Metamaterial Comprising Plasmonic Nanolasers

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Abstract— We consider plasmonic nanoantennas immersed in active host medium. Specifically shaped metal nanoantennas can exhibit strong magnetic properties in the optical spectral range due to the excitation of magnetic plasmon resonance. We propose plasmonic nanolaser, where the metal nanoantenna operates like a resonator. The size of the proposed plasmonic laser is much smaller than the wavelength. Therefore, it can serve as a very effective source of EM radiation.

1. INTRODUCTION

Extending the range of electromagnetic properties of naturally occurring materials motivates the development of artificial metamaterials. For example, it has been demonstrated recently that metamaterials may exhibit such exotic properties as artificial magnetism [1, 2], negative dielectric permittivity (see, for example [1–3]), negative magnetic permeability [4, 5], and even both [6–8]. The double-negative case of $\text{Re}\varepsilon < 0$ and $\text{Re}\mu < 0$ is often referred as a lefthanded material (LHM) because the electric and magnetic fields and the wavevector are left system in a plane wave propagating in material as it was first found by Veselago [6]. Situations when a negative refractive index can be realized in practice are particularly interesting because of "perfect" lens with subwavelength resolution [8]. Negative refraction and subwavelength imaging has been demonstrated in the microwave range [7, 9–11]. For microwave LHMs, artificial magnetic elements providing $\text{Re}\mu < 0$ are the resonators of the split ring type [7, 11, 12].

For the optical range, LHM with a negative refractive index were first demonstrated in [13] and [14–16]. The authors of [14–16] observed the negative real part of the refractive index at the telecommunication wavelength of 1550 nm. In [14–16] the authors experimentally verified their earlier theoretical prediction of negative refraction in the array of parallel metal nanorods [17–19]. The first experimental observations of negative n in the optical range were followed by another successful experiment [20]. Very recently a macroscopic, far field image of the subwavelength object was obtained from cylindrical superlens [21]. Note that the losses become progressively important with decreasing the wavelength towards the optical range [22]. Moreover, finite losses inside the LHM superlens could dramatically reduce the resolution of such lens [23] and made a dream of a full-scale superlens unattainable.

To reduce the loss we suggest filling the metal horseshoe resonator, shown in Fig. 1, with an active medium. We consider the interaction of such nanoantenna with *a two-level amplifying system* (TLS), which can be represented by quantum dot in the semiconductor host, quantum well, dye molecules, or another high gain medium. Horseshoe-shaped subwavelength nanoantennas were first suggested in [24, 25]. These structures support strong magnetic moments at frequencies higher than microwave-mid-IR range for which traditional split ring resonators (see [4, 5, 12, 26–28] for details) were sufficient.

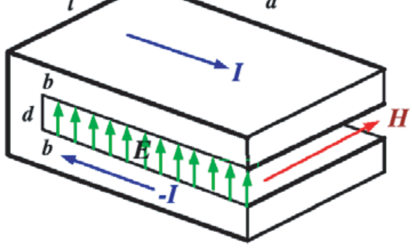

Figure 1: Horseshoe optical nanoantenna is excited by magnetic field **H**; arm length a, arm thickness b, lateral size l, gap width d; electric current flows in metal arms; displacement currents in the gap short the circuit; magnetic moment of the current is either parallel ($\mu > 0$) or antiparallel ($\mu < 0$) to **H**.

2. L-C-R MODEL OF HORSESHOE RESONATOR FILLED WITH ACTIVE MEDIUM

The main features of horseshoe electrodynamics can be understood in terms of a simple equivalent L-C-R model shown in Fig. 2 [29, 30]. We consider the metal horseshoe nanoantenna, which is excited by the magnetic component H of the impingent electromagnetic field. The length a, width l and the thickness b of the metal arms as well as the gap d between the arms are supposed to be much smaller than the wavelength. The electric current $I = \dot{q}$, flowing in the metal arms of the antenna, is shorted by the displacement currents in the gap.

The metal part of the nanoantenna can be presented as an inductance L. The gap between two arms is modeled as capacitance C. Thus the horseshoe antenna can be presented as L-R-C circuit, shown in Fig. 2. The inductance $L = 8\pi a/(k^2|\varepsilon_m|bl)$, where waevector $k = \omega/c$, simulate the metal since metal's permittivity is typically negative in the optics and IR range and it is proportional to ω^{-2}. The resistance R equals to $R = R_1 + R_2$, where $R_1 > 0$ presents the losses in the metal, and R_2 stands for the losses in the dielectric, which fills the space between the two arms of the nanoantenna. For the ordinary dielectric material $R_1 > R_2 > 0$. EMF "generator" $V = V_0 \cos(\omega t)$ presents the electromotive force induced by the external magnetic field H according to Faraday law $V \sim -c^{-1}ad(\partial H/\partial t)$.

