Uncertainty Estimation in Vector Wind Retrievals from Satellite-based Polarimetric Microwave Radiometer Measurements

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Abstract— The main characteristic of wind vector is circularity in any reference direction, which creates problems when analyzing the data statistically. Due to this periodicity, standard statistical techniques cannot be used to analyze circular data. For example, (1) in the regions including upwind/downwind ambiguity, the STD obtained from the linear statistics approach is about 60–80°. However, we couldn't deduce that it is upwind/downwind ambiguity from the STD. (2) The Cramer-Rao bound (CRB) has been widely used as sensitivity estimation. The precondition of the CRB approach is the possibility distribution is normal one. Since the model of wind direction reveres is non-linear, the angel distribution is not normal. A problem appeared for wind-vector sensitivity estimation. The STD obtained from closest solution is lower than the sensitivity obtained from CRB. In this paper, we introduce the directional statistics approach for evaluating the uncertainty. The circular STD is always between zero and one inclusive, making it very different from its linear analog, which can have any positive value. It is indicative of the spread in a data set. If all samples point into the same direction, the circular STD will be small. If the samples are spread out evenly, the circular STD will be close to one. Importantly, however, a circular variance of 1 does not imply a uniform distribution around the circle. If all samples either point towards 0° or 180°, the circular variance is 1, yet the data is not distributed uniformly around the circle. The sensitivity is reassessed based on the directional statistics approach, and compared with CRB. The sensitivities of CRB are high where the derivatives of the brightness temperatures changing rapidly. The incorrect results come from the effect of nonlinearity. When the directional statistics approach is used, the peak disappeared where the derivatives of the brightness temperatures changing rapidly. In addition, the directional statistics approach can be used to estimate the characterization of uncertaintys in wind vector retrievals from Scatterometry.

1. INTRODUCTION

The uncertainty of the wind-vector estimator is critical for a variety of applications in Radiometer design [1], improving the accuracy of wind direction retrieval [2], and demonstration the capacity to obtain wind-direction over the global oceans from passive microwave measurements. The main characteristic of wind vector is circularity in any reference direction, which creates problems when analyzing the data statistically. Physical quantities referring to angles are called angular, directional, or circular data. Due to this periodicity, standard statistical techniques cannot be used to analyze circular data [3, 4]. For example, the samples mean of a data set on the circle is not the usual sample mean [5].

2. CIRCULAR DISTRIBUTIONS

The *sample mean direction* x_{mean} of observations θ_1, θ_2, $\ldots$, θ_n is given by

$$\bar{x} = \mathrm{arc}\frac{\sum_{i=1}^{n} \sin\theta_i}{\sum_{i=1}^{n} \cos\theta_i} \tag{1}$$

Circular variance is defined as

$$S_0 = 1 - \left[\left(\sum_{i=1}^{n} \cos\theta_i\right)^2 + \left(\sum_{i=1}^{n} \sin\theta_i\right)^2\right] \tag{2}$$

It is important to note that the circular variance S_0 is always between zero and one inclusive, making it very different from its linear analog, which can have any positive value [3]. The circular variance S_0 thus indicates the dispersion of a sample. It is indicative of the spread in a data set. If all samples point into the same direction, the circular variance will be small. If the samples are spread out evenly, the circular variance will be close to maximal. Importantly, however, a circular variance of 1 does not imply a uniform distribution around the circle. If all samples either point towards 0° or 180°, the circular variance is 1, yet the data is not distributed uniformly around the circle.

3. POLARIMETRIC RADIOMETRY

Research in polarimetric radiometry modeling and measurements has demonstrated that usable wind direction information can be obtained by combining the vertical and horizontal polarizations with the cross correlation of those two polarizations [6, 7].

