Wiener-Hopf Analysis of Patch Antennas Printed on Anisotropic Substrates

George A. Kyriacou
Department of Electrical & Computer Engineering, Democritus University of Thrace, GR-67100 Xanthi, GREECE,
E-mail: gkyriac@ee.duth.gr

Abstract

This work contributes to the analytical study of patch antennas printed on anisotropic substrate with an also anisotropic superstrate. It particularly focuses on the asymptotic evaluation of the radiated-far field based on the saddle point technique. First, a review of the analysis leading to the scattered field expressions when a TEM wave obliquely incidents upon an infinitely extending patch edge is performed. Then the analysis is devoted to the clarification of the saddle point, the surface waves and Leaky waves as well as the involved branch cut effects on the radiation pattern.

I. Introduction

A lot of low losses substrates employed in printed antenna technology presents a dielectric anisotropy. This is either an inherent property (e.g. sapphire and quartz) or artificially acquired (e.g. Epsilam-10) during the fabrication process. The electromagnetic analysis must take into account this anisotropy, since is has significant effects on the antenna characteristics. The introduction of magnetized ferrite and magnetized plasma substrates offers exceptional features, including wideband electronic tuning, beam steering and the possibility of surface wave as well as RCS reduction. Our research effort is directed toward the analytical study of antennas printed on anisotropic substrates aiming at a clear physical insight and an integral exploitation of the offered features. At a second phase, engineering design formulas are extracted from the approximate evaluation of the Sommerfeld type integrals involved in the final analytical expressions. A review of the previous work concerning the uniaxially anisotropic substrate/superstrate structures, e.g. [1], will be first presented.

The analysis considers a parallel plate waveguide with a semi-infinite upper conductor, which is loaded with the above-described anisotropic material. The dominant TEM wave propagating in the parallel plate region is assumed to incident upon on the edge defined by the truncated upper conductor. The Wiener-Hopf technique is then employed to obtain the scattered field. A reflection coefficient for the scattered dominant mode was established in our previous works. This was in turn exploited in conjunction with the geometrical optics approach for the analysis of patch antenna characteristics, including the resonant frequency, the edge admittance and the input impedance. The current research effort is directed toward the study of the edge as well as the patch antenna radiation characteristics. The radiation field is obtained via a steepest descent path integration, which is greatly affected by the surface wave as well as the leaky wave poles, when the observation angle is varied.

II. Formulation

As noted above, the present effort is focused on the effects of the anisotropic substrate on the radiation of printed antennas. In this paper the analysis will be restricted to the radiation of an infinitely extending edge, for which the scattered field was found in our previous works, e.g. [1-3]. The analysis is based on the Wiener-Hopf technique, which is carried out in a spatial-Fourier transformed domain, resulting in the field spectral solution-expressions. The actual field expressions (in the spatial domain) can be obtained via an inverse Fourier transform. In the usual manner the field expressions in the air region satisfies the radiation condition, which in turn involves a saddle point. A classical and convenient way to obtain the radiated-far field is to deform the integration path of the inverse Fourier transform into the corresponding steepest path (SDP), which passes through the saddle point. The saddle point along with the SDP path is moved as the observation angle is varied from the broadside $\theta=0$ (perpendicular to the substrate) to the end-fire $\theta=\pi/2$ (parallel to the substrate). Also, the original integration path and the SDP are connected with arcs of infinite radius forming a closed integration path. The poles enclosed-captured by the closed integration path as well as any intercepted branch cuts contribute to the radiated-far field. Especially the poles are captured beyond a critical observation angle ($\theta_c$) and from that toward the end-fire ($\theta_c \leq \theta \leq \pi/2$). A similar approach applies for any possible interception of branch-cuts. So, it is very helpful to repeat here the important remarks stated by Felsen and Marcuvitz [4, p.469]: "... The field due to the saddle point (space or sky wave)
is observed everywhere, while the pole or branch-point fields are generally confined to special regions. The poles account for the surface wave and leaky waves, the branch points yield lateral waves, and the saddle points furnish the direct, reflected and refracted fields of geometrical optics. The present work aims at the discrimination of the corresponding contributions and the evaluation of the associated fields.