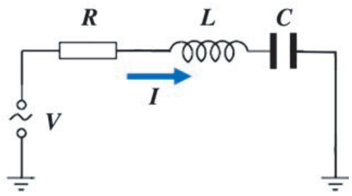

Figure 2: Equivalent L-C-R circuit of the horseshoe nanoantenna.

For the equivalent circuit we obtain the following Kirchhoff's equation, which we write in terms of the electric charge q of the metal arms

$$c^{-2}L\ddot{q} + U + R\dot{q} = V, \tag{1}$$

where $U = Ed = 4\pi d(q/S - P_1 - P_2)$ is the potential drop in horseshoe gap, $S = la$ is the gap area, $P_1 = \chi_1 E$ and P_2 are the regular and resonance (gain) polarization of the medium in the horseshoe (we use CGS units throughout the notes.) We introduce the real capacitance $C = \varepsilon_d S/(4\pi d)$, ($\varepsilon_d = 1 + 4\pi\chi_1$); then the potential drop equals to $U = q/C - SNp/C$, where N and p is the density and dipole moment of TLS (e.g., quantum dots) placed inside the gap. Substituting U in Eq. (1), we obtain

$$c^{-2}L\ddot{q} + (q - SNp)/C + R\dot{q} = V. \tag{2}$$

It is the closed equation for the charge (current) oscillation in the horseshoe resonator in the presence of gain medium. Note that the TLS dipole moment p in Eq. (2) is the quantum operator, which works as a driving force for the self-oscillation of the horseshoe resonator.

3. INTERACTION OF NANOANTENNAS WITH GAIN MEDIUM AND PLASMONIC LASER

In order to understand the origin of the lasing in the gain medium we will consider a microscopic model following the consideration [29, 30]. We use the quantum-mechanical derivation of the equations of motion for the system shown in Figs. 1 and 2, but will neglect quantum correlations and fluctuations in our analysis. The Hamiltonian of the nanoantenna interacting with a TLS is given by the expression:

$$H = H_0 + H_{TLS} + V_{\text{int}} + \Gamma, \tag{3}$$

where H_0 and H_{TLS} describe respectively the horseshoe nanoantenna and TLS. The operator $V_{\text{int}} = -pE$ gives the interaction between the TLS and the nanoantenna, where E is the electric field in the horseshoe. The term Γ describes dissipation and pump effects.

Electrons in the horseshoe nanoantenna couple to the local electric field and oscillate with the frequency ω, close to the plasmon resonance frequency $\omega_r = c/\sqrt{LC}$. Thus, such oscillation presents the surface plasmon in nanoantenna. We will treat the electric charge $q(t) = q_1(t)\exp(-i\omega t) + q_1^*(t)\exp(i\omega t)$ as classical object. Let us introduce the "slow" operator $b(t) = \eta(t)\exp(i\omega t)$ that corresponds to the transition operator $\eta(t)$ between the excited $|e\rangle$ and ground $|g\rangle$ state of the TLS, i.e., $|g\rangle = \eta|e\rangle$ and $\eta|g\rangle = 0$. Then the operator of the dipole moment in Eq. (3) can be represented as

$$p = \Pi b \exp(-i\omega t) + \Pi^* b^+ \exp(i\omega t), \tag{4}$$

where $\Pi = \langle g|re|e\rangle$ is the dipole matrix element. We will also introduce the population inversion operator

$$D(t) = n_g(t) - n_e(t), \tag{5}$$

where $n_e(t) = b^+ b$ and $n_g(t) = bb^+$ are the populations of the excited and ground states of the TLS. We assume that the TLS oscillates between the upper and lower level with the frequency ω close to frequency ω_2, where $\hbar\omega_2$ is the energy distance between the levels in the TLS. Neglecting the fast oscillating terms, $\sim \exp(\pm 2i\omega t)$ we can express the Hamiltonian (3) in terms of the following operators:

$$H_{TLS} = \hbar\omega_2 n_e, \tag{6}$$

$$V_{\text{int}} = -pE = -(Cd)^{-1}\left(\Pi^* q_1 b^+ + \Pi q_1^* b\right) + |\Pi|^2 SN, \tag{7}$$

where the electric field E in the gap of the horseshoe estimates as $E = U/d$. The last term in r.h.s. of Eq. (7) is a constant and, therefore, does not influence the dynamic of the system.

