_{m=-L}^L$ measured/calculated over the equator, and the $(L+1)^2$ coefficients $\{F_{\ell,m}\}_{m=-\ell,\ell=0}^{\ell}$ of the spherical harmonic development that we are looking for. The harmonics with $\ell - m$ even are absent, which have one of their nodal circles over the equator. As an example we consider the case with L = 3 Then we have the development with 16 spherical harmonics:

$$Y_{0,0,} \{Y_{1,m}\}_{m=-1}^{1}, \{Y_{1,m}\}_{m=-2}^{2}, \text{ and } \{Y_{1,m}\}_{m=-3}^{3}$$

with their corresponding coefficients $F_{\ell,m}$.

On the other side, we have the 7 measured/calculated coefficients $\{G_m^0\}_{m=-3}^3$, $\{G_m^0\} = \sqrt{2\pi/7} \widetilde{F}_m^0$. The equations (9) are then:

$$G_{3}^{0} = F_{3,3} y_{3,3} \qquad m = 3 \qquad \bullet \qquad \\ G_{2}^{0} = F_{2,2} y_{2,2} \qquad m = 2 \qquad \bullet \qquad \circ \qquad \\ G_{1}^{0} = F_{1,1} y_{1,1} + F_{3,1} y_{3,1} \qquad m = 1 \qquad \bullet \qquad \circ \qquad \bullet \qquad \\ G_{0}^{0} = F_{0,0} y_{0,0} + F_{2,0} y_{2,0} \qquad m = 0 \qquad \bullet \qquad \circ \qquad \bullet \qquad \circ \qquad \longrightarrow \ell \qquad (10) \\ G_{-1}^{0} = F_{1,-1} y_{1,-1} + F_{3,-1} y_{3,-1} \qquad m = -1 \qquad \bullet \qquad \circ \qquad \\ G_{-2}^{0} = F_{2,-2} y_{2,-2} \qquad m = -2 \qquad \bullet \qquad \circ \qquad \\ G_{-3}^{0} = F_{3,-3} y_{3,-3} \qquad m = -3 \qquad \bullet \qquad \end{aligned}$$

The right diagram shows the structure of the present terms \bullet in the truncated series, and the absent ones \circ .

The equations used to calculate the 16 [that is, the $(L+1)^2$] coefficients of the harmonic series $F_{\ell,m}$ are divided in three groups: are divided in three groups:

- Determined: $F_{3,\pm 3}$ and $F_{2,\pm 2}$ Always are 4: $F_{L,\pm L}$ and $F_{L-1,\pm (L-1)}$.
- In linear combination: $F_{1,1} \leftrightarrow F_{3,1}, F_{0,0} \leftrightarrow F_{2,0}, F_{1,-1} \leftrightarrow F_{3,-1}$. Generally For $|m| \leq L-2$, there are $F_{\ell,m} \leftrightarrow F_{\ell',m}$ with $0 \leq \ell \leq \ell' \leq L$. In the horizontal line m of the diagram (10), there are a total of $\frac{1}{2}[(L-|m|)]+1$ coefficients in linear combination. There are a total of $\frac{1}{2}(L^2+3L-6)$ $F_{\ell,m}$ coefficients that we known only inside of linear combinations.
- Undetermined: $F_{3,\pm 2}$, $F_{2,\pm 1}$, $F_{1,0}$, and $F_{3,0}$. Generically, they are the known $F_{\ell,m}$ with ℓm odd, whose spherical harmonics are zero in the equator, and the number of them is $\frac{1}{2}L(L+1)$.

3. Development over the Circle $C^{(\alpha,\beta,\gamma)}$

In this section, we rotate this maximal circle as it is presented in equation (1) in order to obtain the generic circle $C^{(\alpha,\beta,\gamma)}$.

