Shaping the CPML Absorbing Boundary Condition to Eliminate Impinging Light at a Specific Position inside Electromagnetic Simulations

Sergio Cantero, Shuai-Hsun Lee, and Snow H. Tseng
Graduate Institute of Photonics and Optoelectronics
National Taiwan University, Taipei 10617, Taiwan

Abstract— In this research we explore the possibility to implement an optical target that eliminates incoming wave in finite-different time-domain (FDTD) simulations. This is achieved by a modified morphology of an absorbing boundary condition (ABC). In particular, our model is based on convolutional perfectly matched layers (CPML) ABC. To validate this method, a cylindrical CPML region is constructed as artificial absorbing material and simulated using a two-dimensional FDTD algorithm. Its performance is analyzed for focused beam wave source conditions. Varying the spatial size of the circular CPML, we compute the absorption efficiency for a wide range of applications and demonstrate its performance as a numerical optical target in FDTD simulations. We demonstrate that the reported simulation construct can effectively eliminate impinging light wave for a macroscopic light scattering simulation.

1. INTRODUCTION

The Finite-difference time-domain (FDTD) algorithm is widely used for electromagnetic wave propagation and light scattering problems. Based upon numerical solutions of Maxwell's equations, the electromagnetic wave characteristics can be accurately analyzed. This is of particular interest in modeling macroscopic scattering media, due to the difficulty to find analytical solutions [1]. The complexity of the problem relies on the large number of variables and the inhomogeneity of the material properties, such as index of refraction. To model light propagation through a macroscopic scattering medium to a target position, an absorber is required to eliminate impinging light. In modeling a physical problem in open space, absorbing boundary conditions (ABC) are applied to terminate the computational grid. ABCs are designed to absorb outgoing electromagnetic energy, allowing modeling an optical system isolated in free space with limited computation memory. The introduction in 1994 by Berenger of perfectly matched layers (PML) ABC effectively reduces the artificial numerical error caused by the simulation boundary [2].

Several modifications of the original PML formulation have been developed since its introduction. The numerical model presented utilizes convolutional perfectly matched layers (CPML) to efficiently embed the electromagnetic loss defined in a PML medium into the FDTD algorithm [3]. The advantage of CPML over other formulations is its independency on the host medium. For this reason, a CPML medium can be delimited in a defined region of interest within the simulation grid, allowing faster computations and less memory storage; as well as utilizing different boundary conditions, if necessary. Furthermore, no modification of the CPML algorithm is required due to the complexity of the medium, which is particularly useful in highly scattering media. In this research we construct a numerical tool capable to absorb light with efficacy in FDTD simulations.

2. NUMERICAL METHODS

A more detailed explanation of the CPML formulation can be found elsewhere [3] and therefore will not be discussed here. In a perfectly-matched medium as ABC the electric and magnetic conductivities, which are responsible for the energy losses, are increased across the boundary layers for effective absorption. Likewise in a cylindrical model, the conductivities are tapered down from the center of the target to the interface with the medium. Note that a different geometry can be formulated, as long as the conductivity is smoothly staggered from the boundary with the medium. In general, a given node within the PML has assigned a conductivity:

$$\begin{cases} \sigma(x,y,z) = \sigma_0 \left(\frac{L_{\max}-L(x,y,z)}{L_{\max}}\right)^m \\ \sigma_0 = 0.8\frac{m+1}{\eta\Delta} \end{cases} \tag{1}$$

Each node (x, y, z) of the FDTD grid is associated to a certain layer, with index L, depending on the morphology of the CPML region $L_{\max}$ is therefore the number of layers in the CPML construct and $L = L_{\max}$ at the interface with the rest of the simulation region: Inner boundary if the CPML is the absorbing boundary condition or outer boundary if it is placed within the simulation. For a circular geometry, $L_{\max}$ is simply the radius of the region and $L(r)$ is the distance of the layer to the outer boundary. The parameters m and σ are the slope and maximum of the conductivity staggering. In PML formulations, m typically varies between 3 and 4 and σ is function of m, the wave impedance η, and the grid size Δ. Spatially sampling a circular shaped object into a rectangular grid is cause of numerical errors. Therefore a validation of the model was extensively discussed in previous research using two dimensional FDTD simulations [4]. Error analysis of the absorption efficiency compared the optical target under cylindrical incident source with external ABC, and to a planar-geometry target using Gaussian beam incidence. In this research, two applications of the optical target as numerical tool for scattering simulations are introduced. The model is presented first, as a method to eliminate light within the simulation and second, as an ideal absorber in modeling of optical imaging techniques.

