Microwave Radiation Image Reconstruction Method Based on Adaptive Multi-structural Dictionary Learning

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Abstract— Due to the complicated structure of microwave radiometric imaging system and the massive amount of data collection in one snapshot, it is difficult to achieve the high spatial resolution image by conventional microwave radiation imaging method based on the Nyquist sampling. In this paper, according to the priori information of the compressible multi-structural information of microwave radiation image, we adopt the Fourier random observation matrix to sparsely project the microwave radiation image, reducing the amount of data collection, and lowering the complexity of the system. Considering the multi-structural information of microwave radiation image, such as multi-sparsity in different domains, piecewise smoothness, etc, it is difficult to sparsely represent microwave radiation image in complex scene by the traditional orthogonal basis, but the piecewise smoothness ingredients of microwave radiation image meet the constraint condition of total variation, and the mixed orthogonal basis can sparsely represent the sparsity information of microwave radiation image. We make use of the sparse representation of the mixed orthogonal basis and the constraint condition of total variation regularization to construct the reconstruction model of microwave radiation image based on adaptive multi-structural sparsifying dictionary learning. We integrate dictionary learning technique to adaptively learn the multi-structural sparsifying dictionary, and the sparsifying dictionary is adapted to sparsely represent the microwave radiation image, and solve the convex programming problem of microwave radiation image reconstruction, and design the reconstruction method. The simulation results show that reconstructing microwave radiation image by the proposed algorithm achieves better reconstruction performance than that by the DLMRI algorithm.

1. INTRODUCTION

The characteristics obtained from soil microwave radiation image using microwave radiometers mainly depend on soil moisture. We can get precise soil moisture data by inverting the obtained microwave radiation image, and enhance the accuracy of weather forecasts by analyzing the soil moisture data, and effectively monitor geological disasters, such as drought, flood, etc.. Interferometric synthetic aperture microwave radiometry (ISAMR) [1] integrates the small-aperture array into large observation aperture, it doesn't need a mechanical scanning for imaging, and can solves the disadvantages of real aperture microwave radiometers. However, with the development of refinement and structuralization of the image, ISAMR has evolved into an enormous and complex system for the demand of high resolution, and it can easily reach tens of millions when collecting data in one snapshot. Taking example by the principle of the infrared and visible light, microwave radiation focal plane array imaging system adopts the stared working condition, and it improves the real-time performance of the system and reduces the complexity of the later image processing. However, in order to obtain high spatial image, we need to increase the complexity of the system [2]. So it is difficult to achieve the high spatial resolution image based on the conventional sampling and imaging method.

Compressed Sensing (CS) [3–5] changes the traditional concept of information acquisition. Combined with the sparse prior knowledge of signal for signal reconstruction, it utilizes the random sparse sampling to compress data, and adopts the nonlinear algorithm to achieve the signal, so as to effectively reduce the complexity of the sampling system. In the CS method, the sparse representation of the image is the key aspect for image reconstruction. Generally, the microwave radiation image contains additional structures, such as piecewise smoothness [6], and sparsity in transform domain, etc. [7]. Therefore, the single orthogonal basis is difficult to sparsely represent microwave radiation image in complex scene. Yang et al. [8] develop a reconstruction model of total variation (TV) l_1-l_2 norm, using RecPF algorithm which only consider a single orthogonal basis to fast reconstruct magnetic resonance image (MRI). Ravishankar et al. [9] use a reconstruction model based on dictionary learning, adopt DLMRI algorithm and only consider dictionary learning in the pixel domain to reconstruct MRI. Liu et al. [10] adopt GradDLRec algorithm which just takes into account learning dictionary on TV domain to reconstruct MRI. On the basis of previous studies, we integrate the TV and wavelet transform into the reconstruction model. According to the multi-structural characteristics of the microwave radiation image, we realize the sparsely sample by the Fourier random observation matrix, and sparsely represent the image by adaptive learning multi-structural dictionary, and solve the convex optimization problem of the image reconstruction to achieve high spatial resolution image.

