

Synthesis of the Sparse Conformal Arrays with Convex Optimal Method

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Abstract— In this paper, the convex optimal technique is applied to synthesis of sparse conformal arrays for fitting the predefined patterns using as few elements as possible. The original synthesis problem can be formulated by minimizing the number of active elements subjected to the constraints on the pattern requirements. Unfortunately, it needs to solve a NP-hard problem since the objective function of the related problem is nonconvex. The minimization of the l_1 -norm is presented to relax the above constraint into a convex way. Towards this end, the proposed method can determine both the corresponding locations and amplitudes of the sparse elements based on l_1 -norm of the sparsest excitation set. The method for conformal array optimization addressed in this paper is easy to be implemented and has low computational load. Numerical simulations validate the effectiveness and high accuracy of the proposed synthesis method.

1. INTRODUCTION

Conformal antenna arrays with low profile have been widely used in the fields of space-borne, air-borne and missile-borne radar, space vehicles and sonar due to its low Radar Cross-Section (RCS) and no extra aerodynamic drag [1]. In the case of conformal array, the well-known Fourier transform relationships between element excitations and far field pattern breaks down, and on the other hand, array factor theory does not hold any more since each element pattern depends on its respective orientation, which make the corresponding synthesis problem significantly difficult. Up to now, iterative least square techniques [2] and stochastic optimization such as genetic algorithms (GA) [3], simulated annealing (SA) [4], particle swarm (PS) [5] and differential evolution algorithm (DEA) [6] have been widely applied to the synthesis of conformal array. However, few methods have been proposed to thin out the conformal array matching the desired pattern.

Recently, an effective and robust technique, convex optimal method, has been proposed to design the maximally sparse linear arrays fitting the desired patterns [7]. This method can find the minimum number of active elements and their corresponding positions and excitations by using the reference pattern samples. In this paper, we will extend this method to the sparse conformal array. Notably, the convex optimal method must be carefully described in the case of conformal array due to the complexity of synthesis dimensions and the definition of each element pattern in the synthesis processing. Therefore, in Section 2, the addressed synthesis problem is formulated based on convex optimal method in detail. Numerical tests are presented in Section 3. Finally, some conclusions are drawn in Section 4.

2. PROBLEM FORMULATION

For a conformal array consisting of N elements with arbitrary geometry distribution, the far-field beam pattern in the generic direction (θ, φ) can be expressed as

$$F(\theta, \varphi) = \sum_{n=0}^{N-1} \omega_n g_n(\theta, \varphi) e^{j \frac{2\pi}{\lambda} (x_n \sin \theta \cos \varphi + y_n \sin \theta \sin \varphi + z_n \cos \theta)} \quad (1)$$

in which θ and φ are the elevation and azimuth angles, ω_n is the corresponding excitation of the n -th element at (x_n, y_n, z_n) in global Cartesian coordinate system, $g_n(\theta, \varphi)$ is the element pattern in the global coordinate system, where $n = 0, 1, \dots, N-1$, λ is the wavelength. The position vector of the n -th element is

$$\vec{r}_n = (x_n, y_n, z_n) \quad (2)$$

while the unit vector in direction (θ, φ) is

$$\vec{r} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \quad (3)$$

Thus, Equation (1) can be further written in a matrix form as

$$\mathbf{F}(\theta, \varphi) = \mathbf{V}(\theta, \varphi) \mathbf{W} \quad (4)$$

where

$$\mathbf{V}(\theta, \varphi) = \left(g_0(\theta, \varphi) e^{j \frac{2\pi}{\lambda} \vec{r}_0 \cdot \vec{r}}, g_1(\theta, \varphi) e^{j \frac{2\pi}{\lambda} \vec{r}_1 \cdot \vec{r}}, \dots, g_{N-1}(\theta, \varphi) e^{j \frac{2\pi}{\lambda} \vec{r}_{N-1} \cdot \vec{r}} \right) \quad (5)$$

$$\mathbf{W} = [\omega_0, \omega_1, \dots, \omega_{N-1}]^T \quad (6)$$

and $\mathbf{W}$ is the weighting vector, $\mathbf{V}(\theta, \varphi)$ is the steering vector, T denotes the transpose. It must be pointed out that the element pattern $g_n(\theta_n, \varphi_n)$ in the local coordinate system need to be transformed to $g_n(\theta, \varphi)$ in the global coordinate system before calculating the far-field pattern [1]. Since the elements in conformal array generally direct their radiation beams toward different directions.

