Optimization of Nonlinear Coefficient Map in Back-propagation

Yanru Cao and Junhe Zhou
Department of Electronics Science and Engineering, Tongji University, Shanghai 200092, China

Abstract— In this paper, an optimized back-propagation algorithm for nonlinear compensation is proposed. Two parameters are proposed to control the nonlinear phase shift in BP algorithm. The parameters alter the nonlinear phase shift linearly with respect to the span number and are optimized in different dispersion maps.

1. INTRODUCTION

In optical communication systems, chromatic dispersion (CD) and fiber nonlinearities are the main effects that limit the system performance. Compensation of these effects is essential to realize high speed and ultra-long distance fiber transmission.

Back-propagation (BP) algorithm is one of the most extensively used methods which can jointly compensate the chromatic dispersion and the fiber nonlinearities. The algorithm can virtually inverse the fiber transmission by adopting the digital signal processing (DSP) technique. During the application of the algorithm, the dispersion and nonlinearity interplay with each other. The amount of the dispersion should be exactly compensated, while the nonlinearity should be partially compensated [1]. The nonlinear coefficient used in the BP algorithm should be between 0 and the actual fiber nonlinear coefficient. It was further proposed in [2] that the nonlinear parameter used in the simulation can be optimized with respect to the fiber locations in the span, and this optimal nonlinear compensation map should change with respect to dispersion. However, no systematic approaches have been proposed to optimize the nonlinear compensation map in the BP algorithm.

In this paper, we propose to use two parameters to control the value of the nonlinear coefficient map. The nonlinear coefficient in the BP algorithm changes linearly with respect to the span number and is evaluated as $an + b$, where a and b are two constants to be optimized, n the span number. Numerical simulation shows that by adopting this optimized BP algorithm, the BER performance can be greatly improved.

2. BACK-PROPAGATION THEORY

The evolution of the signal envelope in the optic fiber link is described by the well-known nonlinear Schrodinger equation (NLSE) [3]:

$$\frac{\partial A}{\partial z} = \left(\hat{D} + \hat{N}\right) A \tag{1}$$

$$\hat{D} = -\frac{i\beta_2}{2}\frac{\partial^2}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3}{\partial T^3} - \frac{\alpha}{2}, \quad \hat{N} = i\gamma\left|A\right|^2 \tag{2}$$

where $\hat{D}$ is the linear operator which accounts for the fiber dispersion and attenuation, $\hat{N}$is the nonlinear operator which represents the effect of fiber nonlinearities. α,β_2, β_3 and γ are the fiber attenuation coefficient, group velocity dispersion coefficient, dispersion slope, and nonlinear coefficient, respectively.

Based on the theory of BP, the inverse process of fiber transmission is realized by solving the inversed-NLSE [1]

$$\frac{\partial A}{\partial z} = \left(-\hat{D} - \hat{N}\right) A \tag{3}$$

This equation can be numerically solved by using the split step Fourier method (SSFM) which divides the whole fiber length into a large number of segments and calculates the signal envelope step by step [3, 4]

$$A_{BP}(z,t) \approx \exp\left(-h\hat{D}\right)\exp(-hN(z+h))A_{BP}(z+h,t) \tag{4}$$

where h is the step size. While in the presence of noise, the optical signal can't be exactly recovered. It has been demonstrated that by partially compensate the nonlinear section, the system performance can be improved [1]. The equation above thus can be modified as

$$A_{BP}(z,t) \approx \exp(-h\hat{D})\exp(-h\xi N(z+h))A_{BP}(z+h,t) \tag{5}$$

where $0 \leq \xi \leq 1$ is the nonlinear compensation coefficient which represents the fraction of the nonlinearity compensated.

The length of the step size can be altered according to the computational capacity. In the case of the real time applications, the step size h is usually set to be the span length of the transmission system.