2. MICROWAVE RADIATION IMAGING SCHEME BASED ON CS

2.1. The Basic Method of CS

CS reconstructs the unknown $x \in C^P$ from the measurements $y \in C^m$, or equivalently solves an underdetermined system of linear equations $F_u x = y$ by minimizing the quasi norm of the sparsified image Ψx, where $\Psi \in C^{T \times P}$ represents a global, typically orthonormal sparsifying transform for the image. For example, Ψ may be the wavelet transform, so that Ψx corresponds to the wavelet coefficients of x, assumed to be sparse (mostly zero). The corresponding optimization problem is given by

$$\min_x \|\Psi x\|_0 \quad s.t.\ F_u x = y \tag{1}$$

$\|\cdot\|_0$ denotes the l_0 quasi-norm which counts the number of nonzero coefficients of the vector, $F_u \in C^{m \times P}$ represents the undersampled Fourier encoding matrix. This l_0 problem is NP-hard, there are greedy algorithms to solve this problem such as orthogonal matching pursuit (OMP). We can replace the l_0 quasi-norm with the l_1 relaxation, it can be solved via linear programming in this real case, or via second order cone programming in the complex case [9].

$$\min_x \|F_u x - y\|_2^2 + \lambda \|\Psi x\|_1 \tag{2}$$

2.2. Microwave Radiation Imaging Based on CS

According to the characteristics of complex structure and low imaging resolution for microwave radiometric imaging system, in this paper, we adopt the CS to realize microwave radiation imaging, and solve the above shortcomings. The specific method is to mine the priori information of the compressible multi-structural information of microwave radiation image, and adopt the Fourier random observation matrix to sparsely sample the image, reducing the amount of data collection and lowering the complexity of the system. We integrate the dictionary learning technique into adaptively learn the multi-structural sparsifying dictionary which is adopted to sparsely represent the image, and solve the convex programming problem of the image reconstruction model to achieve the high spatial resolution image.

3. PROPOSED IMAGE RECONTRUCTION MODEL

Taking example by GradDLRec algorithm, we add the TV and wavelet transform to the reconstruction model which recovers more details, and use the Daubechies wavelet-4, and propose a new model based on adaptive multi-structural dictionary learning as follows:

$$\min_{u, D^{(i)}, \Gamma^{(i)}} \left\{ \sum_{i=1}^{2} \sum_{l} \left\| D^{(i)} \alpha_l^{(i)} - R_l \left(\lambda_1 \nabla^{(i)} u + \lambda_2 \Psi u \right) \right\|_2^2 + \frac{v_1}{2} \|F_p u - f\|^2 \right\} s.t. \left\| \alpha_l^{(i)} \right\|_0 \le T_0, \forall l, i. \tag{3}$$

where $(\nabla_x, \nabla_y) = (\nabla^{(1)}, \nabla^{(2)})$ denotes the difference operators in horizontal and vertical directions, Ψ denotes the wavelet transform, The first term in the cost function captures the sparse prior of the sparsified image patches with respect to dictionaries $\{D^{(i)}\ i = 1, 2\}$, while the second term enforces data fidelity in k-space. The weight v_1 is set as $\nu_1 = (\lambda/\sigma)$ like the DLMRI algorithm does, where λ is a positive constant, λ_1 and λ_2 are positive scalars that balance the two sparse domain.

By introducing auxiliary variables $w^{(i)}$, $i = 1, 2$, the problem in (3) can be rewritten as follows:

$$\begin{aligned} &\min_{u, \omega, D^{(i)}, \Gamma^{(i)}} \left\{ \sum_{i=1}^{2} \sum_{l} \left\| D^{(i)} \alpha_l^{(i)} - R_l \left(\omega^{(i)} \right) \right\|_2^2 + v_1 \|F_p u - f\|^2 \right\} \\ &\qquad s.t. \quad \left\| \alpha_l^{(i)} \right\|_0 \le T_0, \forall l, i; \omega^{(i)} = \lambda_1 \nabla^{(i)} u + \lambda_2 \Psi u, \forall i. \end{aligned} \tag{4}$$

