Conductivity Estimation of Breast Cancer Using Stochastic Optimization

A. Jeremic[1] **and E. Khosrowshahli**[2]

[1]Department of Electrical and Computer Engineering, McMaster University, Canada
[2]School of Biomedical Engineering, McMaster University, Canada

Abstract— Breast cancer detection is one of the most important problems in health care as it is second most frequent cancer according to WHO. Breast cancer is among cancers which are most probably curable, only if it is diagnosed at early stages. To this purpose it has been recently proposed that microwave imaging could be used as a cheaper and safer alternative to the commonly used combination of mammography. From a physical standpoint breast cancer can be modelled as a scatterer with a significantly (tenfold) larger conductivity than a healthy tissue. In our previous work we proposed a maximum likelihood based method for detection of cancer which estimates the unknown parameters by minimizing the residual error vector assuming that the error can be modelled as a multivariate (multiple antennas) random variable. In this paper we utilize stochastic optimization technique and evaluate its applicability to the detection of cancer using numerical models. Although these models have significant limitations they are potentially useful as they provide insight in required levels of noise in order to achieve desirable detection rates.

1. INTRODUCTION

According to breast cancer society , breast cancer is the most frequently diagnosed cancer in women with over 232,670 new cases expected in 2014 [1]. Due to the progressive nature of the disease early detection is extremely important and can potentially significantly improve survival of patients. Currently clinical procedures are commonly based on mammography which is routinely prescribed for older women who tend to be more susceptible to the disease [2]. Although mammography is extremely important diagnostic technique, it suffers from some limitations such as false negative and positive results, using ionizing radiation and patients discomfort [2, 3]. The number of false positives is rather significant in the case of so called dense breasts in which healthy tissue may be mistaken for malignant and as a consequence unnecessary biopsies are prescribed. Furthermore, complicating the matter is the fact that mammography is a two-dimensional technique and hence the reconstruction techniques which are needed to obtain three-dimensional images can lead to false positives.

Microwave imaging has been recently proposed as an additional medical imaging technique which can potentially overcome some of the shortcomings of the mammography. Essentially the technique is based on illuminating breast with electromagnetic-wave(s) in microwave range. From the physical point of view this can be represented as a wave propagation in medium (breast) that contains scatterers (both healthy and malignant tissue). Due to the fact that malignant tissue has larger conductivity the measurements obtained by receiving array of antennas will be different if the scatterers are present. Therefore once the wave propagates through the breast the received signal is analyzed in order to obtain permittivity map using appropriately selected image reconstruction technique [4]. Most of the image reconstruction techniques minimize a particular cost function (e.g., mean-square error). In most cases the number of unknowns (e.g., number of pixels in the map) is much larger than the number of available measurements which requires an additional constraint.

In this paper we propose a simplified parametric inverse 3D model which enables us detection of tumour presence and estimation of its size and/or position. Most of the existing solutions [7] employ non-parametric image reconstruction techniques. In our previous work we proposed a maximum likelihood based method for detection of cancer which estimates the unknown parameters by minimizing the residual error vector assuming that the error can be modelled as a multivariate (multiple antennas) random variable. In this paper we utilize stochastic optimization which is based on the minimization of the variance of the expected value rather than on the gradient of the partial-differential equations based model. Namely the scattering models utilize multiple PDEs in order to calculate scattering parameters from a transmitting to receiving antenna. We propose to utilize recently proposed algorithm for stochastic optimization which achieves smaller computational time but can suffer from estimation accuracy. However detection performance is often somewhat robust to estimation results and to this purpose we investigate the accuracy of detection algorithms based on stochastic optimization results. In Section 2 we present computational models of electromagnetic wave propagation in breast when tumours are absent and present and discuss how the aforementioned models were implemented using COMSOL finite element solver. In Section 3 we present the statical models and present estimation algorithm. In Section 4 we present simulation results and discuss their potential use for inverse problems. Finally, Section 5 concludes the paper.

2. PHYSICAL MODEL

In this Section we develop mathematical model describing the measured signals. The imaging system consists of several antennas which can both serve as transmitting and receiving antennas. These antennas in principle can be distributed over the breast surface thus allowing for a three-dimensional scan whose resolution depends only on number of antennas. Obviously the number of antennas is determined by their size which may be constrained by technical requirements such as antenna-to-antenna noise (interference). Once the microwave is generated it propagates through the volume of the breast according to Maxwell's equations which in this particular case can be reduced to the phasor form since microwave antennas operate in a single-frequency mode.