3. OPTIMIZED BACK-PROPAGATION ALGORITHM

During the fiber transmission, chromatic dispersion and fiber nonlinearities interact with each other. As the dispersion accumulates along the optic fiber link, the influence of the dispersion on fiber nonlinearity will vary along the fiber link accordingly. So the nonlinear coefficient map should be optimized with respect to the dispersion map and the fiber location. For the receiver side BP, the accuracy of the signal reduces as it traces back to the transmitter, so ξ should be larger for the spans nearer to the receiver and be smaller for the spans further away [2]. Without loss of generality, we consider the case that ξ changes linearly with the back-propagation span number n $(1 \leq n \leq N)$

$$\xi = an + b \tag{6}$$

where a and b are two constants to be optimized. Since ξ is preferred to be located between 0 and 1, in order to control the range of ξ during the simulation, we use the value of ξ at the starting point ξ_1 and the ending point ξ_N of the transmission link to control the variation of ξ

$$\frac{\xi - \xi_1}{\xi_N - \xi_1} = \frac{n-1}{N-1} \tag{7}$$

$$a = \frac{\xi_N - \xi_1}{N-1}, \quad b = \xi_1 - \frac{\xi_N - \xi_1}{N-1} \tag{8}$$

where ξ_1 and ξ_N are the values of ξ at span 1 and span N respectively, and satisfy the condition $\min(\xi_1, \xi_N) \leq \xi \leq \max(\xi_1, \xi_N)$.

4. NUMERICAL SIMULATION

In this section we compare the performance of the BP algorithm with constant nonlinear coefficient and the proposed optimized BP algorithm with linearly changed nonlinear coefficient for the optical transmission system shown in Fig. 1.

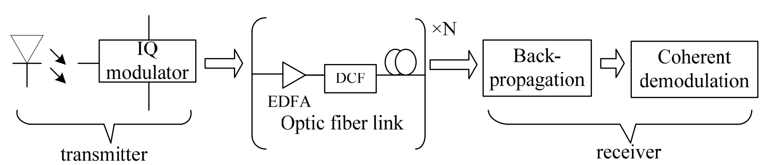

Figure 1: Block diagram of optical transmission system with BP.

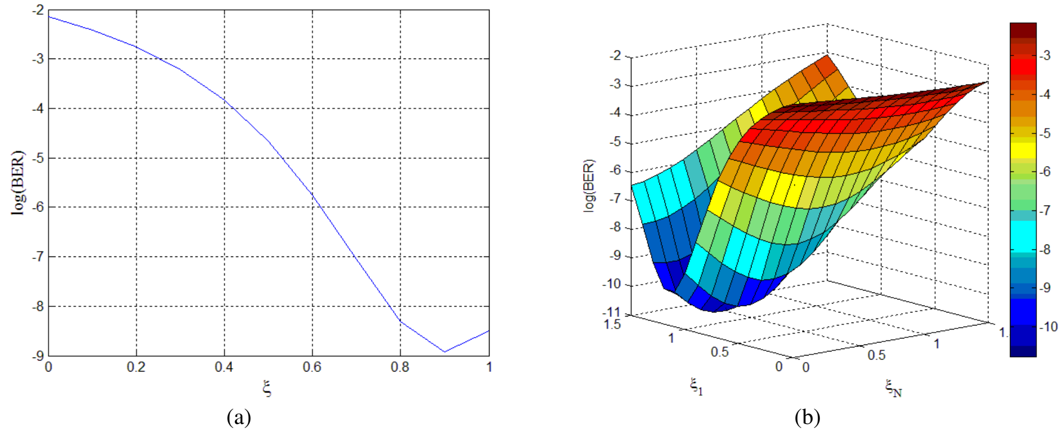

Figure 2: (a) BER vs. ξ obtained in BP algorithm with constant phase shift parameters and (b) BER vs. ξ_1 and ξ_N obtained in BP algorithm with linearly varying phase shift parameters.

In the simulation, 20 Gb/s QPSK signals are transmitted through 20 fiber spans. The fiber is the standard single mode fiber (SMF) whose α, β_2 and γ are 0.2 dB/km, $-20\,\mathrm{ps}^2/\mathrm{km}$ and $2 \times 10^{-3}\,\mathrm{km}^{-1} \cdot \mathrm{W}^{-1}$ respectively. The span length is 100 km. In each fiber span, attenuation is fully compensated by an erbium doped fiber amplifier (EDFA), CD is 90% undercompensated by the dispersion compensation fiber (DCF) and the residual dispersion is 10%. At the receiver side BP algorithm is applied to compensate the nonlinearity and the residual CD. The signals after compensation are demodulated by the coherent QPSK detector. The bit error rate (BER) is calculated depending on the eye pattern of the received signal. The BER performance of the BP algorithm versus the nonlinear phase shift parameters is shown in Fig. 2.

