Optimization of Nanostructured Lüneburg Lens Based on the Transformation Optics Method

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Abstract— In this work, we present the design of the nanostructured Lüneburg lens by using the electromagnetic field simulation and gradient optimization method that based on Transformation Optics (TO) technique. In the proposed method, the nanostructured Lüneburg lens is simulated with an equivalent model, in which the original dielectric rods with varying size are transformed into uniform and fixed sized rods, with parameterized permittivity and permeability distributions, and then the nanostructured Lüneburg lens is optimized with the gradient optimization method. By using the TO modeling method, mesh refinement around the smaller rods can be avoided, and repetitions of regenerating the geometry and mesh grids during the optimization iterations can be eliminated, thus the computation efficiency is improved. The proposed design method opens up a new avenue to design and optimize a variety of photonic and optical devices with large number of complex structures.

1. INTRODUCTION

Lüneburg lens [1] is a spherical lens with gradient refractive index (GRIN) structure, it can focus incoming plane wave from any direction into a focal point at the surface of the lens on the opposite direction. However, as a GRIN device, Lüneburg lens is difficult to be fabricated at optical wavelengths. In recent years, nanostructured Lüneburg lens [2–4] that composed of Graded Photonic Crystals (GPC) has been proposed. The nanostructured Lüneburg lens can be designed using the homogenization method of periodic structures [2], the ray tracing method [3], and the effective medium theory [4]. In our previous work [5], we used the electromagnetic field simulation and optimization method that based on Transformation Optics (TO) [6, 7] to design Photonic Crystal (PhC) devices, including the PhC coupler and PhC waveguide bend that consists of dielectric rods with varying sizes. In this work, we use the TO-based simulation and optimization method to design the nanostructured Lüneburg lens, and compare the optimized nanostructured Lüneburg lens with the GRIN Lüneburg lens.

2. DESIGN OF NANOSTRUCTURED LÜNEBURG LENS

Figure 1(a) shows a nanostructured Lüneburg lens which contains 95 dielectric rods in circular lattice. The radius of the lens is $R = 6a$, where a is the lattice constant. In this work, we set the lattice constant as $a = 0.25\,\mu\text{m}$. The nanostructured Lüneburg lens is modeled with the TO method, namely, the original dielectric rods with varying sizes are transformed into an equivalent model that consists of uniform and fixed sized rods, with parameterized permittivity and permeability distributions [5]. The permittivity and permeability of the transformed model can be calculated with the TO formulae. By using the TO modeling method, mesh refinement around the smaller rods can be avoided, and repetitions of regenerating the geometry and mesh grids during the optimization iterations can be eliminated, thus the computation efficiency is improved [5, 8].

The nanostructured Lüneburg lens is optimized with a gradient optimization package minConf [9], The optimization object is defined as the average electric field intensity around the focal point as

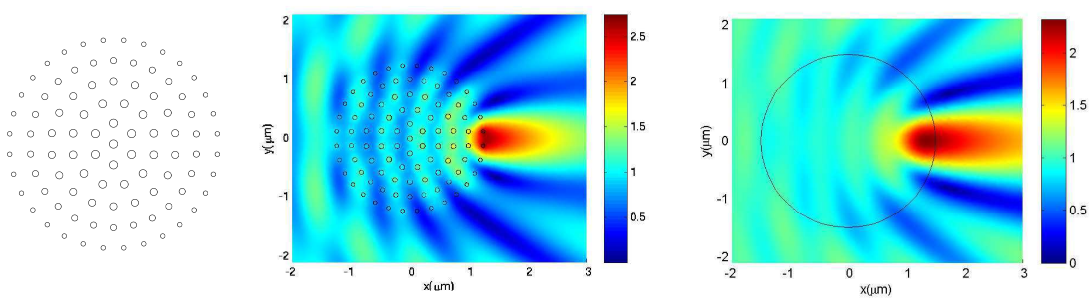
$$E_{obj} = \frac{\iint_{\Sigma} |\mathbf{E}|\,\mathrm{d}x\,\mathrm{d}y}{\iint_{\Sigma} \mathrm{d}x\,\mathrm{d}y} \tag{1}$$

where Σ is a small "observation" zone that enclosing the focal point. For nanostructured Lüneburg lens in Fig. 1(a), the "observation" zone Σ is defined as a rounded region of radius a, with its center at $(x, y) = (6a, 0)$. The wavelength of incident light is set as $\lambda = 1.55\,\mu\text{m}$, and the refractive index of the dielectric rod is $n_{rod} = 3.4$. To ensure the symmetry of the optimized nanostructured Lüneburg lens, the gradient of rods at the same distance from the center of the lens are averaged. In addition, to mimic the gradual change of refractive index of the GRIN material, a constraint is applied in the optimization that the sizes of the inner rods of the nanostructured lens should be larger than the outer rods. At the begining of the optimization process, the initial rods radii are set as

$$r_j = r_0 \left(1 - \alpha \frac{R_j}{R}\right) \tag{2}$$

where r_j is the initial radius of the jth rod, r_0 is the initial radius of the rod at lens center, α is a parameter to control the inital size of rods, R_j is the distance of the jth rod from the lens center, R is the radius the lens. For nanostructured Lüneburg lens of $R = 6a(\sim \lambda)$, we set the initial value parameter as $r_0 = 0.16a$ and $\alpha = 0.5$. The incident light is set as a plane wave propagating along x axis, polarized along z axis. After 21 steps of optimization iterations, the average electric field intensity around the focal point converges to $E_{obj} = 2.432$, the program running time is 857.8 seconds on a computer with Xeon E5645 CPU (2.4 GHz). The optimized structure and electric field distribution is shown in Fig. 1(b).

As a comparison, the GRIN Lüneburg lens of radius $R = 1.5\,\mu\text{m}$ is simulated, the calculated field distribution is shown in Fig. 1(c). The averaged field intensity around focal point is 2.238, which is a bit lower than the optimized nanostructured Lüneburg lens. Comparing Figs. 1(b) and 1(c), the focal spot of the GRIN Lüneburg lens is a bit larger than the optimized nanostructured Lüneburg lens.



(a) (b) (c)

Figure 1: (a) Schematic diagram of the nanostructured Lüneburg lenses in circular lattice, with lens radius of $R = 6a$. (b) The structure and field distribution of the optimized nanostructured Lüneburg lenses. (c) The field distribution of GRIN Lüneburg lens of $R = 1.5\,\mu\text{m}$.

3. CONCLUSIONS

In this work, the design of the nanostructured Lüneburg lens using the TO-base simulation and optimization method is presented. The proposed TO-base simulation and optimization method is an effective and efficient method for graded photonic crystal devices, it opens up a new avenue to design and optimize a variety of photonic and optical devices with large number of complex structures.

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