On an Application of the Hypothesis for the Identity of the $L_2(c,\rho,n)$ and $\hat{L}_2(\hat{c},\hat{\rho},\hat{n})$ Numbers

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Abstract— The theory of waveguides is pointed out as a field of application of the mathematical hypothesis, asserting that the finite real positive numbers $L_2(c,\rho,n)$ and $\hat{L}_2(\hat{c},\hat{\rho},\hat{n})$, advanced through some positive purely imaginary, resp. real zeros in x, resp. in $\hat{x}$ of a special function, constructed by two complex $\Phi(a,c;x)$ and $\Phi(a,c;\rho x)$, resp. real $\hat{\Phi}(\hat{a},\hat{c};\hat{x})$ and $\hat{\Phi}(\hat{a},\hat{c};\hat{\rho}\hat{x})$ Kummer, and two complex $\Psi(a,c;x)$ and $\Psi(a,c;\rho x)$, resp. real $\hat{\Psi}(\hat{a},\hat{c};\hat{x})$ and $\hat{\Psi}(\hat{a},\hat{c};\hat{\rho}\hat{x})$ Tricomi confluent hypergeometric ones of appropriately picked out parameters and variable, are identical, provided it is fulfilled: $c=\hat{c}$, $\rho=\hat{\rho}$ and $n=\hat{n}$, (a — complex; $\hat{a}$, c, $\hat{c}$, $\hat{x}$, ρ and $\hat{\rho}$ — real; ρ and $\hat{\rho}$ — positive, less than unity; x — positive purely imaginary; n — natural and $\hat{n}$ — positive integer; n and $\hat{n}$ — numbers of the zeros). It is indicated that the truthfulness of statement formulated entails the coincidence in case $c=\hat{c}=3$ of certain (envelope) curves in the phase diagrams of the normal TE_{0n} and of the one of the two possible slow $\hat{TE}_{0\hat{n}}$ modes — the $\hat{TE}^{(1)}_{0\hat{n}}$ wave ($n=\hat{n}$) in the coaxial waveguide with ferrite, magnetized azimuthally in negative (clockwise) direction of equations, written in terms of the quantities $L_2(c,\rho,n)$, resp. $\hat{L}_2(\hat{c},\hat{\rho},\hat{n})$. The curves mentioned restrict the areas of propagation of the first (second) mode from the side of higher (lower) frequencies. Besides, the physical sense ascribed to parameter ρ ($\hat{\rho}$) is a relative thickness of the central conductor of the structure and to n ($\hat{n}$) — an order of the propagating mode.

1. INTRODUCTION

Recently, it has been established that if the limited arbitrary real numbers c and $\hat{c}$, the real positive ones ρ and $\hat{\rho}$, ($0<\rho<1$ and $0<\hat{\rho}<1$), and the natural number n and the positive integer $\hat{n}$ are equal, the same holds for the finite real positive numbers $L_2(c,\rho,n)$ and $\hat{L}_2(\hat{c},\hat{\rho},\hat{n})$, defined like the attained under certain conditions common limits of some infnite sequences of real numbers, as well. The terms of the latter are proportional to the positive purely imaginary, resp. real zeros of a complex, resp. real transcendental function, involving four complex, resp. real Kummer and Tricomi confluent hypergeometric functions $\Phi(a,c;x)$ and $\Psi(a,c;x)$ [8] of appropriately chosen parameters and variable [17]. (Hats "ˆ" are put above the symbols, relevant to the second class of numbers [2, 4]. In view of this, the quantities c and $\hat{c}$ are the second parameters of confluent functions, ρ and $\hat{\rho}$ are parameters, multiplying the independent variable of functions $\hat{x}$, resp. $\hat{x}$, while n and $\hat{n}$ are the numbers of the zeros in question of the general function. This assumption has been called Hypothesisi for the identity of $L_2(c,\rho,n)$ and $\hat{L}_2(\hat{c},\hat{\rho},\hat{n})$ numbers [1].