In the BP with constant phase shift parameters, the optimal ξ is 0.9 and the minimum BER is 1.18×10^{-9}. While in BP algorithm with linearly varying phase shift parameters, the minimum BER obtained is 1.70×10^{-11} with the optimal ξ_1 and ξ_N to be 1.1 and 0.4. The BER has been reduced about 18 dB as compared to the BP algorithm with constant phase shift parameter. The constellation diagrams of the received signals without and with the nonlinear compensation are shown in Fig. 3. The comparison of the constellations demonstrates that by using the optimized BP algorithm, the system performance is greatly improved.

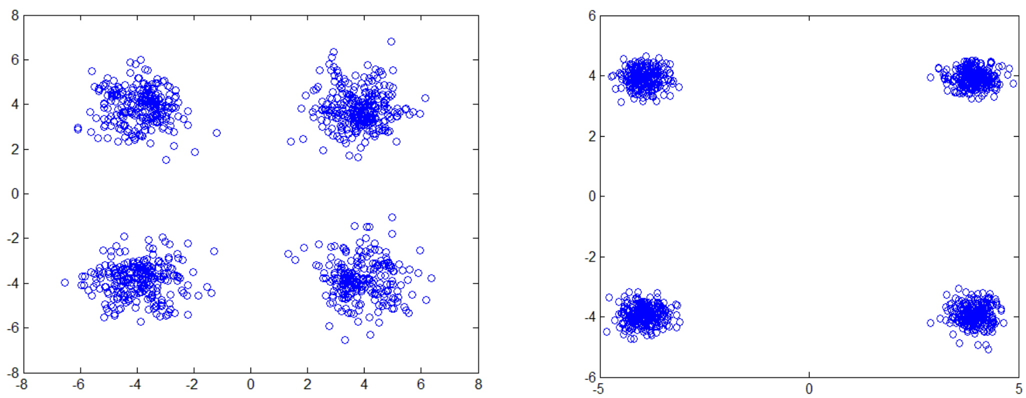

Figure 3: (a) Constellation obtained without nonlinear compensation and (b) constellation obtained with optimized BP algorithm when $\xi_1 = 1.1$, $\xi_N = 0.4$.

The performance of the systems with different values of dispersion compensation map is shown in Table 1 and Table 2.

The results show that the optimal nonlinear coefficient map strongly depends on the dispersion

Table 1: The performance of the system using BP algorithm with constant phase shift parameters.

Residual CD per span	Optimal ξ	BER
100%	0.5	3.02×10^{-6}
80%	0.5	1.65×10^{-6}
60%	0.5	1.49×10^{-6}
40%	0.6	5.98×10^{-7}
20%	0.8	7.13×10^{-9}

Table 2: The performance of the system using BP algorithm with linearly varying phase shift parameters.

Residual CD per span	Optimal ξ_1	Optimal ξ_N	BER
100%	0.7	0.3	1.74×10^{-6}
80%	0.7	0.3	9.69×10^{-7}
60%	0.7	0.3	6.83×10^{-7}
40%	0.8	0.3	2.46×10^{-7}
20%	1	0.4	1.13×10^{-9}

map. The comparison of Table 1 and Table 2 demonstrates that for all different dispersion maps, the proposed BP algorithm with linearly varying phase shift parameters is superior to the BP algorithm with constant phase shift parameters.

5. CONCLUSION

This paper propose to optimize the nonlinear coefficient map in BP by using two parameters to control the value of the nonlinear coefficient and make it change linearly with respect to the span number. The simulation results show that the BER performances of the systems with different dispersion maps have all been improved by using the BP algorithm with linearly varying phase shift parameters, and when the residual CD is 10%, the BER of the system can reduce about 18dB as compared to the system using BP algorithm with constant phase shift parameters.

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REFERENCES

1. Ip, E. and J. M. Kahn, "Compensation of dispersion and nonlinear impairments using digital backpropagation," *IEEE Journal of Lightwave Technology*, Vol. 26, 3416–3425, 2008.
2. Ip, E. and J. M. Kahn, "Nonlinear impairment compensation using backpropagation," *Optical Fiber New Developments*, Intech, 2009.
3. Agrawal, G. P., *Nonlinear Fiber Optics*, 3rd Edition, Academic, London, U.K., 2001.
4. Binh, L. N., "Matlab simulink simulation platform for photonic transmission systems," *I.J. Communications, Network and System Sciences*, Vol. 2, 91–168, 2009.