3. SIMULATION RESULTS

In this research we first complement previous results with an evaluation of the near-field scattering distribution from the cylindrical optical target. An incident continuous-wave (cw) Gausian beam with wavelength 638.32 nm is embedded as soft source [2]. The FDTD grid resolution used for this simulation setup is 15.8×15.8 nm. After the incident beam is scattered by the target, the E_z distribution is stored at an equidistant contour from the target. The decrease in reflection of a Gaussian beam from targets with increasing radius is shown in Figure 1. Results show that the energy is mostly diffracted by small-sized absorbers (dotted lines). For effective absorption, the cross-section of the target must be increased. Approximately −80 dB and −100 dB maximal scattered amplitude can be achieved with radii three and four times the beam waist respectively (dashed lines). The limit is observed in larger absorbers (solid lines), in which the back-scattered energy is larger than the energy in the forward direction, meaning that the behavior is closer to the planar CPML behavior. For effective light absorption in simulations of scattering through macroscopic random media, the target should cover as much portion of the beam as possible, if additional scattering from the target should not affect the remaining field.

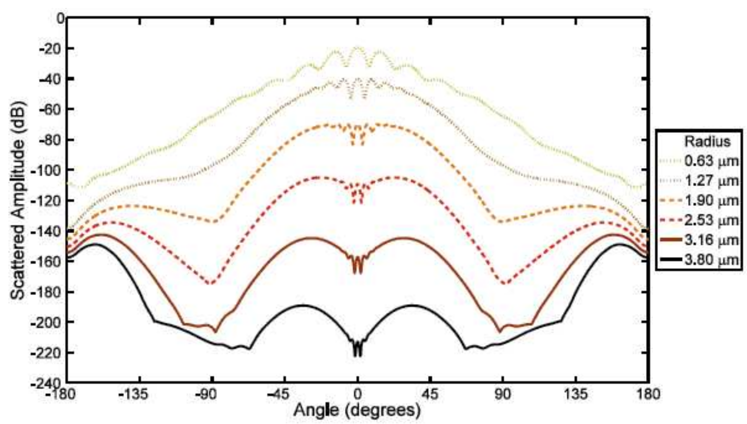

Figure 1: Angular distribution of a Gaussiam beam with 1.27 μm waist scattered by an optical target with different sizes. 0° corresponds to forward propagation in the z-axis and 180° to back-reflections.

An immediate application of the proposed model is to extract and isolate portions of this scattered field. To illustrate the problem, let us consider the light scattered by a known object, a small optical target (Figure 1). Note that due to a beam waist is larger than the object, it presents low absorption is low and can be contemplated as a strong scatterer. The E_z amplitude of this object after the incident source is in a steady state is shown in Figure 2(a). On the left side the Gaussian beam is incident and the scattered field propagates on the right side. In Figure 2(b), the same source is eliminated by a target large enough to cover most of the impinging energy. This data is subtracted to (a) in order to obtain Figure 2(c).

The concept is similar to the total-field/scattered-field (TF/SF) formulation, typically used to implement plane wave source conditions into electromagnetic FDTD simulations. The TF/SF main advantage is the possibility to separate the scattered field from the incident source, which is