By using the well-known commutation rules for the operators b, b^+ and $n_{e,g}$, we derive the equations of motion for the operators: $i\hbar\dot{b} = [b, H]$ and $i\hbar\dot{D} = [D, H]$. We first consider the lasing as a natural oscillation of the electric charge in the horseshoe resonator in the absence of external field ($V = 0$ in Eq. (2).) For simplicity we neglect the loss in the dielectric, i.e., $R_2 = 0$ in Eq. (2). We suppose that the amplitudes of the resonator oscillation do not change ($\dot{q}_1 = \dot{b} = \dot{D} = 0$) so the resonator moves over its limit cycle. Thus, the lasing is the auto-oscillating in the system of nanoantenna+gain medium. Then Eq. (2) and equations for b and D can be rewritten the following form

$$(i\delta + \gamma)q_2 - ib = 0,\ (i\Delta + \Gamma)\, b - iADq_2 = 0,\ (D - D_0)/\tau - 2iA\left(q_2^* b - q_2 b^+\right) = 0, \tag{8}$$

where $q_2 = q_1/(SN\Pi)$, $\delta = 1 - (\omega/\omega_r)^2$, $\gamma = (\varepsilon''_m/|\varepsilon'_m|)\,(\omega/\omega_r)^2 \simeq \varepsilon''_m/|\varepsilon'_m|$, $\Delta = (\omega_2 - \omega)/\omega_r$ [29, 30]; the terms involving Γ and τ account for the relaxation and pump processes in TLS correspondingly. D_0 would be the stationary value of the TLS population if it were not interacted with the horseshoe. We assume $D_0 < 0$ because the pumping process provides the initial population inversion in the TLS. By neglecting quantum fluctuations and correlation, D and b can be treated as complex variables with operators b, and b^+ being replaced by complex variables b and b^* respectively. The dimensionless constant A equals to $A = 4\pi N|\Pi|^2/(\omega_r\hbar n^2) > 0$, where $n = \sqrt{\varepsilon_d}$ is the regular, i.e., nonresonant refractive index of the medium inside the horseshoe. Eqs. (8) define the lasing condition in the horseshoe plasmonic resonator, namely, they have nonzero solution when

$$\Delta/\Gamma = -\delta/\gamma, \quad (\delta/\gamma)^2 + 1 + AD_0/(\Gamma\gamma) = 0. \tag{9}$$

First condition (9) gives the lasing frequency $\omega_L = \omega_r + \gamma(\omega_2 - \omega_r)/(\gamma + 2\Gamma)$, which is always in-between the magnetic plasmon resonance frequency ω_r and TLS resonance frequency ω_2. All terms are positive but the population D_0 in the second lasing condition (9). Therefore, this condition holds only in the inverted medium $n_e > n_g$ when $D_0 < 0$. The population D_0 cannot be smaller than -1, which corresponds to the case when all TLSs are excited. Thus we obtain the lasing condition for the horseshoe nanolaser $A/(\Gamma\gamma) > 1$. As soon as this condition is fulfilled the interaction between TLS and the plasmonic nanoantenna leads to the coherent oscillations of electric charge and the magnetic moment of the horseshoe, even in the absence of the external electromagnetic field.

The same lasing condition can be expressed in terms of the gain G in the active medium inside of the horseshoe [29, 30]

$$G\lambda/(2\pi n\gamma) > 1, \tag{10}$$

where $\gamma = \varepsilon'_m/|\varepsilon''_m| \ll 1$ is the metal loss factor and the refractive index $n \sim 1$. Note that the lasing condition depends on the gain in the active medium and the loss in the metal only. We assume that Eq. (10) holds for any geometry of the subwavelength sized plasmonic laser. For instance, the silver horseshoe shaped nanolaser would lase at wavelength 1.5 μm if the gain medium can maintain optical gain larger than the critical gain $G_c \approx 3\cdot 10^3\,\text{cm}^{-1}$. Such gain is typical for "good" quantum well medium (see, e.g., [31, 32] and references therein). In the recent study very high material gain $G \simeq 2 \cdot 10^5\,\mathrm{cm}^{-1}$, was obtained for *InAs* quantum dot laser created inside the photonic crystal [33]. Less than five quantum dots were enough to get lasing.

Consider now the pumping of the nanolaser, which is excited by the magnetic component H of the impingent electromagnetic wave with frequency ω. The high frequency magnetic field excites currents in the horseshoe and operates as a driving force. Without the driving force the plasmonic nanolaser, which is a nonlinear oscillator, makes auto-oscillations and moves over its limit cycle given by Eq. (8) with lasing frequency ω_L (see Eq. (9)). When the driving force is applied the plasmonic nanolaser still moves over the limit cycle but with frequency ω of the driving force. The well-known synchronization takes place: the nanolaser generates with the frequency ω of the driving force (see e.g., [34].) The impingent em wave retunes the nanolaser. Therefore, the metamaterial comprised by plasmonic nanolasers operates as material with zero or negative loss. Moreover, the nanolaser metamaterial can change the frequency and direction of the emitted coherent light under the action of external light beam. As result we obtain not only the plasmonic nanolaser but also metamaterial for the macroscopic superlens with unlimited resolution since the loss is compensated by the gain. Authors acknowledge stimulating discussions with A. N. Lagarkov and V. G. Veselago.

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