Under the rotation by means of the Euler angles (α, β, γ) , the spherical harmonics showed in equation (4) of each ℓ order, is transformed as:

$$Y_{\ell,m}(\theta',\phi') = [R(\alpha,\beta,\gamma):Y_{\ell,m}](\theta,\phi) = \sum_{m'=-\ell}^{\ell} D_{m,m'}^{\ell}(\alpha,\beta,\gamma):Y_{\ell,m'}(\theta,\phi),$$
(11)

where the rotation of the polar coordinates in the sphere is:

$$\begin{pmatrix} \sin\theta'\cos\phi'\\ \sin\theta'\sin\phi'\\ \cos\theta' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \times \begin{pmatrix} \cos\gamma & -\sin\gamma & 0\\ \sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta \end{pmatrix}.$$
(12)

And the coefficients of the linear combination are the functions D of Wigner, $D_{m,m'}^{\ell}(\alpha,\beta,\gamma)$, which is factorized as:

$$D^{\ell}_{m,m'}(\alpha,\beta,\gamma) = e^{-im\alpha} d^{\ell}_{m,m'}(\beta) e^{-im'\gamma}.$$
(13)

In terms of phases, for α and γ , and the little-*D* of Wigner, $d_{m,m'}^{\ell}(\beta)$, given by:

$$d_{m,m'}^{\ell}(\beta) = \sqrt{(\ell+m)!(\ell-m)!(\ell+m')!(\ell-m')!} \sum_{k} (-1)^{m-m'+k} \frac{(\sin\frac{1}{2}\beta)^{2k+m-m'}}{(k+m-m')!k!} \frac{(\cos\frac{1}{2}\beta)^{2\ell+m'-m-2k}}{(\ell-m-k)!(\ell+m'-k)!}.$$
 (14)

The k index of the addition takes the integer values among the values: $\max(m'-m, 0) \le k \le \min(\ell - |m|, \ell - |m'|)$. The little-d coefficients satisfy many relationships, such as:

$$d_{m,m'}^{\ell}(\beta) = d_{m,m'}^{\ell}(-\beta) = d_{-m',-m}^{\ell}(\beta) = (-1)^{m-m'} d_{-m,-m'}^{\ell}(\beta),$$
(15)

and they are related with the spherical harmonics by:

$$Y_{\ell,m}(\theta,\phi) = \sqrt{\frac{2L+1}{4\pi}} d^{\ell}_{m,0}(\theta) e^{jm\phi}.$$
 (16)

They satisfy recurrence S of three terms in m and m':

$$\sqrt{(\ell - m')(\ell + m' + 1)} \sin\beta d^{\ell}_{m,m'+1}(\beta) + 2(m - m' \cos\beta) d^{\ell}_{m,m'}(\beta) + \sqrt{(\ell + m')(\ell - m' + 1)} \sin\beta d^{\ell}_{m,m'-1}(\beta) = 0 \quad (17)$$

$$\sqrt{(\ell - m)(\ell + m + 1)} \sin\beta d^{\ell}_{m+1,m'}(\beta) - 2(m' - m\cos\beta) d^{\ell}_{m,m'}(\beta) + \sqrt{(\ell + m)(\ell - m + 1)} \sin\beta d^{\ell}_{m-1,m'}(\beta) = 0 \quad (18)$$

The harmonics $Y_{\ell,m}$ are transformed as a column vector. Under the rotation matrix $||D_{m,m'}^{\ell}||$, then, the coefficients $F_{\ell,m}$ of the series (4), that we are trying to find, are transformed as a row vector:

$$F'_{\ell,m} = [R(\alpha_j, \beta_j, \gamma_j) : F]_{\ell,m} = \sum_{m'=-\ell}^{\ell} F_{\ell,m'} D_{m',m}(\alpha, \beta, \gamma).$$
(19)