Let $\nabla = [(\nabla^{(1)})^T, (\nabla^{(2)})^T]^T$, $b = [(b^{(1)})^T, (b^{(2)})^T]^T$ and $w = [(w^{(1)})^T, (w^{(2)})^T]^T$. Combined with the Bregman technique, we obtain a sequence of constrained subproblems as follows:

$$\begin{aligned}&\left\{u^{k+1}, \omega^{k+1}, \left(D^{(i)}\right)^{k+1}, \left(\alpha_l^{(i)}\right)^{k+1}\right\}\\ &= \arg\min_{u,\omega,D,\Gamma} \left\{\begin{array}{l} \sum_{i=1}^{2}\sum_{l}\left\|D^{(i)}\alpha_l^{(i)} - R_l\left(\omega^{(i)}\right)\right\|_2^2 + v_1\left\|F_p u - f\right\|^2 \\ +v_2\left\|b^k + \lambda_1\nabla u + \lambda_2\Psi u - w\right\|_2^2 \quad s.t. \ \left\|\alpha_l^{(i)}\right\|_0 \le T_0, \forall l, i \end{array}\right\}\end{aligned} \tag{5}$$

$$b^{k+1} = b^k + \lambda_1\nabla u^{k+1} + \lambda_2\Psi u^{k+1} - \omega^{k+1} \tag{6}$$

where v_2 denotes the positive penalty parameter. We use the alternating direction method (ADM) to solve the minimization of Eq. (5) with respect to u, w, D and α. This technique carries out approximation via alternating minimization with respect to one variable while keeping others fixed.

1) Updating the Solution u: At the k-th iteration, we assume w, $D^{(i)}$ and $\alpha^{(i)}$ to be fixed with their values denoted as w^k, $(D^{(i)})^k$, and $(\alpha^{(i)})^k$ respectively. After eliminating the constant variables, the objective function for updating $u$is given as

$$u^{k+1} = \arg\min_u \left\{v_1\left\|F_p u - f\right\|_2^2 + v_2\left\|b^k + \lambda_1\nabla u + \lambda_2\Psi u - w^k\right\|_2^2\right\} \tag{7}$$

Considering that Eq. (7) is a simple least squares problem, we can update u with its analytic solution:

$$u^{k+1} = F^{-1}\left(\frac{F\left[v_1 F_p^T f + v_2\left(\lambda_1\nabla^T + \lambda_2\Psi^T\right)\left(\omega^k - b^k\right)\right]}{v_1 F F_p^T F_p F^T + v_2 F\left(\lambda_1\nabla^T + \lambda_2\Psi^T\right)F^T F\left(\lambda_1\nabla + \lambda_2\Psi\right)F^T}\right) \tag{8}$$

2) Updating the Sparsified Image Variables$w^{(i)}$, $i = 1$, 2: The minimization in Eq. (5) with respect to $w^{(1)}$ and $w^{(2)}$ is decoupled, and then it can be solved separately. It yields:

$$(\omega^{(i)})^{k+1} = \arg\min_{\omega^{(i)}}\left\{\sum_l\left\|(D^{(i)})^k(\alpha_l^{(i)})^k - R_l(\omega^{(i)})\right\|_2^2 + v_2\left\|(b^{(i)})^k + \lambda_1(\nabla^{(i)}u)^{k+1} + \lambda_2(\Psi u)^{k+1} - w^{(i)}\right\|_2^2\right\} \tag{9}$$

The least squares solution to (9) is written as:

$$(\omega^{(i)})^{k+1} = \frac{v_2\left[\left(b^{(i)}\right)^k + \lambda_1\left(\nabla^{(i)}u\right)^{k+1} + \lambda_2\left(\Psi u\right)^{k+1}\right] + \sum_l R_l^T\left(D^{(i)}\right)^k\left(\alpha_l^{(i)}\right)^k \Big/ \beta}{v_2 + 1} \tag{10}$$

3) Sparse Representation for Sparsified Patches with Respect to Variables $D^{(i)}$ and $\alpha_l^{(i)}$, l=1, $2\ldots$, L: The minimization (5) with respect to dictionary and coefficient variables of the sparsified images is also decoupled, and thus can be solved separately. It yields:

$$\left\{\left(D^{(i)}\right)^{k+1}, \left(\alpha_l^{(i)}\right)^{k+1}\right\} = \arg\min_{D^{(i)},\Gamma^{(i)}}\left\{\sum_l\left\|D^{(i)}\alpha_l^{(i)} - R_l\left(\omega^{(i)}\right)^{k+1}\right\|_2^2 s.t. \left\|\alpha_l^{(i)}\right\|_0 \le T_0, \forall l, i = 1, 2\right\} \tag{11}$$

The strategy to solve (11) is to alternatively update the dictionary $D^{(i)}$ and coefficient matrix $\alpha_l^{(i)}$, which is the same as that used in K-SVD [11] The K-SVD algorithm has two steps: sparse coding and dictionary updating, the sparse coding is solved by the greedy algorithm-OMP algorithm, and the dictionary updating is solved by singular value decomposition (SVD) to minimize the approximation error.

