Analysis of Optimal Panel Geometry for Self-illustration Corner Reflector

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Abstract— Self-illuminating corner reflector is the corner reflector that all the rays that enter the reflector's cavity experience the triple reflection on the panel and return to the radar. It has the advantage of reducing the unexpected coherent ground interaction and thus improving the accuracy of RCS and benefiting SAR calibration and image quality assessment. The optimal panel geometry is the one that has the minimal edge length for a given panel area in order to alleviate edge diffraction. The panel geometry had been previously designed to be square, pentagon and hexagon. In this paper, the general expressions for the panel area and panel external edge length of arbitrarily self-illuminating corner reflectors were described by parameter equation firstly. Then the edges of reflector panel were assumed as circular arc and further elliptical arc for analysis. Through a numerical approximation approach, the external edge lengths of those self-illuminating trihedral corner reflectors are derived and compared. The results showed that the self-illuminating trihedral corner reflector with circular arc had the minimal edge length and was the optimal panel geometry among the panel geometries under analysis.

1. INTRODUCTION

SAR calibration is becoming more and more important for quantitative earth observation application [1–3]. The mostly widely used corner reflector as radar bright point target is the triangular trihedral corner reflectors. It has the advantage of large radar cross section (RCS), extremely wide RCS pattern, light weight, cheap and simple to manufacture. The panel geometry for trihedral corner reflector has been traditionally chosen as triangular shape [4–6]. However, not all the panel area is the effective area which contributes to the nominal RCS of the reflector. The additional 'tip' reflecting area, if interacting with ground plane, will yield an increase of RCS. This problem will affect SAR radiometric and phase calibration accuracy. The self-illuminating corner reflectors are proposed to solve this problem [7]. All the rays that enter the reflector's cavity experience the triple reflection on the panel and return to the radar. There are many types of self-illuminating corner reflectors, e.g., square, hexagon. Note that the nominal RCS of trihedral corner reflector is calculated by the panel area and radar wavelength, however this is true only when the leg length is very large compared to the radar wavelength. Otherwise, the contribution from edge diffraction becomes an issue. Since edge diffraction is proportional to the panel external edge length, the optimal panel geometry is the one that has the minimal external edge length for the same panel area. In [7], the panel geometry was assumed as polygon and optimum hexagon panel geometry was obtained. Based on the above approach, this paper analyzes more types of self-illuminating corner reflectors. For this purpose, the general expressions for the panel area and panel external edge length of arbitrarily self-illuminating corner reflectors are described by parameter equation firstly. Then the edges of reflector panel are assumed as straight line, circular arc, and elliptical arc. Through a numerical approximation approach, the external edge lengths of those self-illuminating trihedral corner reflectors are derived and compared.

2. TYPES OF SELF-ILLUSTRATION CORNER REFLECTOR AND ITS MATHEMATIC EXPRESSION

2.1. Expression of Self-illustration CR

Figure 1 shows the geometry of a self-illuminating corner reflector. Assuming curve $\mathbf{C}$ has the following form:

$$\mathbf{C}(\tau) = [0, f(\tau), g(\tau)], \quad \tau \in [0, t] \tag{1}$$

where $f(0) = 0$, $f(t) = l$, $g(0) = l$, $g(t) = l$ denote that the curve passes point A and B'. Arbitrary point p on YOZ plane will be mapped onto point p'. According to the mapping relationship, the coordinate of p' could be expressed by coordinate of p as

$$\begin{cases} x_{p'} = z_p \\ y_{p'} = z_p - y_p \end{cases} \tag{2}$$

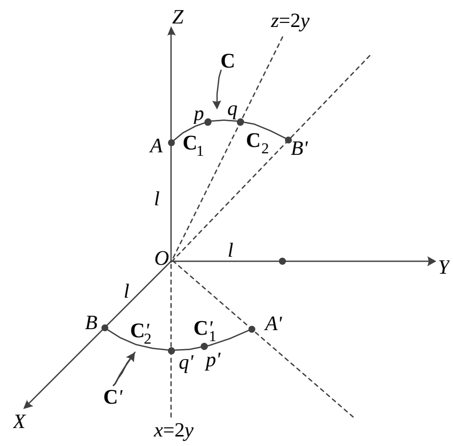

Figure 1: Mapping geometry.