The equator has been transformed in the circle $C^{(\alpha,\beta,\gamma)}$, nd over it, we make the measurements/calculations of the corresponding coefficients $G_m^{(\alpha,\beta,\gamma)}$, in the same way as in G_m^0 , given by equation (10). So, we have, as in (9):

$$G_{m}^{(\alpha,\beta,\gamma)} = \sum_{\substack{\ell=|m|\\\ell-m \,\text{even}}}^{L} F_{\ell,m}' y_{\ell,m} = \sum_{\substack{\ell=|m|\\\ell-m \,\text{even}}}^{L} y_{\ell,m} \sum_{m'=-\ell}^{L} F_{\ell,m'} D_{m',m}^{\ell}(\alpha,\beta,\gamma).$$
(20)

The arrangement of the two additions can not be made directly, but in generic form, it can be written in terms of the column vectors:

$$F_{0,0}, \ F_{1,\cdot} := \begin{pmatrix} F_{1,1} \\ F_{1,0} \\ F_{1,-1} \end{pmatrix}, \ F_{2,\cdot} := \begin{pmatrix} F_{2,2} \\ F_{2,1} \\ F_{2,0} \\ F_{2,-1} \\ F_{2,-2} \end{pmatrix}, \ \dots \ F_{\ell,\cdot} := \begin{pmatrix} F_{\ell,\ell} \\ F_{\ell,\ell-1} \\ F_{\ell,\ell-2} \\ \vdots \\ F_{\ell,-\ell} \end{pmatrix},$$
(21)

and the row vectors:

$$D^{\ell}_{\cdot,m} := (D^{\ell}_{\ell,m} \ D^{\ell}_{\ell-1,m} \ D^{\ell}_{\ell-2,m} \ \dots \ D^{\ell}_{-\ell,m}).$$
(22)

The equations (20), which related to the spherical harmonic coefficients $F_{\ell,m}$ that we are trying to find, and the measured/calculated coefficients $G_m \equiv G_m^{(\alpha,\beta,\gamma)}$ over the maximal circle, are obtained from equation (10) changing $F_{\ell,m}$ by $F'_{\ell,m}$ in accordance with equation (19). The sub-matrix representation can be compared with the case L = 3:

$$\begin{pmatrix} G_{3}^{(\omega)} \\ G_{2}^{(\omega)} \\ G_{1}^{(\omega)} \\ G_{0}^{(\omega)} \\ G_{-1}^{(\omega)} \\ G_{-2}^{(\omega)} \\ G_{-2}^{(\omega)} \\ G_{-2}^{(\omega)} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & y_{3,3}D_{\star,3}^{3} \\ 0 & 0 & y_{2,2}D_{\star,2}^{2} & 0 \\ 0 & y_{1,1}D_{\star,1}^{1} & 0 & 0 \\ y_{0,0} & 0 & y_{2,0}D_{\star,0}^{2} & 0 \\ 0 & y_{1,-1}D_{\star,-1}^{1} & 0 & y_{3,-1}D_{\star,-1}^{3} \\ 0 & 0 & y_{2,-2}D_{\star,-2}^{2} & 0 \\ 0 & 0 & 0 & y_{3,-3}D_{\star,-3}^{3} \end{pmatrix} \begin{pmatrix} F_{0,\star} \\ F_{1,\star} \\ F_{2,\star} \\ F_{4,\star} \end{pmatrix}.$$
(23)

The matrix is not square, it has 7 rows and its 4 columns represent the 1 + 3 + 5 + 7 = 16 columns of the developed matrix, which generically is of $(2L + 1) \times (L + 1)^2$.

When we calculate the elements of the matrix (23), we use the properties $y_{\ell,m} = y_{\ell,-m}$ and the recurrence property mentioned in equation (15).

4. Equation Systems

Over the equator and in respect to the Greenwich meridian, the equation system (23) obtains again it most simple representation, as in equation (10), since $D_{m,m'}^{\ell}(0,0,0) = \delta_{m,m'}$ After each rotation, however, the number of determined, in linear combination and the undetermined coefficients are the same (for L = 3, 4, 6 and 6). And $2L + 1 < (L + 1)^2$ for L < 0.