II.1. Field Expressions in the Spectral Domain

In order to clarify the far field estimation procedure the particular structure of Fig.1 will be considered. This geometry presents a parallel plane waveguide with a semi-infinite upper conductor, which is loaded with uniaxially anisotropic substrate-superstrate layers. A TEM wave propagating in the parallel plane region obliquely incidents (at an angle $\alpha$) upon the edge-slot defined by the truncated upper conductor. The scattered field in the spectral domain was obtained by the aid of a Wiener-Hopf technique, in our previous works, e.g. [1]. As the far field is concerned, it is interesting to observe two types of surface (or leaky) waves in Fig.1.: A single layer surface wave propagating above the truncated conductor and toward the positive x-direction ($x>0$), as well as double layer one propagating along the grounded two-layer substrate toward the negative x-direction. The corresponding characteristic equations are currently investigated for complex roots, which give rise to Leaky waves. These surface waves, the Leaky waves along with the possible branch cuts, associated with the semi infinitely extending dielectric layers, contribute to the radiated field at specific observation angles.

**Figure 1** A parallel plane waveguide with a semi-infinite upper conductor, loaded with uniaxially anisotropic substrate/superstrate layers.

Recall also, that the major contribution to the radiated field comes from the saddle point (sky wave) involved in the expressions and is present at all observation angles. For the presentation of the above phenomena we have to lay down some spectral field expressions at specific areas of the structure to be adapted from [1]. The scattered field spectral expressions in the air region $z>d_{12}$ are:

$$
\begin{align*}
\vec{E}_{z_0} (\lambda) &= E_0 (\lambda) e^{-k_0 u_0 (z-d_{12})} \\
\vec{H}_{z_0} (\lambda) &= H_0 (\lambda) e^{-k_0 u_0 (z-d_{12})}
\end{align*}
$$

**Figure 1** A parallel plane waveguide with a semi-infinite upper conductor, loaded with uniaxially anisotropic substrate/superstrate layers.

Recall also, that the major contribution to the radiated field comes from the saddle point (sky wave) involved in the expressions and is present at all observation angles. For the presentation of the above phenomena we have to lay down some spectral field expressions at specific areas of the structure to be adapted from [1]. The scattered field spectral expressions in the air region $z>d_{12}$ are:

$$
\begin{align*}
\vec{E}_{z_0} (\lambda) &= E_0 (\lambda) e^{-k_0 u_0 (z-d_{12})} \\
\vec{H}_{z_0} (\lambda) &= H_0 (\lambda) e^{-k_0 u_0 (z-d_{12})}
\end{align*}
$$

where:

$$
\begin{align*}
\vec{E}_{z_0} (\lambda) &= E_0 (\lambda) e^{-k_0 u_0 (z-d_{12})} \\
\vec{H}_{z_0} (\lambda) &= H_0 (\lambda) e^{-k_0 u_0 (z-d_{12})}
\end{align*}
$$

The spectral functions $E_0(\lambda)$ and $H_0(\lambda)$ are given in [1] as :

$$
\begin{align*}
E_0 (\lambda) &= \frac{\vec{E}_{0} (\lambda)}{\left\{ D_{LS} (\lambda) \cosh(k_0 u_{n1} (d_{12})) \right\}} \\
H_0 (\lambda) &= \frac{\vec{H}_{0} (\lambda)}{\left\{ D_{L} (\lambda) \sinh(k_0 u_{n1} (d_{12})) \right\}}
\end{align*}
$$

where:

