Mutual Coupling Evaluation within Waveguide Slotted Antennas

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Abstract— Mutual coupling in an array of different sized narrow rectangular waveguide slots embedded within an infinite PEC plane is estimated by computing the generalized admittance matrix, connecting the coefficients of the modal expansion of the magnetic field to the ones of the electric field. The generic element of the above matrix requires the numerical evaluation of a quadruple integral. In this paper a singularity cancellation approach is used to remove the singularity of the Green's function and the order of integration is reduced by a suitable changes of variables. In this way self and mutual admittances can be estimated by computing double integrals. A further approximation is introduced in order to compute the latter ones by single integrals saving computational time and resources. The results of such approach are then compared with the ones provided by a commercial numerical simulation tool showing good agreement.

1. INTRODUCTION

Planar arrays of waveguide fed slotted antennas [1] find many applications in telecommunication because they show low cost and weight, do not change the structure aerodynamic properties and have a very narrow beam. The principal limit is the relatively narrow band. For both analysis and synthesis purposes, evaluation of the mutual coupling between the slots is fundamental since it affects in a non-negligible way the array factor. In literature many works offer a phenomenon approximate analysis while a more accurate one could be carried out by using a commercial numerical simulation tool. However, both computational time and resources rise considerably when the array dimensions increase and such tools become unusable. So, the problem arises of both an efficient and accurate analysis.

We assume that the slots are embedded within an infinite PEC plane and mutual coupling is estimated by computing the generalized admittance matrix (GAM) [2, 3]. We start from the integral relationship connecting the magnetic equivalent currents to the magnetic field that can be obtained by means of the vector potential. Then tangential fields are expanded into rectangular waveguide mode functions and the GAM is built by connecting the coefficients of the modal expansion of the magnetic field to the ones of the electric field. The resulting integral relationship are simplified and numerically evaluated accounting exactly for the singular behavior of self-admittance.

2. FORMULATION

N rectangular slots are considered within an infinite PEC plane at $z = 0$ as shown in Fig. 1 (right panel highlights two of them for the sake of illustration). The infinite PEC plane assumption is well verified because the antenna radiates mainly around the direction orthogonal at the plane so the electrical field tangential components can be assumed vanishing everywhere at $z = 0$ except than within the slots.

By invoking the equivalence theorem and introducing the vector potential, the following integral relationship connecting the tangential magnetic field over the i-th slot with the tangential electric field over the k-th slot is established:

$$\underline{H}_t^{(i)}(\underline{r}) = j\frac{1}{2\pi\omega\mu_0}\left(k^2 + \nabla_t\nabla_t\right)\cdot\left(\hat{z}\times\sum_{k=1}^{N}\iint_{S_k}\underline{E}_t^{(k)}(\underline{r}')G\left(\left|\underline{r}-\underline{r}'\right|\right)ds'\right) \tag{1}$$

where $G(\cdot)$ is the free space scalar Green's function and $\underline{r} \in S_i$.

As mentioned before, the tangential fields are expanded into M modes over each slot:

$$\begin{aligned}\underline{H}_t^{(i)}(\underline{r}) &= \sum_{m=1}^{M} I_{mi}\underline{h}_{mi}(\underline{r})\\ \underline{E}_t^{(k)}(\underline{r}) &= \sum_{p=1}^{M} V_{pk}\underline{e}_{pk}(\underline{r})\end{aligned} \tag{2}$$

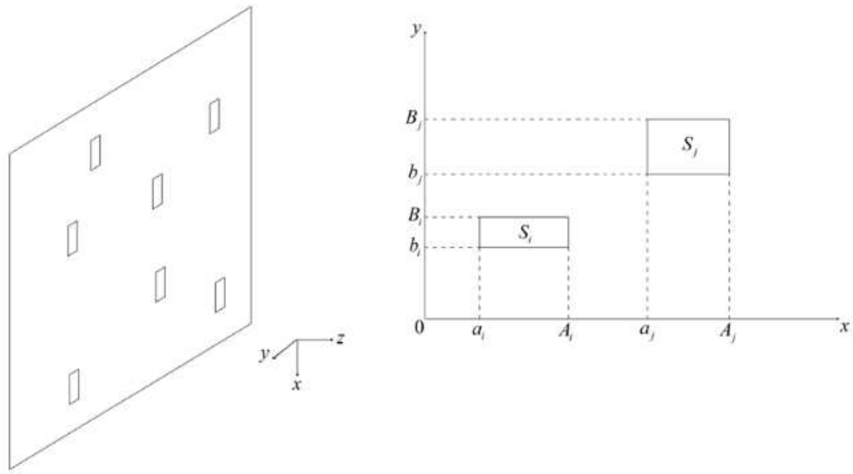

Figure 1: Problem geometry.