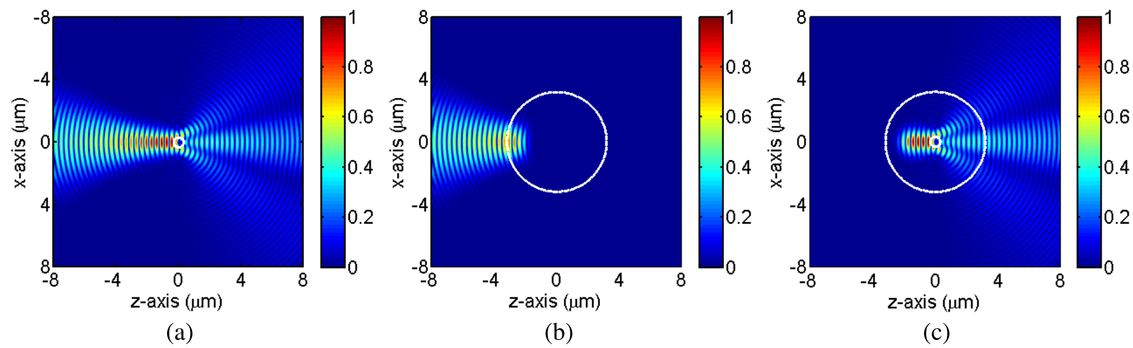

Figure 2: (a) Near-field computation of a Gaussian beam impinging a strongly scattering object: a 0.63 nm diameter CPML absorber. (b) Source eliminated by an optical target. (c) E_z field in (a) minus E_z field in (b).

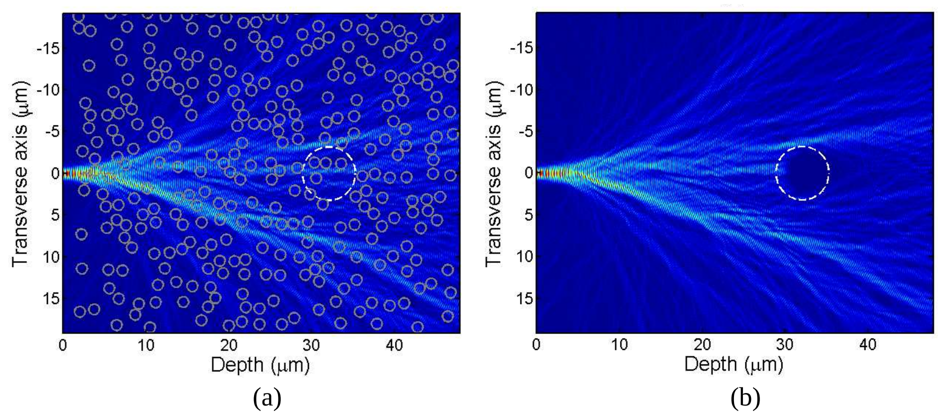

Figure 3: (a) Propagated light through a medium composed of a random aggregate of scattering cylinders. (b) Placement of an optical target inside the medium, centered at $z = 32.64\,\mu\text{m}$ with radius $3.19\,\mu\text{m}$.

useful for retrieving properties of the problem object, such as its scattering cross-section. However, it expects the incident source to be known, and errors arise when the known source does not match plane wave propagation [5]. The optical target, on the other hand, can be applied to an arbitrary incident source, and its exact formulation is not necessary. The only requirement is the additional computation of the same medium with the optical target covering the incidence area. The equivalence of the suggested method with the TF/SF formulation is shown in Figure 2: The inner region of the optical target in (c), *total-field* includes both the incident field, computed in (b), and the scattered field; while at the external, *scattered-field*, region the source (b) is effectively subtracted.

In addition to the application of the presented numerical construct to extract portions of the electromagnetic energy within the problem, the proposed method can be used to model an absorbing object in FDTD simulations. Note that not only perfectly-matched conductivities can be implemented with the CPML formulation, but more general conductivity profiles are allowed, permitting different absorbing media to be modeled. For this purpose, the optical target is placed under a simple source-detector scheme. Figure 3 shows the placement of an optical target in a highly scattering medium. The medium consists of 300 randomly placed scatterers with index of refraction 1.44 and diameter 1.28 μm on a background homogeneous medium with index of refraction 1.33. An incident source of wavelength 638.32 nm is applied on a $38.3 \times 47.9\,\mu\text{m}$ region with resolution $31.9 \times 31.9\,\text{nm}$. As expected, strong scattering defocuses the electromagnetic energy at the random medium. At the same time, strongly focused paths materialize as previously studied by [6]. In Figure 3(b) the optical paths formed by the propagated light are ostensibly deformed after inserting an absorber.