Let us define the aperture of the conformal array as S and the reference pattern as $F_{ref}(\theta, \varphi)$, respectively. Firstly, K measurements will be obtained by sampling on the reference pattern, the sampling directions are as [8]

$$\theta = \pi \frac{\text{mod}(k, K_\theta)}{K_\theta - 1} \quad \varphi = 2\pi \frac{\lfloor k/K_\theta \rfloor}{K_\varphi - 1} \quad (7)$$

where $k = [0, 1, \dots, K]$, $K = K_\theta \times K_\varphi$. Meanwhile, the column vector consisting of the aforementioned samplings is denoted as $\mathbf{F}_{ref}$. Moreover, let us assume that a P -element conformal array can approximately radiate the reference pattern with a matching error of ε . The P elements are selected from N predefined candidate locations within the aperture S . In order to achieve the minimum number of active element P , the related sparse conformal array synthesis is formulated mathematically as

$$\min \|\mathbf{W}\|_{l_0} \quad s.t. \quad \|\mathbf{F}_{ref} - \mathbf{V}\mathbf{W}\|_{l_2} \leq \varepsilon \quad (8)$$

where $\|\mathbf{W}\|_{l_0}$ denotes the number of nonzero elements of the vector $\mathbf{W}$. As we know, the objective function in (8) is nonconvex. It is worth noting that l_0 -norm should be replaced with its convex approximation, i.e., l_1 -norm, to avoid the NP-hard problem [7]. Towards to this end, the synthesized problem is relaxed to a convex problem as following

$$\min \|\mathbf{W}\|_{l_1} \quad s.t. \quad \|\mathbf{F}_{ref} - \mathbf{V}\mathbf{W}\|_{l_2} \leq \varepsilon \quad (9)$$

An iterative weighted l_1 -norm algorithm as shown in [7] is also herein applied to enhancing the sparsity of the solution $\mathbf{W}$ (see [7] for the details). In addition, the iterative convex optimization problem is convenient to solve optimally because useful software, such as CVX is readily available with only a few iterations (less than 5 for the synthesis problem as following).

3. NUMERICAL EXAMPLES

In this section, two numerical simulations are carried out to verify the effectiveness and accuracy of the aforementioned method for sparse conformal array synthesis. In order to describe the approximation degree of the synthesized pattern to the reference pattern, the matching error is defined as

$$\varepsilon = \frac{\int_0^{2\pi} \int_0^\pi |F_{ref}(\theta, \varphi) - F(\theta, \varphi)|^2 d\theta d\varphi}{\int_0^{2\pi} \int_0^\pi |F_{ref}(\theta, \varphi)|^2 d\theta d\varphi} \quad (10)$$

As the first example, the target pattern is from a uniform circular array composed of 30 isotropic elements with the radius $R = 15\lambda/2\pi$. The elements are equally spaced with $\lambda/2$ along the periphery of the circle. By using the proposed method, a sparse array with only 22 elements conformal to the herein circle is achieved, i.e., 8 elements are saved. The sparse array layout and excitations are shown in Fig. 1. The minimum inter-element spacing is 0.63λ and the maximum inter-element spacing is 0.72λ in the synthesized sparse array. The synthesized pattern, as shown in Fig. 2(a), has a total matching error of 1.862×10^{-4} in 3-D form as compared to the target (see Fig. 2(b)). The comparison between the synthesized pattern and the target pattern is also presented by the

plots in the planes $\varphi = 0^\circ$ (Fig. 3(a)) and $\varphi = 90^\circ$ (Fig. 3(b)). It is clear that the pattern matching error is negligible.

