# Investigation on Slow Light of Nonuniform Photonic Crystal Fiber Bragg Gratings

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**Abstract**— Based on the coupled-mode theory and transmission matrix method, the influences of Gaussian apodization coefficient and linear chirp coefficient on group velocity of photonic crystal fiber Bragg gratings are demonstrated. The results indicate that the minimum value of group velocity first decreases and then increases with the increase of apodization coefficient when the 'dc' index change value is big enough, which means that the minimum value of group velocity can be optimized at a certain apodization coefficient. But when the 'dc' index change value is very small, the minimum value of group velocity increases with the increase of apodization coefficient. A good linear area of the group velocity can be obtained by linear chirp and the linear area can be broadened by increasing the absolute value of chirp coefficient.

## 1. INTRODUCTION

In recent years, the researches on controllable slow light and its applications to optical memories, optical buffers and optical sensors have become the frontier and hot issue in optical field [1–4]. As fiber grating slow light technology has many advances, such as simple experiment conditions, easy modularization and distortionless transmission pulse [5], it has an promising future in practical applications. In 2005, a theoretical and experimental analysis of group velocity reduction in periodic superstructure Bragg gratings was presented by D. Janner [6] et al. Experimental demonstration of group velocity reduction of sub-nanosecond pulses at the 1.5 μm wavelength of optical communications for the first time was reported using a moiré fiber grating. And in 2012, He Wen [7] et al. showed theoretically for the first time that an FBG with a large index modulation and an optimized length and apodization can support light with a much lower group velocity than previously anticipated. But they did not show how does the apodization influence the group velocity of photonic crystal fiber Bragg gratings (PCFBGs).

Compared with ordinary fiber gratings, PCFGs have many advances [8–11], such as larger design freedom, endless single-mode transmission and adjustable dispersion characteristics. Therefore, PCFGs have wide potential applications in Optical fiber communication, optical fiber sensing and optical information processing field. In 2012, Bi Weihong et al. [12] theoretically analyzed the reflection spectrum characteristics of grapefruit-type photonic crystal fiber chirped grating by finite element method and transfer matrix method. The results indicated that both chirped parameters and photonic crystal fiber (PCF) parameters had influences on the reflection spectrum characteristics. The emergence of PCFG provides a new research direction for fiber grating slow light technology.

In this paper, the influences of gaussian apodization coefficient and linear chirp coefficient on slow light for PCFBGs are numerically simulated by improved full-vectorial effective index method and transfer matrix method. The paper is organized as follows: In Section 2, we show the convergence of two theoretical frameworks, the improved full-vectorial effective index method and transfer matrix method. In Section 3, detailed simulation results are presented; in Section 4, we discuss their merits and demerits.

## 2. THEORY MODEL

### 2.1. Improved Full-vectorial Effective Index Method

Improved full-vectorial effective index method can be used to simply, rapidly and accurately analyze the mode characteristics of PCF [13–15]. Its fundamental principle is as follows: first, suppose the cladding region of PCF as hexagon two-dimensional photonic crystal structure with numerous periodically spaced airholes and without central defects. Next, substitute hexagon unit cells with circular unit that has the equal area as the former. Then, obtain the effective index of the cladding fundamental mode by the vector theory of electromagnetic propagation. At last, PCF is equivalent to conventional step fiber with an effective core radius.

### 2.2. Transfer Matrix Method

Gaussian apodization function can be expressed as

$$\Delta n(z) = \Delta n \exp[-G(z/L)^2] \tag{1}$$

where $\Delta n$ is the peak value of the 'dc' effective index change, $G$ is the apodization coefficient and $L$ is the length of fiber Bragg grating.

Chirped fiber Bragg grating is a kind of fiber grating with inhomogeneous period that changes with the position $z$. The periods of linear chirped fiber gratings can be expressed as

$$\Lambda(z) = \frac{\Lambda_0}{1 + Cz} \tag{2}$$

where $\Lambda_0$ is the period at the centre of fiber Bragg grating and $F$ is is the linear chirp coefficient.