It should be observed that any non-homogeneity in the object can be modelled as a scatterer and thus in the presence of multiple scatterers the resulting electromagnetic field becomes very complex superposition of reflected and refracted waves. Since malignant tissue has significantly larger permittivity than healthy tissue it can also be modelled as a scatterer (see Figure 1). The reflection/refraction from scatterers can then be modelled as described in Figure 2.

In the remainder of the paper we will assume that in the absence of cancer the scattering in the breast is due only to small non-homogeneities which will be included as the modelling noise. Of course if a particular patient is submitted to continuous monitoring in a regular intervals the previous images can be used a reference signal and thus "healthy scattering" can be properly recorded and modelled. In this paper we assume that the tumours can be modelled as spheres and therefore are uniquely defined by location vector and radius. In general, arbitrarily shaped tumours can be represented by spatial Fourier transform and corresponding spatial frequency amplitudes and phases. As we stated before, the electromagnetic properties of the malignant tissue is significantly different than breast tissue and thus proper boundary conditions must be considered in order to ensure continuity (see Figure 2).

In order to solve the above equations we utilize finite-element method by developing three-dimensional model using RF module in COMSOL Multiphysics software. In this paper we model the breast as a sphere with radius of 100 mm as shown in Figure 3 immersed into the cubical structure representing microwave imaging system. Antennas are equally spaced on all the sides of the imaging structure. These antennas are modelled as slim cubes which are centred on the surface of the sphere. Three boundary conditions are used to send waves in the medium. Electric

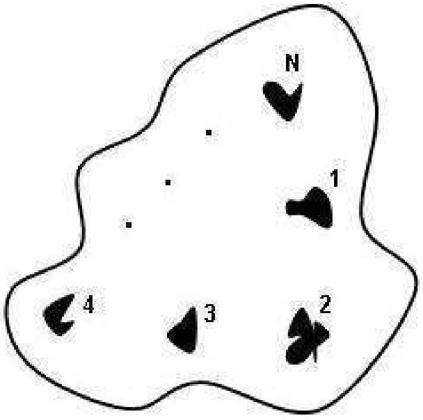

Figure 1: Medium with multiple scatterers.

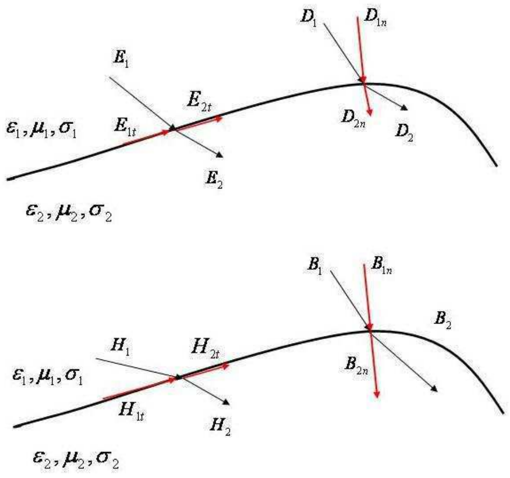

Figure 2: Boundary conditions.

Table 1: Tissue properties.

Tissue	Relative permittivity	Condutcitivty
Immersion Liquid	9	0
Chest Wall	50	7
Skin	36	4
Breast Tissue	9	0.4
Tumour	50	4

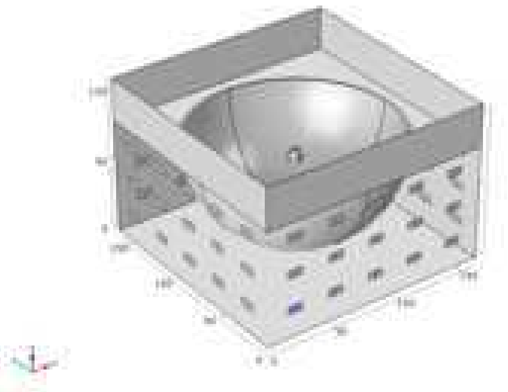

Figure 3: Geometry of the breast.

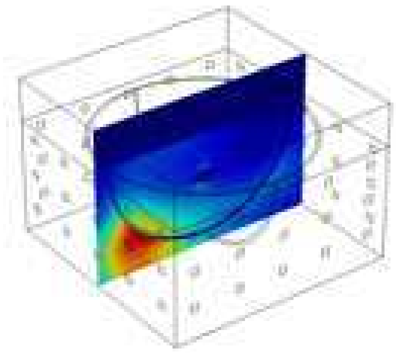

Figure 4: Sample result for the field.

field is applied to transmitter antenna. Perfect electric conductor boundary condition is applied to sides of antennas in order to guide wave through them, and scattering boundary condition is applied to the rest of surfaces to let waves propagate freely. Afterward, for different studies one or multiple tumours with arbitrary shapes can be modelled. In this study tumour is considered as a sphere inside the breast with an arbitrary size and in arbitrary position. The actual values for physiological parameters are outlined in the Table 1.