Next, our method is applied to synthesizing a cylindrical array so as to further test the sparse performance. The target pattern is from a 4-by-24 element array uniformly spaced by 0.5λ and located on a cylindrical surface of radius $R = 15\lambda$, as presented in [9]. The excitation in the 24-element direction is Chebyshev amplitude distribution while uniform excitation in the 4-element direction is used. The corresponding n -th element pattern in the global coordinate system is

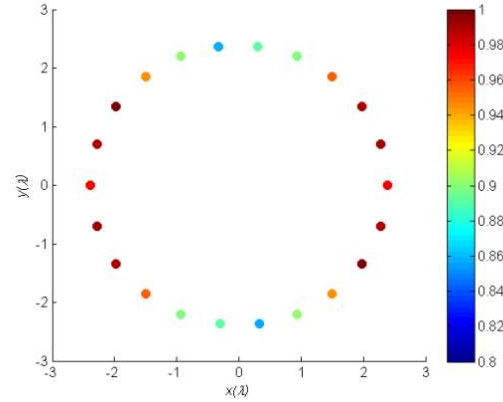


Figure 1: The layout and excitations of the synthesized arrays.

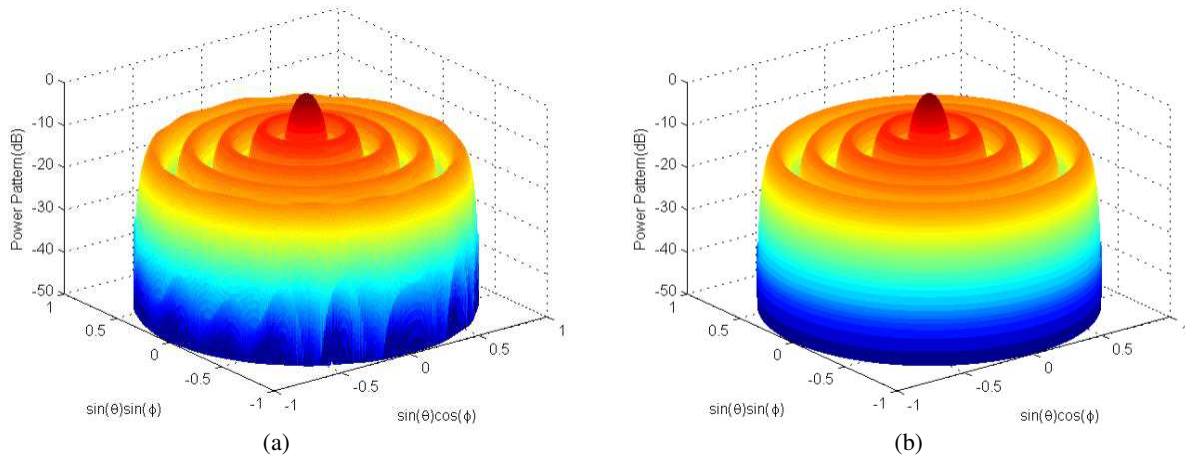


Figure 2: The comparison between the synthesized pattern and the target in 3-D. (a) The reconstructed pattern by sparse array, (b) the target pattern by the uniform circular array.

