An Efficient and Innovative Modelisation for Nanolasers

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Abstract— Nanolaser modelling still presents difficulties stemming from the complexity of the physical problem and lacking a simple and efficient way of predicting the temporal evolution of the laser field. The application of a Stochastic Simulator to the case of a large-β laser shows how this new approach can deliver precious information on the relative fraction of stimulated (and spontaneous) photons in the output, together with the fluctuations intrinsic to the stochasticity of the physical processes (emission by *atoms* and transmission by the cavity). A brief discussion of the open problems in nanolaser modelling and characterization is offered.

1. INTRODUCTION

Progress in laser miniaturisation has been steady and successful since the proposal of the Vertical Cavity Surface Emitting Semiconductor Laser (VCSEL) [1] in 1988. The idea of *turning around* the cavity axis by placing the plane of the active medium (e.g., a quantum well) ortogonally to the direction of propagation of the electromagnetic (e.m.) field and using very high mirror reflectivities has allowed for the realization of the first microcavities. Their length is now limited to half a wavelength (in the material of refractive index n) and their steadily decreasing diameter, to ensure a high-quality transverse mode and very low thresholds of operation (i.e., minimal electric currents), has permitted the reduction of the active volume to a few tens of cubic micrometers.

Modelling efforts, parallel with technological developments, showed that the miniaturisation smoothed out the *kink* normally observed in the laser response (cf. Figure 1), thus leading to difficulties in a clear identification of the transition towards field coherence. In the limit of a cavity consisting of a single e.m. field mode — i.e., one and only mode both for the spontaneous as well as for the stimulated emission ($\beta = 1$, where β represents the fraction of spontaneous emission coupled into the lasing mode) — the kink may entirely disappear, producing what has been called the *thresholdless laser*. In reality, a transition towards field coherence persists, but its identification remains very difficult to achieve [2], even in today's nanolasers [3].

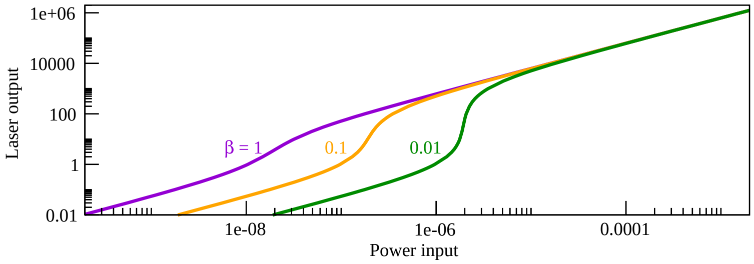

Figure 1: Laser output vs. injected power (arbitrary units) from Eq. (25) in [4] with parameter values: $\gamma = 1 \times 10^{10}\,\mathrm{s}^{-1}$, $\xi = 0.1$, $\tau_{sp} = 1 \times 10^{-9}\,\mathrm{s}$, $\tau_{nr} = 1 \times 10^{-10}\,\mathrm{s}$.

In nanolasers, the problem is further complicated by the extremely low level of optical power which can be recovered from the device and by the technical restriction (still current in most devices) of pulsed operation. Thus, the experimental tools available for testing the degree of coherence of the output field remain quite limited and most investigations offer a characterisation of the laser emission in terms of second (or, at times, higher) order (auto-)correlation functions. Since their interpretation amounts to solving a mathematical inverse problem, good modelling guidance is indispensable. In this paper, we briefly discuss the physical peculiarities of the nanolaser and show how a new modelling approach can contribute to shed new light into the physics of coherence buildup.

2. THE PHYSICAL PECULIARITY OF THE NANOLASER

The principal advantage of the nanolaser is its extremely small cavity size, allowing for: a. integration into nano-optical circuits for on-chip information transmission and processing; and b. extremely low power consumption. Good optical coupling, due to the on-chip integration, reduces spurious losses and improves optical efficiency, while the extremely small cavity volume minimizes the radiative losses through spontaneous emission in the "unwanted" e.m. cavity modes.

However, since the traditional, sharp *threshold* is a direct consequence of the large cavity volume in a standard laser [5, 6], the difficulties arising in the large-β [7] limit are not surprising. In a nanolaser possessing a unique e.m. cavity mode **all** photons end up into this mode, whether they are spontaneous or stimulated, and collectively contribute to the photon output. Thus, experimental measurements must either detect the statistical properties of the different photon (cf. below) or measure directly the phase noise [8]. Models must therefore be able to either account for the phase of the total e.m. field, or describe self-consistently the fraction of spontaneous and stimulated photons and their fluctuations, without ressorting to hypotheses on their statistics (as is done, for instance, when adding Langevin noise to rate equations). This is where the SS provides itself as a very useful tool, since it preserves the simplicity and versatility of population-based models (i.e., rate equations [5]) which can be rigorously proven to be consistent with more sophisticated Maxwell-Bloch techniques [9], while keeping track of all spontaneous photon populations and self-consistently including the stochasticity of the emission and cavity transmission.