In this investigation an application of the above statement in the theory of azimuthally magnetized coaxial ferrite waveguides [3–5], is considered. It has been found out earlier that there are $En_{1-}-(\hat{E}n_{1-}-)$ envelope curves in the the $\bar{\beta}(\bar{r}_0)-[\bar{\hat{\beta}}^{(1)}(\bar{\hat{r}}^{(1)}_0)-]$ phase diagram of the normal TE_{0n} [3–5] (of the the $\hat{TE}^{(1)}_{0\hat{n}}$ wave which is the one of the two possible slow $\hat{TE}_{0\hat{n}}$ [4]) modes of equation $\bar{\beta}_{en-}=\bar{\beta}_{en-}(\bar{r}_{0en-})[\bar{\hat{\beta}}^{(1)}_{en-}=\bar{\hat{\beta}}^{(1)}_{en-}(\bar{\hat{r}}^{(1)}_{0en-})]$, written in parametric form as: $\bar{r}_{0en-}=L(c,\rho,n)/[|\alpha_{en-}|(1-\alpha^2_{en-})^{1/2}]$, $\bar{\beta}_{en-}=(1-\alpha^2_{en-})^{1/2}(\bar{\hat{r}}^{(1)}_{0en-}=\hat{L}_2(\hat{c},\hat{\rho},\hat{n})/\{|\hat{\alpha}^{(1)}_{en-}|[1-(\hat{\alpha}^{(1)}_{en-})^2]^{1/2}\}$, $\bar{\hat{\beta}}^{(1)}_{en-}=[1-(\hat{\alpha}^{(1)}_{en-})^2]^{1/2})$ with $c=\hat{c}=3$ [3–5]. Here $\bar{\beta}$ and $\bar{r}_0$ ($\bar{\hat{\beta}}^{(1)}$ and $\bar{\hat{r}}^{(1)}_0$) are the normalized in a special way phase constant and guide radius, $\rho(\hat{\rho})$ is the relative thickness of the central switching conductor of the structure, $\alpha(\hat{\alpha})$ is the off-diagonal ferrite Polder permeability tensor, $n(\hat{n})$ — order of the mode. All quantities without (with) hats relate to the normal (slow) modes, those with the superscripts (1) — to the slow $\hat{TE}^{(1)}_{0\hat{n}}$ waves and the ones with the subsript "$en-$" — to the envelopes. (Note that both α and $\hat{\alpha}^{(1)}$ are less than unity.) The $En_{1-}-(\hat{E}n_{1-}-)$ line for specific $n(\hat{n})$ restricts from above (below) the phase characteristics for negative (clockwise) ferrite magnetization. It follows from the Hypothesis for identity of numbers that if $\rho=\hat{\rho}$ and $n=\hat{n}$, the envelopes in question for a TE_{0n} and a $\hat{TE}_{0\hat{n}}^{(1)}$ mode of the same order in two waveguides of the same relative thickness of the inner wire (or in given configuration) coincide. Accordingly, it could be regarded as a line at which the normal mode is transformed to a slow one when the normalized radius (i.e., frequency) grows. Graphical results for different parameters $\rho(\hat{\rho})$ are presented and juxtaposed, assuming $n = \hat{n} = 1$.

2. HYPOTHESIS FOR THE IDENTITY OF THE $L_2(c, \rho, n)$ AND $\hat{L}_2(\hat{c}, \hat{\rho}, \hat{n})$ NUMBERS

Definition 1: *The $L_2(c, \rho, n)$ numbers in which c is a restricted arbitrary real, positive, negative or zero, ρ is real, positive, $0 < \rho < 1$ and n is a natural number, are finite real positive ones, specified in the following way:*

i) In case $c \neq l$, $l = 0, -1, -2, \ldots$:

$$L_2(c, \rho, n) = \lim_{k_- \to -\infty} K_{2-}(c, \rho, n, k_-) = \lim_{k_- \to -\infty} M_{2-}(c, \rho, n, k_-), \tag{1}$$

where $K_{2-}(c, \rho, n, k_-) = |k_-|\,\chi_{k_-,n}^{(c)}(\rho)$, $M_{2-}(c, \rho, n, k_-) = |a_-|\,\chi_{k_-,n}^{(c)}(\rho)$ and $\chi_{k_-,n}^{(c)}(\rho)$ is the nth positive purely imaginary zero of the function $F_2(a, c; x, \rho) = \Phi(a, c; x)\,\Psi(a, c; \rho x) - \Phi(a, c; \rho x)\,\Psi(a, c; x)$ in x, $(n = 1, 2, 3, \ldots)$ in which $\Phi(a, c; x)$ and $\Psi(a, c; x)$ are the Kummer and Tricomi confluent hypergeometric functions with $a = a_-$ $a_- = c/2 - jk_-$ — complex, $c = 2\mathrm{Re}a$ $(c = 2\mathrm{Re}a_-)$, $x = jz$ — positive purely imaginary, z — real, positive, k_- — real, negative, $-\infty < k_- < 0$, ρ — real, positive, $(c$, n — fixed).