3. STATISTICAL MODEL

Most of the existing methods describe the above physical model using Maxwell equations in order to model the electromagnetic wave propagation. Consequently for the transmitting antenna located at position $\mathbf{r}_0$ and the receiving antenna located at location $\mathbf{r}_i$ the scattering parameter can be obtained either by solving Maxwell's equation with adequately defined boundary conditions related to the transmitting antenna and finding the solution at the location of the receiving antenna. Due to irregular geometry and non-homogeneus parameters the solution can be obtained only numerically using finite-element method. In order to define non-linear transfer functions we utilize COMSOL finite-element solver and consequently describe the measurements as

$$y_{ij} = f(i, j, \theta_s, \theta_g) + e_{ij} \tag{1}$$

where y_{ij} is scattering parameter when the i-th antenna is transmitter and the j-th antenna receiver. The parameters θ_s and θ_g are scatterers parameters (number, radiuses and permittivity/conductivity parameters) and θ_g are the geometry parameters. In this paper we simplify the problem by removing the geometry parameters, i.e., they are fixed for a particular breast model. The residual errors/noise e_{ij} is assumed to be zero-mean Gaussian uncorrelated in space. Note the correlated noise can be easily addressed using any of the existing techniques based on the growth-curve model by Pothoff and Roy [9]. Our main focus in this paper is to demonstrate applicability of stochastic optimization and hence we choose the simplest possible noise model.

It has been recently proposed [8] that a set of multiple PDE equations can be inverted, i.e., the unknown parameters can be estimated without the need of solving multiple PDEs at each iteration. Let q_i be the power of the transmitting antenna and let y_{ij} be the $n-1$ scattering parameters obtained on all the other antennas (note that each antenna can serve as transmitting and receiving but we need n PDEs in order to utilize all n antenna's as transmitters). The main idea of stochastic optimization is based on so called simultaneous random sources in which a new source is obtained using a linear combination of the existing sources, i.e., $\hat{q} = \Sigma w_i q_i$ and new data are obtained as $\hat{y_{i\cdot}} = \sum w_j y_{ij}$ in which the weighting coefficients are chosen from appropriately defined probability density function, i.e., they are random. In that case it can be shown that the optimization problem reduces to minimizing the expected value

$$\frac{1}{2} E_w \left\| \left(A^{-1}(\theta_s)\mathbf{q} - \mathbf{y} \right) w \right\|^2 \tag{2}$$

where A is the transfer matrix obtained from non-linear functions f which represent finite-element solutions in which the discretized/linear formulation results in antenna-to-antenna transfer matrix, $\mathbf{y}$ and $\mathbf{q}$ are the lumped vectors of scattering parameters and sources respectively.

In order to solve the above stochastic optimization problem there are two general techniques: a) sample average approximation in which the expected value is approximated by a Monte-Carlo sum with the constraint that the number of realizations should be smaller than the number of PDEs and b) stochastic approximation technique in which a single realization is used to compute the stochastic gradient. In order to reduce the computational complexity and due to a smaller number of antennas (16) in this paper we choose the stochastic approximation technique and investigate its applicability to our problem.

The structure of SA approximation is given by:

- Initialize the solution, i.e., set the scatterer parameters to $\theta_s = \theta_s^0$;
- while convergence is not reached do;
- draw a random sample for weighting coefficients;
- approximately solve the optimization problem by calculating the gradient at the given point — note due to random mixing it requires only single solution to PDE;
- obtains the new value of $\hat{\theta}_s^{i+1}$ calculated using the gradient from the previous step;
- average the new value with the previous values and obtain the new value of parameter, i.e., $\theta_s^{i+1} = \frac{1}{i+1}(\sum_{j=1}^{i} \theta_s^j + \hat{\theta}_s^{i+1})$;
- end while.

It has been empirically shown [8] that the above technique shows acceptable performance when the random weight are drawn from binary distribution $(-1, 1)$ as in this case the minimization of the expected value yields mean with the smallest variance. In this preliminary work we use the same pdf for the weights and leave for future work to examine the possibility of selecting the pdf according to a particular problem.

In order to determine whether the estimated scatterer can potentially represent a cancer we need to perform a classification/detection algorithm which determines the probability (likelihood) that the conductivity and permittivity of the tissue are comparable to the values from Table 1. In general the likelihood ratio tests result in ratio of variances (or determinants of covariance matrices in the presence of correlated noise). In this preliminary approach we propose a simplified algorithm in which the estimated value of conductivity is compared to the healthy tissue value (9). Obviously the resulting test is comparable to T-test as it is similar to comparing the sample mean to a priori known value.

