Classification of Chaotic Codes Using Discriminant Analysis Classifiers and Higher Order Statistical Features

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Abstract— This paper investigates the classification of a new type of Pseudo-Random (PN) codes which are generated using noise-like chaotic signals. The classification is made using different discriminant analysis classifiers such as linear, diagonal linear, quadratic, diagonal quadratic, and mahalanobis discriminant analysis classifiers. These classifiers are compared with other classifiers such as neural networks, support vector machines, k-nearest neighbor, and maximum likelihood classifiers. Higher order statistical (HOS) moments and cumulants of the eighth order are used as features. Simulation results illustrates the dependence of the proposed classification method on the type of the used classifier, the type of chaotic map and the initial values used for generating the chaotic code. Two types of one dimensional chaotic maps are considered in this work, namely, the logistic map and the Bended up down map. The performance, considered as the probability of correct decision, is shown to differ according to the type of the used classifier and is dependent on the signal-to-noise ratio. The results show that the quadratic discriminant analysis classifier outperforms the other discriminant analysis classifiers. Also, the performance of two codes generated from one logistic map with two different initial values outperforms the performance of two codes generated from one Bended up down map with two different initial values, two codes generated from logistic map with initial value and Bended up down map with the same initial value, and two codes generated from logistic map with one initial value and Bended up down map with another initial value.

1. INTRODUCTION

Chaotic signals have several properties which make them attractive candidate for communications. They have wideband spectrum, they do not accurately repeat themselves, they have a random-like appearance, and they are relatively simple to be implemented [1]. The mathematical definition of chaos is an unpredictable long time behavior arising in a deterministic dynamical system because of the sensitivity to initial conditions. So, these signals are very sensitive to initial conditions and hence one can generate a large number of codes using the same chaotic map. Chaotic codes are used in a variety of applications in communications such as code-division multiple-access (CDMA).

The discrete chaotic signal is generated by a chaotic map $M(x)$ which is a nonlinear function describing the relation between the previous states and the next state. The general form of m-dimensional chaotic map is defined by Equation (1). where x_n and x_{n+1} are successive iterations of the output x and M is the forward transformation mapping function.

$$x_{n+1} = M(x_n, x_{n-1}, \ldots, x_{n-m}) \tag{1}$$

One dimensional chaotic maps are the simplest form of chaotic maps and the commonly used in communication applications are logistic map, bended up down map, chebyshev map. The general form of the logistic map and bended up down map are defined by Equation (2) in Figure 1 and (3) in Figure 2 respectively.

$$x_{n+1} = \mu x_n(1 - x_n), \quad 0 < x_n < 1 \quad \text{and} \quad 0 < \mu < 4 \tag{2}$$

$$x_{n+1} = \begin{cases} \frac{9x_n}{2x_n+3} & \text{for } 0 \le x_n < \frac{1}{3} \\ \frac{13x_n-3}{3x_n+3} & \text{for } \frac{1}{3} \le x_n < \frac{3}{5} \\ \frac{29x_n-15}{7x_n+3} & \text{for } \frac{3}{5} \le x_n < \frac{9}{11} \\ \frac{99x_n-81}{25x_n-3} & \text{else} \end{cases} \tag{3}$$

The output of chaotic map is quantized to convert to a stream of ones and zeros using threshold level. The optimum threshold is suggested to be the median of the invariant probability density

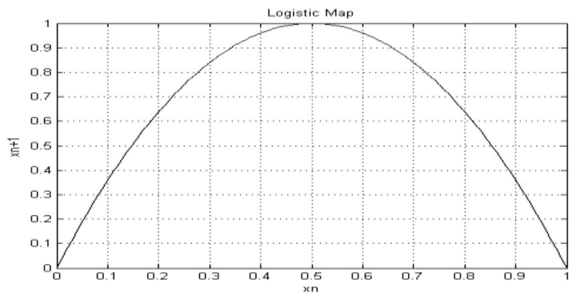

Figure 1: Logistic map.