ii) On the understanding that $c = l$ and ε is an infinitesimal positive real number:

$$L_2(c, \rho, n) = \lim_{\varepsilon \to 0} L_2(l - \varepsilon, \rho, n) = \lim_{\varepsilon \to 0} L_2(l + \varepsilon, \rho, n) = L_2(2 - l, \rho, n), \tag{2}$$

where $L_2(l - \varepsilon, \rho, n)$ and $L_2(l + \varepsilon, \rho, n)$ are finite real positive quantities, determined in the sense of point i).

iii) When $c = g$, $g = 2 - l (g = 2, 3, 4, \ldots)$:

$$L_2(g, \rho, n) = \lim_{\varepsilon \to 0} L_2(g - \varepsilon, \rho, n) = \lim_{\varepsilon \to 0} L_2(g + \varepsilon, \rho, n). \tag{3}$$

[$L_2(g - \varepsilon, \rho, n)$, $\lim_{\varepsilon \to 0} L_2(g + \varepsilon, \rho, n)$ and ε are specified like in the previous item.]

Besides it is carried out too:

$$L_2(c, \rho, n) = L_2(2 - c, \rho, n), \quad c \neq l, \tag{4}$$

$$L_2(1 + h, \rho, n) = L_2(1 - h, \rho, n), \quad h \neq l, \tag{5}$$

$$L_2(1 + l, \rho, n) = L_2(1 - l, \rho, n), \tag{6}$$

h — real number.

Definition 2: *The $\hat{L}_2(\hat{c}, \hat{\rho}, \hat{n})$ numbers in which $\hat{c}$ is a restricted arbitrary real positive, negative or zero, $\hat{\rho}$ is real, positive, $0 < \hat{\rho} < 1$ and $\hat{n}$ is a natural number are finite positive real ones determined by means of* **Definition 1** *provided:*

i) The meaning of notations is unchanged;

ii) The form of equalities is preserved the same;

iii) Hats are put above all symbols;

iv) All quantities are assumed real;

v) In point i) $\hat{\chi}_{\hat{k}_-,\hat{n}}^{(\hat{c})}(\hat{\rho})$ is the $\hat{n}$th real positive zero of the function $\hat{F}_2(\hat{a}, \hat{c}; \hat{x}, \hat{\rho}) = \hat{\Phi}(\hat{a}, \hat{c}; \hat{x})\hat{\Psi}(\hat{a}, \hat{c}; \hat{\rho}\hat{x}) - \hat{\Phi}(\hat{a}, \hat{c}; \hat{\rho}\hat{x})\hat{\Psi}(\hat{a}, \hat{c}; \hat{x})$ in $\hat{x}$ in which $\hat{\Phi}(\hat{a}, \hat{c}; \hat{x})$ and $\hat{\Psi}(\hat{a}, \hat{c}; \hat{x})$ are the Kummer and Tricomi confluent hypergeometric functions with $\hat{a} = \hat{c}/2 + \hat{k}_-$ — real, negative, $(\hat{a} \neq -\hat{m}$, $\hat{m} = 1, 2, 3, \ldots)$, $\hat{c} > 0$ or $\hat{c} < 0$ (in both cases $\hat{a} < \hat{c}$), $\hat{x}$ — real, positive, $\hat{k}_- = \hat{a} - \hat{c}/2$ — real, negative, $\hat{n} = 1, 2, \ldots, \hat{t}$, $\hat{t}$ — finite positive integer, whose numerical equivalent is determined by the parameters of $\hat{F}_2$ $(\hat{a}_- \equiv \hat{a})$, $(\hat{c}, \hat{n}$ — fixed).

vi) Item iii) might be dropped.

