Aharonov-Bohm Control of Optical Properties in System of Parallel Coupled Quantum Wells

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Abstract— In this paper we consider the system of parallel coupled identical one-dimensional quantum wells (such a system can be presented as a multi-arm quantum ring) with a given properties, namely, a width and a depth. Addition of the Aharonov-Bohm flux at origin of such a "ring of quantum wells" allows to change the distance between the bound states as well as their positions just only by changing the magnetic flux. Thereby, the Aharonov-Bohm flux can be considered as a "strong" driven parameter for the optical properties of the system under consideration.

1. INTRODUCTION

In recent time various one-dimensional (quantum wires, quantum rings [1] etc.) and two-dimensional systems (such as graphene [2]) have particular interest in connection with the development of low-dimensional technologies. In particular, such systems is widely used (or proposed to be used) in different fields, such as optics [3], spintronics [4], quantum interferometry [5] etc.. It should be noted that the charge transport process in such structures is of purely quantum nature. A special kind of such a low-dimensional systems are quantum interference devices (such as quantum graphs and quantum rings). Using these devices it is possible to observe the "delicate" quantum effects that are connected with the changes in the electron's wave function phase.

In this work we consider (our consideration is based on the scattering theory which allow to investigate not only the continuous spectrum of charge carriers, but also the discrete spectrum) the system of parallel coupled identical one-dimensional quantum wells (such a system can be considered as a multi-arm quantum ring). State of an electron in such a system has an interesting properties, namely, if one change the number of parallel coupled quantum wells the new bound states in such a system will not appear (as is known, when the quantum wells are arranged in series, i.e., the width of the resulting quantum well increases, there are many bound states in such a system and addition of new wells leads to increasing of the number of bound states) just only shift to the limiting value which determines by the parameters of a single well. In the case of quantum wells with two bound states we have the same result, however the distance between the bound states (it should be noted that the distance between the bound states corresponds to THz frequencies) decreases when the number of wells in a system increases. As a result, the characteristics of such a system, e.g., the optical properties (namely, the frequencies of laser transitions), can be changed by addition of new wells only. Addition of the Aharonov-Bohm flux at origin of such a "ring of quantum wells" allows to change the distance between the bound states as well as their positions just only by changing the magnetic flux. As a result, the Aharonov-Bohm flux can be considered as a "strong" driven parameter for the optical properties of the system under consideration.

2. MAIN FORMALISM

Most of the problems of one-dimensional quantum mechanics can be formulated in terms of quantum graphs. Moreover, the solution of these problems in the frame of the graphs formalism could be more efficient and "elegant" in comparison with the traditional methods of quantum mechanics. A simple and clear example which demonstrates the "elegancy" of the graphs formalism is the problem of the one-dimensional quantum well.

There are various ways to solve the Schrödinger equation in the graph [6–8]. In the presented work we use the *vertex amplitudes* method [9]. The main idea of this method is to express the parameters of the problem through the values of the wave function at the vertex of graph Ψ_1 and Ψ_2. The considered method allows to solve the scattering problem in the quantum graph, to find the energy spectrum and to construct the wave functions. The classical analog of the *vertex amplitudes* method is the Kirchhoff's nodal potentials method in the theory of electrical circuits.

Let us consider the one-dimensional scattering problem for a quantum graph with the potentials that are placed in the graph's edges. We assume that the quasi-one-dimensional dynamics takes place[1]. The considered quantum graph consists of a compact part connected with the reservoirs of the charge carries by the semi-infinite edges. We denote these edges as the *in, out*-edges. In this edges the asymptotic conditions for the electron wave functions with respect to the compact part of the graph are realized.

The wave functions of an electron in the *in, out*-edges are:

$$\psi_{in} = a_{in}\exp(ikx) + b_{in}\exp(-ikx), \tag{1}$$
$$\psi_{out} = a_{out}\exp(ikx) + b_{out}\exp(-ikx), \tag{2}$$

where k is the electron wave-number. As it follows from (1) and (2) the elastic scattering takes place. This assumption facilitates further mathematics.