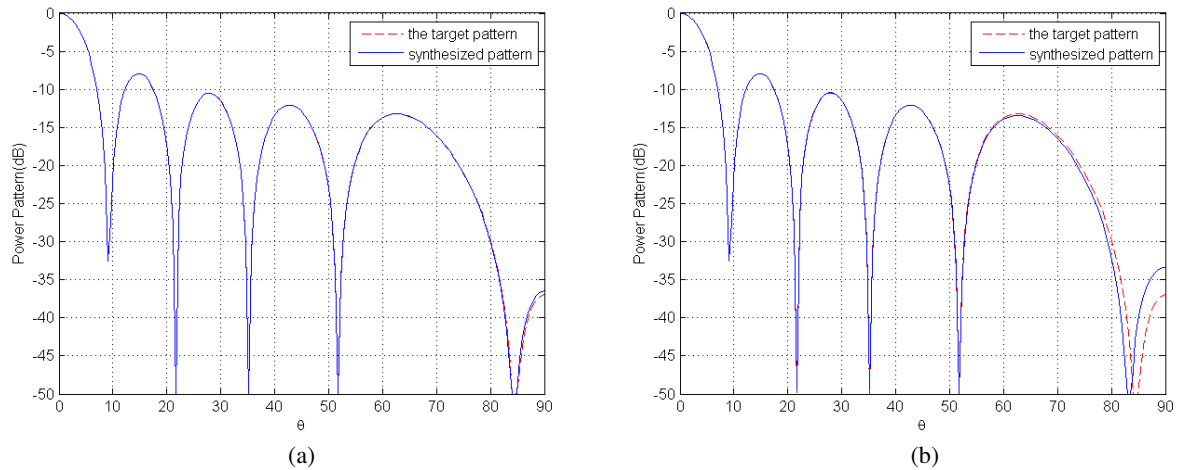


Figure 3: The synthesized pattern as compared to the target. (a) $\varphi = 0^\circ$ plane, (b) $\varphi = 90^\circ$ plane.

expressed as following

$$g_n(\theta, \varphi) = \sin \theta \cos(\varphi - \varphi_n) \quad (11)$$

where φ_n is the corresponding element's azimuthal position. The aperture of the herein cylindrical array is discretized only along the arc as

$$\begin{cases} x_n = R \cos \left(-\frac{11}{90}\pi + \frac{22 \bmod(n, n_{xy})}{n_{xy} - 1} \right) \\ y_n = R \sin \left(-\frac{11}{90}\pi + \frac{22 \bmod(n, n_{xy})}{n_{xy} - 1} \right) \\ z_n = -\frac{3}{4}\lambda + \frac{3}{2}\lambda \frac{\lfloor n/n_{xy} \rfloor}{n_z - 1} \end{cases}, n = 0, \dots, N - 1 \quad (12)$$

where $N = n_{xy} \times n_z$, n_{xy} is defined as the element number along the azimuthal direction, n_z denotes the elements number along the axial direction. As expected, a sparse cylindrical array consisted of 79 elements (Fig. 4(a)) is obtained to reproduce the target pattern with the matching error $\varepsilon = 1.047 \times 10^{-2}$, i.e., 18% of the total elements are saved as compared to the reference array presented in [9]. It means the cost can be further reduced and the feeding network can be further simplified in practical applications by the proposed method. The comparison between the synthesized pattern and the target in the plane $\theta = 90^\circ$ (Fig. 4(b)) clearly indicate that the sparse