Hypothesis 1: *On condition that $c = \hat{c}$, $\rho = \hat{\rho}$ and $n = \hat{n}$, where c and $\hat{c}$ are restricted arbitrary real, ρ and $\hat{\rho}$ are real, positive, less than unity, n is a natural number and $\hat{n}$ — a positive integer (n and $\hat{n}$ — restricted), the $L_2(c, \rho, n)$ numbers in the sense of* **Definition 1** *and the $\hat{L}_2(\hat{c}, \hat{\rho}, \hat{n})$ ones in that of* **Definition 2**, *are identical. Accordingly, it is fulfilled:*

$$L_2(c, \rho, n) \equiv \hat{L}_2(\hat{c}, \hat{\rho}, \hat{n}). \tag{7}$$

If beside the above assumptions it holds also that $c = \hat{c} \neq l$ ($l = \hat{l}$), $c = g$, $g = 2 - l$ ($g = 2, 3, 4, \ldots$), $k_- = \hat{k}_-$ is large negative and $\varepsilon = \hat{\varepsilon}$ is an infinitesimal positive real number, it is true:

$$\chi_{k_-,n}^{(c)}(\rho) \approx \hat{\chi}_{\hat{k}_-,\hat{n}}^{(\hat{c})}(\hat{\rho}), \tag{8}$$

$$K_{2-}(c, n, \rho, k_-)\, M_{2-}(c, n, \rho, k_-) \approx \hat{K}_{2-}\left(\hat{c}, \hat{n}, \hat{\rho}, \hat{k}_-\right) \approx \hat{M}_{2-}\left(\hat{c}, \hat{n}, \hat{\rho}, \hat{k}_-\right). \tag{9}$$

(The meaning of all tokens is as in **Definitions 1** and **2**.).

3. APPLICATION IN THE THEORY OF WAVEGUIDES

The propagation of normal TE_{0n} (slow $\hat{TE}_{0\hat{n}}$) modes of phase constant $\beta(\hat{\beta})$ in the coaxial waveguide of outer and inner conductor radii r_0 and r_1, resp., uniformly filled with azimuthally magnetized lossless remanent ferrite, described by a Polder permeability tensor $\overleftrightarrow{\mu} = \mu_0[\mu_{ij}]$, $i, j = 1, 2, 3$, $\mu_{12} = \mu_{21} = \mu_{23} = \mu_{32} = 0$, $\mu_{ii} = 1$, $\mu_{13} = -\mu_{31} = -j\alpha$, $\alpha = \gamma M_r/\omega$, ($\gamma$ — gyromagnetic ratio, M_r — remanent magnetization, ω — angular frequency of the wave), and a scalar permittivity $\varepsilon = \varepsilon_0\varepsilon_r$, is governed by the equations:

$$\Phi(a, c; x_0)/\Psi(a, c; x_0) = \Phi(a, c; \rho x_0)/\Psi(a, c; \rho x_0), \tag{10}$$

$$\hat{\Phi}(\hat{a}, \hat{c}; \hat{x}_0)/\hat{\Psi}(\hat{a}, \hat{c}; \hat{x}_0) = \hat{\Phi}(\hat{a}, \hat{c}; \hat{\rho}\hat{x}_0)/\hat{\Psi}(\hat{a}, \hat{c}; \hat{\rho}\hat{x}_0). \tag{11}$$

In Eq. (10) $a = c/2 - jk$, $c = 3$, $x_0 = jz_0$, $k = \alpha\bar{\beta}/\left(2\bar{\beta}_2\right)$, $z_0 = 2\bar{\beta}_2\bar{r}_0$, $\rho = \bar{r}_0/\bar{r}_1$ (k, z_0, ρ — real, $-\infty < k < +\infty$, $0 < |\alpha| < 1$, $z_0 > 0$, $0 < \rho < 1$). It is satisfied, if $\bar{\beta}_2 = \chi_{k,n}^{(c)}(\rho)/(2\bar{r}_0)$ that determines the eigenvalue spectrum of the fields studied. The barred (normalized) quantities are introduced through the relations: $\bar{\beta} = \beta/\left(\beta_0\sqrt{\varepsilon_r}\right)$, $\bar{\beta}_2 = \beta_2/\left(\beta_0\sqrt{\varepsilon_r}\right)$, $\bar{r}_0 = \beta_0 r_0\sqrt{\varepsilon_r}$, $\bar{r}_1 = \beta_0 r_1\sqrt{\varepsilon_r}$, resp. where $\beta_0 = \omega\sqrt{\varepsilon_0\mu_0}$, $\{\bar{\beta}_2 = \left[\omega^2\varepsilon_0\mu_0\varepsilon_r\left(1 - \alpha^2\right) - \beta^2\right]^{1/2}$ — radial wavenumber$\}$. Figs. 1(a)–(c) and 2(a)–(c) portray the region (featured by light green) in which the normal TE_{01} wave may be sustained for positive ($\alpha_+ > 0$) and negative ($\alpha_- < 0$) ferrite magnetization in case $\rho = 0$, 0.05 and 0.1, resp. The relevant normalized $\bar{\beta}(\bar{r}_0)$ — phase characteristics with α as parameter, counted, employing the scheme, worked out earlier [1], are also shown in these pictures by solid and dashed lines, resp. In case $\alpha_+ > 0$, the aforesaid region is bounded from the side of lower frequencies by the $\alpha = 0$ solid curve, concurring to a dielectric-loading and is termless beyond the frequency range above. When $\alpha_- < 0$, however, it is bilaterally limited. Its lower border is more complex and as an upper one serves the En_{1-} — envelope dotted line of equation of equation $\bar{\beta}_{en-} = \bar{\beta}_{en-}(\bar{r}_{0en-})$, written in parametric form as: $\bar{r}_{0en-} = L_2(c, \rho, n)/[|\alpha_{en-}|(1 - \alpha_{en-}^2)^{1/2}]$, $\bar{\beta}_{en-} = (1 - \alpha_{en-}^2)^{1/2}$ (α_{en-} is a parameter).