Algorithm 1: The Proposed Algorithm

1: **Initialization:**

$(\Gamma^{(i)})^0 = 0$, $(D^{(i)})^0$, $(b^{(i)})^0 = 0$, $i = 1, 2$; $u^0 = F_p^T f$;

2: **For** $k = 1, 2, \ldots$ repeat until a stop-criterion is satisfied:

3: $(\omega^{(i)})^{k+1} = \dfrac{v_2[(b^{(i)})^k + \lambda_1(\nabla^{(i)} u)^k + \lambda_2(\Psi u)^k] + \sum_l R_l^T (D^{(i)})^k (\alpha_l^{(i)})^k \big/ \beta}{v_2 + 1}, i = 1, 2$

4: Updating $\{(D^{(i)})^{k+1}, (\mathrm{G}^{(i)})^{k+1}\}$ from sparse images $(w^{(i)})^{k+1}$ by Eq.(11), $i = 1, 2$

5: $u^{k+1} = F^{-1}\left(\dfrac{F[v_1 F_p^T f + v_2(\lambda_1 \nabla^T + \lambda_2 \Psi^T)(\omega^{k+1} - b^k)]}{v_1 F F_p^T F_p F^T + v_2 F(\lambda_1 \nabla^T + \lambda_2 \Psi^T) F^T F(\lambda_1 \nabla + \lambda_2 \Psi) F^T}\right)$

6: $(b^{(i)})^{k+1} = (b^{(i)})^k + \lambda_1(\nabla^{(i)} u)^{k+1} + \lambda_2 \Psi u^{k+1} - (\omega^{(i)})^{k+1}$, $i = 1, 2$

7: **END**

8: **Output** u^{k+1}

4. EXPERIMENT

In this section, we use CS to reconstruct the moon microwave radiation image, the size of image is 256×256, we adopt the Fourier random observation matrix to sample the image. Our proposed algorithm is compared with the DLMRI algorithm. The parameters for DLMRI algorithm are set as the values: patch size $n^{1/2} = 6$, dictionary size $K = n = 36$, the patch overlap $r = 1$, $\beta = 36$, $\lambda = 140$, the learning stage (K-SVD) employed 10 iterations, $200 \cdot K$ patches, and a fixed sparsity $T_0 = 5$. The proposed algorithm followed the same setup, with additional choices of $\nu_2 = 3$, $\lambda_1 = 1$ and $\lambda_2 = 0.1$. We reconstructed the moon microwave radiation image using DLMRI algorithm and the proposed algorithm from reduced k-space data with the undersampling ratios of 30%–70%.

Table 1: The reconstruction PSNR (dB) values with different undersampling ratios.

Algorithm	Undersampling ratio				
	30%	40%	50%	60%	70%
DLMRI	42.4374	42.0343	41.0898	39.5656	37.3270
The proposed algorithm	45.6674	44.1393	42.2118	40.1677	37.6708

To evaluate the reconstruction performance under different sampling ratios at the same experimental setting, we adopt five undersampling ratios (0.3, 0.4, 0.50, 0.60 and 0.7) to acquire the partial

(a) (b) (c)

Figure 1: Visual comparison of image reconstructions using two methods with 70% undersampling. (a) The original image, (b) (c) Reconstructions using DLMRI and the proposed algorithm, respectively.

Fourier measurements respectively The reconstruction PSNRs are given in Table 1. Figures 1(b) and (c) show the images reconstructed from 70% 2D undersampled measurements by DLMRI and the proposed algorithm. With the comparison of the PSNR values with different undersampling ratios, the results show that the proposed algorithm exhibits more accurate reconstruction with larger PSNR values than DLMRI for specified undersampling ratios.

5. CONCLUSION

In this paper, we propose a new CS reconstruction model based on adaptive multi-structural dictionary learning and an efficient algorithm to reconstruct the microwave radiation image. The model aims to learn multi-structural adaptive dictionary in the sparsified image for better reconstruction. We use a multi-structural adaptive learning dictionary from the sparsified image to reconstruct the image. The simulation results show that the proposed algorithm achieves better reconstruction performance than the DLMRI algorithm. Further studies will be conducted to improve the proposed algorithm, such as the parameter setting.

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