Analysis on the Aperture Averaging Weight Factor for Equidistant Dual-aperture Receiver

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Abstract— Free-space optical communication receiver aperture is usually a single circular aperture. When the aperture size is large, its cost will be high. This paper proposes a new receiver aperture structure: equidistant dual-aperture receiver. The authors analyze its performance. Aperture Averaging Weight Factors of the two kinds of receiver structures are compared. The analysis results show that: Equidistant dual-aperture receiver can obviously decrease the Aperture Averaging Weight Factor, and reduce the optical scintillation.

1. INTRODUCTION

The research on free-space optical(FSO) communication has been sustained for a very long time [1, 2]. In the design of a FSO system, usually it only uses a singular large aperture. When the receiver aperture diameter increases, it can receive more optical flux, and the aperture averaging effect is also more obvious [3, 4]. However, as the receiver aperture size gets larger, its cost gets higher. For the civil FSO, cost factor is of great importance. When the receiver aperture is small, the optical signal can be collected by a single lens. When the receiver aperture increases, because of the influence of aberrations produced by gravity, an aspherical reflecting mirror must be used as the primary mirror. But the aspherical reflecting mirror is very expensive. Generally speaking, when the receiver aperture diameter is larger than 15 cm, it must use an aspherical mirror as the primary mirror. However, if we use 2 or 3 10-cm diameter lens to replace it, the cost reduces significantly. This paper concerns on such an issue: when use an equidistant dual-aperture receiver, how does the aperture averaging effect change?

2. PRINCIPLE OF APERTURE AVERAGING

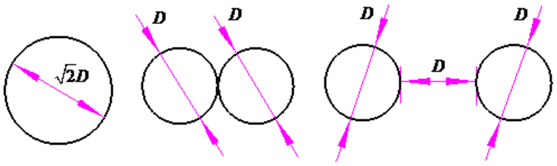
At the end of the FSO receiver, the normalized log-signal variance is shown as below [3]:

$$\begin{aligned}\sigma_\vartheta^2 &= \frac{1}{S_{receiver}^2}\int d\boldsymbol{\rho}\int d\boldsymbol{\rho}' W\left(\frac{1}{2}\boldsymbol{\rho}+\boldsymbol{\rho}',D\right)W\left(-\frac{1}{2}\boldsymbol{\rho}+\boldsymbol{\rho}',D\right)\sigma_l^2(\boldsymbol{\rho})\\ &= \frac{1}{S_{receiver}^2}\int_0^\infty \rho d\rho\int_0^{2\pi} d\varphi K_0(\rho,\varphi,D)\,\sigma_l^2(\rho)\\ &= \frac{1}{S_{receiver}^2}\int_0^\infty \rho d\rho\int_0^{2\pi} d\varphi\cdot S_{overlap}\cdot\sigma_l^2(\rho)\,. \end{aligned}\tag{1}$$

where, $W(\mathbf{r},D)$ is a function used to depict the aperture. When $|\mathbf{r}|\le D/2$, $W=1$; when $|\mathbf{r}|>D/2$, $W=0$. $\boldsymbol{\rho}=\mathbf{r}-\mathbf{r}'$, $\boldsymbol{\rho}'=(\mathbf{r}+\mathbf{r}')/2$, $\rho=|\boldsymbol{\rho}|$ (ρ means the distance between two arbitrary points at the wave-front). $K_0(\rho,\varphi,D)$ is a result that integrates over $\boldsymbol{\rho}'$. It is function including variables $\boldsymbol{\rho}$, D and φ. It is called as the K function. It means the overlap area when moves the aperture centroid to a distance $\boldsymbol{\rho}$. Fried gave its result [3]. In the case of singular aperture, because of the rotational symmetry, $K_0(\rho,\varphi,D)$ is independent of the angle φ. We define a parameter $A(\rho,D)=\int_0^{2\pi} d\varphi\cdot S_{overlap}$. Function A is called as Aperture Averaging Weight Factor. Thus

$$\sigma_\vartheta^2=\frac{1}{S_{receiver}^2}\int_0^\infty \rho d\rho\cdot A(\rho,D)\cdot\sigma_l^2(\rho)\approx\frac{1}{S_{receiver}^2}\sum_{i=1}^\infty \rho_i A(\rho_i,D)\,\sigma_l^2(\rho_i)\,\Delta\rho_i.\tag{2}$$

As a result, the log-signal variance is changed to a sigma summation. It consists of products at many points. $A(\rho_i,D)$ is seemed to be the weight factor of $\sigma_l^2(\rho_i)$.



(a) (b) (c)

Figure 1: Three kinds of aperture configurations.

