Microwave Radiation Interferometry High Resolution Reconstruction Based on Mixed Orthogonal Basis

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Abstract— Microwave radiometry is the primary means of soil moisture remote sensing and capable of observation soil moisture all-time, all-weather. But the imaging method of traditional microwave radiometry has complex structure and low resolution, which limited largely the application in regional soil moisture remote sensing. In this paper, we propose a new inversion method for microwave radiation interferometry imaging based on compressed sensing (CS), by mining the sparse and compressible information of the microwave radiation image. The proposed method can break through the limit of the inherent spatial resolution of the imaging system which based on the Nyquist sampling, achieving the spatial resolution image closely obtained by the expensive large aperture imaging system, and reducing the complexity and the cost of the hardware of the imaging system configuration. Due to the complexity of microwave radiation image scene, it is difficult to sparsely represent the microwave radiation image by single orthogonal basis, so we make use of the sparse representation in mixed orthogonal basis and the constraint conditions of total variation regularization of microwave radiation image, and establish the optimal reconstruction model of microwave radiation image. We solve the optimal solution of reconstruction model by adopting the alternating direction method (ADM), achieve high resolution microwave radiation image reconstruction from low-dimensional measurements to high -dimensional data, and solve the contradiction between the high resolution and the complexity of the system, and establish evaluation methods of spatial resolution. The simulation results show that, compared with orthogonal the matching pursuit (OMP) algorithm, the proposed algorithm can achieve higher spatial resolution.

1. INTRODUCTION

Interferometric synthetic aperture microwave radiometry (ISAMR) integrates small aperture antenna array into large observation aperture, without mechanical scanning, can overcome the shortcomings of real aperture microwave radiometer. However, for L-Band space borne ISARM, in order to achieve a spatial resolution of 50 km, it still needs 9 meters in diameter antenna array. And with soil moisture sensing area and the fine structure in the direction of development, need increase the diameter of antenna array to satisfy the high spatial resolution requirement, the resolution of radiometer data can be enhanced by using either image processing techniques or special reconstruction algorithms. ISAMR has evolved into a large and complex system that need hundreds of millions data to imaging. In this regard, interferometry and conventional microwave radiation imaging method based on spatial Nyquist is difficult to achieve.

Compressed sensing (CS) theory [1] brought a huge breakthrough in the field of information processing in recent years,and it has changed the way people access information. The CS model is shown in Figure 1.

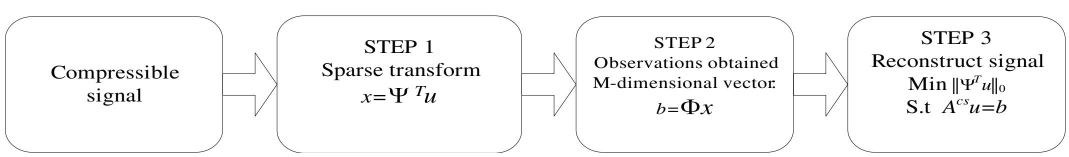

Figure 1: Compressed sensing theoretical framework.

For encoders, recent results indicate that stable reconstructionfor both K-sparse and compressible signals canbe ensured by *restricted isometry property* (RIP). However, it is extremely difficult to verify the RIP propertyin practice. Fortunately, Candes et al. show that RIP holds with high probability when the measurement matrix are random. Being under determined, the equation $b = \Phi\Psi^T u$ usually has infinitely many solutions. If we know in advance that b is acquired from a highly sparse signal, a reasonable approach would be adopted to seek the sparsest one among all solutions, i.e.,

$$\min_{x}\{\left\|\Psi^T u\right\|_0 : Au = b\} \tag{1}$$

Unfortunately, this l_0 decoder is generallyNP-hard and impractical in computation for almost all realproblems. Candes and other studies show that we can replace the l_0 with $l_1$as a signal sparse approximation measure, i.e.,

$$\min_{x}\{\left\|\Psi^T u\right\|_1 : Au = b\} \tag{2}$$

We can see that the problem (2) is constrained. According to the optimization theory, it can be transformed into the following unconstrained problem.

$$\min_{x} \left\|\Psi^T u\right\|_1 + \mu \left\|Au - b\right\|_2^2 \tag{3}$$

where $\mu > 0$.

In literature [3], the CS theory was applied to microwave radiation interference measurement inversion imaging. By using sparse sampling methods to reduce the sampling rate to optimize the number of antenna array, and decrease the complexity of the imaging system in some certain extent. However, for image restoration,recent research [5] has confirmed that the use of total variation (TV) regularization instead of the l_1 term in CS problems makes the recovered image quality sharper, and the edges or boundaries which is essential to characterize images can be preserved accurately. Based on the disadvantage of minimum l_1 norm in computing speed and matching pursuit algorithm in the reconstruction accuracy, and according to the microwave radiation image's characteristic of multi-structure, in this paper, we propose a new model MixTV-W based on the TV and wavelet basis combined with alternating direction method (ADM) algorithm, which can overcome the above disadvantages.