In Eq. (11) $\hat{a} = \hat{c}/2 + \hat{k}$, $\hat{c} = 3$, $\hat{x}_0 = 2\hat{\beta}_2\hat{r}_0$, $\hat{k} = \hat{\alpha}\bar{\hat{\beta}}/(2\bar{\hat{\beta}}_2)$, $\hat{\rho} = \bar{\hat{r}}_0/\bar{\hat{r}}_1$, $\bar{\hat{\beta}} = \hat{\beta}/(\beta_0\sqrt{\varepsilon_r})$, $\bar{\hat{\beta}}_2 = \hat{\beta}_2/(\beta_0\sqrt{\varepsilon_r})$, $\bar{\hat{r}}_0 = \beta_0\hat{r}_0\sqrt{\varepsilon_r}$ and $\bar{\hat{r}}_1 = \beta_0\hat{r}_1\sqrt{\varepsilon_r}$, $\{\hat{\beta}_2 = [\hat{\beta}^2 - \omega^2\varepsilon_0\mu_0\varepsilon_r(1 - \hat{\alpha}^2)]^{1/2}$ — radial wavenumber$\}$. All quantities relevant to the slow waves are real and are distinguished by hats "^". They are transmitted, provided $\bar{\hat{\beta}}_2 = \hat{\chi}_{\hat{k}_-,\hat{n}}^{(\hat{c})}(\hat{\rho})/(2\bar{\hat{r}}_0)$, assuming $\hat{k}_- < \hat{k}_{-th}(\hat{\rho})$, where $\hat{k}_{-th}(\hat{\rho}) < 0$ is certain threshold value of parameter $\hat{k}_-$, depending on $\hat{\rho}$, i.e., for $\hat{\alpha}_- < 0$ only [5]. The $\bar{\hat{\beta}}^{(1)}(\bar{\hat{r}}_0^{(1)})$ — dashed phase curves of the $\hat{TE}_{0\hat{n}}^{(1)}$ mode (the one of the two possible $\hat{TE}_{0\hat{n}}$ waves — $\hat{TE}_{0\hat{n}}^{(1)}$ and $\hat{TE}_{0\hat{n}}^{(2)}$ [4]), supported for $|\hat{\alpha}_-^{(1)}| < 1$ are depicted in Figs. 2(a)–(c). They have a lower limit — the $\hat{E}n_{1-}$ — envelope of equation $\bar{\hat{\beta}}_{en-}^{(1)} = \bar{\hat{\beta}}_{en-}^{(1)}(\bar{\hat{r}}_{0en-}^{(1)})]$, presented parametrically like: $\bar{\hat{r}}_{0en-}^{(1)} = \hat{L}_2(\hat{c}, \hat{\rho}, \hat{n})/\{|\hat{\alpha}_{en-}^{(1)}|[1 - (\hat{\alpha}_{en-}^{(1)})^2]^{1/2}\}$, $\bar{\hat{\beta}}_{en-}^{(1)} = [1 - (\hat{\alpha}_{en-}^{(1)})^2]^{1/2}$ ($\hat{\alpha}_{en-}^{(1)}$ is a parameter). The domain in that the slow wave in question might get excited (presented by dark green) is restricted from below and unlimited from above. (Throughout the paper the subscripts "+" ("−") and "*en*−"