One point which remains unclear for the moment is the true nature of the photon statistics in the extended transition (or threshold) *region*. Class A-based [10] photon statistics predicts the existence of a *statistical mixture* of spontaneous and stimulated photons [11], postulated also in a microcavity device [12]. However, cross-correlation measurements performed in a nanolaser prove the existence of strong correlations (i.e., predominantly stimulated photons) but over short times [13] (compatible results are obtained in a microcavity [14]). Since the statistical distributions do not give access to time-resolved information, the modelling requires efficient stochastic predictions on temporal sequences of events to compare to experiments.

3. FLUCTUATIONS, CORRELATIONS AND PHOTON STATISTICS

The coherence of laser output is characterized by a spectrum of fluctuations which crucially differ from that of thermal radiation [15]. Photon statistics, developed in Class A lasers as a tool to answer this question [11], has been very successfully used on a wide gamut of laser devices. However, most lasers currently in use belong to the Class B [10], due to their much higher efficiency in storing energy in the internal (quantum) states with almost negligible losses (which instead dominate in Class A devices).

Semiconductor micro- and nanocavities belong to the Class B: their temporal dynamics is predicted to be more complex [16, 17] and experimental results suggest that their statistical features present a more complex evolution across the threshold region [14]. This may be the basis for the difficulties in interpreting the observed convergence (or lack thereof) in the (auto)correlation functions [18, 19], it could explain the inconsistencies observed in the convergence of higher-order correlations [20], it would match the observation of coherent oscillations [21, 22] and may also be in agreement with the short correlation times observed in cross-correlation analysis [13]. For all these reasons, fast access to predictions relying fluctuation-related information, even in the temporal domain [17], becomes crucial. The advantage of the SS is that using a minimal amount of assumptions (usual in semiclassical modeling of lasers) it is possible to have access to the intrinsic stochastical features of the physics of laser emission.

4. LASER MODELLING WITH THE STOCHASTIC SIMULATOR

The Stochastic Simulator (SS), detailed elsewhere [16], offers predictions based on the estimate of the probability of occurrence of the competing physical processes which take place when *atoms* (where the term here denotes any kind of physical element storing energy in its internal state) relax from an excited state with emission of a photon. Only the discrete physical processes and their probabilities are included, thus excluding all hypotheses necessary for writing differential models. The scheme is illustrated in Figure 2.

The introduction of photon classes (three colors in the figure) is the crucial tool for the success of this modelling approach based on dynamically following the photon evolution from the entirely uncorrelated (spontaneous) to the predominantly correlated (stimulated) regime *across* the thresh-

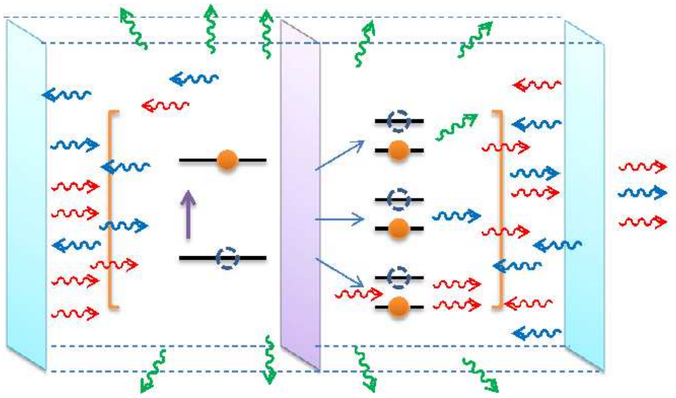

Figure 2: Illustration of the Stochastic Simulator principle. Each *atom* (represented by the two quantum levels) is externally pumped and may decay radiatively in one of the three channels — 1. the *off-axis spontaneous "mode"*, grouping all modes other than the lasing one (green photons); 2. the *on-axis spontaneous mode* (blue photons); 3. the (on-axis) *stimulated mode* (red photons). On-axis photons are recycled by the cavity (light turquoise-green planes) and are transmitted by the coupling mirror (right); the off-axis (green) photons exit the interaction volume laterally. The full pumping process is represented by the upward magenta arrow. The large bracket collects all interactions occurring, e.g., in the grey plane. Absorption and nonradiative relaxations are neglected, their effect being limited to raising the laser threshold.