For slots, only the TE_{no} modes are considered because $B_k - b_k \ll \lambda$ and, so, the others are in cut-off. By inserting (2) into (1) we obtain the formulation of the GAM matrix

$$\sum_{m=1}^{M} I_{mi}\underline{h}_{mi}(\underline{r}) = j\frac{1}{2\pi\omega\mu_0}\left(k^2 + \nabla_t\nabla_t\right)\cdot\left(\sum_{k=1}^{N}\iint_{S_k}\sum_{p=1}^{M} V_{pk}\underline{h}_{pk}(\underline{r}')Gds'\right) \tag{3}$$

whose elements are provided by:

$$y_{mn}^{(ij)} = \frac{I_{mi}}{V_{nj}} = j\frac{\omega\varepsilon}{2\pi}\left[\iint_{S_i}\iint_{S_j}\underline{h}_{mi}(\underline{r})\cdot\underline{h}_{nj}(\underline{r}')Gds'ds - \frac{k_{tmi}k_{tnj}}{k_z^2}\iint_{S_i}\iint_{S_j}\Psi_{mi}(\underline{r})\Psi_{nj}(\underline{r}')Gds'ds\right] \tag{4}$$

As it can be appreciated, the numerical evaluation of a quadruple integral would be required. However the slots are narrow, so, to save computational time, after a first change of variables, we can approximate (4) by three single integral:

$$\begin{aligned} y_{mn}^{(ij)} &= j\frac{\omega\varepsilon_0 K_i K_j}{4\pi}(B_i - b_i)(B_j - b_j)\left[\int_{A_1}^{B_1} F_1\left(\sigma, \frac{B_i - b_i}{2}, \frac{B_j - b_j}{2}\right)\gamma_1(\sigma)d\sigma\right. \\ &\left. + \int_{A_2}^{B_2} F_1\left(\sigma, \frac{B_i - b_i}{2}, \frac{B_j - b_j}{2}\right)\gamma_2(\sigma)d\sigma + \int_{A_3}^{B_3} F_1\left(\sigma, \frac{B_i - b_i}{2}, \frac{B_j - b_j}{2}\right)\gamma_3(\sigma)d\sigma\right] \end{aligned} \tag{5}$$

where γ_1, γ_2 and γ_3 are linear combinations of sinusoidal functions and F_1 is an exponential function.

Such approach can be pursued for the mutual admittances but it cannot be used for the self-admittance because the Green's function is singular. In this paper a singularity cancellation approach [4] is used to overcome such problem. After other two changes of variables we obtain:

$$\begin{aligned} y_{mn}^{(ii)} &= j\frac{\omega\varepsilon_0 K_i K_j}{4\pi}\left\{\int_{-D_{ai}}^{0}\gamma_1(v)\left[\int_{\frac{-D_{bi}}{v}}^{0} F_3(u,v) * (\nu u + D_{bi})du + \int_{0}^{\frac{D_{bi}}{v}} F_3(u,v)(D_{bi} - \nu u)du\right]\mathrm{sign}(v)dv\right. \\ &\left. + \int_{0}^{D_{ai}}\gamma_3(v)\left[\int_{\frac{-D_{bi}}{v}}^{0} F_3(u,v)(\nu u + D_{bi})du + \int_{0}^{\frac{D_{bi}}{v}} F_3(u,v)(D_{bi} - \nu u)du\right]\mathrm{sign}(v)dv\right\} \end{aligned} \tag{6}$$

where F_3 is again an exponential function.

3. NUMERICAL RESULTS

The results of such approach are now compared with the ones provided by a commercial numerical simulation tool (CST).

First a single slot of dimension $0.5\,\mathrm{m} \times 0.1\,\mathrm{m}$ in an infinite PEC plane is considered and $y_{11}^{(11)}$, $y_{22}^{(11)}$ and $y_{12}^{(11)}$ are computed and shown in Figs. 2–6.