Considering now, several measurement circles $C^{(\omega_j)}$, $j = 1, 2, ..., N_c$, with orientation $\omega_j = (\alpha_j, \beta_j, \gamma_j)$. For each N_c and L, we will have a set of equations of the form (23)–(20):

$$G_m^{(\omega_j)} = \sum_{\substack{\ell = |m| \\ \ell - m \text{ even}}}^L y_{\ell,m} \, D_{\bullet,m}^{\ell}(\omega_j) \, F_{\ell,\bullet} \,.$$

$$\tag{24}$$

They can be written using the double indexes (j and m) and $(\ell \text{ and } m')$ for enumerate the arrows and columns of the matrix $N_c(2L+1)\times(L+1)^2$:

$$G = MF \tag{25}$$

where $G = \|G_{j,m}\|$, $G_{j,m} = G_m^{(\omega j)}$, $M = \|M_{j,m;\ell,m'}\|$, $M_{j,m;\ell,m'} = y_{\ell,m}D_{m',m}^{\ell}(\omega j)$, $F = \|F_{\ell,m'}\|$.

5. Conclusions

The problem of determining the spherical harmonics of the field $F(\theta, \phi)$ over the sphere can be considered as an optimization problem, where there are N_c systems of 2L + 1 simultaneous equations with $(L+1)^2$ variables.

If the observations of $F(\theta_n, \phi_n) \Rightarrow F_m^{(\omega j)} \Rightarrow G_m^{(\omega j)}$ are not accurate, we will need to adjust to the harmonic coefficients $F_{\ell,m}$ by minimal square or other algorithm.

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A Study of RF Absorber for Anechoic Chambers Used in the Frequency Range for Power Line Communication System

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Abstract—Power line communication (PLC) system is spreading as a new communication system without providing new infrastructure for network communication. However, It is needed to investigate the EMI problem caused by PLC, and it is often tested in an anechoic chamber and an open area test site (OATS). The semi anechoic chamber lined ferrite tiles used for EMC testing is not generally designed in the frequency range from 2 MHz to 30 MHz used for PLC system. This paper presents characteristics of conventional ferrite absorber which are used for a semi anechoic chamber (SAC) and site attenuation of the semi anechoic chamber in the frequency range used for PLC system.

1. Introduction

Recently, power line communication (PLC) system is spreading as convenient communication without providing new infrastructure for network communication. PLC system is using utility-owned power lines in the low-voltage mains grid to provide broadband Internet access in areas that are mostly residential. PLC uses unshielded, low-voltage power line distribution cables inside and outside of buildings as transmission media with high speed rates. Because PLC system uses the frequency range from 2 MHz to 30 MHz in order to communicate information, it becomes a subject of discussion to influence other electrical or electronic device. Therefore RF emissions from PLC system have been investigated in an anechoic chamber or an open area test site (OATS). Ferrite absorber with thickness from about 6 mm to 7 mm without some kind of pyramidal absorbers is used for 3 m or compact anechoic chamber (CAC), because the ferrite absorber has excellent absorption in the frequency range from 30 MHz to 1 GHz. In this paper, we discussed the characteristics of conventional ferrite absorber which is used for a 3 m or a CAC from 2 MHz to 30 MHz utilized for PLC system. Site attenuation characteristics of the conventional 3 m anechoic chamber were investigated.



Figure 1: Semi anechoic chamber lined ferrite tiles. (Ferrite tiles are installed back of the white interior finishing panel. Pyramidal ferrite absorbers are installed back of the black color rea).

