Influence of an Intense Electromagnetic Wave on Magnetoconductivity and Hall Coefficient in Compositional Semiconductor Superlattices: Optical Phonon Interaction

B. D. Hoi[1,2], **H. V. Ngoc**[1], **and N. Q. Bau**[1]

[1]Department of Physics, College of Natural Science, Vietnam National University, Hanoi, Vietnam
[2]Department of Physics, National University of Civil Engineering, Hanoi, Vietnam

Abstract— We theoretically study the influence of an intense electromagnetic wave (EMW) on the Hall effect in compositional semiconductor superlattices (CSSLs) with a periodical superlattice potential in the z direction, subjected to a crossed dc electric field $\vec{E}_1 = (E_1, 0, 0)$ and magnetic field $\vec{B} = (0, 0, B)$. By considering the electron-optical phonon interaction at high temperature, we obtain analytical expressions for the magnetoconductivity (MC) and the Hall coefficient (HC) with a dependence on external fields, the temperature of the system, and characteristic parameters of the CSSL. These expressions are fairly different in comparison to those obtained for bulk semiconductors. The influence of the EMW is interpreted by using the dependencies of the MC and the HC on the amplitude and the frequency of the EMW. The analytical results are computationally evaluated for GaAs/AlGaAs CSSL. The optically detected magnetophonon resonance conditions are obtained. The HC reaches saturation when the magnetic field or the EMW frequency increases.

1. INTRODUCTION

The propagation of an electromagnetic wave (EMW) in materials leads to some interesting effects that can be applied experimentally to determine materials information. For example, the cyclotron effect, magneto-phonon-photon resonance, optically detected electron-phonon resonance, and so on. The Hall effect in bulk semiconductors under the influence of EMWs has been studied in much details [1–5]. In Refs. [1, 2] the odd magnetoresistance was calculated when the nonlinear semiconductors are subjected to a magnetic field and an EMW with low frequency, the nonlinearity is resulted from the nonparabolicity of distribution functions of carriers. In Refs. [3, 4], the magnetoresistance was derived in the presence of a strong EMW for two cases: the magnetic field vector and the electric field vector of the EMW are perpendicular [3], and are parallel [4]. The existence of the odd magnetoresistance was explained by the influence of the strong EMW on the probability of collision, i.e., the collision integral depends on the amplitude and frequency of the EMW. This problem was also studied in the presence of both low and high frequency EMWs [5]. Throughout these problems, the quantum kinetic equation method have been seen as a powerful tool. In low-dimensional semiconductor systems, the carrier confinement leads to unusual behaviors in comparison with bulk semiconductors under external stimuli. So, in recent works, we have used this method to study the influence of an intense EMW on the Hall effect in rectangular and parabolic quantum wells [6, 7], in doped semiconductor superlattices [8].

In this work, by using the quantum kinetic equation method we study the Hall effect in a compositional semiconductor superlattice (CSSL) subjected to a crossed dc electric field and magnetic field in the presence of an EMW. We only consider the case of high temperature when the electron-optical phonon interaction is assumed to be dominant and electron gas is nondegenerate. We derive analytical expressions for the conductivity tensor and the Hall coefficient (HC) taking account of arbitrary transitions between the energy levels. The analytical result is numerically evaluated and graphed for the GaAs/AlGaAs CSSL.

2. MODEL OF THE PROBLEM AND ANALYTICAL RESULTS

In this model, we consider a compositional semiconductor superlattice composed of N_0 layers of the semiconductor I (the layer thickness is d_I) and N_0 layers of the semiconductor II (the layer thickness is d_{II}) arranged alternatively along the z-direction. The energy-gap difference between these two materials is U. If a static magnetic field $\vec{B}$ is applied in the z-direction and a dc electric field $\vec{E}_1$ is applied in the x-direction, then the one-electron normalized eigenfunctions and eigenvalues in the Landau gauge for the vector potential $\vec{A} = (0, Bx, 0)$ are, respectively, given by [9, **?**]

