

# Silicon Microspheres in Metrology

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**Abstract**— The Système International unit of mass is the kilogram. The present definition of the kg is based on a prototype dating back to 1880s. New approaches to define the unit of mass are being investigated. Avogadro Project uses 10 cm diameter single crystal silicon spheres. The technique commonly observed to measure the radius of the silicon sphere is optical interferometry. Here, we propose an alternate method of measuring the diameter of the single crystal silicon sphere using near-infrared spectroscopy. We demonstrate our approach by numerically simulating the electromagnetic coupling of a silicon microdisk of radius  $5\ \mu\text{m}$  to an optical waveguide of width  $0.5\ \mu\text{m}$ , thereby approximating the coupling of a microsphere to a rectangular optical waveguide. It might be possible to have a precise technique for determining the radius of the sphere, which can be used for the definition of the kilogram.

## 1. INTRODUCTION

The kilogram (kg) is defined as the mass of international prototype made of platinum-iridium alloy kept at Bureau International des Poids et des Mesures (BIPM) [1]. The kg is the only base unit, which uses a material artifact in Système International (SI) [2]. Three other base units are affected by the definition of mass: the ampere, whose definition depends on force (newton) acting between current carrying wires; the mole, whose definition refers to 12 grams of carbon-12, and the candela, whose definition refers to the watt.

Currently, there are efforts to replace the definition of the kg based on the mass artifact by a definition based on physical constants. The kg can be defined as the mass of certain number of silicon atoms. For the definition of a base unit, the quantity used should be a true invariant of nature i.e., invariant under translation in space and time [3, 4]. The need for redefinition surfaced because of the long-term stability problem with the old platinum–iridium artifact [5, 6]. The appropriate quantities for the redefinition are the Planck constant,  $h$  or the Avogadro number,  $N_A$ . Perfect silicon single crystals are used [7] to find  $N_A$ . Calculating  $N_A$  requires an accurate measurement of the radius of the silicon crystal spheres. For this aim, interferometric methods are used to measure the diameter [8] of single crystal silicon spheres. Spheres possess whispering gallery modes (WGMs) with narrow linewidths [9] and quality factors (Q) as high as  $8 \times 10^9$  [10]. Exciting the WGMs of the spheres with a tunable laser results in the measurement of the spectral parameters such as the resonant wavelengths, Q's, and mode spacing. We propose the measurement of the radius of the silicon microspheres by using the optical resonance condition  $x < n < xm_{sphere}$ , where  $x = 2\pi a/\lambda$  is the size parameter,  $n$  the angular mode number,  $\lambda$  the resonant wavelength,  $a$  the radius of the sphere, and  $m_{sphere}$  the refractive index of the sphere. In our approach, we can calculate  $N_A$  by measuring the silicon spheres radius with near-infrared (near-IR) spectroscopy. Here we demonstrate our approach by performing a simulation of the out of plane electric field strength of an evanescently coupled silicon microdisk (the 2D analogue of the microsphere) and various slab waveguides (the 2D analogue of optical waveguide) [11]. We utilize finite difference time domain (FDTD) method by using the MIT electromagnetic equation propagation (MEEP) tool for the simulations [12].

## 2. SPECTROSCOPIC GEOMETRY

The geometry of the simulations for the spectroscopic measurements is given as follows: a slab waveguide with a width  $w = 0.5\ \mu\text{m}$  is coupled to a silicon microdisk with a radius of  $a = 5\ \mu\text{m}$  at a wavelength of 1550 nm. The materials for the slab waveguides are silicon ( $m_{silicon} = 3.4757$ ), diamond ( $m_{diamond} = 2.4$ ), sapphire ( $m_{sapphire} = 1.746$ ) and silica ( $m_{silica} = 1.44$ ). The silicon microdisk is placed at an impact parameter  $b = 5.35\ \mu\text{m}$ . The waveguide width is chosen to operate in single mode at a wavelength of  $\lambda_{laser} = 1550\ \text{nm}$ , where the left end of the waveguide is chosen to be the excitation port. Ideally, the waveguide should have a core and a cladding layer for evanescent coupling to the microdisk, but for the sake of simplicity the cladding part is omitted.

The overall proposed experimental setup diagram is given in Figure 1 where a silicon microsphere will be used instead of a microdisk. A distributed feedback (DFB) diode laser in the near-IR