2. Characteristics of Ferrite Absorber

2.1. Measurement Method

The Reflection characteristics, relative permittivity and relative permeability of ferrite absorbers were measured using the coaxial line with a diameter of 39 mm (The 39 D Coax.) in Figure 2, and the square coaxial line with a section of $300 \text{ mm} \times 300 \text{ mm}$ (The Square Coax.) in Figure 3. The 39 D Coax is used for measurement

complex permittivity and permeability. In this paper, it was used to measure characteristics of ferrite material without small gaps. It is well known that minute air gaps between ferrite tiles reduce absorption of ferrite absorber, and the Square Coax is able to evaluate the reflection of ferrite tiles including the air gaps. This measurement procedure of the Square Coax is same way as the $1.8 \text{ m} \times 1.8 \text{ m}$ Large Square Coax which had been developed and presented [1]. The Square Coax is an outer conducting line with a section of $302 \text{ mm} \times 302 \text{ mm}$ and an inner conducting line with a section of $98 \text{ mm} \times 98 \text{ mm}$, and eight pieces of ferrite tiles were arranged. A special feature is to set two ports in order to measure full complex S parameter (S11, S22 and S21) of a test sample, and it is possible to calculate complex permittivity and permeability from the measured S parameter of the test sample. As it is well known, it is very important to know the complex permittivity and permeability. Because the characteristics impedance of the Coax. To solve this mismatching problem, a serial resistance was inserted at the end of inner conducting line, and it could reduce inner reflection. On the other hand, the mismatching at the port of viewpoint from outside increased by the inserted serial resistance. The complex S parameters which removed the redundant reflections are driven by quotation (1), (2), and (3).

$$S11_{cor} = \frac{S11_{sample} - S11_{air}}{1 - S11_{air}} \quad \text{RPR}_{S11} \tag{1}$$

$$S22_{cor} = \frac{S22_{sample} - S22_{air}}{1 - S22_{air}} \quad \text{RPR}_{S22} \tag{2}$$

$$S21_{cor} = \frac{S21_{sample}}{S21_{air}} \tag{3}$$

Where

$$\operatorname{RPR}_{S11} = e^{J \cdot \frac{2\pi \cdot 2(EL_{s11} - d)}{\lambda}}$$
(4)

$$\operatorname{RPR}_{S22} = e^{J \cdot \frac{2\pi \cdot 2 \cdot EL_{S22}}{\lambda}}$$
(5)

$$RPR_{S21} = e^{J \cdot \frac{2\pi \cdot d}{\lambda}} \tag{6}$$

Where

d: Thickness of test sample

 λ : Wave length

 EL_{S11} : Electric Length between the calibration point of port 1 and the sample set point EL_{S22} : Electric Length between the calibration point of port 2 and the sample set point





Figure 2: The 39D Coax and test sample.

Figure 3: The 300 Square Coax.

2.2. Fundamental Characteristics of Ferrite Absorber

At first the fundamental characteristics of the Ni–Zn–Cu ferrite absorber with 6.3 mm thick was investigated below 30 MHz, in order to confirm the characteristics which is influenced by minute air gaps between ferrite tiles. The 39 D Coax and the Square Coax were each used for obtaining the fundamental characteristics of without or with the influence of minute air gaps. Figure 4 shows the shape of test sample for 39 D Coax, and Figure 5 shows the eight piece of ferrite tiles are inserted in the Square Coax.

Figure 6 shows chart of the reflectivity of conventional ferrite absorber which was measured using the 39 D Coax. In this chart, this ferrite absorber does not have sufficient absorbing characteristics in the frequency range from 3 MHz to 30 MHz used for PLC system. The absorbing characteristics of eight piece of ferrite tile including minute air gaps were reduced compared from the measured data of ferrite absorber with no air gap.

These air gaps were not artificially given between ferrite tiles, ferrite tiles were arranged to keep minimum air gaps. The complex permittivity and permeability are shown in Figures 7 and 8, and the value of the imaginary permeability which is loss paragraph is reduced by minute air gaps.





Figure 4: Test sample for 39D Coax.

Figure 5: Arrangement of Ferrite Tiles in the 300 Square Coax.



Figure 6: Measured permittivity and permeability of ferrite absorber with short end.