A good agreement between the two results is observed. (Numerical evaluation of $y_{12}^{(11)}$ provides zero).

Mutual coupling is now evaluated. Two horizontally and vertically, respectively, aligned slots

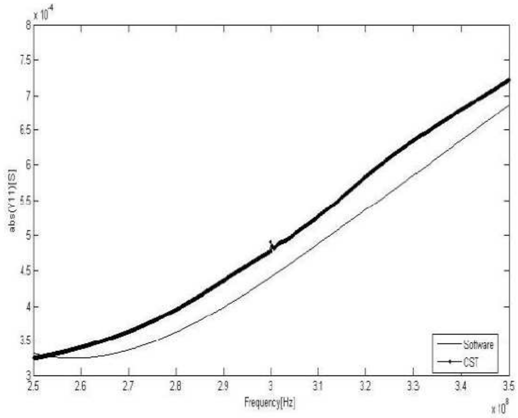

Figure 2: Module of $Y_{11}^{(11)}$.

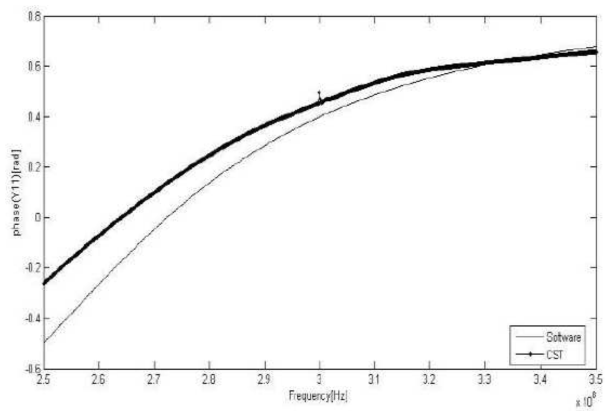

Figure 3: Phase of $Y_{11}^{(11)}$.

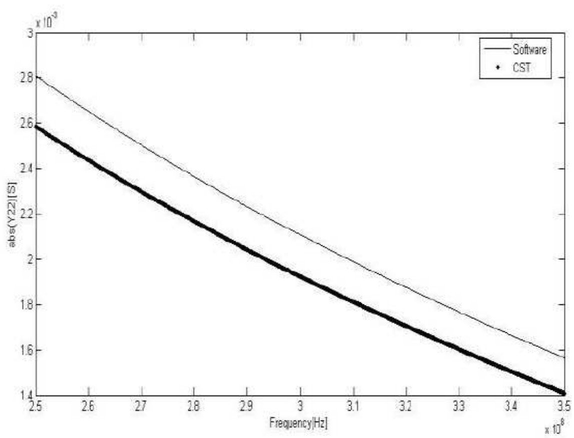

Figure 4: Modulus of $Y_{22}^{(11)}$.

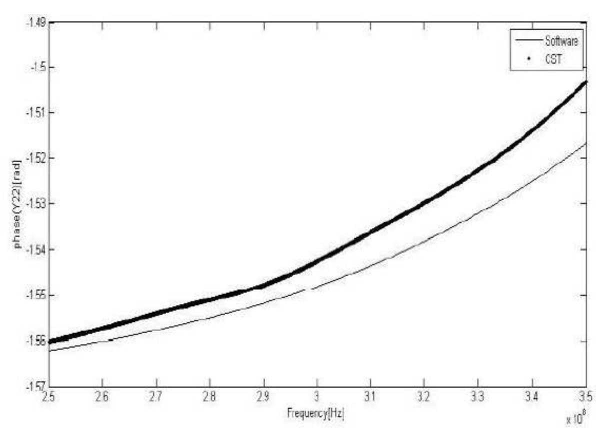

Figure 5: Phase of $Y_{22}^{(11)}$.

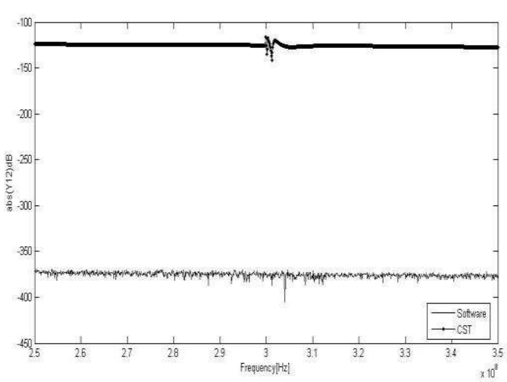

Figure 6: Module of $Y_{12}^{(11)}$.