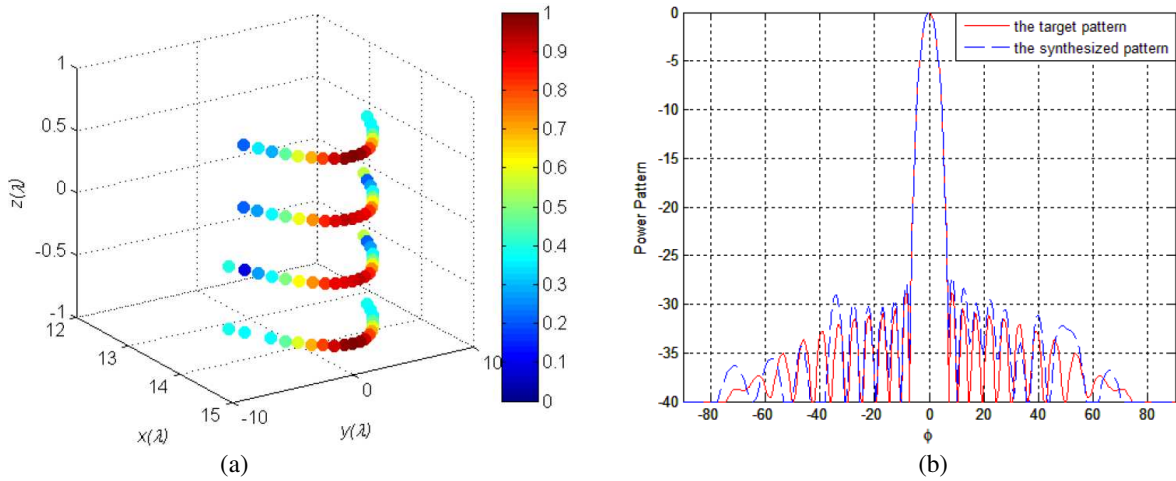


Figure 4: (a) The layout and excitations of the synthesized cylindrical conformal array, (b) the synthesized pattern along with the target in plane $\theta = 90^\circ$.

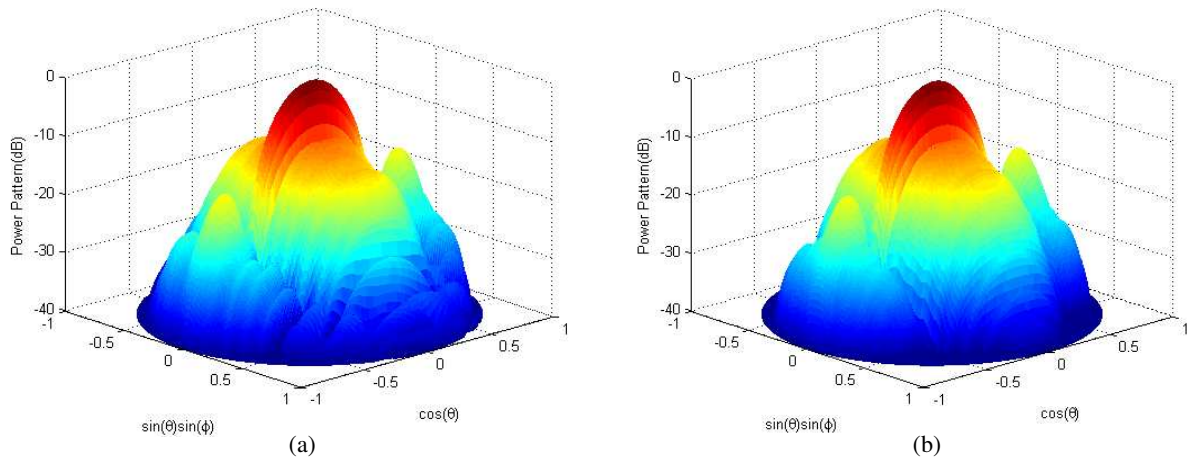


Figure 5: The comparison between the synthesized pattern and the target in 3-D. (a) The reconstructed pattern, (b) the target pattern.

conformal solution realizes an accurate reconstruction of the target main-beam while with only some minor mismatching in low side-lobes. The related 3-D radiation patterns are shown in Fig. 5. Moreover, the reconstructed array would be sparser if we discrete the cylindrical aperture in axial direction further.

4. CONCLUSION

In this paper, the convex optimal method has been introduced to design sparse conformal arrays in match of the reference patterns. The synthesis problem can be formulated by minimizing the number of active elements subjected to the constraints on the pattern requirements. To form the optimization problem, much dense elements are first assigned to equally spaced dense positions, which compose the conformal array aperture. The convex optimal method is aimed to find the active elements from the predefined candidate positions within the conformal array. In addition, this method can derive the element positions and weights simultaneously. Compared with the stochastic algorithms, the proposed method is very simple and easy to carry out, the computational burden is acceptable. Numerical experiments are conducted to validate the effectiveness and flexibility of the proposed method.

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