# Transient 3-D Eddy Current Analysis in a Superconducting Magnet System Using an Integral Approach

**J. Junak** [1)]
Institute of General Electrical Engineering
Rostock University
A.-Einstein-Str. 2
D-18051 Rostock, GERMANY
**e-mail: jacek.junak@etechnik.uni-rostock.de**

**U. van Rienen**
Institute of General Electrical Engineering
Rostock University
A.-Einstein-Str. 2
D-18051 Rostock, GERMANY
**e-mail: ursula.van-rienen@etechnik.uni-rostock.de**

## Abstract

The Wendelstein 7-X is a thermonuclear fusion experiment being built at the Greifswald Branch Institute of the Max-Planck-Institut für Plasmaphysik (IPP). The eddy current distribution is investigated during a transient process in the superconducting coils of the magnet system. Application of volume-grid methods (e.g. FEM) for calculations of the eddy currents in the magnet system is difficult due to the large number of unknowns and the complicated mesh in the air region. Therefore, the investigation of effective methods for that computational problem is useful. We apply the method based on the integral approach, where only active regions without the air embedding are taken into consideration. Such approach leads to the full coefficients matrix and conseqentially to large computational requirements. There are techniques that allow that problem to be overcome. In that approach the coupling between elements are distinguished as near and far in order to achieve the less dense resulting matrix. For far interactions the equivalent current in the coil is introduced. The principles of the applied method and example results are presented in the paper.

## Introduction

The Wendelstein 7-X is a thermonuclear fusion experiment being built at the Greifswald Branch Institute of the Max-Planck-Institut für Plasmaphysik (IPP) [1]. Intensive research on thermonuclear fusion led to the construction of stellarator type reactors. In the stellarator the required distribution of the magnetic field is achieved by a set of non-planar coils (the magnet system) – Fig. 1.a. In order to increase the efficiency of the magnet system superconducting coils are used.

Each coil embraces the winding pack and the steel encasing – Fig. 1.b [2]. During an emergency discharge of the magnet system the excitation current in winding pack $J_L$ decreases, as it is shown in Fig. 1.c. Due to changes of the magnetic field eddy currents ($J_E$) are generated in the steel encasing. In order to assure the reliability of the overall system the knowledge of the eddy current distribution during the transient process is extremely important.

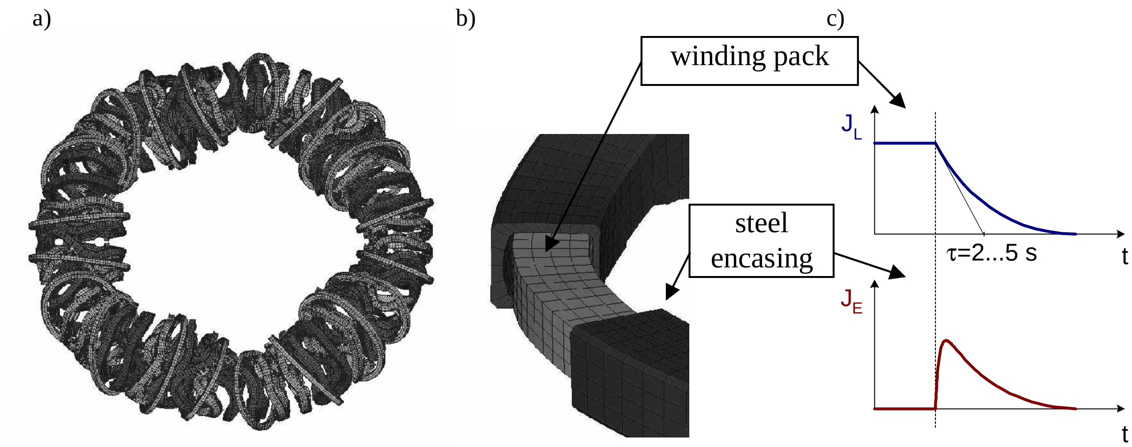


**Figure 1**. The magnet system of the stellarator (a); Cross-section of a coil (b); Turn-off of the magnet system (c).

Due to the non-planar construction of the magnet system computations of the eddy currents by means of analytical or approximate circuit methods are hardly possible. We applied the Finite Element

Method (FEM) using ANSYS software to calculate eddy currents' distribution in the steel encasing [3]. Such volume-grid method required complicated mesh in air regions between coils. Moreover, the problem of eddy currents in the stellarator should be treated as open boundary. It results in additional numerical problems for the application of FEM.