In each edges γ_n of length l_{γ_n} the proper coordinates are used $\xi_{\gamma_n} \in [0, l_{\gamma_n}]$. In the *in, out*-edges coordinates are defined in a different way: $\xi_{in} \in (-\infty, 0)$, $\xi_{out} \in (0, \infty)$. These coordinates are just a natural parameter in differential geometry [10]. The electron wave function Ψ in the graph is represented by a set of components $\psi_{\gamma_n}(\xi_{\gamma_n})$, $n = 1, 2, \ldots, M$, where M is the number of edges in graph. Each element $\psi_{\gamma_n}(\xi_{\gamma_n})$ of the set is governed by the Schrödinger equation:

$$\begin{aligned} &H_{\gamma_n}\psi_{\gamma_n}(\xi_{\gamma_n}) = \varepsilon_{\gamma_n}\psi_{\gamma_n}(\xi_{\gamma_n}),\\ &H_{\gamma_n} = \left[-\frac{\hbar^2}{2}\frac{d}{d\xi_{\gamma_n}}\left(\frac{1}{m_{\gamma_n}}\frac{d}{d\xi_{\gamma_n}}\right) + V_{\gamma_n}(\xi_{\gamma_n})\right],\\ &D(H_{\gamma_n}) = \left\{\psi_{\gamma_n} : \psi_{\gamma_n} \in C_0^\infty[\gamma_n]\right\}, \end{aligned} \tag{3}$$

where m_{γ_n} is an effective mass of an electron in the edge γ_n, V_{γ_n} is the real and locally measurable potential placed in the edge γ_n.

Let us consider the edge γ_n with a potential which is localized somewhere in this edge. The wave function ψ_{γ_n} can be determined by the following pairs of coefficients: $(a_{\gamma_n}, b_{\gamma_n})$ and $(c_{\gamma_n}, d_{\gamma_n})$. They have the following meaning: the wave function in every edge outside the potential can be taken in the form $a\exp(ikx) + b\exp(-ikx)$, thereby, the pair $(a_{\gamma_n}, b_{\gamma_n})$ are the coefficients of ψ_{γ_n} before the potential and the pair $(c_{\gamma_n}, d_{\gamma_n})$ are the coefficients of ψ_{γ_n} beyond the potential.

An application of the continuity and hermiticity conditions leads to the system of linear algebraic equations for the *vertex amplitudes* $\mathbf{\Psi} = (\Psi_{in} \equiv \Psi_1, \Psi_2, \ldots, \Psi_{out} \equiv \Psi_n)^T$. This system contains the coefficients of functions ψ_{γ_n}. Hence to solve the obtained system for $\mathbf{\Psi}$ it is needed to express a_{γ_n}, b_{γ_n}, c_{γ_n} and d_{γ_n} in terms of $(\Psi_1, \Psi_2, \ldots, \Psi_n)$. In order to do this we use the following obvious relations:

$$\begin{pmatrix} a_{\gamma_n} \\ b_{\gamma_n} \end{pmatrix} = \mathbf{\Gamma}^{(1,\gamma_n)} \begin{pmatrix} \Psi_n \\ \Psi_{n+1} \end{pmatrix}, \quad \begin{pmatrix} c_{\gamma_n} \\ d_{\gamma_n} \end{pmatrix} = \mathbf{\Gamma}^{(2,\gamma_n)} \begin{pmatrix} \Psi_n \\ \Psi_{n+1} \end{pmatrix}. \tag{4}$$

The pairs of coefficients $(a_{\gamma_n}, b_{\gamma_n})$ and $(c_{\gamma_n}, d_{\gamma_n})$ are related by the transfer-matrix $\mathbf{M}$ [11]:

$$\begin{pmatrix} a_{\gamma_n} \\ b_{\gamma_n} \end{pmatrix} = \mathbf{M}^{(\gamma_n)} \begin{pmatrix} c_{\gamma_n} \\ d_{\gamma_n} \end{pmatrix}. \tag{5}$$

The elements of the $\mathbf{M}$-matrix are

$$\mathbf{M}^{(\gamma_n)} = \begin{pmatrix} 1/t_{\gamma_n} & r_{\gamma_n}/t_{\gamma_n} \\ r^*_{\gamma_n}/t^*_{\gamma_n} & 1/t^*_{\gamma_n} \end{pmatrix}, \tag{6}$$

$t_{\gamma_n}(r_{\gamma_n})$ is the transmission (reflection) amplitude for the potential placed in the edge γ_n. Using the relations (4), (5) and hermiticity condition one get:

$$\begin{aligned} &\mathbf{\Gamma}^{(1,\gamma_n)} = \frac{1}{\zeta - \zeta^*} \begin{pmatrix} \zeta & -1 \\ -\zeta^* & 1 \end{pmatrix},\\ &\text{where} \quad \zeta = \exp(-ik\xi_{v_{n+1}})\mathrm{M}_{11}^{(\gamma_n)} - \exp(ik\xi_{v_{n+1}})\mathrm{M}_{12}^{(\gamma_n)}. \end{aligned} \tag{7}$$

[1]Let us note, that the quasi-one-dimensional dynamics may be realized in the lowest sub-band of a very narrow quantum wire. The wire width in the real experiments can not be infinitely narrow. But, for the quantum wire under low temperature the electron dynamics is quasi-one-dimensional because the higher transverse levels can not be excited.