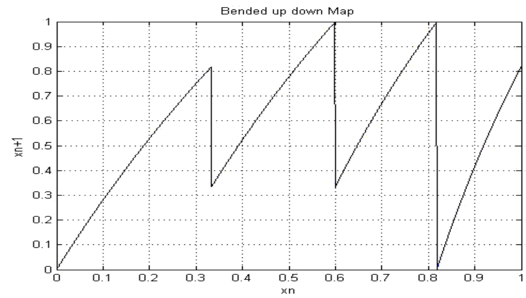

Figure 2: Bended up down map.

function (pdf) of the used chaotic map; the invariant pdf is the histogram of the long generated sequence using any initial conditions, where the invariant pdf is independent of the used initial conditions and hence independent of the generated sequence. Then the threshold of the logistic map and bended up down map is 0.5 and 0.639 respectively [2].

Classification using higher order statistics and linear discriminant analysis classifiers was considered in [3] to classify mental tasks from EEG signals, in [4] to classify digital modulated signals and in [5] to classify electrical power quality signals. Quadratic discriminant analysis classifiers were used to classify electrical power quality signals in [6].

The paper is organized as follows. Section 2 describes features extraction and Section 3 describes the discriminant analysis classifiers. Section 4 shows simulation results. Finally, Section 5 concludes the paper.

2. FEATURES EXTRACTION

In this paper, we used two features for classification and compare its performance with the cases of only two features and only four features are used. These features are the eighth order moment and cumulant of the higher order statistics. Even order moments and cumulants expressions up to eighth order are found in the appendix of [7]. The used features are those which show significant differences between the different chaotic codes.

3. DISCRIMINANT ANALYSIS CLASSIFIERS

We use five discriminant analysis classifiers such as linear discriminant analysis classifier, diagonal linear discriminant analysis classifier, quadratic discriminant analysis classifier, diagonal quadratic discriminant analysis classifier, and mahalanobis discriminant analysis classifier [8] and compare the results with the classifiers used in [7] such as maximum likelihood classifier, k-nearest neighbor classifier, multilayer perceptron neural networks, and support vector machine classifiers.

4. SIMULATION RESULTS

In this section, we evaluate the performance of classification of two codes of chaotic maps using different types of discriminant analysis classifiers and higher order statistics for features extraction. We used four groups of two different codes generated using four chaotic signals. The two codes of the first group are generated using different types of chaotic maps and using the same phase or initial value 0.3, the second two codes are generated using the same one type of chaotic map and using different initial values 0.3 and 0.45, the third two codes are generated using the other type of chaotic map and using different initial values 0.3 and 0.45, and the fourth two codes are generated using two different types of chaotic maps and two different initial values 0.3 and 0.45. The four chaotic signals used in generating these codes are shown in Figure 3. Each signal is

Table 1: The eighth order features for Logistic map with different phase codes.

	Logistic map with 0.3 phase	**Logistic map with 0.45 phase**	**Bended up down map with 0.3 phase**	**Bended up down map with 0.45 phase**
M8	−243.8503	−243.4009	−243.2175	−243.9946
C8	1.0025	1.0101	1.0132	1.0001

chosen to have 150 realizations and 4096 samples (1 second). They are then divided into 100 realizations for training and 50 realizations for testing data sets. White Gaussian noise is added to these signals and features are extracted using eight order moment and cumulant using equations in [7]. The two features used are M8 and C8 and are shown in Table 1 under the constraints of zero mean, unit variance and noise free. This method is compared with classifiers in [7]. The performance considered is the probability of correct decision. using Figure 4, Figure 5, Figure 6, and Figure 7 show the performance using maximum likelihood classifier, nearest neighbor classifier for $k = 3$, multilayer perceptron neural network with MSE is taken to be 10-6, and support vector machine classifier. Figure 8, Figure 9, Figure 10, Figure 11, and Figure 12 show the performance using linear discriminant analysis classifier, diagonal linear discriminant analysis classifier, quadratic discriminant analysis classifier, diagonal quadratic discriminant analysis classifier, and mahalanobis discriminant analysis classifier. The comparison between different discriminant analysis classifiers are shown in Figure 13. From Figure 13, we note the performance of quadratic discriminant analysis classifier is the best and compare the performance of this with the classifiers used in [7] as shown in Figure 14.