Figure 7: Measured permittivity and permeability of ferrite absorber. (Compare of 39 D and 300 Square Coax.)

2.3. Various Measurement Results

The following methods were investigated in order to improve the absorption of the ferrite tiles with minute air gaps.

- (1) Change the thickness of the ferrite.
- (2) Put overlap tile onto the joint between ferrite tiles as shown in Figure 5.
- (3) Combine a carbon material board on the ferrite absorber.
Figure 8 shows the reflectivity of the ferrite tiles with a thickness of 6.3, 8, 10 and 12 mm thick. The reflectivity became to reduce with to thicken the ferrite below 15 MHz, and it became larger over 10 mm thick in the frequency range above 15 MHz. To put overlap tile onto the ferrite tile reduced the reflectivity of the ferrite in Figure 9.



Figure 8: Relation reflectivity and thickness of ferrite.

Figure 9: Reflectivity of using overlap tile.

It was studied that combining carbon material was reduced the reflectivity of the conventional ferrite from 3 MHz to 30 MHz used PLC system. Figure 10 shows the relative complex permittivity of the carbon material (Polypropylene dispersed carbon powder) for combining to the ferrite absorber. Figure 11 shows the measured reflectivity of the double layer absorber composed the ferrite with 12 mm thick and carbon material. The resonance frequency was lower with to thicken the carbon material.



Figure 10: Relative permittivity of carbon material Figure 11: Reflectivity of combining carbon material.

3. Calculated Results of Site Attenuation of Semi Anechoic Chamber

Measurement layout for testing leaked E-field from PLC system is shown Figures 12 [2]. Figures 13 shows a layout of two 80 MHz tuned dipole antennas in order to calculate classical site attenuation (CSA) of a typical 3 m semi anechoic chamber lined ferrite tiles with 6.0 m wide $\times 9.0 \text{ m}$ length $\times 5.5 \text{ m}$ high. Test distance was 3 m, and the transmitting and the receiving antenna were each 1.5 m and 1.0 m high. The CSA of the SAC was calculated by ray tracing method, and the electro motive force (EMF) method was utilized for analysis of antenna [3]. The characteristics of the ferrite absorber which measured by the Square Coax was adopted for calculation.

Figure 14 shows the calculated of CSA of the SAC lined the ferrite tiles. To take notice of the CSA deviation of the 6.3 mm thick conventional ferrite without overlap tile, there were two large peak points at about 21 MHz and 48 MHz at horizontal polarization, and there was one large peak point at about 16 MHz at



Figure 12: Layout for leaked E-field from PLC.



Figure 13: Model for CSA calculation of anechoic chamber.

vertical polarization. These peak points were caused by the resonance of the chamber and insufficient absorbing characteristics of the conventional ferrite absorber. The CSA deviation was reduced to put the overlap tile onto air gap between ferrite tiles. Furthermore CSA deviation was improved to about $\pm 4 \, dB$ by adopting the double layers absorber composed the 12 mm thick ferrite tile and the 80 mm thick carbon material. The results showed that there was a possibility to evaluate PLC system in the SAC lined conventional ferrite absorber without long pyramidal absorber.





4. Conclusion

The conventional ferrite absorber had not excellent absorption below 30 MHz, and it was able to improve by changing thickness of ferrite tile, adding overlap tile onto air gap or combining the carbon material with the ferrite tile. The calculated CSA deviation was improved to about $\pm 4 \,\mathrm{dB}$ by adopting the double layers absorber composed the 12 mm thick ferrite tile and the 80 mm thick carbon material, it showed that there was a possibility to evaluate PLC system in the SAC lined conventional ferrite absorber without long pyramidal absorber. However the ray tracing method does not have sufficient accuracy below 100 MHz [4]. We will calculate the site attenuation by FDTD method, and will measure the actual site attenuation of the SAC which lined the conventional ferrite tiles.

