Accurate Statistical Modeling Method for Dynamic RCS

Ya-Qiang Zhuang, Chen-Xin Zhang, and Xiao-Kuan Zhang
Air and Missile Defense College, Air Force Engineering University, Xi'an 710051, China

Abstract— The dynamic RCS of an aircraft target varies dramatically with target geometry and motion information (position and velocity), the values fluctuate and have no pattern. So the fluctuation models are often adopted to describe the statistical nature of dynamic RCS. In this paper, we first introduce a simulation method of dynamic RCS for aircraft target. Secondly, a brief introduction of Chi2 distribution and Lognormal distribution model is given, and Gaussian Mixture distribution is proposed in order to describe the fluctuation more accurately. The parameter estimation method is discussed in detail to find the best fitting result. As an example, the dynamic RCS data of an aircraft target are analyzed with Chi2 distribution, Lognormal distribution model and Gaussian Mixture distribution. The fitting results show Gaussian Mixture distribution can provide a better approximation of the dynamic RCS than the other two distributions in accordance with error-of-fit and Kolmogorov-Smirnov fitting goodness test.

1. INTRODUCTION

RCS (Radar Cross Section) is an important parameter reflects target scattering ability for incident EM waves. It is often applied in the radar detection and target recognition field [1]. The dynamic RCS of aircraft in the movement has practical significance which can be acquired by field measurement and simulation method. Due to the high cost and uncertainty of field measurement, the latter is often adopted. The combinations of PO (Physical Optics) method with PTD (Physical Theory of Diffraction) [2, 3] are often used to evaluate the static RCS of the target in the simulation process.

RCS of aircraft depends on geometry and orientation whose values are often time-varying during its movement [4]. It has been advantageous to treat the target RCS as a statistic items for the dynamic RCS with random fluctuation. Statistical distributions are used for RCS characterization. Much work has been done over the years in statistical modeling of RCS, including conventional distribution such as Chi2 distribution, Rice distribution, Lognormal distribution and some new distributions such as Weibull distribution, NCGG (Non Central Gamma Gamma) distribution [5] and Two State Rayleigh-Chi distribution [6].

The EM calculation model of an aircraft is modeled and all-attitude static RCS database is calculated firstly in this paper. The dynamic RCS of spiral flying is calculated by linear interpolation based on static database. Then, the brief introduction of three distributions is given, and parameter estimation procedure of GM (Gaussian Mixture) distribution is provided. In addition, statistical characterization of dynamic RCS using three fluctuation models was investigated. The comparison results show that GM distribution can provide a better approximation of the dynamic RCS.

2. SIMULATION METHOD OF AIRCRAFT'S DYNAMIC RCS

2.1. Static RCS Calculation

In this section, the EM calculation model was modeled and the all-attitude static RCS database was calculated in FEKO as shown in Figure 1. The EM model was modeled by the size of scale target after multiplying its scale ration, and static RCS value was calculated by PO method at the steps of 1° under L band (165 MHz) for HH polarization. The target coordinate center is located at the geometrical center of the target. The attitude angle was defined as shown in Figure 1. The pitch angle is from X axis round to Z axis in the XOZ plane, and the azimuth angle is from X axis round to Y axis in the XOY plane.

2.2. Dynamic RCS Series

After the all-attitude static RCS database was calculated, the dynamic RCS series of the special flight path can be acquired by following procedures. Firstly, we predestined path the aircraft flies with and radar position as shown in Figure 2. Given the aircraft make a spiral flying with radius 40 km under speed 1.4 Ma at height 9 km level plane. From the flight path, the aspect angles of radar line of sight in radar coordinate system can be calculated easily. A coordinate transformation need to be done in order to acquired the aspect angles in target coordinate system. Then, the dynamic RCS can be acquired according to aspect angles using linear interpolation method. Plot of dynamic RCS for the aircraft in given flight path is provided in Figure 3.