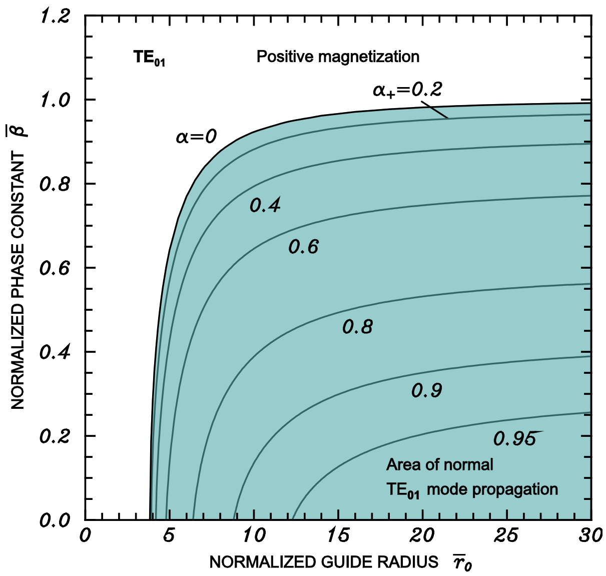

Figure 1: Area of normal TE_{01} mode propagation in case of positive magnetization of the azimuthally magnetized circular ferrite waveguide.

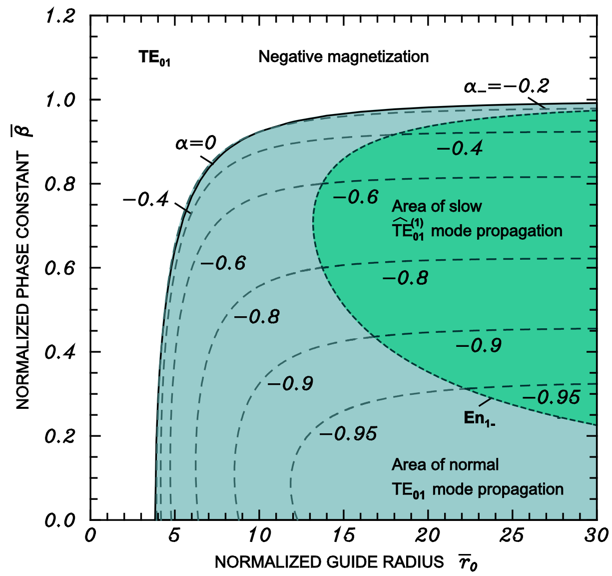

Figure 2: Areas of normal TE_{01} and slow $T\hat{E}_{01}^{(1)}$ mode propagation in case of negative magnetization of the azimuthally magnetized circular ferrite waveguide.

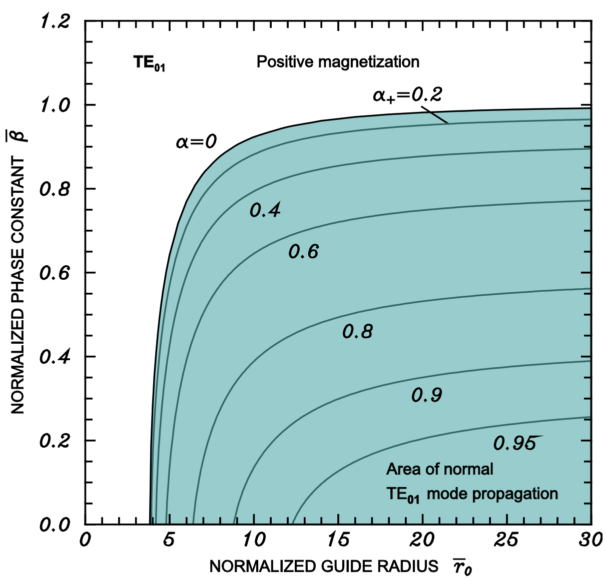

Figure 3: Area of normal TE_{01} mode propagation in case of positive magnetization of the azimuthally magnetized coaxial ferrite waveguide, $\rho = 0.05$.