In the paper we present a method where only active regions are modelled and magnetic coupling is done using an integral approach based on Green's function. The integral method leads to dense matrices, which are difficult to handle. There are techniques which allow the storage requirements of such matrices to be reduced (e.g. Fast Multipole Method [4], [5]). We applied a technique based on an equivalent current flowing in a cross-section of the coil.

The integral method was applied for one tested coil and can be extended to the full magnet system.

## Numerical method

The eddy currents flow mainly in the direction tangential to the surface of the encasing and their density along the encasing's thickness is constant. This is due to the low thickness of the steel encasing and the slow varying turn-off process. Therefore, the encasing can be modelled using the surface edge elements - shown in Fig. 2. In the model with each edge *k* is associated current density $J_k$. and with each node *n* is associated the scalar potential $\varphi_n$. The winding pack is replaced by a filament centrally situated in a cross-section of the coil.

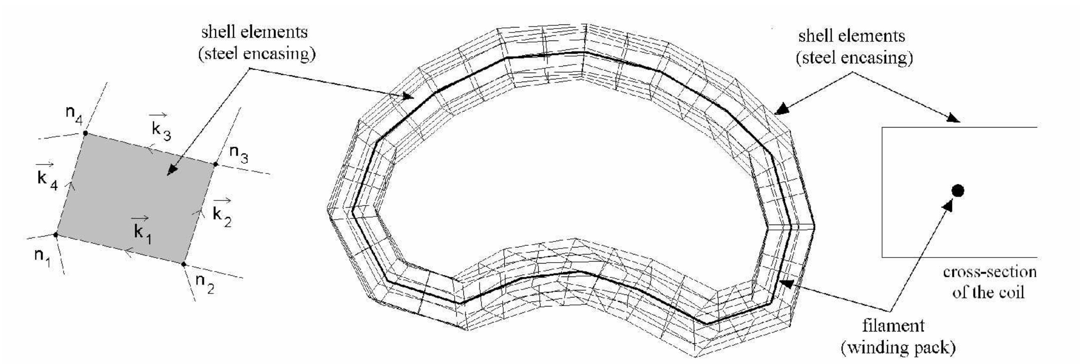


**Figure 2**. The model of the tested coil containing the surface elements

Assuming thickness $g^e$ of each surface element, it is possible to define the cross-section surface for each edge *k* in the element $e - S_k^e$ as illustrated in Fig. 3.a. The edge current $I_k$ is approximated by (1).

$$I_k = \iint_{S_k} \overset{\circ}{J} \cdot d\overset{\circ}{S} \approx J_k \cdot S_k = J_k \cdot \left(\sin(\alpha_1)\cdot S_k^{e1} + \sin(\alpha_2)\cdot S_k^{e2}\right) \qquad (1)$$

Where *e1* and *e2* are elements including edge *k*.

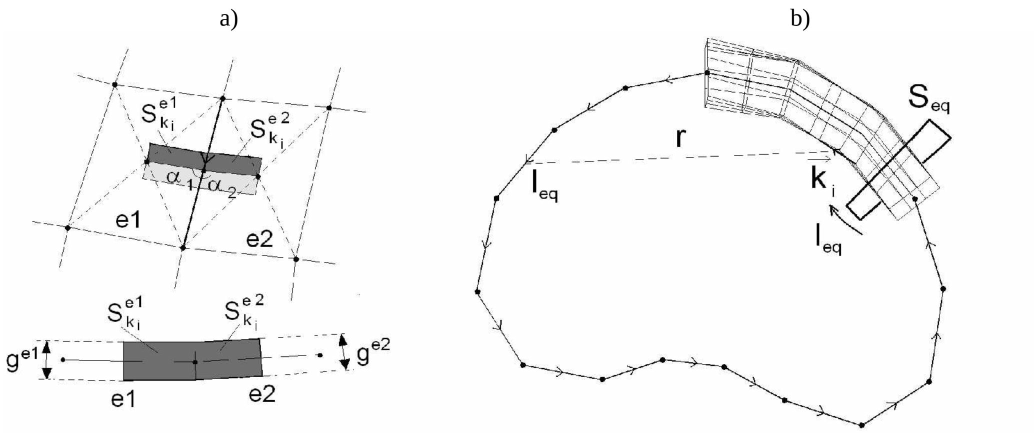


**Figure 3**. Definition of the surface associated with edge (a); The model of the encasing with the equivalent current (b)