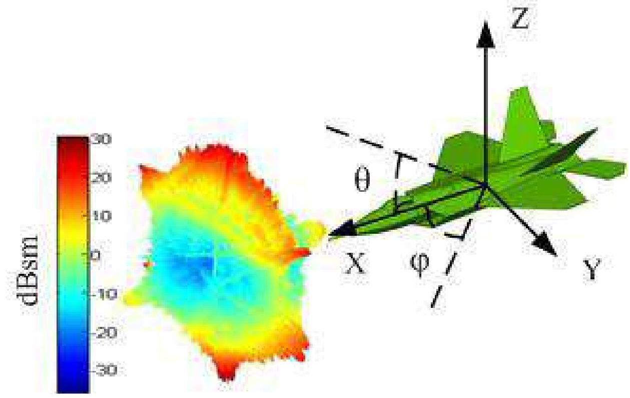


Figure 1: Aircraft EM model and all-attitude static RCS database.

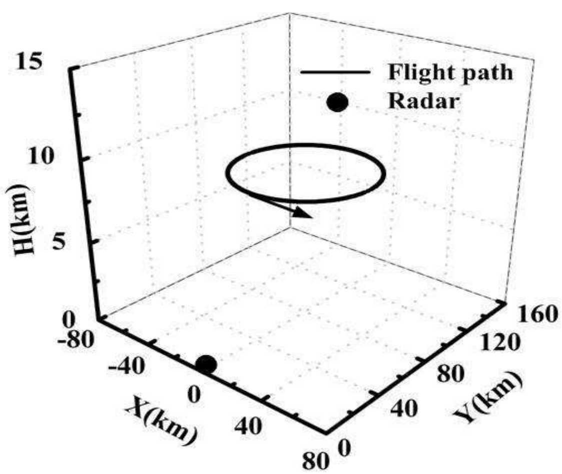


Figure 2: Aircraft flight path and radar position.

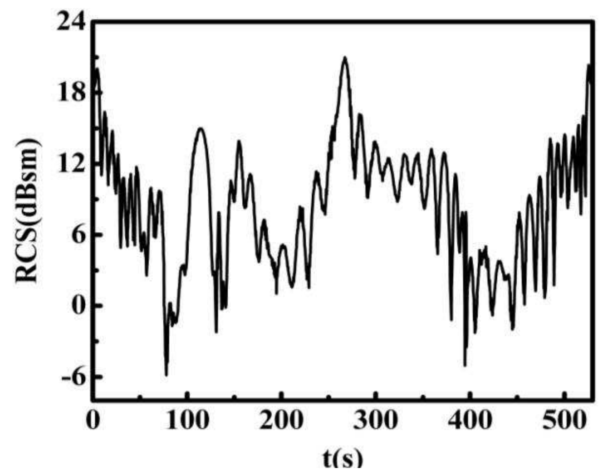


Figure 3: Dynamic RCS series.

3. TARGET FLUCTUATION MODELS

Among the existing RCS distribution models, Chi2 distribution and Lognormal distribution are worthwhile to promote. The parameter analysis has been discussed detailedly in [7, 8]. The statistical modeling method of dynamic RCS using Gaussian Mixture distribution is proposed.

3.1. Chi2 Distribution l

This model was introduced by Swerling and assumes that the PDF (Probability Density Function) of the target RCS is a generalized Chi2 distribution is as Eq. (1)

$$p(\sigma) = \frac{k}{\Gamma(k)\,\bar{\sigma}}\left(\frac{k\sigma}{\bar{\sigma}}\right)^{k-1}\exp\left(-\frac{k\sigma}{\bar{\sigma}}\right), \quad \sigma > 0 \tag{1}$$

In Eq. (1), σ is RCS variable and $\overline{\sigma}$ is the mean of RCS value. k named double-degrees of freedom is equal to $\overline{\sigma}^2$/variance of σ. Parameter k can be positive integer or non-integer. So it's fitting to different distributions is very good. $\Gamma(k)$ is a gamma function with parameter k. The PDF (1) subsumes as a special case the Swerling I-IV and Marcum distribution, corresponding to $k = 1,\ 2,\ N,\ 2N$ and ∞ [9].

