A Novel Mutual Coupling Matrix Monitoring Method in Two Dimensional Rectangle Antenna Array

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Abstract— In this paper, mutual coupling in two-dimensional rectangle antenna array is investigated. It is discovered that the mutual coupling matrix for such an antenna array can be decomposed into the product of a unitary matrix, a diagonal matrix and the Hermitian transpose of the unitary matrix. If the coupling between the non-adjacent elements is ignored, the unitary matrix will be the two dimensional discrete sine transform matrix and the diagonal matrix can be evaluated analytically. When the external excitation voltage of the array is set to be eigen vectors of the mutual coupling matrix, the resulting voltages on the array elements will also be the eigen vector of the mutual coupling matrix, except for a scaling factor. By analyzing the scaling factor, the coupling between the adjacent elements can be calculated.

1. INTRODUCTION

Mutual coupling is caused by the electro-magnetic interactions between the antenna elements inside the array. It will greatly impact the performance of the beamforming algorithms and the directional finding algorithms [1, 2].

It is generally accepted that the mutual coupling effect mainly exists between the closely arranged antenna elements. To monitor the coupling effect, one should inject independent excitation voltage vectors and measure the voltages on all of the antenna elements. Furthermore, a matrix inversion is necessary during the final calculation of the coupling coefficients. In [3], a calibration approach is proposed by using known sources from certain direction. This is, however, not practical for the real time monitoring of the mutual coupling matrix.

In this work, a novel method to characterize the mutual coupling matrix of the rectangle antenna array is proposed. The coupling matrix of the array can be decomposed into the product of the discrete sine transform matrix, a diagonal matrix and the transpose of the discrete sine transform matrix provided that only the coupling between the neighboring elements in the array is considered.

Based on this fact, the measuring of the mutual coupling between the neighboring elements can be accomplished by monitoring only one element within the array. It is accomplished as follows: the excitation voltage vector of the array is set to be the eigen vector of the coupling matrix, i.e., the row vector of the discrete sine transform matrix. It can be derived that the voltages on the antenna terminals will also be the row vector of the discrete sine transform matrix, albeit with a constant, which relates to the mutual coupling effect. By altering the voltage with respect to the row vectors of the discrete sine transform matrix and measuring the ratio of the excitation voltage over the load voltage on the monitoring antenna element, one is able to determine the mutual coupling coefficients.

2. MATHEMATICAL MODEL FOR THE MUTUAL COUPLING MATRIX IN A LINEAR ANTENNA ARRAY

The mutual coupling matrix for a linear antenna array can be formulated as

$$\mathbf{Z} = \begin{pmatrix} Z_1 & Z_2 & \cdots & 0 \\ Z_2 & Z_1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & Z_2 \\ 0 & \cdots & Z_2 & Z_1 \end{pmatrix} \tag{1}$$

where Z_1 is the self impedance, Z_2 the mutual impedance resulting from coupling between neighboring elements, while all the coupling for non-neighboring elements is neglected.

$\mathbf{Z}$ can be decomposed by [4]

$$\mathbf{Z} = \mathbf{QDQ}^H \tag{2}$$

where $\mathbf{Q}$ is the discrete sine transform (DST) matrix and $\mathbf{Q}^H$ is its Hermitian transpose. Matrix $\mathbf{Q}$ has the elements as [4]

$$Q_{mn} = \sqrt{\frac{2}{N}} \sin\left(\frac{\pi mn}{N+1}\right) \tag{3}$$

The diagonal elements of matrix $\mathbf{D}$ can be formulated as [4]

$$D_{nn} = Z_1 + 2Z_2 \cos\left(\frac{n\pi}{N+1}\right) \tag{4}$$

The antenna voltage vector and the antenna current vector are related by [5]

$$\mathbf{U} = \mathbf{ZI} \tag{5}$$

The external excitation voltage $\mathbf{U}_E$ is applied to the antenna elements. However it is not directly proportionally related to the antenna voltage $\mathbf{U}$, the actual relationship is [5]

$$\mathbf{U} = \mathbf{U}_E - \mathbf{Z}_L\mathbf{I} \tag{6}$$

where $\mathbf{Z}_L$ is the load impedance matrix, which is diagonal when the load impedances are the same for every antenna elements. Hence, we have [5]

$$\mathbf{U} = \mathbf{Z}\left(\mathbf{Z} + \mathbf{Z}_L\right)^{-1} \mathbf{U}_E \tag{7}$$

Combining with Eqs. (3)–(5), we have [5]

$$\mathbf{U} = \mathbf{QGQ}^H\mathbf{U}_E \tag{8}$$

where G is a diagonal matrix with the diagonal elements of [5]

$$G_{nn} = \frac{D_{nn}}{D_{nn} + Z_L} \tag{9}$$

Therefore, when the excitation voltage of the antenna elements are assumed to be the row vectors of matrix $\mathbf{Q}$, the real voltage that is applied to the antenna elements will also be the row vectors of matrix $\mathbf{Q}$, albeit with the factor of G_{nn}. By measuring the ratio of the real voltage and the excitation voltage on only one antenna element, the value of G_{nn} can be obtained. With two different values of G_{nn}, one is able to determine the final value of Z_1 and Z_2.

3. EXTENSION OF THE CONCLUSION TO THE TWO-DIMENSIONAL RECTANGLE ANTENNA ARRAY

When the antenna array is extended from the one dimensional linear array to the two dimensional rectangle array, the matrix $\mathbf{Z}$ can be formulated by

$$\mathbf{Z} = \left(\mathbf{Q} \otimes \mathbf{Q}\right) \mathbf{E} \left(\mathbf{Q} \otimes \mathbf{Q}\right)^T \tag{10}$$

where $\otimes$ denotes the Kronecker product. The matrix Q is the same matrix as the one in the one dimensional array. The diagonal matrix E has its diagonal elements as

$$E_{nN+m,nN+m} = Z_1 + 2Z_{2x} \cos\left(\frac{n\pi}{N+1}\right) + 2Z_{2y} \cos\left(\frac{m\pi}{N+1}\right) \tag{11}$$

There are three unknown variables in the formula, namely Z_1 Z_{2x} and Z_{2y}, which are related to the self impedance and the mutual impedance between the two adjacent elements in the x and y directions. The impedances can also be obtained by measuring the ratio of the external excitation voltage over the real antenna element voltage while setting the excitation voltage vector to be the row vector of

$$\mathbf{Q} \otimes \mathbf{Q}$$

In such a configuration, measuring of only one elements' excitation voltage and real antenna voltage is adequate. By measuring the ratio three times and setting up the three linear equations, one is able to calculate Z_1 Z_{2x} and Z_{2y}.

4. CONCLUSION

We have proposed a novel method to monitor the mutual coupling matrix in linear and rectangle antenna arrays. The method is based on the analysis of the mutual coupling without considering the non-adjacent coupling. The method is useful for the application in massive MIMO systems.

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