To confirm the detectability of the target within the medium, a detector is placed along the

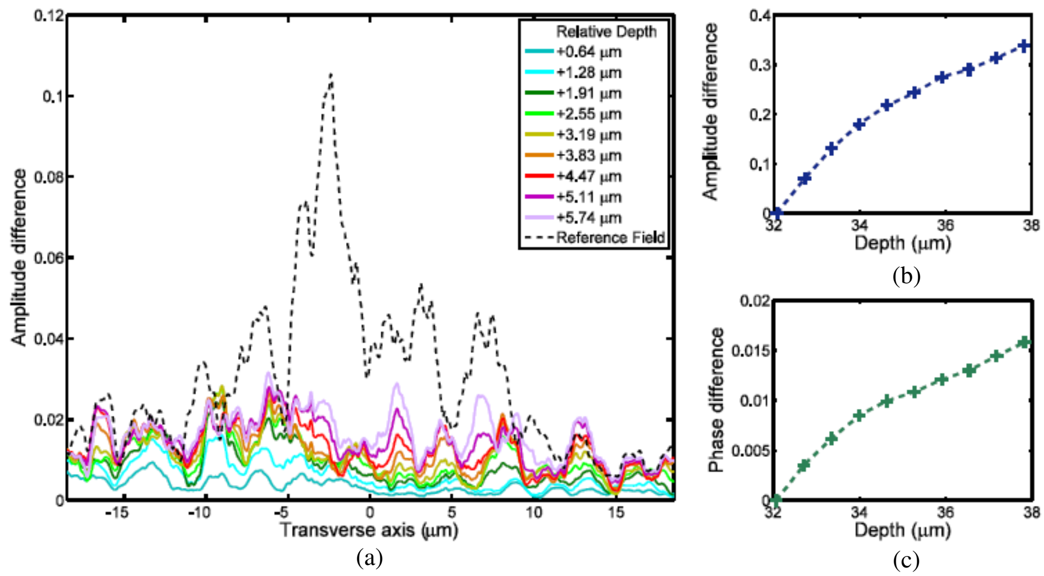

Figure 4: Amplitude difference at the detector, relative to the initial position ($z = 32.64\,\mu\text{m}$, $x = 0$). To smoothen the peaks, a local average is numerically integrated over the 40 nearest points. The difference between the field without absorber and the initial position in dashed line. Average difference in (b) amplitude and (c) phase at every point of the detector array.

side opposite to the source. The amplitude and phase of the electric field is then measured at the detector points. The difference in amplitude and phase of a moving target with respect to an initial position is presented in Figure 4. As we can observe, the differences are scattered through the medium and spread along the detector. However, as the target shifts its position from the original placement, the differences increase. Figure 4 shows the increasing average deviation in both amplitude (b) and phase (c) as the target moves towards the detector.

4. CONCLUSION

In summary, the possibility to shape the CPML ABC formulation to design an optical absorber within the simulation region is demonstrated. Instead of a planar geometry such as implemented in the boundary condition, a circular CPML with radial dependency is constructed. Results show reduction of the spurious scattering with increasing sizes up to the accuracy of CPML ABC. To determine the feasibility of the model, we introduced two applications under both focused light and omnidirectional light incidence. First, a method to extract scattered and incident field from a region is provided. Unlike other approaches, the mathematical formulation is not subject to the incident source formulation. And second, we confirm detection and tracking of a CPML target utilizing a very simple source-detector scheme.

Additionally, the CPML formulation is equally valid for a different value of the conductivity, not only perfectly matched layers. In large scale FDTD simulations, the computation time plays an important role and resolution and size are strongly limited. For this reason, the CPML absorber model reported in this manuscript presents the possibility to integrate absorption loss at specific regions of the simulation. This is possible without defining the electric or magnetic conductivity in every single grid point and thus reducing the computation time and memory resources required. An ideal absorber is of interest for scattering simulations in biomedical applications as tool to design and validate future and current biomedical optical techniques, such as in medical diagnosis. Scattering simulations confirm the effective light elimination by the optical target model.

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