3.2. Lognormal Distribution

The fluctuation RCS of targets such as ships and missiles is often well modeled as a log-normal random variable. The PDF of Lognormal distribution can be written as Eq. (2)

$$p(\sigma) = \frac{1}{\sigma s\sqrt{2\pi}}\exp\left(-\frac{(In\sigma - \mu)^2}{2s^2}\right), \quad \sigma > 0 \tag{2}$$

In Eq. (2), σ is RCS variable. μ is the mean of σ and s is the standard deviation of σ. The normal and Lognormal distributions are closely related. If the RCS value σ is distributed log normally with parameters μ and s, then $In\sigma$ is distributed normally with mean μ and standard deviation s.

3.3. Gaussian Mixture Distribution

Gaussian Mixture distribution is a semi parametric density estimation method, and it combines the advantages of parameter estimation method and non-parametric estimation method [10]. Gaussian Mixture distribution has been used widely in the field of voice recognition, image processing, and microarray gene expression data and so on. The PDF of Gaussian Mixture distribution is as Eq. (3)

$$f\left(x,\Theta\right)=\sum_{i=1}^{M}\frac{a_i}{\sqrt{2\pi s_i}}\exp\left(-\frac{\left(x-\mu_i\right)^2}{2s_i}\right) \tag{3}$$

where $\Theta=(a_1,a_2,\ldots,a_M;\mu_1,\mu_2,\ldots,\mu_M;s_1,s_2,\ldots,s_M)$, a_i is the weight of number i component, and it meets $\sum\limits_{i=1}^{M}a_i=1$. The variable μ_i and s_i are the mean and variance of number i component respectively. If the component number M is enough, the Gaussian Mixture distribution will approximates any continuous distribution with perfect accuracy.

The Expectation Maximization (EM) method is often adopted to estimate the parameter of Gaussian Mixture distribution. Given that

$$p\left(x\right|\Theta)=\sum_{i=1}^{M}a_i p_i\left(x\left|\theta_i\right.\right) \tag{4}$$

In the Eq. (4), $\theta=(\mu,s)$, $p_i(x|\theta_i)$ is the number i Gaussian component with θ_i.

The iterative formula to calculate parameter by EM method is as Eq. (5):

$$\begin{cases} a_l^{new}=\frac{1}{N}\sum\limits_{i=1}^{N}p(l\left|x_l,\Theta^g\right.) \\ u_l^{new}=\sum\limits_{i=1}^{N}x_i p(l\left|x_i,\Theta^g\right.)\Big/\sum\limits_{i=1}^{N}p(l\left|x_i,\Theta^g\right.) \\ s_l^{new}=\sum\limits_{i=1}^{N}p(l\left|x_i,\Theta^g\right.)\left(x_i-u_l^{new}\right)^2\Big/\sum\limits_{i=1}^{N}p(l\left|x_i,\Theta^g\right.) \end{cases} \tag{5}$$

In the Eq. (5), $p(l|x_i,\Theta^g)=\frac{a_l^g p_l(x_i|\theta_l^g)}{p(x_l|\Theta^g)}=\frac{a_l^g p_l(x_i|\theta_l^g)}{\sum_{j=1}^{M}a_j^g p_j(x_i|\theta_j^g)}$.

4. RESULTS

4.1. Fitting Results

The statistical modeling of dynamic RCS distribution and fitting method has been discussed for many years. We choose the Chi2 distribution, Lognormal distribution and two-order Gaussian Mixture distribution to fit the PDF (probability density function) and CDF (Cumulative Distribution Function) of dynamic RCS. The fitting parameters of Chi2 distribution and Lognormal distribution can be estimated by non-linear least squares algorithm. The distribution parameters of Gaussian Mixture distribution are estimated by EM method. The fitting results are shown in Figure 4.