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Author Index

Abrishamian M. S., 163 Abubakar A., 214, 223, 485 Akimoto M., 157 Aksun M. I., 35 Alumbaugh D., 214 Archambeault B., 297 Arridge S., 48 Auvinen A., 197 Avdeev D. B., 210, 219 Avdeeva A. D., 210, 219 Aziz J. A., 461 Bajalan D., 455, 459, 461 Bala U. B., 270 Barabanenkov M. Y., 10 Barabanenkov Y. N., 10 Barrile V., 153, 528 Bartušek K., 412 Baup F., 94 Bindu G., 110 Bodrov V. V., 15 Brańczyk A. M., 442 Cacciola M., 153, 528 Callebaut D. K., 403, 406, 409 Cao S., 347 Caorsi S., 385 Cevini G., 385 Chênerie I., 94 Chenn A. C., 150 Collard B., 399 Collins L. M., 302 Cotroneo F., 153, 528 Dauvignac J. Y., 60 de Cogan D., 336 de Oliveira R., 227 de Vivo B., 236 Dorn O., 43, 48 Drabkin V., 120 Drewniak J. L., 15, 297 Druskin V., 214 DuBroff R. E., 297 Dudley D. G., 425 Dusséaux R., 227 El-Zein G., 137 Elkhetali S. I., 251 Eremina E., 447 Escobedo-Alatorre J., 177, 533 Fang A., 292 Fares B., 399

Farschtschi A., 465 Fasenfest B. J., 420 Feng Q., 357 Fernandes B. G., 322 Fiala P., 412 Fikioris G., 275 Fikioris J. G., 275 Gao J., 347 Gashinova M. S., 133 Ghorbani A., 168 Godin Y. A., 53 Golovanov O. A., 143, 416 Gonzàlez R. G., 495 Gorodetsky D. A., 510 Greiff M., 270 Grimalsky V., 177 Guillon M., 437 Høgda K. A., 182 Habashy T. M., 214, 223, 485 Haji-Hashemi M. R., 69 Hakkak M., 352 Hakula H., 260 Hamran S. E., 182 Heckenberg N. R., 442 Helden L., 447 Helfert S. F., 172 Himed B., 380 Hosseini M., 352 Houghton C., 120 Hruška J., 283 Hsiang T. Y., 232 Hu J., 292 Huang J. J., 57 Huang J. Y., 15, 297 Huang T.-Y., 475 Ikuta A., 317 Ilyin N. V., 288 Irishina N., 43 Ishizuka K., 538 Jha A. K., 322 Jin J., 64 Kantorovich I., 120 Karan B. M., 312 Karugila G. K., 409 Kawahara M., 157 Kawakami H., 148 Kawamoto K., 74 Khalatbari S., 104

Khater A. H., 403, 409 Kishore A., 312, 322 Kitazawa T., 31, 38 Knizhnerman L., 214 Kokkorakis G. C., 275 Koledintseva M. Y., 15, 241, 297 Kolehmainen V., 48 Kondratiev I. G., 288 Korolkov V. D., 130 Koshevaya S., 177 Kotik N., 514 Kurttio P., 197 Kuznetsov V. L., 22 Lahkola A., 197 Lang S., 192 Langley K. A., 182 Langley R., 368 Lanteri J., 60 Larsen Y., 182 Laskar J., 483 Laurent J. St., 120 Lee C.-H., 483 Li H., 380 Li J., 306, 374, 390, 490, 505 Li J. H., 505 Liao G. S., 374 Lin Y.-D., 31, 38 Linnikov O. N., 187 Liu H. Q., 374 Liu J. X., 256 Liu W., 368 Liu Y. Z., 64 Loke V. L. Y., 442 Lonappan A., 110 Lu D., 357 Müller K., 182 Macháč J., 283 Makeeva G. S., 143, 416 Malnes E., 182 Mathew K. T., 110 Mathis W., 270 Matsubayashi H., 157 Matsumoto M., 157 Mehboodi Z., 99 Migliaccio C., 60 Mir-Mohammad H., 69 Mirzaee A. A., 104 Miyagawa H., 38 Moghtadai V. M., 69 Molchanov S., 53 Monsiváis A., 94