$$|\xi\rangle \equiv |N, n, k_y, k_z\rangle = \frac{1}{\sqrt{L_y}} \exp(ik_y y)\phi_N(x - x_0) \otimes |n, k_z\rangle, \quad (1)$$

$$\varepsilon_\xi\left(\vec{k}_y\right) \equiv \varepsilon_{N,n,k_z}\left(\vec{k}_y\right) = \left(N + \frac{1}{2}\right)\hbar\omega_c + \varepsilon_{n,k_z} - \hbar v_d k_y + \frac{1}{2}v_d^2, \quad N, n = 0, 1, 2, \ldots, \quad (2)$$

where N is the Landau level index and n denotes level quantization in the z-direction; k_y (k_z) and L_y are the wave vector and normalization length in the y (z)-direction, respectively; $v_d = E_1/B$ being the drift velocity and $\omega_c = eB/m_e$ is the cyclotron frequency in which e is the charge of a conduction electron and m_e is its effective mass. Also, $\phi_N(x)$ represents harmonic oscillator wave functions, centered at $x_0 = -\ell_B^2(k_y - m_e v_d/\hbar)$ where $\ell_B = (\hbar/(m_e\omega_c))^{1/2}$ is the radius of the Landau orbit in the x-y plane; $|n, k_z\rangle$ is the wave function in the z-direction. Moreover, in the tight-binding approximation we have [9, 11]

$$\varepsilon_{n,k_z} = \varepsilon_n - t_n \cos(k_z d), \quad (3)$$

where $d = d_I + d_{II}$ is the superlattice period, $\varepsilon_n = \hbar^2\pi^2(n+1)^2/(2m_e d_I^2)$, t_n is the half-width of the nth mini-band given by [12, 13]

$$t_n = -4(-1)^n \frac{d_I}{d - d_I}\varepsilon_n \frac{\exp(-2\sqrt{2m_e(d - d_I)^2 U/\hbar^2})}{\sqrt{2m_e(d - d_I)^2 U/\hbar^2}}. \quad (4)$$

If the specimen is subjected to an intense EMW with the electric field vector $\vec{E} = (0, E_0 \sin\omega t, 0)$ (E_0 and ω are the amplitude and frequency, respectively), the Hamiltonian of the electron-phonon system, in the second quantization representation, can be written similarly to the one obtained in Refs. [6–8]. From this Hamiltonian, the Hall conductivity and the HC are derived in the single (constant) relaxation time approximation. In this calculation, we also consider only the electron-optical phonon interaction at high temperature, so the electrons system is nondegenerate and obeys the Boltzmann distribution function. After some manipulation, we obtain the expression for the conductivity tensor as

$$\sigma_{im} = \frac{e^2\tau}{\hbar(1+\omega_c^2\tau^2)}\left(\delta_{ij} - \omega_c\tau\epsilon_{ijk}h_k + \omega_c^2\tau^2 h_i h_j\right)\left\{a\delta_{jm} + b\delta_{jl}\left(\delta_{lm} - \omega_c\tau\epsilon_{lmp}h_p + \omega_c^2\tau^2 h_l h_m\right)\right\}, \quad (5)$$

where δ_{ij} is the Kronecker delta; ϵ_{ijk} being the antisymmetric Levi-Civita tensor; the Latin symbols i, j, k, l, m, p stand for the components x, y, z of the Cartesian coordinates;

$$a = -\frac{\hbar\beta v_d L_y I}{2\pi m_e}\sum_{N,n} e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \quad (6)$$

with ε_F is the Fermi level; and

$$b = \frac{\beta A N_0 L_y I}{8\pi^2 m_e^2}\frac{\tau}{1+\omega_c^2\tau^2}\sum_{N,N'}\sum_{n,n'} I(n,n')\left(b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8\right), \quad (7)$$

$$b_1 = \frac{1}{M}\frac{eB\bar{\ell}}{\hbar}e^{\beta(\varepsilon_F - \varepsilon_{N,n})}\left[\frac{(N+M)!}{N!}\right]^2 \delta(X_1),$$