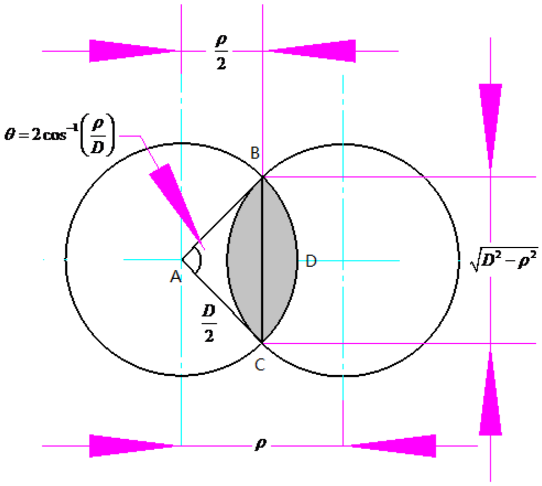


Figure 2: Diagram for calculating the K function.

Because $\sigma_l^2(\rho_i)$ cannot be controlled, if want to reduce σ_ϑ^2, $A(\rho_i, D)$ should be reduced. If want to reduce $A(\rho_i, D)$, the overlapping area of the aperture function should be reduced.

As shown in Figure 1(a), we can use a receiver aperture with diameter $\sqrt{2}D$. As shown in Figure 1(b), we also can use 2 receiver apertures with diameter D. Figure 1(a) and Figure 1(b) have the same light receiving area. Assuming that the receiver aperture D is 10 cm, diameter $\sqrt{2}D$ is too large to use lens as the primary mirror. It can only use aspherical mirror. So the cost is too high. As a result, we can use the aperture layout in Figure 1(b). But is it the best choice? No. We find out from our research that the layout in Figure 1(c) is much better that in Figure 1(b). Let's go back to the calculation process of K function. It is actually gotten through by calculating the overlap area of two apertures when two graphics center spacing is ρ. The calculation process is given in Figure 2 and Equation (3).

$$S_{shade} = 2\left(S_{arcABDC} - S_{\Delta ABC}\right) = \frac{D^2}{2}\left\{\cos^{-1}\left(\frac{\rho}{D}\right) - \frac{\rho}{D}\left[1-\left(\frac{\rho}{D}\right)^2\right]^{1/2}\right\}. \tag{3}$$

3. APERTURE AVERAGING WEIGHT FACTOR

In the next part the authors will analyze the k function of that in Figure 1(c). As shown in Figure 3, when $0 < \rho < D$, the moving region of the stacking ring is limited in the green line area. No one arbitrary stacking ring will stack two stacked rings at the same time.

As shown in Figure 4, when $\rho > 3D$, there is not any possibility of overlapping. K function is

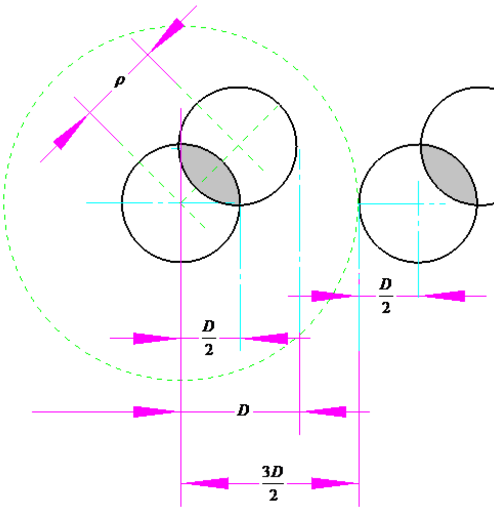


Figure 3: Diagram for calculating the K function in the case of $0 < \rho < D$.

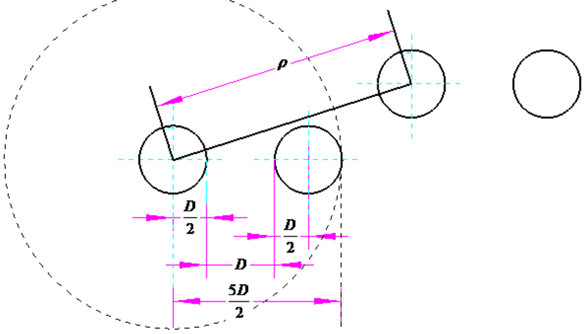


Figure 4: Diagram for calculating the K function in the case of $\rho > 3D$.

0. As shown in Figure 5, when $D < \rho < 3D$ and $\pi/6 < \varphi < 11\pi/6$, there is not any possibility of overlapping. K function is 0.