Mori A., 74 Moscoso M., 43 Mougin E., 94 Moussakhani K., 168 Mudaliar S., 88 Mukhopadhyay R., 483 Nakata T., 31 Negishi T., 13 Nickel J., 519 Nieminen T. A., 442 Nishikawa T., 38 Ogawa H., 317 Ohta M., 317 Onal T., 35 Padvanama S., 483 Pardavi-Horvath M., 143, 416 Parnes M. D., 130 Perumana B., 483 Petrarca C., 236 Pichot C., 60 Pinel S., 483 Pingenot J., 425 Pirhadi A., 352 Prasad R. C., 312 Praveen Kumar A. V., 110 Purkait P., 330 Pursiainen S., 260 Qin Y. L., 292 Ravva P. C., 15, 297 Remis R. F., 206 Richter T., 465 Riddolls R. J., 26 Rieben R., 265 Rieben R. N., 425 Riefler N., 450 Roberts T. M., 395 Roscher H.-J., 465 Rozanov K. N., 297 Rubinsztein-Dunlop H., 442 Sabirov M., 15 Sadafi H. A., 99, 104 Sagnard F., 137

Saltykov E. G., 344 Sanchez-Mondragon J., 177, 533 Sapogova N. V., 288 Sarabandi K., 94 Sardari D., 99, 104 Sarkar S., 483 Sasaki T., 200 Schurmann H. W., 519 Scott I. J. G., 336 Serov V. S., 519 Sevgi L., 430 Shahvarpour A., 163 Shan F. Q., 57 She J. Z., 57 Sheng K., 347 Shestopalov Y., 514 Shimada K., 538 Shu W., 246 Sikora J., 48 Skoupil T., 412 Smirnov A. I., 288 Sohn K. J., 380 Song J. M., 115, 246 Souny B., 399 Sourkov V. I., 15 Sourkova I. V., 15 Speciale R. A., 279, 470, 478 Sriramakavacham R. S., 330 Storvold R., 182 Stowell M. L., 420 Sugimoto K., 13 Suzuki T., 13 Tang J., 347 Tango C. L., 150 Tantum S. L., 302 Tecpoyotl-Torres M., 177, 533 Thomas V., 110 Tokola K., 197 Tokuda M., 74, 81, 200, 538 Torres-Cisneros M., 533 Trappeniers D., 495 Tucci V., 236 Uluışık Ç., 430

van de Capelle A., 495

van den Berg P. M., 223, 485 van Lil E., 495 Vendik O. G., 126, 130, 133 Vitelli M., 236 Vojtek T., 412 Wakino K., 38 Wakino, K., 31 Wallin B., 265 Wang H. L., 64 Wang L., 347 Wang X., 347 Wang Y.-Q., 302 Wang Z., 347 Watanabe Y., 13, 74, 81, 200 White D., 265 White D. A., 420, 425 Wilsey P. A., 510 Wilt M., 214 Wolf B., 533 Wriedt T., 447, 450 Wu C. Y., 150 Wu R., 357 Wu R. B., 256, 363, 368 Wu R.-B., 475 Xie F., 306, 390, 490, 505 Xie G., 306, 390, 490 Xie G. Q., 505 Xie L., 490 Xu L. F., 363 Yamagami Y., 13 Yatsyk V. V., 524 Yoshitake H., 31 Yu Y.-C., 475 Yufryakov B. A., 187 Zacharopoulos A., 48 Zehentner J., 283 Zhang B., 256 Zhang J., 232 Zhang L., 115 Zhang M., 292 Zhang P., 214 Zhao W. Y., 363 Zhou W. W., 292 Zhu W. M., 64