An empirical geophysical model function relating the amplitude of U and V to ocean-surface wind speed and direction has been derived from the Jet Propulsion Laboratory aircraft wind radiometer (WINDRAD) [8] as well as WindSat measurements. The directional dependence of the third and fourth Stokes parameters can be described [12] by

$$\begin{aligned} T_h &= T_{h0} + T_{h1}\cos\phi + T_{h2}\cos 2\phi \\ T_v &= T_{v0} + T_{v1}\cos\phi + T_{v2}\cos 2\phi \\ T_U &= T_{U1}\sin\phi + T_{U2}\sin 2\phi \\ T_V &= T_{V1}\sin\phi + T_{V2}\sin 2\phi \end{aligned} \tag{3}$$

where $\phi = \phi_0 - \phi_w$ is the relative angle between the observation azimuth angle ϕ_w and the wind direction ϕ_0. The coefficients of the first harmonics account for upwind and downwind asymmetric surface features, while those of the second harmonics account for the upwind and crosswind asymmetry.

In order to analyze the dependence of ambiguous solutions on the true wind vector, we plot the errors associated with global cost function minima, along with the Monte Carlo simulation results.

The results of the uncertainty calculation are plotted in Fig. 1.

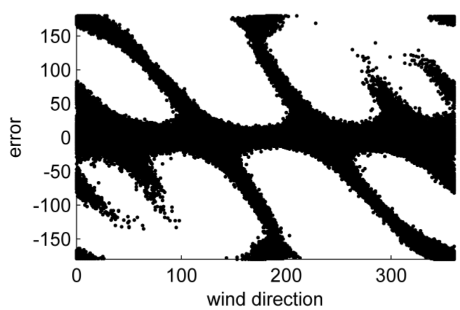


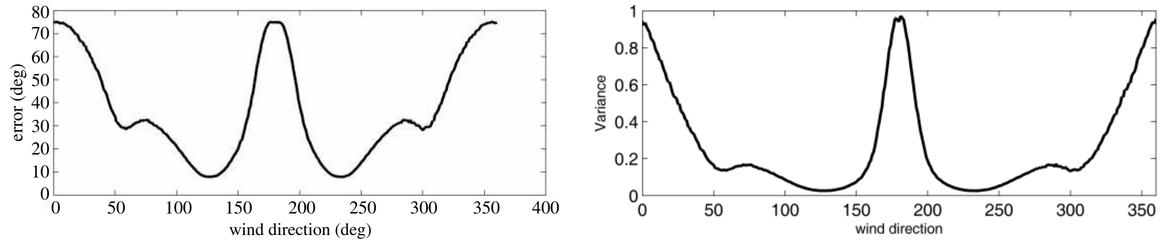
Figure 1: Black dots indicate errors associated with global cost function minima observed in Monte Carlo simulation results.

In the regions including upwind/downwind ambiguity, the STD obtained from the linear statistics approach is about 60–80°. It can not correctly indicate the dispersion of the wind vector. We can see from the Fig. 2(b) that the circular variance can produce the meaningful indication of the dispersion of the wind vector.

4. SENSITIVITY ANALYSIS

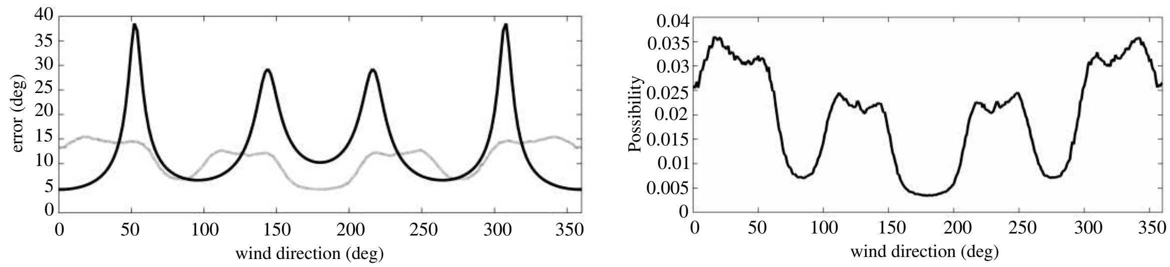
The sensitivity was used as an important parameter to character the capacity of wind-direction reverse. We use the directional statistics approach to calculate the sensitivity in wind vector inversions. The result was compared with the Cramer-Rao minimum variance bound and the closest cost function minimum to the true solution [9].