The current in the surface should satisfy continuity condition: div J=0. It means that at each node *n* of the model, Kirchoff's law should be satisfied. This is given by (2).

$$\sum_{k} \delta_{n,k} \cdot I_k = 0 \tag{2}$$

Where $\delta_{n,k}=\pm 1$ when current $I_K$ flows to (or from) node *n*, and $\delta_{n,k}=0$ when node *n* doesn't belong to edge *k*.

The magnetic coupling in the model is done using an integral approach based on Green's function. With each edge is associated the magnetic vector potential $A_k$. The value of the magnetic vector potential along edge *k* as a result of the current flowing in edge *q* can be expressed by the formula:

$$A_k = \frac{\mu_0}{4\pi} \frac{\breve{k} \cdot \breve{q}}{l_k \cdot r_{k,q}} I_q \tag{3}$$

In equation (3) $l_k$ is a length of the edge *k* and $r_{k,q}$ is a distance between the edges *k*, *q*. The edge *q* can belong to the encasing or to the filament of the winding pack. The distance $r_{k,q}$ is calculated between the centers of the edges. In order to achieve higher accuracy the Gauss quadrature for numerical integration can be applied. When $k=q$ equation (3) should be calculated analytically.

On the other hand, using the Faraday's law, the electric field intensity associated with the edge *k* can be given by (4).

$$E_k = -\left( \frac{\Delta A_k}{\Delta t} + \text{grad}\, \phi_k \right)$$
$$\text{grad}\, \phi_k = \frac{1}{l_k} \cdot \sum_{n} \delta_{n,k} \cdot \phi_n \tag{4}$$

Where $\Delta t$ is a time step of the transient analysis.

The relationship between the edge current $I_k$ and the electric field $E_k$ is given by:

$$I_k = \sigma_{enc} \cdot S_k \cdot E_k \tag{5}$$

Where $\sigma_{enc}$ is conductivity of the steel encasing.

Combining (3), (4) and (5) and taking into account (2) it's possible to form the following matrix equation:

$$\begin{bmatrix} \mathbf{H} & \mathbf{G}^T \\ \mathbf{G} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \end{bmatrix} \tag{6}$$

The matrix **H** includes magnetic coupling between edges (3) and the matrix **G** results from Kirchoff's law – (2). The vector **I** contains unknown edge currents and the vector **Φ** contains unknown scalar potentials at the nodes.

The matrix **G** is sparse but **H** is full coefficients matrix. In the case of large models, a full matrix leads to large computational and storage requirements. There are techniques, which allow sparsity in such matrix to be introduced e.g. [4], [5]. We apply a technique based on the equivalent current in the steel encasing of the coil – Fig. 3.b. The equivalent current $I_{eq}$ is current flowing in the encasing through e.g. the surface $S_{eq}$. It can also be a surface in another part of the coil because Kirchoff's law (2) ensures continuity of the current flow in the encasing. Therefore the unknown equivalent current $I_{eq}$ can be expressed by:

$$I_{eq} = \sum I_{qs} \tag{7}$$

Where $I_{qs}$ is a current of the edge crossing the surface $S_{eq}$.

Next for each edge (e.g. $k_i$ in Fig. 3.b) influence of other edges can be divided into two parts: short-range and long-range interactions. For the long-range interactions the group of edges are replaced by equivalent edge (or edges) with the current $I_{eq}$. It allows sparsity in the matrix **H** to be achieved.

## Results

Calculations of the tested coil using the presented method were done for three cases: 1) the full matrix **H;** 2) the matrix with the equivalent current $I_{eq}$ (the sparse matrix **H)** and 3) with the matrix **H** containing only short-range interactions. The model of the steel encasing consists of 768 edges and 384 nodes. The full matrix **H** contains 592888 non-zero elements (7.1 MB). By introducing the equivalent current, it is possible to reduce the number of non-zero elements in matrix **H** to 114437 (1.34 MB). Similar compression is achieved for the third case (only short-range interactions). Results of that method are compared to results of the FEM model (volume tetrahedral elements) using ANSYS software. The FEM model consists of 1400 elements and 460 nodes in the encasing and 14000 elements in the air region.