$$b_2 = -\frac{\theta}{2}\left(\frac{eB\bar{\ell}}{\hbar}\right)^2 b_1,$$

$$b_3 = \frac{\theta}{4M}\left(\frac{eB\bar{\ell}}{\hbar}\right)^3 e^{\beta(\varepsilon_F - \varepsilon_{N,n})}\left[\frac{(N+M)!}{N!}\right]^2 \delta(X_2),$$

$$b_4 = \frac{\theta}{4M}\left(\frac{eB\bar{\ell}}{\hbar}\right)^3 e^{\beta(\varepsilon_F - \varepsilon_{N,n})}\left[\frac{(N+M)!}{N!}\right]^2 \delta(X_3),$$

$$
\begin{aligned}
b_5 &= \frac{1}{M}\frac{eB\bar{\ell}}{\hbar}e^{\beta(\varepsilon_F-\varepsilon_{N,n})}\left[\frac{N!}{(N+M)!}\right]^2\delta\left(X_4\right),\\
b_6 &= -\frac{\theta}{2}\left(\frac{eB\bar{\ell}}{\hbar}\right)^2 b_5,\\
b_7 &= \frac{\theta}{4M}\left(\frac{eB\bar{\ell}}{\hbar}\right)^3 e^{\beta(\varepsilon_F-\varepsilon_{N,n})}\left[\frac{N!}{(N+M)!}\right]^2\delta\left(X_5\right),\\
b_8 &= \frac{\theta}{4M}\left(\frac{eB\bar{\ell}}{\hbar}\right)^3 e^{\beta(\varepsilon_F-\varepsilon_{N,n})}\left[\frac{N!}{(N+M)!}\right]^2\delta\left(X_6\right), \quad M=|N-N'|=1,2,3,\ldots,\\
X_1 &= (N'-N)\hbar\omega_c+\varepsilon_{n',\pi/d}-\varepsilon_{n,0}-eE_1\bar{\ell}-\hbar\omega_0, \quad X_2=X_1+\hbar\Omega, \quad X_3=X_1-\hbar\Omega,\\
X_4 &= (N-N')\hbar\omega_c+\varepsilon_{n',\pi/d}-\varepsilon_{n,0}+eE_1\bar{\ell}+\hbar\omega_0, \quad X_5=X_4+\hbar\Omega, \quad X_6=X_4-\hbar\Omega,
\end{aligned}
$$

where ω_0 is the frequency of the longitudinal optical phonons, $\beta = 1/(k_BT)$ with k_B being Boltzmann constant, $\theta = e^2E_0^2/(m_e^2\Omega^4)$, $A = 2\pi e^2\hbar\omega_0(\chi_\infty^{-1}-\chi_0^{-1})/\epsilon_0$, $\bar{\ell}=(\sqrt{N+1/2}+\sqrt{N+1+1/2})\ell_B/2$, $I = a_1(\alpha\beta)^{-1}[\exp(\alpha\beta a_1)+\exp(-\alpha\beta a_1)]-(\alpha\beta)^{-2}[\exp(\alpha\beta a_1)-\exp(-\alpha\beta a_1)]$, $a_1 = L_x/2\ell_B^2$, $\alpha = \hbar v_d$, $\varepsilon_{N,n} = (N+1/2)\hbar\omega_c+\varepsilon_{n,0}+m_ev_d^2/2$, and

$$
\begin{aligned}
I(n,n') &= \int_{-\infty}^{\infty}|I_{n,n'}(0,\pi/d,q_z)|^2dq_z, && (8)\\
I_{n,n'}(k_z,k_z',q_z) &= \frac{1}{2}\frac{\sin\{[q_z\pm(k_{n'}\pm k_n)]d_I/2\}}{[q_z\pm(k_{n'}\pm k_n)]d_I/2}\exp\{i[q_z\pm(k_{n'}\pm k_n)]d_I/2\}, && (9)
\end{aligned}
$$

where $k_n = (2m_e\varepsilon_{n,k_z}/\hbar^2)^{1/2}$.