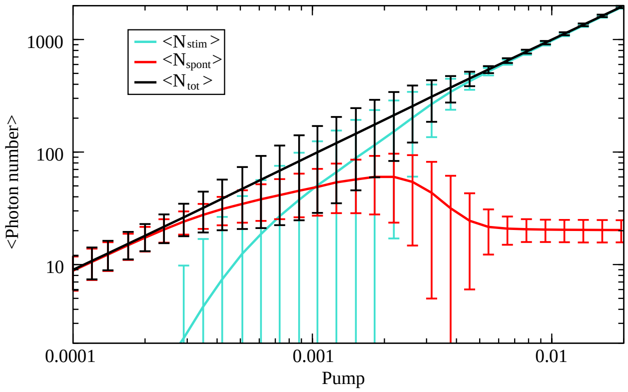

Figure 3: Average photon number as a function of pump for a laser with $\beta = 0.9$. Red line: spontaneous photon fraction emitted on-axis; blue line: stimulated fraction of photons. The black line represents the total number of photons transmitted by the output coupler. The error bars represent the fluctuations and are computed, for the black curve, on the sum of the two photon populations.

old region. This way, it is possible to see how the "phase transition" takes place for different values of β [16], since its value reflects the presence of off-axis spontaneous modes. A clear threshold can be identified in macroscopic (low β) devices [16] and a progressively smoother transition naturally appears as the cavity volume is reduced. Of course, this representation is a semi-classical approximation of the actual quantum-mechanical description and cannot reproduce interference effects and the collapse of the phase of the e.m. field. The strong computational efficiency of the scheme [16], however, offers the possibility of easily repeating the calculations for averaging and for statistical information.

Figure 3 shows the computed average photon numbers, together with their fluctuations (error bars), when varying the pump strength. The blue curve reproduces the growth of the stimulated fraction of photons, whose contribution in the left part of the graph is entirely negligible (notice the logarithmic scale). The spontaneous photons (red) entirely dominate the low pump interval and slow their growth until they drop to a saturated value as the pump grows sufficiently, while the stimulated fraction eventually takes a linear growth. The average total number of photons emitted on-axis — i.e., the combination of spontaneous and stimulated which cannot be discerned when measuring the output power with a detector — show a practically linear response (black curve). The fluctuations of the total output, however, display a marked increase in the central region of the figure to decrease to a level almost indistiguishable from the line in the upper pumping range. This is the signature of the transition to lasing, and we can reasonable consider that the laser output is coherent starting from pump values approaching 0.01. No clear threshold, however, can be assigned, contrary to the low-β situation [16]. Notice that the asymmetry (top-bottom) in the error bars in the graph is due to the representation in logarithmic scale, rather than to a true asymmetry in the fluctuations.

The information obtained from the SS goes well beyond what rate equations can provide. While on the one hand, the latter can easily predict the shape of the *input-output* curve (average photon number vs. pump — cf., e.g., Figure 1), the rate-equation-approach is entirely incapable of providing any meaningful information about the minimum value of pump above which the laser output can be considered coherent. Indeed, even when introducing noise (e.g., in Langevin form) to the model, the rate equations for a very small system will at best reflect the noise characteristics of the noise scheme which has been chosen, rather than those of the stochastic processes which lead to *atomic* relaxation and then – under suitable conditions — lasing.

The predictions of the SS instead, given their intrinsic stochastic nature, provide meaningful information about the fluctuations around the average and, given the explicit inclusion of the three photon classes, can be used to follow the transition between the spontaneous and the coherent regime (within the approximations of the model). This information can be used to follow the relative fluctuations, compute the Fano factor, etc., thus allowing for a more complete characterization of the predicted laser response. External fluctuations can be added, e.g., to the pump, to further investigate their influence on the device's operation. Here we have exclusively focussed on the internal noise.

5. CONCLUSION

In conclusion, we have given a quick overview of the features of nanolasers which still pose problems and of some of the techniques which have and can be used for their characterization. Since numerous problems remain open in the description of these nanosources and modelling presents difficulties, either related to the inadequacy of simplified approaches, or the complexity of advanced techniques, we have presented the advantages offered by the Stochastic Simulator and shown how its use can efficiently provide useful information about the operation of very small lasers.