$$
\begin{align*}
u_{n1}^{(i)} &= \sqrt{\alpha^2 + \lambda^2 - n_{1i}^2}, \quad u_{n1}^{(i)} = \frac{n_{1i}}{n_{j1}} \sqrt{\alpha^2 + \lambda^2 - n_{j1}^2}, \quad \text{Re}(u_{n1,2}^{(i)}) \geq 0 \quad \text{and} \quad i = 1, 2
\end{align*}
$$
The spectral functions $\mathcal{P}(\lambda)$ and $\mathcal{Q}(\lambda)$ are considered analytic in the lower $\lambda$ half plane and they are associated to the charge and current densities induced on the truncated conductor placed at $z=d_i$. They could be defined as:

$$\mathcal{P}(\lambda) = -\frac{1}{k_0 e^{i z_0}} \frac{\partial \mathcal{P}_0}{\partial z}(z=d_i^-) \quad \text{and} \quad \mathcal{Q}(\lambda) = -j\epsilon_0 \frac{\partial \mathcal{P}_0}{\partial z}(z=d_i^-) \quad (5)$$

The evaluation of these functions constitutes the major task of [1] and their explicit expressions are given therein. The particular interest in these expressions is that their denominator constitutes the characteristic equation of surface/Leaky wave modes, as:

$$\mathcal{P}_0(\lambda) \propto Q_{e}(\lambda) \quad \text{and} \quad \mathcal{Q}(\lambda) \propto Q_{m}(\lambda) \quad \text{where}:$$

$$Q_{e}(\lambda) = u_{n1}^{(i)} u_{n2}^{(i)} \tan [k_0 u_{n1}^{(i)} d_1] D_{\text{LSE}}(\lambda) \quad \text{and} \quad Q_{m}(\lambda) = \mu_{n1} \mu_{n2} D_{\text{LSM}}(\lambda) \quad (6)$$

The surface/Leaky wave poles of the single layer (superstrate $x>0$) can be calculated from:

$$D_{\text{LSE}}(\lambda) = 0, \quad D_{\text{LSM}}(\lambda) = 0 \quad (7)$$

While, those of the double layer ($x<0$) from:

$$D_{\text{LSE}}(\lambda) = 0, \quad D_{\text{LSM}}(\lambda) = 0 \quad (8)$$

The possible branch points involved in the $E_0(\lambda), H_0(\lambda)$ spectral functions can be defined from:

$$u_{n1} = 0, \quad u_{n1}^{(i)} = 0 \quad \text{and} \quad u_{n2}^{(i)} = 0, \quad i = 1,2 \quad (9)$$

The branch point/cut at $u_{n1} = 0$ is eliminated through the transformation, [4, p.462]: $\zeta = k \sin \omega$, which herein takes the form:

$$\lambda = \zeta \sin \omega, \quad \zeta = \begin{cases} \sqrt{1 - \alpha^2} & \text{for } \alpha < 1 \\ -j \sqrt{\alpha^2 - 1} & \text{for } \alpha > 1 \end{cases} \quad (10)$$

The transformation of Eq.(10) is necessary in order to map the proper and improper Riemann sheets of $u_{n1}$ to corresponding strips in the $w$-complex plane and subsequently deform the steepest descent path, as shown in Fig.2. However, any branch points $u_{n1}^{(i)}, u_{n2}^{(i)} = 0$ would not be eliminated in the $w$-plane.

A question that still remains is the examination of these spectral functions to whether they are “even” or “odd” functions of $u_{n1}^{(i)}, u_{n2}^{(i)}$. As it is logical but also explained in [4, p.457] an even function in terms of a squared root ($u_{n1}$) is regular at the $\lambda_m$ point where $u_{n1}(\lambda_m) = 0$. A detailed examination of all the spectral functions shows that indeed $\mathcal{P}_0(\lambda), \mathcal{Q}(\lambda)$ and finally $E_0(\lambda)$ and $H_0(\lambda)$ are “even” functions of $u_{n1}^{(i)}, u_{n2}^{(i)}$; $i = 1,2$. This is actually an expected phenomenon, since according to [4, p.544] the presence of the metallic ground plane at $z = 0$ (Fig.1) prevents the existence of lateral waves by making the $z = \pm \infty$ inaccessible (prohibits the possible lateral waves to propagate toward $z = \pm \infty$ in the denser media).