The divergence of the delta functions is avoided by replacing them by the Lorentzians as [14]

$$\delta(X)=\frac{1}{\pi}\left[\frac{\Gamma}{X^2+\Gamma^2}\right], \tag{10}$$

where Γ is the damping factor associated with the momentum relaxation time τ by $\Gamma \approx \hbar/\tau$. The HC is given by the formula [15]

$$R_H=\frac{\rho_{yx}}{B}=-\frac{1}{B}\frac{\sigma_{yx}}{\sigma_{yx}^2+\sigma_{xx}^2}, \tag{11}$$

where σ_{yx} and σ_{xx} are given by Eq. (5).

Equations (5) and (11) show the complicated dependencies of the conductivity tensor and the HC on the external fields, including the EMW. It is obtained for arbitrary values of the indices N, n, N' and n'. In the next section, we will give a deeper insight into these results by carrying out a numerical evaluation and a graphic consideration by the computational method.

3. NUMERICAL RESULTS AND DISCUSSION

In this section, we present detailed numerical calculations of the conductivity and the HC for the GaAs/AlGaAs CSSL with the parameters [16]: $\varepsilon_F = 50\,\text{meV}$, $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $\hbar\omega_0 = 36.25\,\text{meV}$ (optical phonon frequency), and $m_e = 0.067\times m_0$ (m_0 is the mass of a free electron). For the sake of simplicity, we also choose $N = 0$, $N' = 1$, $n = 0$, $n' = 0 \div 1$, $\tau = 10^{-12}\,\text{s}$, and $L_x = L_y = 100\,\text{nm}$.

In Figure 1, the solid curve describes the dependence of the magnetoconductivity on the cyclotron energy in the case of absence of the EMW. We can see that this curve has three maximum peaks and the values of conductivity at the peaks are very much larger than they are at others. Physically, the existence of the peaks can be explained in details as follows by using the computational method to determine their positions. All the peaks correspond to the conditions

$$\left(N'-N\right)\hbar\omega_c=\hbar\omega_0+eE_1\bar{\ell}\pm\Delta_{n,n'}, \quad \Delta_{n,n'}=\varepsilon_{n',\pi/d}-\varepsilon_{n,0}. \tag{12}$$

This condition is generally called the intersubband magnetophonon resonance (MPR) condition under the influence of an dc EF (all the peaks now may be called resonant peaks). However, the values of the term $eE_1\bar{\ell}$ are very small in comparison to the optical phonon energy and can be neglected. For instance, if we take $B = 20\,\mathrm{T}$ (approximately $\hbar\omega_c = 34.59\,\mathrm{meV}$), then $eE_1\bar{\ell} = 0.0277\,\mathrm{meV}$.

The dashed curve in Figure 1 shows the dependence of on the cyclotron energy in the presence of a strong EMW. It is seen that besides the main resonant peaks as in the case of absence of the EMW, the subordinate peaks appear. The appearance of the subordinate peaks is due to the contribution of a photon absorption/emission processes that satisfy the conditions $\hbar\omega_c = \hbar\omega_0 \pm \Delta_{0,1} \pm \hbar\Omega$. It is also seen that the main peaks are much higher than the subordinate peaks. This means that the possibility of processes without photon is much larger than it is for the processes with one photon absorption/emission. The above conditions with the presence of the EMW are actually the optically detected MPR conditions.

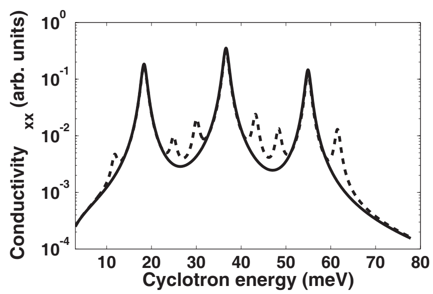

Figure 1: The magnetoconductivity (arb. units) as functions of the cyclotron energy for two cases: absence (solid line) and presence (dashed line) of the EMW. Here, $E_1 = 5\times10^3\,\mathrm{V\cdot m^{-1}}$, $E_0 = 10^5\,\mathrm{V\cdot m^{-1}}$, $d = 35\,\mathrm{nm}$, and $T = 270\,\mathrm{K}$.