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REFERENCES

1. Iga, K., F. Koyama, and S. Kinoshita, "Surface emitting semiconductor lasers," *IEEE J. Quantum Electron.*, Vol. 4, No. 9, 1845–1855, 1988.
2. Chow, W. W., F. Jahnke, and C. Gies, "Emission properties of nanolasers during the transition to lasing," *Light: Science & Applications*, Vol. 3, No. 8, e201(1–8), 2014.
3. Strauf, S. and F. Jahnke, "Single quantum dot nanolaser," *Laser Photonics Rev.*, Vol. 5, No. 5, 607–633, 2011.
4. Björk, G. and Y. Yamamoto, "Analysis of semiconductor microcavity lasers using rate equations," *IEEE J. Quantum Electron.*, Vol. 27, No. 11, 2386–2396, 1991.
5. Siegman, A. E., *Lasers*, University Science Books, Mill Valley, CA, 1986.
6. Dohm, V., "Nonequilibrium phase transition in laser-active media," *Solid State Commun.*, Vol. 11, No. 9, 1273–1276, 1972.
7. Björk, G., A. Karlsson, and Y. Yamamoto,"Definition of a laser threshold," *Phys. Rev. A*, Vol. 50, No. 2, 1675–1680, 1994.
8. Piels, M., W. Xue, C. Schäffer, Y. Yu, E. Semenova, L. Ottaviano, K. Yvind, I. Tafur Monroy, J. Mørk, and D. Zibar, "Highly sensitive photonic crystal cavity laser noise measurements using Bayesian filtering," *Optical Fiber Communication Conference*, W2A-28(1–3), Anaheim, CA, USA, Mar. 2015.
9. Narducci, L. M. and N. B. Abraham, *Laser Physics and Laser Instabilities*, World Scientific, Singapore, 1988.
10. Tredicce, J. R., F. T. Arecchi, G. L. Lippi, and G. P. Puccioni, "Instabilities in lasers with an injected signal," *J. Opt. Soc. Am. B*, Vol. 2, No. 1, 173–183, 1985.

11. Arecchi, F. T. and V. Degiorgio, "Statistical properties of laser radiation during a transient buildup," *Phys. Rev. A*, Vol. 3, No. 3, 1108–1124, 1971.
12. Aßmann, M., F. Veit, M. Bayer, C. Gies, F. Jahnke, S. Reitzenstein, S. Höfling, L. Worschech, and A. Forchel, "Ultrafast tracking of second-order photon correlations in the emission of quantum-dot microresonator lasers," *Phys. Rev. B*, Vol. 81, No. 16, 165314(1–5), 2010.
13. Lebreton, A., I. Abram, R. Braive, I. Sagnes, I. Robert-Philip, and A. Beveratos, "Unequivocal differentiation of coherent and chaotic light through interferometric photon correlation measurements," *Phys. Rev. Lett.*, Vol. 110, No. 16, 163603(1–5), 2013.
14. Wang, T., G. P. Puccioni, and G. L. Lippi, in Preparation.
15. Mandel, L. and E. Wolf, *Optical Coherence and Quantum Optics*, Cambridge University Press, Cambridge, 1995.
16. Puccioni, G. P. and G. L. Lippi, "Stochastic simulator for modeling the transition to lasing," *Opt. Express*, Vol. 23, No. 3, 2369–2374, 2015.
17. Wang, T., G. P. Puccioni, and G. L. Lippi, "An efficient and innovative modelisation for nanolasers," *PIERS Proceedings*, Paper 150204094555, Prague, Czech Republic, Jul. 2015, in Preparation.
18. Ulrich, S. M., C. Gies, S. Ates, J. Wiersig, S. Reitzenstein, C. Hofmann, A. Löffler, A. Forchel, F. Jahnke, and P. Michler, "Photon statistics of semiconductor microcavity lasers," *Phys. Rev. Lett.*, Vol. 98, No. 4, 043906(1–4), 2007.
19. Wiersig, J., C. Gies, F. Jahnke, M. Aßmann, T. Berstermann, M. Bayer, C. Kistner, S. Reitzenstein, C. Schneider, S. Höfling, A. Forchel, C. Kruse, J. Kalden, and D. Hommel, "Direct observation of correlations between individual photon emission events of a microcavity laser," *Nature*, Vol. 460, No. 7252, 245–249, 2009.
20. Elvira, D., X. Hachair, V. B. Verma, R. Braive, G. Beaudoin, I. Robert-Philip, I. Sagnes, B. Baek, S. W. Nam, E. A. Dauler, I. Abram, M. J. Stevens, and A. Beveratos, "Higher-order photon correlations in pulsed photonic crystal nanolasers," *Phys. Rev. A*, Vol. 84, No. 6, 061802(R)(1–4), 2011.
21. Lebreton, A., I. Abram, N. Takemura, M. Kuwata-Gonokami, I. Robert-Philip, and A. Beveratos "Stochastically sustained population oscillations in high-β nanolasers," *New J. Phys.*, Vol. 15, No. 3, 033039(1–11), 2013.
22. Takemura, N., J. Omachi, and M. Kuwata-Gonokami, "Fast periodic modulations in the photon correlation of single-mode vertical-cavity surface-emitting lasers," *Phys. Rev. A*, Vol. 85, No. 5, 053811(1–5), 2012.