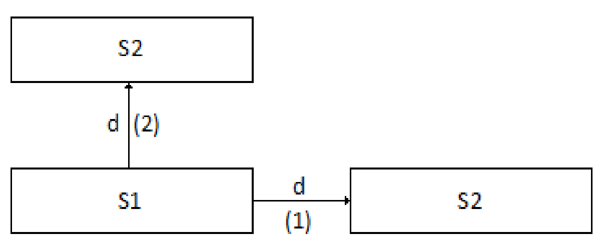

Figure 7: Example geometry.

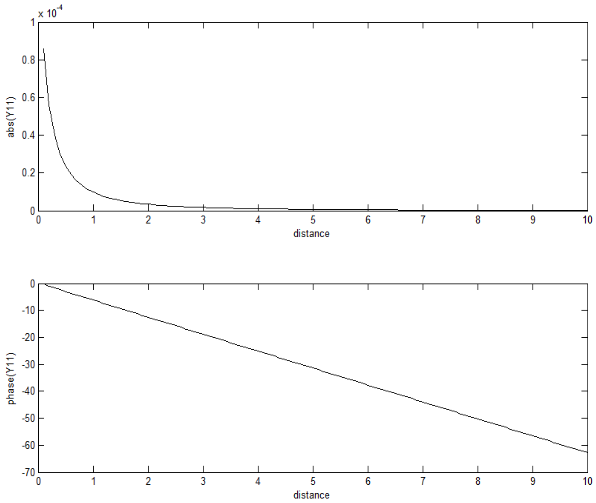

Figure 8: Modulus and phase of $Y_{11}^{(12)}$.

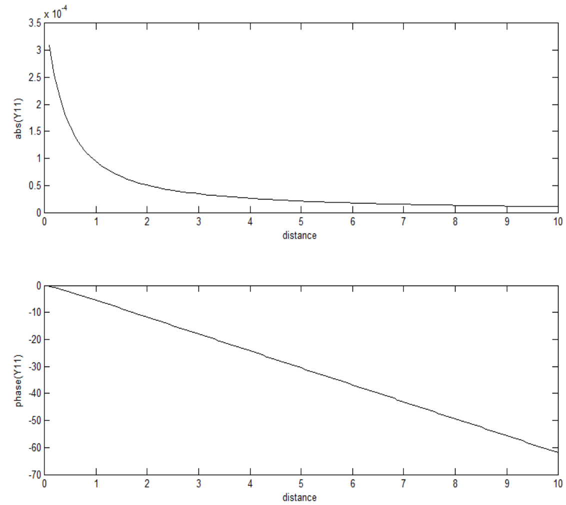

Figure 9: Module and phase of $Y_{11}^{(12)}$.

Slot	Auto-admittance	Active admittance
S1	6.46e-4 + i8.64e-4	5.58e-4 + i7.04e-4
S2	2.78e-4 + i4.45e-4	2.71e-4 + i4.46e-4
S3	1.05e-3 + i1.14e-3	9.57e-4 + i9.90e-4

Table 1: Slots admittances.

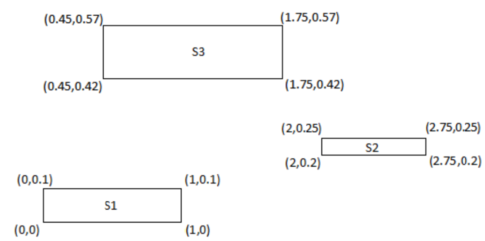

Figure 10: Geometry.

are considered (Fig. 7) and $y_{11}^{(12)}$ has been computed versus relative distance function and shown in Fig. 8, exhibiting the typical decay.

Finally in order to estimate the mutual coupling effect we consider the geometry of Fig. 10, composed of 3 slots of different dimensions where we assume only the fundamental mode on each slot to exist.

The following table shows the auto and active admittances values for each slot when the equivalent voltage is assumed equal to 1 V on each slot.

So, without taking into account the mutual coupling, a radiated power of $P_{\mathrm{i}} = 0.99\,\mathrm{mW}$ would be obtained. In turn, the mutual coupling leads to a value of 0.89 mW with about 10% decrease.

ACKNOWLEDGMENT

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