The transfer matrix method to model nonuniform gratings is based on identifying $2 \times 2$ matrices for each uniform section of the grating. And then multiplying all of these together to obtain a single $2 \times 2$ matrix that describes the whole grating [16]. We divide the grating into $M$ uniform sections and define $R_i$ and $S_i$ to be the field amplitudes after traversing the section $i$. The propagation through each uniform section $i$ is described by a matrix $F_i$ defined such that

$$\begin{bmatrix} R_i \\ S_i \end{bmatrix} = F_i \begin{bmatrix} R_{i-1} \\ S_{i-1} \end{bmatrix} \tag{3}$$

$F_i$ is the transmission matrix of $i$th section and can be expressed as

$$F_i = \begin{bmatrix} \cosh(\gamma_B \Delta z) - i\frac{\sigma}{\gamma_B}\sinh(\gamma_B \Delta z) & -i\frac{\kappa}{\gamma_B}\sinh(\gamma_B \Delta z) \\ i\frac{\kappa}{\gamma_B}\sinh(\gamma_B \Delta z) & \cosh(\gamma_B \Delta z) + i\frac{\sigma}{\gamma_B}\sinh(\gamma_B \Delta z) \end{bmatrix} \tag{4}$$

where $\Delta z$ is the length of $i$th section fiber grating, the 'ac' coupling coefficient $\kappa$ and 'dc' self-coupling coefficient $\sigma$ are the local values in the $i$th sectionand $\gamma_B = \sqrt{(\kappa^2 - \sigma^2)}$. Once all of the matrices for the individual sections are known, we find the output amplitudes from

$$\begin{bmatrix} R_M \\ S_M \end{bmatrix} = F_M F_{M-1} \cdots F_1 \begin{bmatrix} R_0 \\ S_0 \end{bmatrix} \tag{5}$$

For Bragg gratings we start with the boundary conditions $R_0 = R(L/2) = 1$ and $S_0 = S(L/2) = 0$, and the amplitude reflection coefficient $\rho = S_M/R_M$ can be obtained. Then the group delay can be shown as

$$\tau = \frac{d\theta}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{d\theta}{d\lambda} \tag{6}$$

where $\theta = \arctan(\rho)$ is the phase of reflection coefficient $\rho$.

So we can derive the relationship between the group velocity and the group delay

$$v_g = \frac{c}{n_g} = \frac{L}{\tau} \tag{7}$$

From the above equation we can conclude that adding the group delay can decrease the group velocity when the length of fiber grating is constant.

## 3. NUMERICAL SIMULATION AND RESULT DISCUSSION

### 3.1. Influences of Gaussian Apodization on PCFBG Slow Light

In the simulation, the cladding of PCFG is composed of hexagonal arrangement airholes and germanium is doped in fiber core. We assume that germanium-doped fiber core does not affect the characteristics of the waveguide and PCF meets the single-mode transmission conditions [9, 13] (air filling rate $d/A < 0.4$). The refractive index of air $n_0 = 1$ and the refractive index of silica can be computed by Sellmeier equation [17].

The parameters are chosen as: diameter of airholes $d = 0.616\,\mu\text{m}$, the pitch of airholes $A = 2.2\,\mu\text{m}$, the length of grating $L = 1\,\text{cm}$, the period of grating $\Lambda = 0.5426\,\mu\text{m}$, the fringe of the visibility of the index change $\nu = 1$, the 'dc' index change $\Delta n = 0.0006$ and the apodization coefficient $G = 8$. The simulation result is shown in Fig. 1. Compared with the group velocity spectra of the uniform fiber gratings, apodization can efficiently restrain the side lobes in the long wavelength area, which means that the minimum values of group velocity can only appears in the short wavelength area. In Fig. 1, $V$ represents the rato of the minimum value of group velocity $V_g$ and the vacuum speed of light $c$. In the following simulations, we replace the minimum value of group velocity with $V$. To demonstrate the effects of apodization coefficient $G$ on the minimum value of group velocity.

The influences of gaussian apodization coefficient $G$ on the minimum value of group velocity are detailedly described in Fig. 2. Parameters for Fig. 2 are same to that for Fig. 1 except apodization coefficient $G$. In Fig. 2, the three selected apodization coefficients are all in the are all in the range that the minimum value of group velocity decreases with the increase of apodization coefficient. From Fig. 2, the minimum value of group velocity not only decreases but also moves to smaller wavelength with the increase of apodization coefficient. The wavelength where the minimum value of group velocity appears becomes smaller with the increase of apodization coefficient $G$ . At the same time, the full width at half maximum of the minimum group velocity peak increases with the decrease of minimum value of group velocity.