Generally linear statistics can be used with directional data only if the range of observations is small. Figure 3(a) displays the relationship of the wind direction STD versus the relative wind



(a) (b)

Figure 2: (a) The wind direction error obtained from the linear statistics approach, the solution corresponding to the global cost function minimum. (b) The wind direction variance obtained from the directional statistics approach, the solution corresponding to the global cost function minimum.



(a) (b)

Figure 3: (a) The wind direction sensitivity obtained from the linear statistics approach. (b) The wind direction variance obtained from the directional statistics approach, the closest cost function minimum to the true solution was used.

direction angle. The regions agree well with each other, where the derivatives of the brightness temperatures changing slowly. The sensitivities of CRB are high where the derivatives of the brightness temperatures changing rapidly. The incorrect results come from the effect of nonlinearity. Fig. 3(b) displays the result of the directional statistics approach. The directional statistics approach is suitable to the nonlinear model. Compared to the sensitivities of CRM, the peak disappeared where the derivatives of the brightness temperatures changing rapidly.

Considering the distribution is not normal, the sensitivity described in this paper is more prefer than the one obtained from linear statistics approach.

5. CONCLUSION

In this paper, we introduced a method for evaluating the uncertainty of Vector Wind Retrievals from Satellite-Based Polarimetric Microwave Radiometer Measurements based on the Radiative Transfer Model. The Cramer-Rao bound (CRB) and MLE has been analyzed.

The sensitivity of Vector Wind Retrievals was computed used the new method, and compared with CRB and the closest solution of maximum-likelihood estimation (CS-MLE). The uncertainty method presented here can obtain minimum of sensitivity, according with the sensitivity conception, and can give a preferable explanation to the simulated result.

This paper also computed the different wind-vector ambiguous solutions and corresponding uncertainty, under the based Model method (BM) and maximum-likelihood estimation (MLE). A discrete representation for the wind-vector solutions has be obtained, which can predict the characteristics of errors in wind direction retrievals and their dependence on the true wind direction relative to the observation azimuth angle. It is anticipated that the techniques and strategies developed in this study, and prior related studies, to analyze wind direction uncertainty will be requisite for maximizing the information content of future instruments.

The method commutes over nonlinear operations, is potentially more reliable algorithm to assess radiometer system, Model Function and Retrieval Algorithm's performance.

REFERENCES

1. Racette, P. and R. H. Lang, "Radiometer design analysis based upon measurement uncertainty," *Radio Science*, Vol. 40, 2005.

2. Brown, S. T., C. S. Ruf, and D. R. Lyzenga, "An emissivity-based wind vector retrieval algorithm for the WindSat polarimetric radiometer," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 44, 611–621, 2006.
3. Mardia, K. V. and P. E. Jupp, *Directional Statistics*, Vol. 494, Wiley, 2009.
4. Nikolaidis, N. and I. Pitas, "Nonlinear processing and analysis of angular signals," *IEEE Transactions on Signal Processing*, Vol. 46, 3181–3194, 1998.
5. Ravindran, P. and S. K. Ghosh, "Bayesian analysis of circular data using wrapped distributions," *Journal of Statistical Theory and Practice*, Vol. 5, 547–561, 2011.
6. Piepmeier, J. R. and A. J. Gasiewski, "High-resolution passive polarimetric microwave mapping of ocean surface wind vector fields," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 39, 606–622, 2001.
7. Yueh, S. H., "Modeling of wind direction signals in polarimetric sea surface brightness temperatures," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 35, 1400–1418, 1997.
8. Gaiser, P. W., K. M. St Germain, E. M. Twarog, G. A. Poe, W. Purdy, D. Richardson, et al., "The WindSat spaceborne polarimetric microwave radiometer: Sensor description and early orbit performance," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 42, 2347–2361, 2004.
9. Laws, K. E., D. R. Lyzenga, D. M. Wiberg, and J. F. Vesecky, "Characterization of errors in vector wind retrievals from satellite-based polarimetric microwave radiometer measurements," *IEEE Geoscience and Remote Sensing Letters*, Vol. 3, 45–48, 2006.