4. NUMERICAL RESULTS

In order to evaluate the applicability of the proposed technique we perform the following experiment. For a fixed geometry of the breast we insert 10 scatterers of random size at random locations. In order to simulate so called dense breasts in which the fat tissue can have slightly larger conductivity we choose to select conductivity of scatterers to be uniformly distributed in range (9, 25) where the lower boundary is chosen to equal healthy tissue and the upper one is arbitrarily set to half of the cancerous tissue. Then we insert a tumour located in the center of the breast with a radius of 0.5 cm. For a simplicity we assume that the location is known (note that other imaging techniques such as ultrasound and MRI could be used to detect suspicious regions and hence these locations can be known). Our main goal at this stage is to demonstrate ability to non-invasively estimate unknown conductivity and hence potentially detect breast cancer. An additional unknown parameter would most likely decrease performance but we believe that this issue could be addressed by utilizing more advanced techniques. In this preliminary work we want to examine applicability of model-based scatterer detection. We perform the proposed experiment 2000 times and in half of the experiments we use cancerous tissue as the scatterer in the centre and half of the times we insert healthy tissue in the center location. As discussed previously to account for the errors we add zero-mean Gaussian noise.

In order to evaluate the proposed the applicability of the proposed estimation algorithm we count the number of false positives and negatives in these 2000 runs. In Figure 5 we show the mean square error of the proposed stochastic optimization algorithm as a function of signal-to-noise ratio defined with respect to the scattering parameter values. In future work we will improve the noise models and include additive noise directly to the values of the scattered electromagnetic field. As expected we see that the error decreases as the SNR improves which may be used in order to understand what noise levels are needed in order to have an algorithm that can function in the clinical setting. In Figure 6 we illustrate the detection performance calculated using empirical values of probabilities based on the count of false decisions. The preliminary results indicate that the error is not symmetric and thus we expect to be able to achieve better performance by investigating the properties of error residual vector in more details.

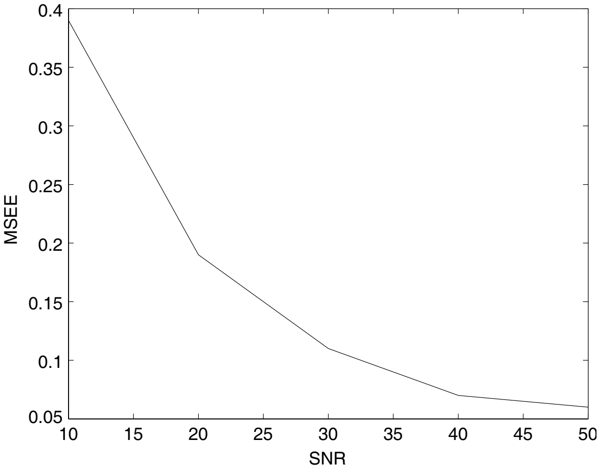

Figure 5: Mean square error for the stochastic optimization algorithm.

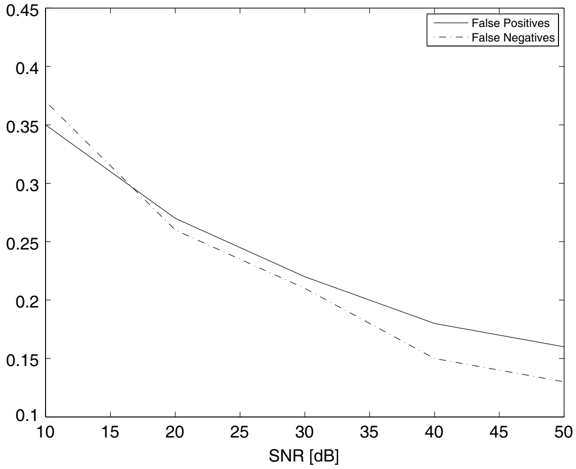

Figure 6: Detection performance as a function of SNR.

5. CONCLUSIONS AND FUTURE WORKS

In this paper we proposed a model based algorithm for estimating the conductivity of unknown scatterer in the breast tissue and implemented the decision making algorithm which can potentially be used for making decisions if the scatterers have conductivity similar to that of cancerous tissue. We developed a computational algorithm based on the stochastic optimization and finite-element model in order to decrease computational time without significant losses in accuracy. In future work we plan to compare the performance of stochastic algorithms and maximum likelihood based algorithms for different noise models. This is especially important in microwave settings as the noise levels can have significant impact. Furthermore we plan to investigate the effects that probability density functions of weighting coefficients have on the overall performance of the proposed algorithms.

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