The calculations were done for the excitation current $I_L$=2 MA and the time constant $\tau$=2 s (Fig. 1.c). The conductivity of the steel encasing is $\sigma_{enc}$=2 MS/m. Fig. 4.a shows the eddy current density over time for each approach. Despite compression the results of the model with the equivalent current (curve 2) correlate well with the results of the model with the full matrix **H** (curve 1). Such accuracy is not achieved in the presence of only the short-range interactions (curve 3). Fig. 4.b shows an example of the eddy current flow in the encasing of the tested coil for the approach with the equivalent current.

The results of the proposed method also correlate well with the results of the FEM model. Differences come mainly from the fact that in case of the FE approach volume elements were applied.

a) b)

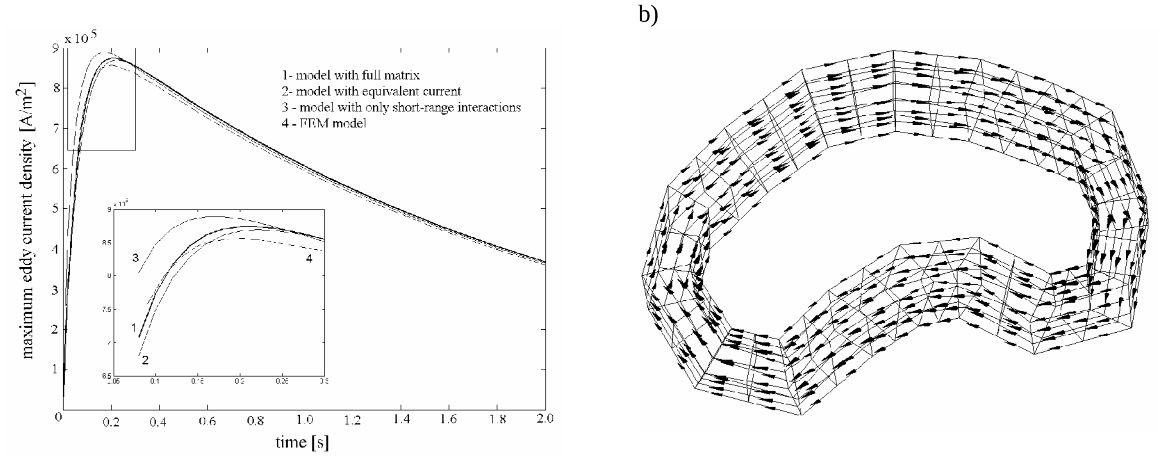


**Figure 4**. The maximum density of eddy current in the steel encasing over time (a) Eddy current flow in the encasing (b)

## Conclusion

The presented numerical procedure can be applied for large-scale eddy currents' calculations in the magnet system of the Wendelstein 7-X stellarator. In the case of the model having several coils of the magnet system included, the equivalent current should be defined separately for each coil (7). In the presented approach only active regions are modelled using shell elements and therefore, the model preparation is easier than for FEM.

**REFERENCES**

1. http://www.ipp.mpg.de
2. J. Sapper and W7-X team, "The Superconducting Magnet System for the Wendelstein 7-X stellarator", *Superconducting Sci. Technology*, No. 13, pp. 516-518, 2000.
3. J. Junak, U. van Rienen: "Calculations of Eddy Currents in the Magnet System of the Wendelstein 7-X Stellarator", *Proceedings of CAD-FEM Users' Meeting,* Potsdam Germany, Nov 12-14 2003.
4. A. Buchau, W. M. Rucker, O. Rain, V. Rischmüller, S. Kurz, S. Rjasanow: „Comparison Between Different Approaches for Fast and Efficient 3-D BEM Computations", *IEEE Trans. on Magnetics*, vo. 39, No. 3, May 2003.
5. A. Buchau, Ch. J. Huber, W. Rieger, W. M. Rucker: „Fast BEM Computations with the Adaptive Multilevel Fast Multipole Method", *IEEE Trans. on Magnetics*, vol. 36, No. 4, July 2000.

[1]) This work is supported by the Max-Planck-Institut für Plasmaphysik (IPP), Germany.