2. BASIC ALGORITHM AND OPTIMALITY

2.1. The Model of MixTV-W

The greatest contribution of this literature is to propose the MixTV-W model. This model is mainly the combination of mixed orthogonal basis and augmented Lagrangian function. In our approach, u is reconstructed based on the following model:

$$\min_{u} \sum_{i} \left\|D_i u\right\|_2 + \tau \left\|\Psi^T u\right\|_1 + (1/2)\mu \left\|Au - b\right\|_2^2 \tag{4}$$

where $\sum_{i}$ is taken over all pixels, $\sum_{i} \left\|D_i u\right\|_2$ is a discretization of the TV of u, and $\tau \left\|\Psi^T u\right\|_1$ is the l_1-norm of the representation of u under Ψ. Ψ is the wavelet basis particularly the Haar wavelet basis in our experiments, A is a random observation matrix and τ, $\mu > 0$ are scalars which are used to balance regularization and data fidelity.

We introduce auxiliary variables $w = [w_1,\ w_2,\ \ldots,\ w_N]$, where each $w_i \in R^2$ and $z \in R^2$ therefore the problem (4) is equivalently transformed to the following form

$$\begin{gathered}\min_{w,z,u} \sum_{i} \left\|w_i\right\|_2 + \tau \left\|z\right\|_1 + (1/2)\mu \left\|Au - b\right\|_2^2 \\ s.t.\ w_i = D_i u,\ for alli\ ;\ z = \Psi^T u\end{gathered} \tag{5}$$

To tackle the linear constraints, we consider the augmentedLagrangian function of (5). Then come to our MixTV-W model:

$$\begin{aligned}\min_{w,z,u,\lambda_1,\lambda_2} \ell_A(w,z,u,\lambda_1,\lambda_2) \ = \ & \sum_{i} \Big\{ \left\|w_i\right\|_2 - (\lambda_2)_i (w_i - D_i u) + (\beta_2/2) \left\|w_i - D_i u\right\|_2^2 + \tau \left\|z_i\right\|_1 \\ & -(\lambda_1)_i (z_i - \psi_i^T u) + (\beta_1/2) \left\|z_i - \psi_i^T u\right\|_2^2 \Big\} + \mu/2 \left\|Au - b\right\|_2^2\end{aligned} \tag{6}$$

where $\beta_1,\ \beta_2 > 0$, for each $i (\lambda_1)_i \in R$, $(\lambda_2)_i \in R^2$, and ψ_i is the ith column of Ψ. For simplicity, we assume $\beta_1 = \beta_2 \equiv \beta$.

2.2. Solving the MixTV-W Model by the ADM

However, each iteration of the augmented Lagrangian method (ALM) is relatively expensive. In contrast, the ADM approach has a much cheaper per-iteration cost. Supposed that w_i, z_i, and u respectively denote the approximate minimizers of (6) at the kth iteration which refers to the inner iteration while solving the subproblem

Firstly, for fixed z_i, u and (here and after $\lambda = (\lambda_1, \lambda_2)$), the minimization of (6) is equivalent to solve the so-called "w-subproblem"

$$\min_{w_i} \sum_i \|w_i\|_2 - (\lambda_2)_i(w_i - D_i u) + (\beta/2)\|w_i - D_i u\|_2^2 \tag{7}$$

According to the theory of two-dimensional shrinkage operator [5], the minimizer w_i is defined by

$$w_i = \max\left\{\left\|D_i u + \frac{(\lambda_2)_i}{\beta}\right\|_2 - \frac{1}{\beta}, 0\right\} \cdot \frac{D_i u + (\lambda_2)_i/\beta}{\|D_i u + (\lambda_2)_i/\beta\|_2} \tag{8}$$

Similarly, according to the theory of one-dimensional shrinkage operator, the minimizer z_i is defined by

$$z_i = \max\left\{\left\|\psi_i^T u + \frac{(\lambda_1)_i}{\beta}\right\|_1 - \frac{\tau}{\beta}, 0\right\} \cdot \frac{\psi_i^T u + (\lambda_1)_i/\beta}{|\psi_i^T u + (\lambda_1)_i/\beta|} \tag{9}$$

In addition, after the values of w_i and u_i is determined, u can be achieved by solving (6) which is equivalent to solve the so-called "u-sub-problem"

$$\begin{aligned}\min_u \vartheta(u) \;=\; & \sum_i -(\lambda_2)_i(w_i - D_i u) + (\beta/2)\|w_i - D_i u\|_2^2 - (\lambda_1)_i(z_i - \psi_i^T) \\ & +(\beta/2)\left\|z_i - \psi_i^T u\right\|_2^2 + (\mu/2)\|Au - b\|_2^2 \end{aligned}\tag{10}$$

The minimizer u can be solved by the least squares method, the value of u is as follows

$$u = (\beta\psi_i^T\psi_i + \beta D_i^T D_i + \mu A^T A)^T \cdot \left[-(\lambda_2)_i D_i^T + \beta D_i^T w_i - (\lambda_1)_i\psi_i + \beta\psi_i z + \mu A^T b\right] \tag{11}$$

In summary, according to the ADM, the (6) is divided into three sub-problems, and these three sub-problems can be solved relatively simple arithmetic operations, thereby avoiding the direct solution more than 1-norm and 2-norm of hybrid optimization question.