From (4), (5) it follows

$$\mathbf{\Gamma}^{(2,\gamma_n)} = \left[\mathbf{M}^{(\gamma_n)}\right]^{-1}\mathbf{\Gamma}^{(1,\gamma_n)}. \tag{8}$$

Finally, (4), (7), (8) together with the hermiticity condition lead to the system of linear equation for unknown vector $\mathbf{\Psi}$. Finding the $\mathbf{\Psi}$ solves the transport problem for graph because the wave functions ψ_{in}, ψ_{out} satisfy the following conditions at the *in,out*-vertices:

$$1 + r(k) = \Psi_{in}, \quad t(k) = \Psi_{out}. \tag{9}$$

3. PARALLEL COUPLED QUANTUM WELLS UNDER AHARONOV-BOHM EFFECT

Using the presented technique, let us consider the interesting system of n parallel coupled quantum wells (for details see [9]). It is interesting to note that if one change the number of quantum wells in such a system the new bound states will not appear (as is known, when the quantum wells are arranged in series, i.e., the width of the resulting quantum well increases, there are many bound states in such a system and addition of new wells leads to the fact that the number of bound states increases) just only shift to the limiting value which determined by the parameters of single well (in the case of identical wells). The problem becomes more complicated if at the center of such a ring-like system there is an infinitely thin solenoid carrying finite magnetic flux Φ (see the Fig. 1). At the same time it is a standard situation in the quantum transport [12].

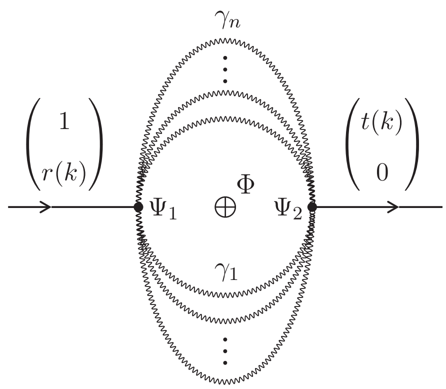


Figure 1: Graph that corresponds to the scattering problem in the system of n parallel coupled quantum wells with the AB flux Φ. The wavy lines represent the potential (in the considered case the potential is the rectangular quantum well).

The magnetic flux modifies the phase of the wave function in the ring[2] while the *in, out*-wave functions are still the same, namely:

$$\begin{aligned}
\psi_{in} &= \exp(ikx) + r(k)\exp(-ikx),\\
\psi_{out} &= t(k)\exp(ikx),\\
\psi_{\gamma_1} &= a_{\gamma_1}\exp(ik^{-}x) + b_{\gamma_1}\exp(-ik^{+}x),\\
\psi_{\gamma_2} &= a_{\gamma_2}\exp(ik^{+}x) + b_{\gamma_2}\exp(-ik^{-}x),
\end{aligned} \tag{10}$$

where $k^- = k - \alpha$, $k^+ = k + \alpha$, $\alpha = \Phi/(\Phi_0 L)$, Φ is the magnetic flux through the loop section area, $\Phi_0 = 2\pi\hbar c/e$ and L is the ring's length.

The explicit expression for the transmission amplitude (in the case of identical wells) can be obtained in the following form[3] (using the technique of graphs [9]):

$$t_n(k) = \frac{4inkq\sin(ql)\cos\left(\frac{\alpha}{2n}\right)}{(2nq)^2\cos^2\left(\frac{\alpha}{2n}\right) + k^2\sin^2(ql)\left(1 + 2inq\cot(ql)\right)^2},$$

where $q = \sqrt{k^2 - V}$, $V = 2m_e U/\hbar^2$, U is the well's depth and l is the well's width, n is a number of quantum wells in a system, α is a dimensionless AB flux. As is known the energies of bound

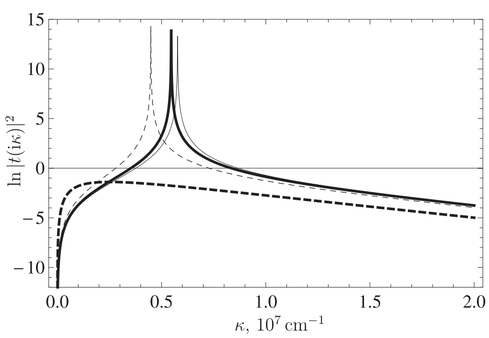
[2]Equation (10) is presented for the case of two-arm ring, for the multi-arm ring the result can be obtained by a similar way.