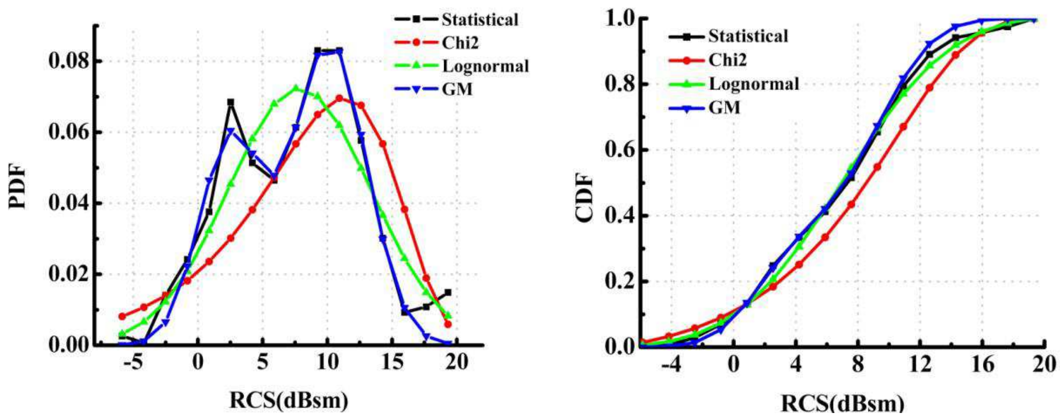
4.2. Result Analysis

The errors-of-fit e_f is used to compared the fitting result which defined as Eq. (6)

$$e_f=\sum_i\left[p_e\left(\sigma_i\right)-p_m\left(\sigma_i\right)\right]^2 \tag{6}$$

In Eq. (6), $p_e(\sigma_i)$ is the RCS statistical distribution and $p_m(\sigma_i)$ is the fitting result of these distributions. The errors-of-fit are provided in Table 1.

The non-parametric fitting goodness test procedure is investigated to evaluate the probability model of the RCS sample data for three distributions applied above. The Kolmogorov-Smirnov test [11] statistically measures the absolute errors between an observed CDF of sample values and



(a) (b)

Figure 4: Fitting result of distribution of dynamic RCS. (a) PDF distribution. (b) CDF distribution.

Table 1: Errors-of-fit and K-S test results.

	Chi2	Lognormal	GM
errors-of-fit	0.43%	0.22%	0.05%
K-S test result	0.1232	0.0417	0.0377

a specified continuous distribution function. The test function is shown in Eq. (7), in which $F(x)$ is the statistical CDF of dynamic RCS data and $F'(x)$ is CDF of the fitting results. If the test result is too large, it can be determined that the distribution used is not applicable for the data. The results of K-S test are shown in Table 1.

$$D = \max \left| F(x) - F'(x) \right| \tag{7}$$

From the fitting curves, errors-of-fit and Kolmogorov-Smirnov test results, it is can be concluded that Gaussian Mixture distribution and Lognormal distribution are better than Chi2 distribution for dynamic RCS of this target. In addition, the fitting result of Gaussian Mixture distribution is a quite better than Lognormal distribution form its extreme little errors-of-fit. Therefore, the dynamic RCS data can be modeled accurately by Gaussian Mixture distribution.

5. CONCLUSION

A new approach to obtain dynamic RCS quickly in the predestined path is proposed firstly in this paper. After time-varying attitude angles of radar line of sight in aircraft coordinate were calculated based on coordinate transformation, the dynamic data was obtained from all-attitude static RCS database using linear interpolation method. A statistical modeling method of dynamic RCS based on Gaussian Mixture distribution is investigated. The comparison on statistical characterization of dynamic RCS under typical paths using Ch2 distribution, Lognormal distribution and Gaussian Mixture distribution is analyzed. The fitting results show that Gaussian Mixture distribution is the best applicable among three distributions to describe the dynamic RCS. The research result of this paper can be applied in statistical characterization of aircraft's dynamic RCS, and can provide a reliable support for accurate simulation and examining of moving target's echo.

ACKNOWLEDGMENT

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