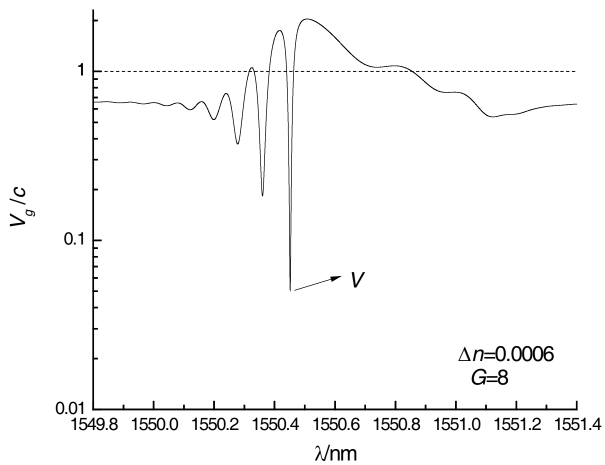


Figure 1: Group velocity spectra for PCF apodized grating.

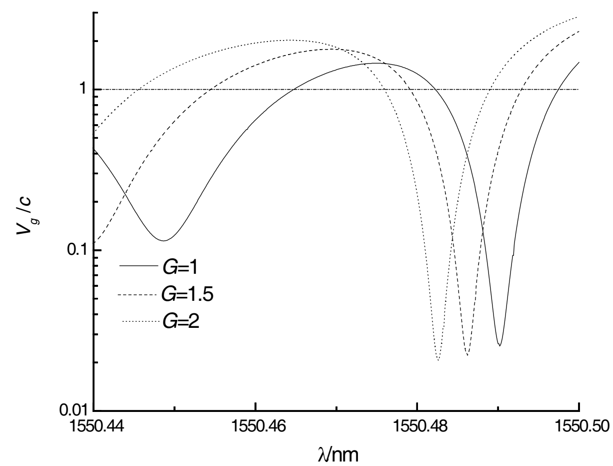


Figure 2: Group velocity spectra for PCF apodized gratings when $G$ is 1, 1.5 respectively.

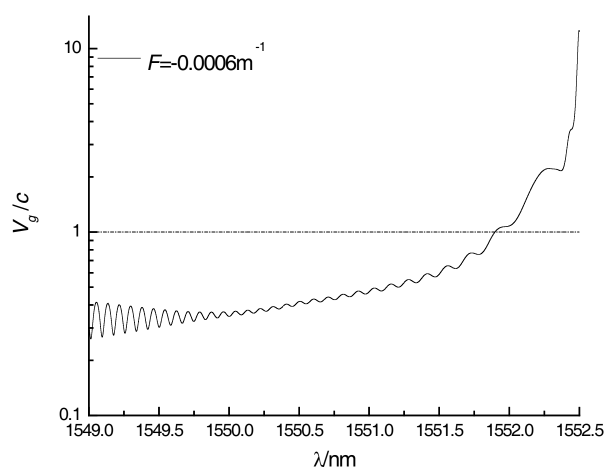


Figure 3: Group velocity spectra for PCF chirped gratings for $F = -0.0006\,\mathrm{m}^{-1}$.

### 3.2. Influences of Linear Chirp on PCFBG Slow Light

To demonstrate the effects of linear chirp on slow light, Fig. 3 shows the group velocity spectra for PCFBGs similar to that described in Fig. 1, only here PCFBGs have linear chirp coefficients $F$ as described by Eq. (2) and have not apodization coefficient $G$. In Fig. 3, the linear chirp coefficient is chosen as $F = -0.0006\,\mathrm{m}^{-1}$. Compared with Fig. 1, the minimum value of group velocity is much bigger, but the group velocity increases linearly with the increase of wavelength. And there is a band that has good linear characteristics.

## 4. CONCLUSION

For PCFBG, parameters of PCF, apodization and chirp have different influences on its characteristics of slow light. Appropriate gaussian apodization can not only decrease the minimum value of group velocity but also can adjust the full width at half height of the minium group velocity peak; although chirp can not make the minimum value of group velocity smaller, it generates a good linear area in the group velocity spectra. On this basis, the influences of different apodization functions and nonlinear chirp on slow light for PCFBGs can be discussed under this method.

## ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China (Nos. 61367007, 61167005), in part by the Natural Science Fund of Gansu Province of China (Nos. 1112RJZA017, 1112RJZA018) and in part by the Research Fund for the Doctoral Program of Lanzhou University of Technology.

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