II.2. Far Field Via a Saddle Point Technique.

Returning back to Eq.(1) any scattered field spectral component can be formulated in a general form:

$$\hat{\mathcal{F}}(\lambda)(\omega) \leftarrow \hat{\mathcal{P}}(\omega)(\lambda), \quad \hat{\mathcal{Q}}(\omega)(\lambda) \quad \text{where}:$$

$$\hat{\mathcal{F}}(\lambda)(\omega) = \hat{\mathcal{P}}(\lambda)(\omega) e^{k_{\omega} d} \quad (11)$$

The corresponding field expressions in the spatial domain can be obtained through the inverse Fourier transform:

$$\psi(\omega, \lambda)(x) = \int_{-\infty}^{\infty} \hat{\mathcal{F}}(\lambda)(\omega) e^{-ik_\omega(x - d_1 \omega)} d\lambda = \int_{-\infty}^{\infty} \hat{\mathcal{F}}(\lambda)(\omega) e^{ik_\omega d} d\lambda \quad (12)$$

In order to apply the steepest descent technique the Cartesian coordinates $(x, z)$ are transformed to Polar ones $(\rho, \theta)$ as in Fig.1:

$$x = \rho \sin \theta \quad \text{and} \quad z - d_{12} = \rho \cos \theta \quad (13)$$

In turn the quantities $\Omega$ and $q(\lambda)$ in Eq.(12) become:

$$\Omega = k_\rho \rho \quad \text{and} \quad q(\lambda) = -u_0 \cos \theta - j \sin \theta \quad (14)$$

The saddle point ($\lambda = \lambda_s$) of the integrand is defined by setting $q'(\lambda) = 0$ as:

$$q'(\lambda) = \partial q / \partial \lambda = 0 \quad \rightarrow \quad \lambda = \lambda_s = \zeta \sin \theta$$

861
The values of the required quantities at the saddle point are:
\[
\begin{align*}
u_0(\lambda_s) &= j\zeta \cos \theta, \\
q(\lambda_s) &= j\tilde{q}(\lambda_s), \\
\tilde{q}(\lambda_s) &= -\zeta \\
q''(\lambda_s) &= -\frac{j}{\zeta \cos \theta} = -j/\zeta \cos \theta
\end{align*}
\] (15)

Considering that there is not any pole singularity near the saddle point the sky wave contribution \(\psi_{sk}\) to the far field can be obtained from the asymptotic approximation [4, p.382, Eq.(1b)] as:
\[
\psi_{sk}(\rho, \theta)\bigg|_{\rho \to \infty} = \frac{2\pi}{k_0 \rho} \phi(\lambda_s)(1 - \alpha^2)^{1/4} \cos \theta \cdot e^{-jk_0\rho\sqrt{1-\alpha^2}}
\] (16)

Moreover, we have to take into account the residue contributions of the surface and Leaky wave poles enclosed by the closed integration path of Fig.2. Especially, for the observation angles resulting in bringing the saddle point \(\lambda = \lambda_s\) close to a pole (roots of Eqs.(7) and (8)), then the asymptotic approximation is given by [4, p.399, Eq.(2) or p.403, Eq.(16)].

Further discussions, analytical details as well as the numerically evaluated far field will be given at the presentation, during the symposium.

### III. Conclusions

The evaluation of the spectral expression of the field scattered by an infinitely extending edge excited by an obliquely incident TEM wave is first reviewed. The particular geometry of uniaxially anisotropic dielectric substrate/superstrate is examined. The surface and leaky wave pole singularities along with four branch cut singularities of the spectral function are identified. The inverse Fourier integral is then considered along with the deformation of the integration path to the steepest descent one. A clarification of the singularities effects on the radiation pattern is then attempted through the examination of which of them captured between the original and the SDP path. A lot of effort is still required in order for this study to be completed, especially when magnetized ferrite and plasma substrates will be considered.

### REFERENCES