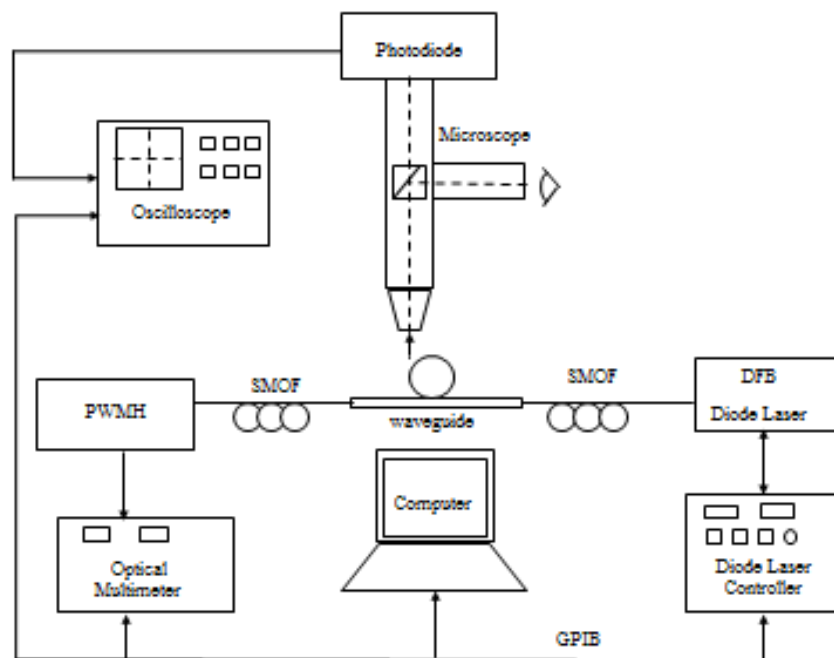


Figure 1: Proposed experimental setup for evanescent coupling of silicon sphere to an optical slab waveguide.

operating at  $\lambda_{laser} = 1550 \text{ nm}$  can be used for the excitation of the WGMs in the microsphere. The input laser is coupled to a single mode optical waveguide. The evanescent coupling of the near-IR laser to the microsphere can be achieved with a rectangular waveguide. The  $90^\circ$  elastic scattering intensity from the microsphere can be collected with an optical microscope and can be measured with a photodiode which is connected to an oscilloscope. The  $0^\circ$  transmission can be measured with an optical multimeter with a power/wavelength measurement head (PWMH).

### 3. SIMULATION RESULTS AND DISCUSSION

Polarization of the source electric field at the left port is vertical, and the propagation of the electromagnetic wave is in the  $x-y$  plane and we are calculating the out of plane component of the electric field strength everywhere on the  $x-y$  plane in  $z$  direction. The impact parameter  $b$  is always set to the radius  $a$  of the microdisk plus the half width of the waveguide, and the air-gap between the resonator and the waveguide. By changing the impact parameter  $b$ , or the wavelength  $\lambda$ , it is possible to address various mode orders ( $l$ ) and mode numbers ( $n$ ) of the WGMs.

Figure 2(a) shows off resonance and Figure 2(b) shows on resonance condition for a silica waveguide and silicon microdisk coupling. In the off-resonance case, we can see that the intensity of the

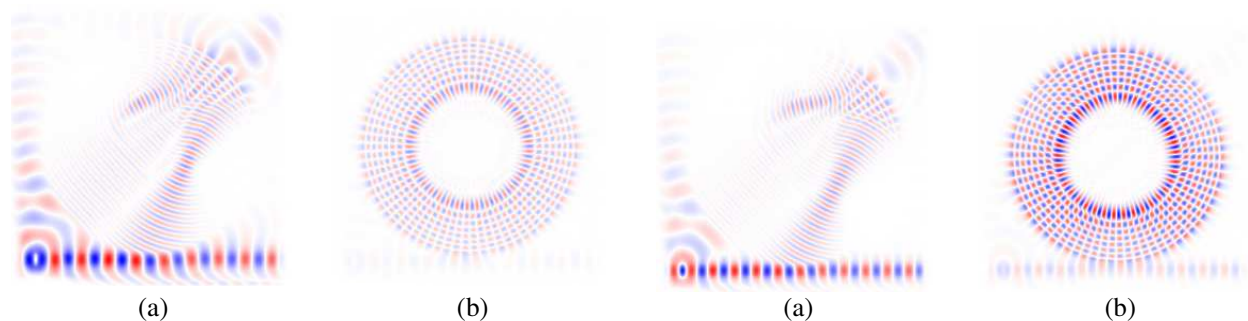


Figure 2: (a) The off-resonance and (b) on-resonance condition for a silicon disk on a glass slab waveguide.

Figure 3: (a) The off-resonance and (b) on-resonance condition for a silicon disk on a sapphire slab waveguide.

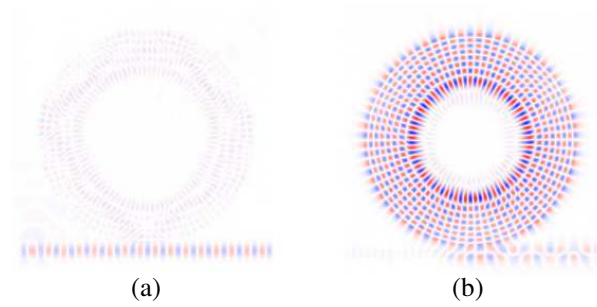


Figure 4: (a) The off-resonance and (b) on-resonance condition for a silicon disk on a diamond slab waveguide.