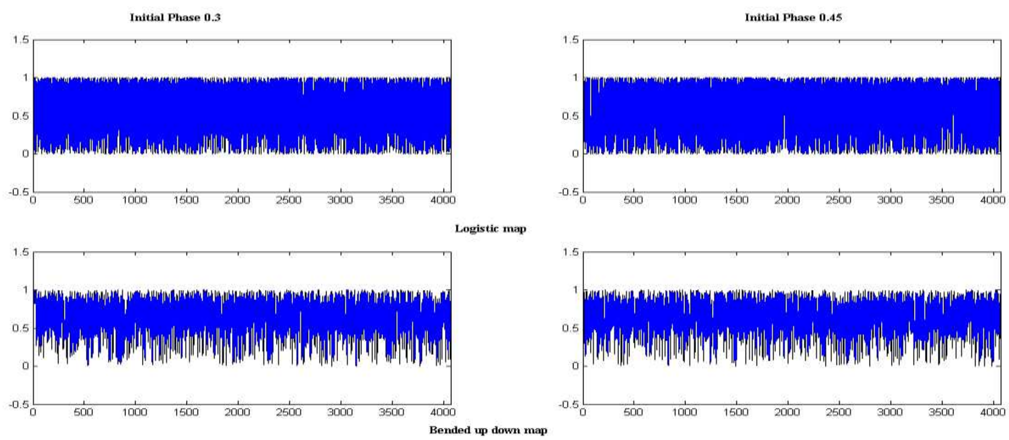

Figure 3: The considered four chaotic signals.

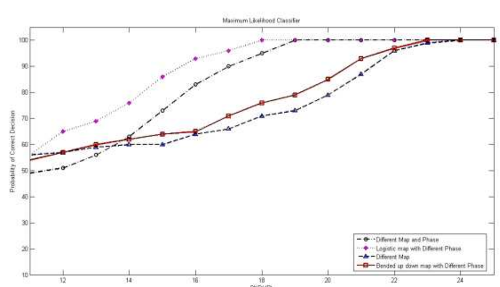

Figure 4: The Performance using ML classifier.

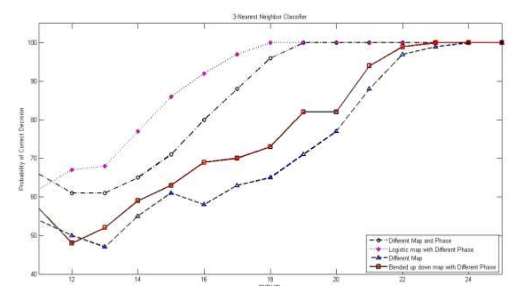

Figure 5: The Performance using 3NN classifier.

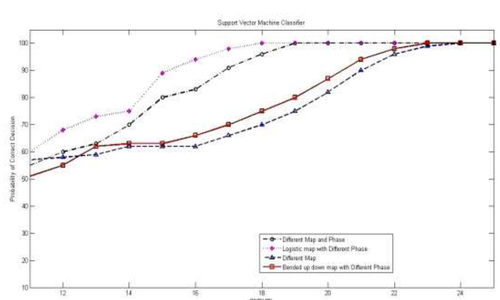

Figure 6: The Performance using NN classifier.

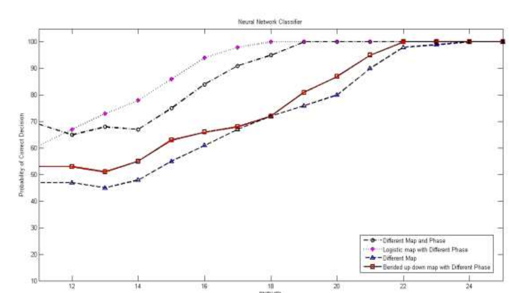

Figure 7: The Performance using SVM classifier.

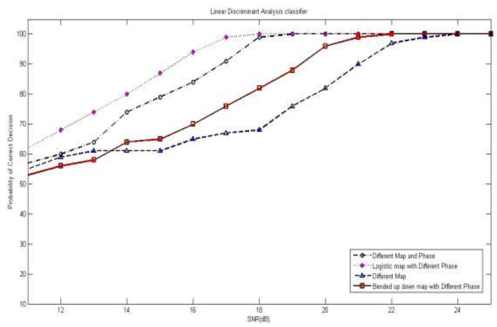

Figure 8: The Performance using LDA classifier.