[3]Here we would like to note that the graph ideology allows to obtain the transmission coefficient not only in the case of identical wells, but in this case only numerical results take place.

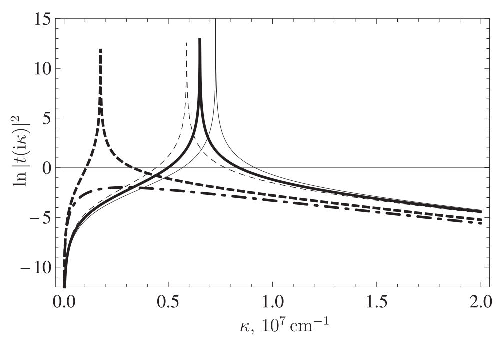
states correspond to simple poles of the transmission amplitude. In order to determine the bound states energies $E_{bound} = -\hbar^2\kappa^2/2m_e$ we construct the analytic continuation of the transmission amplitude $t(i\kappa)$.

In the Fig. 2, we present the results of numerical simulations for the logarithm of the transmission coefficient $\ln|t(i\kappa)|^2$ which determines the bound states positions in the system under consideration.

We see that the presence of the AB flux leads to the fact that the bound state position in such a system becomes dependent on the value of AB flux. As a result also the optical properties that determined by the bound states positions become dependent on the AB flux. Namely for some values of the AB flux the bound state in such a system will not appear due to interference effects. In this paper we present the results for the case when the single well contains single bound state. In the case when the quantum well contains two bound states the positions of these states together with the distance between them depends not only on the number of wells in the system, but also on the value of AB flux (it should be noted also that the distance between the bound states corresponds to THz frequency domain, so the considered system seems promisingly and perspective in connection with development of modern low-dimensional optics). We can conclude that the number of wells in such a system together with the AB flux can be considered as driven parameters for the bound states positions as well as for the distance between the bound states. These facts open a new way for control of the optical properties of ring-like structures by addition of extra wells in the system, as well as by changing of the magnetic flux through the system.



(a)



(b)

Figure 2: Logarithm of the transmission coefficient $\ln|t(i\kappa)|^2$ for various AB fluxes α in the case of two coupled wells ((a) $n = 2$) and four coupled wells ((b) $n = 4$) as a function of the wave number κ ($k = i\kappa$). (a) Thin solid line is $\alpha = 0$, thick solid line is $\alpha = 0.5$, thin dashed line is $\alpha = 1$, thick dashed line is $\alpha = 2$; (b) Thin solid line is $\alpha = 0$, thick solid line is $\alpha = 1.5$, thin dashed line is $\alpha = 2$, thick dashed line is $\alpha = 3.5$, thick dot dashed line is $\alpha = 4$. The parameters of wells are $U = -0.5\,\mathrm{eV}$ and $l = 10^{-7}$ cm.

4. CONCLUSIONS

In this paper we have considered a system of parallel coupled quantum wells with the AB flux at origin of such a system. In the case of similar quantum wells the explicit analytic expression for the transmission amplitude is observed and numerically analyzed. In particular, it is shown numerically that changing of wells number in the system (in the the case of identical wells) does not lead to appearance of a new bound states. However, the addition of new wells leads to the fact that the bound states positions in such a system shift to some limiting value determined by the characteristics of a single well. Addition of the AB flux at origin of such a system leads to the fact that the bound states position become dependent on its values. Moreover, for some values of the AB flux the bound states in such a system will not appear due to quantum interference effects. Thus, we can conclude that the number of quantum wells in the system under consideration together with the AB flux can be considered as driven parameters for the bound states positions as well as for the distance between the bound states. As a result, the optical properties of such a system (frequencies of laser transitions) depend on the number of well as well as on the AB flux. These facts open a new way for control of the optical properties of ring-like structures with quantum wells by addition of extra wells in the system, as well as by changing of the magnetic flux through the system.

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