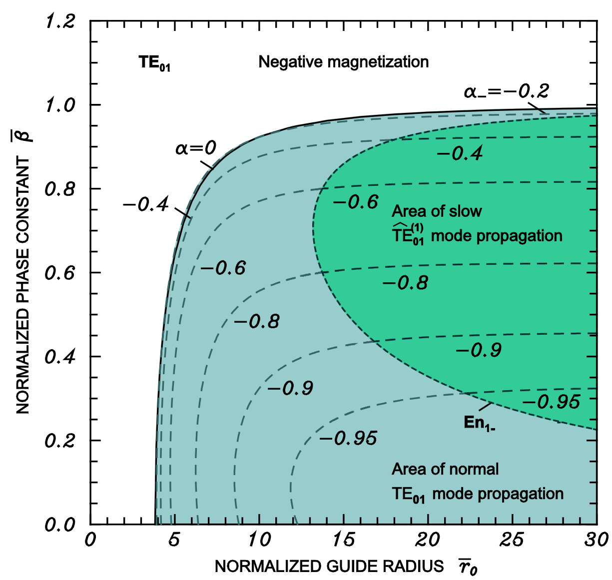

Figure 4: Areas of normal TE_{01} and slow $T\hat{E}_{01}^{(1)}$ mode propagation in case of negative magnetization of the azimuthally magnetized coaxial ferrite waveguide, $\rho = 0.05$.

distinguish the quantities, relevant to positive (negative) magnetization, resp. to the envelopes and the superscript (1) — marks those, having reference to the $T\hat{E}_{0\hat{n}}^{(1)}$ mode.). Since $c = \hat{c} = 3$, with regard to Hypothesis 1 (Eq. (7)), it follows that if $\alpha_{en-} \equiv \hat{\alpha}_{en-}^{(1)}$ and $n = \hat{n}$, then $\bar{r}_{0en-} \equiv \bar{\hat{r}}_{0en-}^{(1)}$ and $\bar{\beta}_{en-} \equiv \bar{\hat{\beta}}_{en-}^{(1)}$, i.e., the $En_{1-}-$ and $\hat{E}n_{1-}-$ curves coincide. Thus, in case of negative magnetization the $En_{1-} - (\hat{E}n_{1-}-)$ line separates the zone of propagation in two sub-domains: left (light green) and right (dark green) in which a normal TE_{0n} and slow $T\hat{E}_{0\hat{n}}^{(1)}$ mode, resp. might be observed. Besides, it is connected with the upper boundary of the region of phase shifter operation of the confguration for the first set of fields [5].

As seen the increase of ρ shifts the envelope(s) to the right. Thus, the area of normal TE_{0n}

mode propagation expands (of the slow $\hat{TE}_{0\hat{n}}^{(1)}$ waves shrinks). Thus, the first (second) ones might be transmitted (might get excited) at higher frequencies when the central conductor becomes thicker.

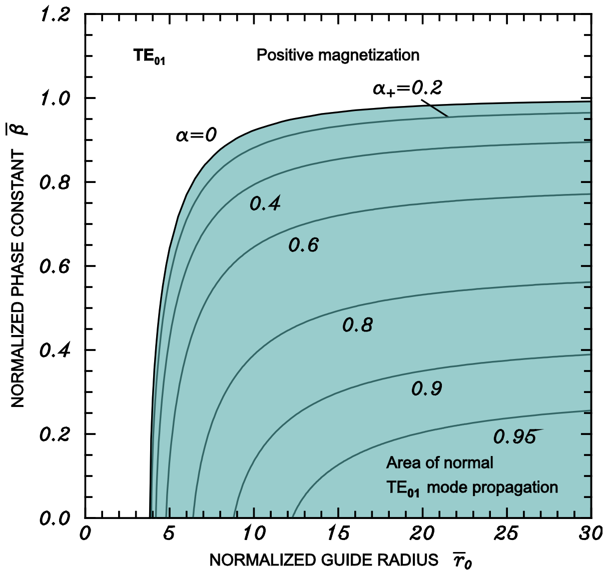

Figure 5: Area of normal TE_{01} mode propagation in case of positive magnetization of the azimuthally magnetized coaxial ferrite waveguide, $\rho = 0.1$.