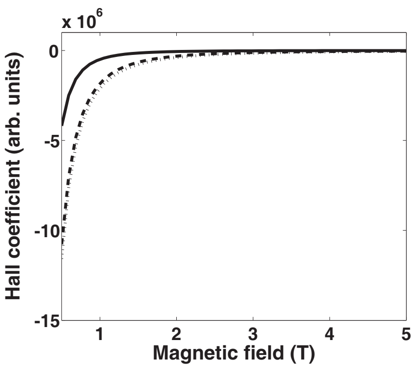

Figure 2: Hall coefficients (arb. units) as functions of the magnetic field at the EMW frequency of 10^{12} Hz (solid line), 4×10^{12} Hz (dashed line), and 7×10^{12} Hz (dotted line). Here, $d = 35\,\mathrm{nm}$, $E_1 = 5\times10^3\,\mathrm{V\cdot m^{-1}}$, $E_0 = 10^5\,\mathrm{V\cdot m^{-1}}$, and $T = 270\,\mathrm{K}$.

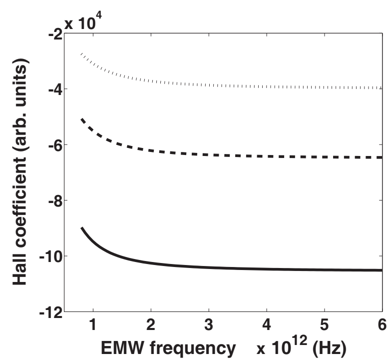

Figure 3: Hall coefficients (arb. units) as functions of the EMW frequency at the CSSL period of 35 nm (solid line), 36 nm (dashed line), and 37 nm (dotted line). Here, $B = 4.2\,\mathrm{T}$, $E_1 = 5\times10^3\,\mathrm{V\cdot m^{-1}}$, $E_0 = 10^5\,\mathrm{V\cdot m^{-1}}$, and $T = 270\,\mathrm{K}$.

In Figure 2 and Figure 3, we show the dependence of the HC on the magnetic field and on the EMW frequency, respectively, at different values of the temperature T; the necessary parameters involved in the computation are the same as those in Figure 1. As the magnetic field increases, the HC increases. When the magnetic field is increased further, the HC reaches saturation at high magnetic fields. This behavior is similar to the case of low temperature in two-dimensional electron systems have been observed before for both the in-plane and perpendicular magnetic fields (see Ref. [17] and references therein). The HC can be seen to decrease slowly with increasing EMW frequency for the region of small values ($\Omega < 2\times10^{12}$ Hz) and reaches saturation as the EMW frequency continues to increase. Moreover, the HC depends strongly on the CSSL period at the chosen values of the other parameters as we can see that the value of the HC raises remarkably when the period increases slightly.

4. CONCLUSION

We have studied the influence of laser radiation on the Hall effect in CSSLs subjected to crossed dc electric and magnetic fields. The electron-optical phonon interaction is taken into account at high temperature, and the electron gas is nondegenerate. We obtain the expressions for the conductivity as well and the HC. The influence of the EMW is interpreted by using the dependencies of the Hall conductivity and the HC on the amplitude E_0 and the frequency Ω of the EMW and by using the dependencies on the magnetic B and the dc electric field E_1 as in the ordinary Hall effect. The most important thing is the appearance of the maximum peaks satisfying the MPR condition optically detected MPR condition.The HC reaches saturation as the magnetic field or the EMW frequency increases. These behaviors are similar to the case of the in-plane magnetic field. Moreover, the HC in this work is always negative while it has both negative and positive values in the case of in-plane magnetic field