Algorithm of MixTV-W:

Input: random observation matrix A, Observation information b, τ, μ, $\beta > 0$, tolerance ε.
Initialize u^0,$(\lambda_1)^0(\lambda_2)^0$, $k = 0$.
While stopping criteria unsatisfied **Do**

1) Compute w^{k+1},z^{k+1} by solving (8) and (9).
2) Compute u^{k+1} by solving (11).
3) Update λ_1 and λ_2 by $(\lambda_1)_i^{k+1} = -(\lambda_1)^k + \beta\left(z_i^{k+1} - \psi_i^T u^{k+1}\right)$

$$(\lambda_2)_i^{k+1} = -(\lambda_2)^k + \beta\left(w_i^{k+1} - D_i u^{k+1}\right)$$

4) $k = k + 1$.

End Do
The stopping criteria is $\left\|u^{k+1} - u^k\right\|_2 \le \varepsilon$.

(a) (b) (c)

Figure 2: The result of reconstruction under the sampling rate 0.4.

(a) (b) (c)

Figure 3: The result of reconstruction under the sampling rate 0.7.

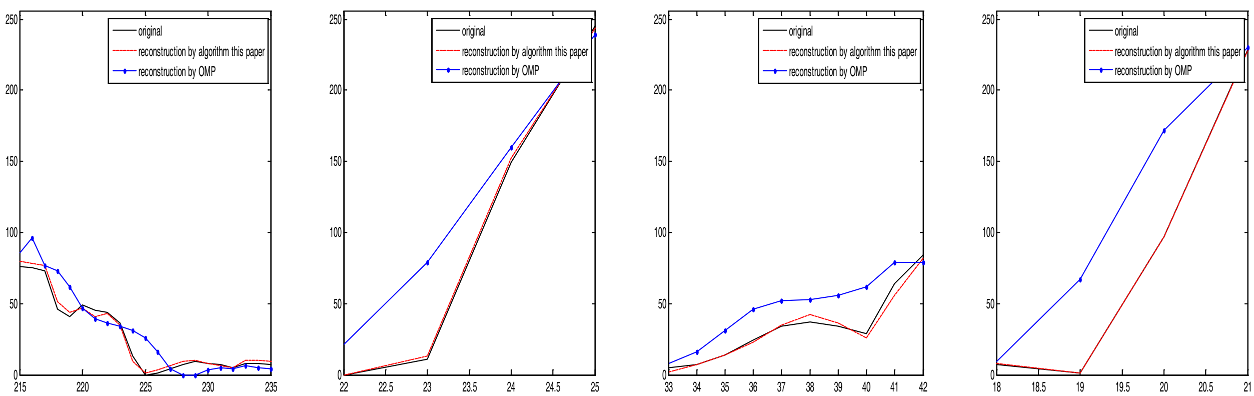

Figure 4: Gray value near the edge portion of the moon.

Table 1: The PSNR (dB) of different sampling rates and different algorithms.

	algorithm this paper		OMP	
sampling rate	0.4	0.7	0.4	0.7
Moon	28.43	33.84	25.02	30.42

3. SIMULATION EXPERIMENT

In the simulation experiment, we simulate the microwave radiation image of the Moon by the proposed algorithm based on the total variation basis and wavelet basis. Also the OMP algorithm based on the wavelet basis is adopted to simulate the result as the comparision. The two images' size are 256×256. Sampling rate is set to 0.4 and 0.7 respectively. The reconstruction results are presented in Figure 2 to Figure 3.

In the two figures, subgraph (a) are the original, subgraph (b) are the results of the reconstruction by the algorithm this paper proposed, subgraph (c) are the results of the reconstruction by the OMP based on the signal basis. We can see that the results of the subgraph (b) in the two figures are better than the subgraph (c). According to the Simulation data in Table 1, the PSNR of subgraph (b) in the two figures are higher 2–4 db than subgraph (c).

In each subgraph of the Figure 4, black solid line represents original, red dotted line represents reconstruction by algorithm this paper, blue dot and solid line represents reconstruction by OMP. In the Figure 4, each subgraph shows the grey value of image border and the change illustrat that the spatial resolution are high. We can see from the figure that the faster the grey value changes which is the steeper the curve changes illustrating that the spatial resolution are higher. In each subgraph of the Figure 4, black solid line represents original, red dotted line represents reconstruction by algorithm this paper, blue dot and solid line represents reconstruction by OMP. So from the Figure 4 denotes that the spatial resolution of reconstruction by our proposed algorithm is higher.

4. CONCLUSION

This paper proposed the model based on the mixed orthogonal basis and the augmented Lagrangian approach, and use the ADM reconstruct the microwave radiation images. The result of simulation of complex microwave radiation image show that the method we proposed is better than the OMP method based on the signal basis. However if the image's structure is too simple, the result used by our proposed algorithm is not as good as the result of OMP. The next step will focus on the reconstruction quality, and optimize the reconstruction algorithm.

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