So the curve $\mathbf{C}'$ on XOY plane could be expressed by: $\mathbf{C}' = [g(\tau), g(\tau) - f(\tau), 0], \tau \in [0, t]$. When τ increase from 0 to t, curve $\mathbf{C}$ traces from A to B' and curve $\mathbf{C}'$ traces from A' to B. On the other hand, according to the self-illuminating symmetry requirements, the curve mapped onto XOY must be identical to the curve mapped to YOZ, therefore there is point $t = t_0$, so that

$$\begin{cases} x_{q'} = z_q \\ y_{q'} = y_q \end{cases} \tag{3}$$

Substituting (3) into (2), then

$$y_{q'} = z_q - y_q = y_q \Rightarrow \begin{cases} z_q = 2y_q \\ x_{q'} = 2y_{q'} \end{cases} \tag{4}$$

Equation (4) means that there must be one point q on lines $z = 2y$ for arbitrary self-illuminating corner reflector. According to [7], the edge curve $\mathbf{C}_1$ and $\mathbf{C}_2$ of the self-illuminating corner reflector has the following forms:

$$\mathbf{C}_1(\tau) = [0, f_1(\tau), g_1(\tau)], \quad \tau \in [0, t_0] \tag{5}$$
$$\mathbf{C}_2(\tau) = [0, g_1(\tau), g_1(\tau) - f_1(\tau)], \quad \tau \in [0, t_0] \tag{6}$$

In Section 3, the types of edge curve $\mathbf{C}_1$ (i.e., straight line, circular, elliptical) will be discussed.

2.2. RCS and Edge Length of Self-illustration Corner Reflector

According to area integration formula, one panel area of the trihedral corner reflector could be expressed as

$$S = 2 \cdot \int_0^{t_0} \left(f_1' g_1 - g_1' f_1\right) d\tau \tag{7}$$

where f' denotes partial derivatives of f, and g' denotes partial derivatives of g.

3. ANALYSIS OF THE MINIMAL EDGE LENGTH OF VARIOUS TYPES OF SELF-ILLUSTRATION CORNER REFLECTOR

According to curve integration formula, edge length L of the trihedral corner reflector could be expressed as

$$L = 2 \cdot \int_0^{t_0} \left\{ \sqrt{f'^2_1 + g'^2_1} + \sqrt{(f'_1 - g'_1)^2 + g'^2_1} \right\} d\tau \tag{8}$$

The purpose is to choose f so that it would minimize (8) subject to constraint (7).

3.1. Curve $\mathbf{C}_1$: Straight Line

For simplicity, firstly assume the curve $\mathbf{C}_1$ is a straight line. According to (6), $\mathbf{C}_2$ is also a straight line, and thus the panel geometry is polygon. The shape of the panel geometry is dependent on the position of point q. If q lies on AB', then the panel geometry is square, and edge length of the panel geometry is $2\sqrt{A}$. If q lies on the position that makes the three points on the same straight line, then the panel geometry is pentagon, and edge length of the panel geometry is $2.1\sqrt{A}$.

In order to obtain the minimal edge length, express the area and edge length of polygon as:

$$S = l^2 + l\left(2y_0 - l\right) \tag{9}$$

$$L = 2\left(\sqrt{y_0^2 + \left(2y_0 - l\right)^2} + \sqrt{\left(l - y_0\right)^2 + \left(2y_0 - l\right)^2}\right) \tag{10}$$

From (9) and (10), the relation between external edge length L and interior leg length l could be derived, as shown in Figure 3. The minimal edge length is $L_{\min} = 1.9441\sqrt{S}$ when $l = 0.9469\sqrt{S}$. It can be concluded that among polygonous panel geometry, hexagon is the optimal panel geometry.