REFERENCES

1. Epshtein, E. M., "Odd magnetophotoresistance effect in semiconductors," *Sov. Phys. Semicond.* [*Fiz. Tekh. Poluprovodn.*], Vol. 10, 1414–1415, 1976 (in Russian).
2. Epshtein, E. M., "Odd magnetoresistance of nonlinear conductors in time-dependent electric fields," *Sov. J. Theor. Phys. Lett.*, Vol. 2, No. 5, 234–237, 1976 (in Russian).
3. Malevich, V. L. and E. M. Epshtein, "Photostimulated odd magnetoresistance in semiconductors," *Sov. Phys. Solid State* [*Fiz. Tverd. Tela*], Vol. 18, 1286–1289, 1976 (in Russian).
4. Pankratov, A. A. and E. M. Epshtein, "Kinetic theory of longitudinal Hall effect in high-frequency electric field," *Sov. Phys. Semicond.* [*Fiz. Tekh. Poluprovodn.*], Vol. 16, No. 9, 1689–1691, 1982 (in Russian).
5. Shmelev, G. M., G. I. Tsurkan, and N. H. Shon, "The magnetoresistance and the cyclotron resonance in semiconductors in the presence of strong electromagnetic wave," *Sov. Phys. Semicond.* [*Fiz. Tekh. Poluprovodn.*], Vol. 15, No. 1, 156–161, 1981 (in Russian).
6. Bau, N. Q. and B. D. Hoi, *Int. J. Mod. Phys. B*, Vol. 28, 1450001, 14 pages, 2014.
7. Bau, N. Q. and B. D. Hoi, "Influence of a strong electromagnetic wave (laser radiation) on the Hall effect in quantum wells with a parabolic potential," *J. Korean Phys. Soc.*, Vol. 60, 59–64, 2012.
8. Bau, N. Q., N. V. Nghia, N. V. Hieu, and B. D. Hoi, "Influence of a strong electromagnetic wave (laser radiation) on the hall coefficient in doped semiconductor superlattices with an in-plane magnetic field," *PIERS Proceedings*, 416–421, Taipei, March 25–28, 2013.
9. Mitra, B. and K. P. Ghatak, "Effect of crossed electric and quantizing magnetic fields on the Einstein relation in semiconductor superlattices," *Phys. Stat. Sol. (b)*, Vol. 164, K13–K18, 1991.
10. Vasilopoulos, P., M. Charbonneau, and C. M. Van Vliet, "Linear and nonlinear electrical conduction in quasi-two-dimensional quantum wells," *Phys. Rev. B*, Vol. 35, No. 3, 1334–1344, 1987.
11. Ploog, K. and G. H. Dohler, "Compositional and doping superlattices in III-V semiconductors," *Adv. Phys.*, Vol. 32, No. 3, 285–359, 1983.
12. Esaki, L. and R. Tsu, "Superlattice and negative differential conductivity in semiconductors," *IBM. J. Res. Develop.*, Vol. 14, 61–65, 1970.
13. Silin, A. P., "Semiconductor superlattices," *Sov. Phys. Usp.*, Vol. 28, 972, 1985.
14. Chaubey, M. P. and C. M. V. Vliet, "Transverse magnetoconductivity of quasi-two-dimensional semiconductor layers in the presence of phonon scattering," *Phys. Rev. B*, Vol. 33, 5617–5622, 1986.
15. Charbonneau, M., K. M. Van Vliet, and P. Vasilopoulos, "Linear response theory revisited III: One-body response formulas and generalized Boltzmann equations," *J. Math. Phys.*, Vol. 23, 318–336, 1982.
16. Lee, S. C., "Optically detected magnetophonon resonances in quantum wells," *J. Korean Phys. Soc.* Vol. 51, No. 6, 1979–1986, 2007.
17. Hwang, E. H. and S. Das Sarma, "Hall coefficient and magnetoresistance of two-dimensional spin-polarized electron systems," *Phys. Rev. B*, Vol. 73, No. 12, 121309(R), 2006.