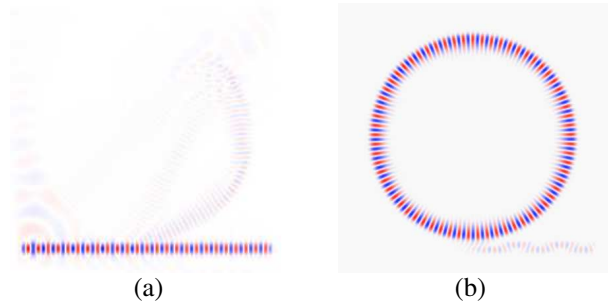


Figure 5: (a) The off-resonance and (b) on-resonance condition for a silicon disk on a silicon slab waveguide.

waveguide is high, but no WGMs can be observed inside the microdisk. The wavelength for the on-resonance case is  $\lambda_{laser} = 1550$  nm, the angular mode number is  $n = 37$  and radial mode order is  $l = 8$ . Figure 3(a) shows the off-resonance and Figure 3(b) shows the on-resonance condition for sapphire waveguide and silicon microdisk coupling. Similar pattern is observed for off-resonance case, while the on-resonance wavelength is  $\lambda_{laser} = 1550$  nm. For the on-resonance case the angular mode number is  $n = 37$  and radial mode order is  $l = 8$ .

Figure 4(a) shows off resonance and Figure 4(b) shows the on resonance condition for diamond waveguide and silicon microdisk coupling. The off-resonance wavelength is 1560 nm while the on resonance wavelength is 1550 nm. The mode number and mode orders observed are same as that of the silica and sapphire waveguides. Figure 5(a) shows the off resonance and Figure 5(b) shows the on-resonance condition for silicon waveguide and silicon microdisk coupling. The off-resonance wavelength is 1552 nm, while the on-resonance wavelength is 1548 nm. For the on-resonance case the angular mode number is  $n = 65$  and radial mode order is  $l = 8$ . As for the resonance condition i.e.,  $x \leq n \leq m_{sphere}x$ ,  $n$  should lie between 22 and 76, and our numerical analysis has confirmed this condition in all of the cases considered above.

#### 4. CONCLUSIONS

We have shown that, in addition to the commonly used interferometric method, near-IR spectroscopy can also be used to measure the radius of the silicon Avogadro sphere. We demonstrated our approach by numerically calculating the electric field strengths of an evanescently coupled silicon microdisk (the 2D analog of the microsphere) and various slab waveguides (the 2D analog of the optical waveguide) of glass, sapphire, diamond, and silicon.

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#### REFERENCES

1. Davis, R., "The SI unit of mass," *Metrologia*, Vol. 40, 299, 2003.
2. Quinn, T. J., "The kilogram: The present state of our knowledge," *IEEE Transactions on Instrumentation and Measurement*, Vol. 40, 81–85, 1991.
3. Mills, I. M., P. J. Mohr, T. J. Quinn, B. N. Taylor, and E. R. Williams, "Redefinition of the kilogram, ampere, kelvin and mole: A proposed approach to implementing CIPM recommendation 1 (CI-2005)," *Metrologia*, Vol. 43, 227–246, 2006.
4. Becker, P., P. D. Bièvre, K. Fujii, M. Glaeser, B. Inglis, H. Luebbig, and G. Mana, "Considerations on future redefinitions of the kilogram, the mole and of other units," *Metrologia*, Vol. 44, 1–14, 2007.
5. Becker, P., "Tracing the definition of the kilogram to the Avogadro constant using a silicon single crystal," *Metrologia*, Vol. 40, 366, 2003.
6. Mills, I. M., P. J. Mohr, T. J. Quinn, B. N. Taylor, and E. R. Williams, "Redefinition of the kilogram: a decision whose time has come," *Metrologia*, Vol. 42, 71–80, 2005.

7. Becker, P., H. Bettin, H. Danzebrink, M. Gläser, U. Kuetgens, A. Nicolaus, D. Schiel, P. De Bievre, S. Valkiers, and P. Taylor, "Determination of the Avogadro constant via the silicon route," *Metrologia*, Vol. 40, 271, 2003.
8. Fujii, K., M. Tanaka, Y. Nezu, K. Nakayama, R. Masui, and G. Zosi, "Interferometric measurements of the diameters of a single-crystal silicon sphere," *Review of Scientific Instruments*, Vol. 63, 5320, 1992.
9. Righini, G. C., Y. Dumeige, P. Féron, M. Ferrari, G. Nunzi Conti, D. Ristic, and S. Soria, "Whispering gallery mode microresonators: Fundamentals and applications," *Rivista del Nuovo Cimento*, Vol. 34, 435–488, 2011.
10. Murib, M. S., E. Yüce, O. Gürlü, and A. Serpengüzel, "Polarization behavior of elastic scattering from a silicon microsphere coupled to an optical fiber," *Photonics Research*, Vol. 2, 45–50, 2014.
11. Eryürek, M., "Hydrogen gas sensor based on a polymer optical microdisk resonator," Koç University, 2013.
12. Oskooi, A. F., D. Roundy, M. Ibanescu, P. Bermel, J. D. Joannopoulos, and S. G. Johnson, "MEEP: A flexible free-software package for electromagnetic simulations by the FDTD method," *Computer Physics Communications*, Vol. 181, 687–702, 2010.