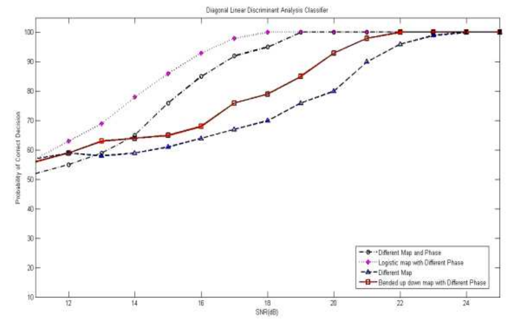

Figure 9: The Performance using diagonal LDA classifier.

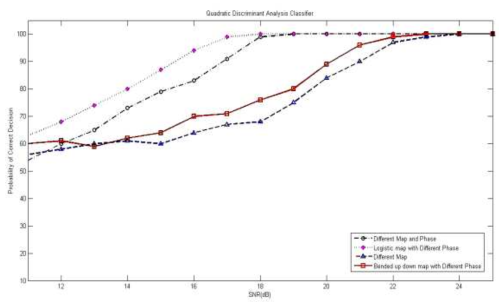

Figure 10: The Performance using QDA classifier.

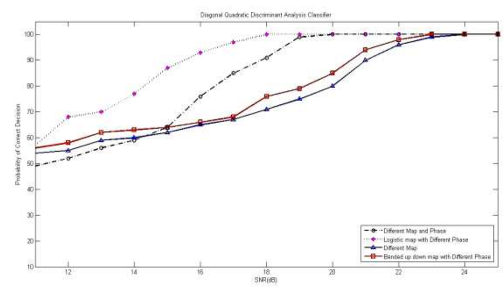

Figure 11: The Performance using diagonal QDA classifier.

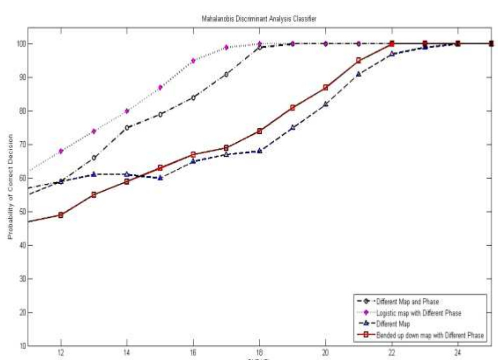

Figure 12: The Performance using Mahalanobis DA classifier.

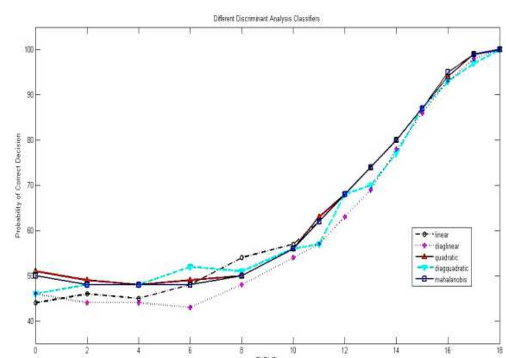

Figure 13: The Performance using different DA classifier.

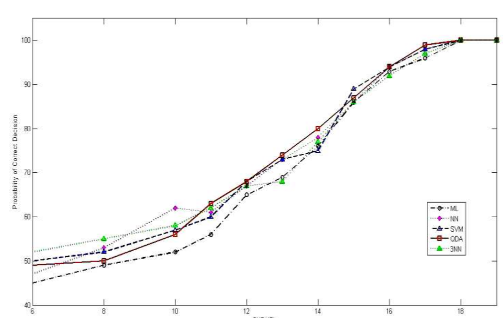

Figure 14: The Performance using different classifiers.

5. CONCLUSION

In this paper, we presented classification of chaotic codes using different classifiers and using eighth orders moment and cumulant as features. We have shown the dependence of the performance on the type of the used classifier, the type of the discriminant analysis classifiers, the type of chaotic maps, the initial values used for generating the chaotic code, and the signal-to-noise ratio.

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