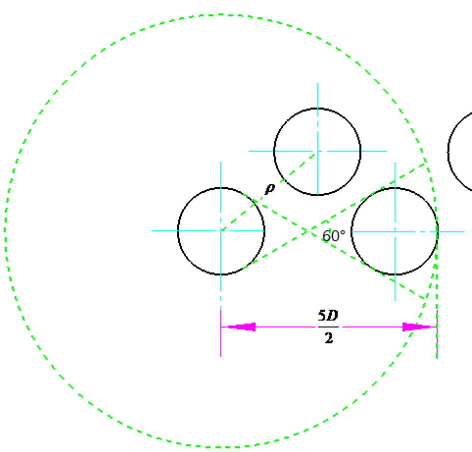


Figure 5: Diagram for calculating the K function in the case of $D < \rho < 3D$ and $\pi/6 < \varphi < 11\pi/6$.

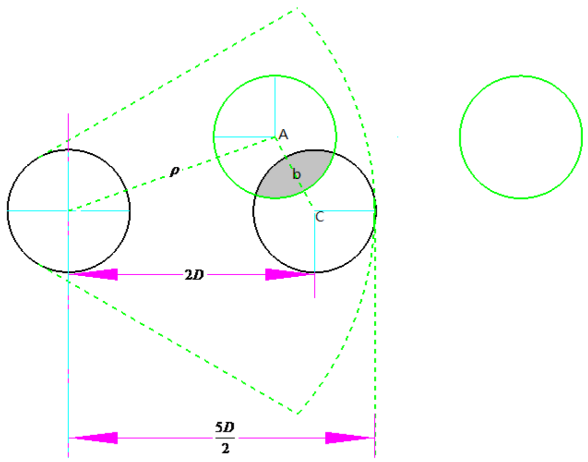


Figure 6: Diagram for calculating the K function in the case of $D < \rho < 3D$ and $-\pi/6 < \varphi < \pi/6$.

As shown in Figure 6, when $D < \rho < 3D$ and $-\pi/6 < \varphi < \pi/6$, the first stacking ring can only move in the green line range. $\overline{AC} = b$. According to a simple trigonometry and a series expansion, it changes into $b \approx |2D - \rho\cos\varphi + \frac{1}{4D}\rho^2\sin^2\varphi|$. To sum it up:

$$A(\rho, D) = \begin{cases} 2\pi D^2 \left\{ \cos^{-1}\left(\frac{\rho}{D}\right) - \left(\frac{\rho}{D}\right)\left[1 - \left(\frac{\rho}{D}\right)^2\right]^{1/2} \right\}, \quad 0 < \rho \le D,\ \varphi \in (0, 2\pi) \\ \frac{D^2}{2}\int_{-\pi/6}^{\pi/6} d\varphi \left\{ \cos^{-1}\left(\frac{b}{D}\right) - \frac{b}{D}\left[1 - \left(\frac{b}{D}\right)^2\right]^{1/2} \right\}, \\ \text{where, } b \approx \left| 2D - \rho\cos\varphi + \frac{1}{4D}\rho^2\sin^2\varphi \right|, \quad D < \rho < 3D, \\ \qquad 0,\ \rho \ge 3D,\ \varphi \in (0, 2\pi) \end{cases} \tag{4}$$

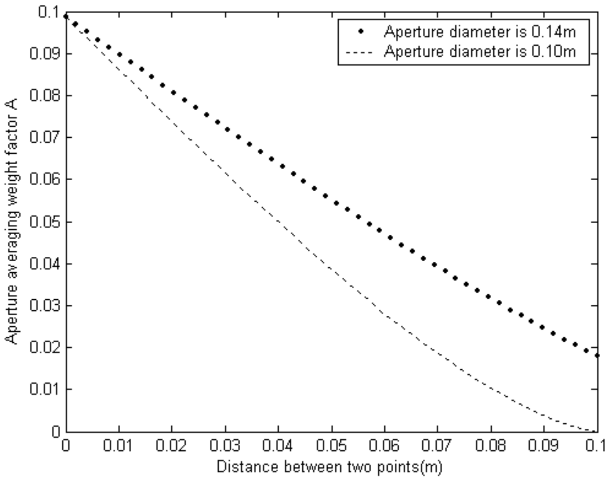


Figure 7: Aperture averaging weight factors of two different receiver structures.

4. CONCLUSIONS

In this paper, the authors propose a new kind of receiver structure for FSO. The comparison result is shown in Figure 7. Obviously the equidistant dual-aperture receiver has a significant decrease of Aperture Averaging Weight Factor. Next step work will concerns on an experiment with this kind of new aperture.

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