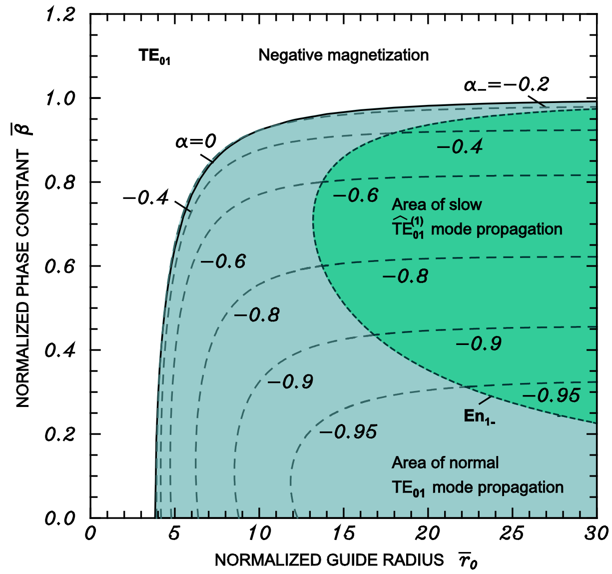

Figure 6: Areas of normal TE_{01} and slow $\hat{TE}_{01}^{(1)}$ mode propagation in case of negative magnetization of the azimuthally magnetized coaxial ferrite waveguide, $\rho = 0.1$.

4. THEOREM FOR THE IDENTITY OF THE $L_2(c, \rho, n)$ AND $\hat{L}_2(\hat{c}, \hat{\rho}, \hat{n})$ NUMBERS

In case $\rho = 0$ (circular waveguide, entirely filled with azimuthally magnetized ferrite) the relevant $En_{1-}-$ and $\hat{E}n_{1-}-$ curves of parametric equations of the form: $\bar{r}_{0en-} = L(c,n)/[|\alpha_{en-}|(1-\alpha_{en-}^2)^{1/2}]$, $\bar{\beta}_{en-} = (1-\alpha_{en-}^2)^{1/2}$ and $\bar{\hat{r}}_{0en-}^{(1)} = \hat{L}(\hat{c},\hat{n})/\{|\hat{\alpha}_{en-}^{(1)}|[1-(\hat{\alpha}_{en-}^{(1)})^2]^{1/2}\}$, $\bar{\hat{\beta}}_{en-}^{(1)} = [1-(\hat{\alpha}_{en-}^{(1)})^2]^{1/2}$, resp are completely indistinguishable. This is a corollary of Theorem 1 [3] for the identity of $L(c,n)$ and $\hat{L}(\hat{c},\hat{n})$ numbers (denoted recently also as $L_1(c,n)$ and $\hat{L}_1(\hat{c},\hat{n})$ ones [5]).

The exact numerical analysis has shown that for some values of parameters $c = \hat{c}$, $\rho = \hat{\rho}$ and $n = \hat{n}$ there is a slight deviation between the calculated outcomes for $L_2(c,\rho,n)$ and $\hat{L}_2(\hat{c},\hat{\rho},\hat{n})$. In particular this is observed in case $c = \hat{c} = 3$, corresponding to the propagation problem considered [5]. Accordingly, the resulting $En_{1-}-$ and $\hat{E}n_{1-}-$ curves should not coincide completely.

This would lead to the appearance of a gap between the existence areas of the normal and slow waves in which no propagation at all is possible or to the springing up of a region in which the areas in question would overlap, i.e., in which a simultaneous propagation of slow and normal wave could be observed for in the same band.

Such effects, however, are not observed in the circular waveguide ($\rho = 0$) which is a partial case of the coaxial structure ($\rho > 0$). There are no physical reasons which could lead to their appearance when the finite central conductor is inserted. Thus, the deviation from the exact coincidence of mathematical outcomes of statement (7) should be ascribed to computational reasons only.

This could be accepted as a physical substantiation of the truthfulness of relation (7). Therefore the Hypothesis for identity of $L_2(c,\rho,n)$ and $\hat{L}_2(\hat{c},\hat{\rho},\hat{n})$ numbers could be reformulated as a Theorem for the identity of $L_2(c,\rho,n)$ and $\hat{L}_2(\hat{c},\hat{\rho},\hat{n})$ numbers.

5. CONCLUSION

It is demonstrated numerically that the positive real numbers $L(c;n)$ and $\hat{L}(\hat{c};\hat{n})$ (the limits of specially constructed sequences of numbers with terms, involving the zeros of certain complex, resp. real Kummer functions) concur, if their parameters c and $\hat{c}$ (arbitrary real), and n and $\hat{n}$ (restricted natural numbers), are the same. It is shown that the peculiarities of the rotationally symmetric TE modes transmission in the circular waveguide, entirely filled with azimuthally magnetized ferrite are directly linked with this mathematical outcome. From a physical point of view its validity when $c = \hat{c} = 3$ for each $n = \hat{n}$ means existence of a curve in the phase portrait of geometry for negative magnetization of the load at which the propagating mode is transformed from normal to slow.

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