3.2. Curve $\mathbf{C_1}$: Circular Arc

According to the isoperimetric theorem, the circle has the shortest perimeter among all planar shapes with the same area, so the external edge length of a circular arc panel should be even smaller than that of hexagon panel, theoretically. Assume that the curve $\mathbf{C}_1$ is part of a circle with the center at $(y_0,\, l - x)$, as shown in Figure 4. From Figure 4, radius of the circle is

$$r = \sqrt{y_0^2 + x^2} = x + (2y_0 - l) \tag{11}$$

and the line segment

$$x = \frac{y_0^2 - (2y_0 - l)^2}{2(2y_0 - l)} \tag{12}$$

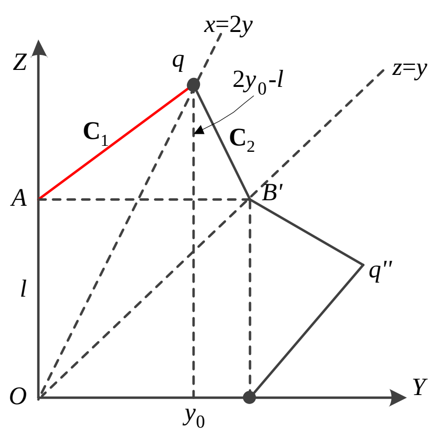

Figure 2: Panel geometry of polygon.

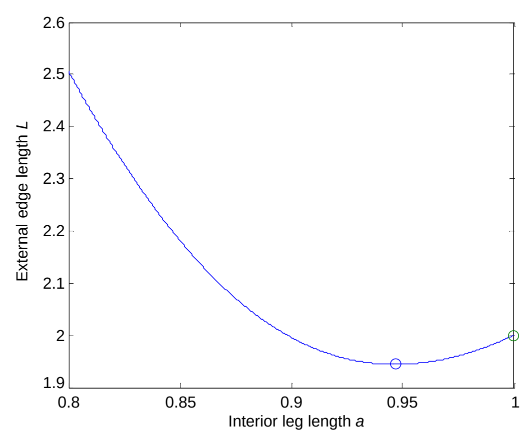

Figure 3: The relation between edge length L with interior edge length l.

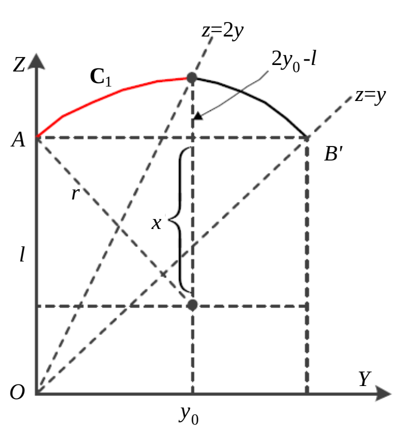

Figure 4: Panel geometry of circular arc.

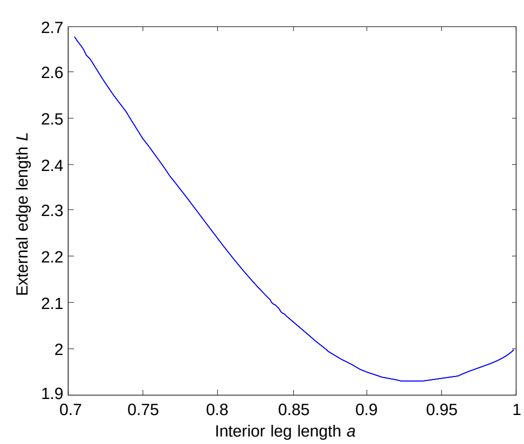

Figure 5: The relation between edge length L with interior edge length l.

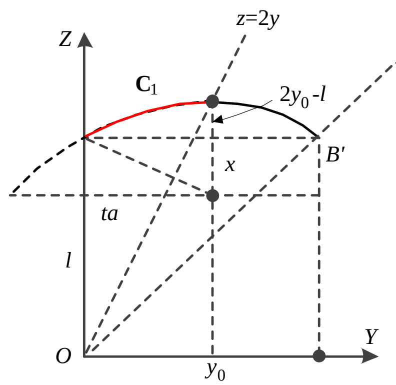

Figure 6: Panel geometry of elliptical arc.

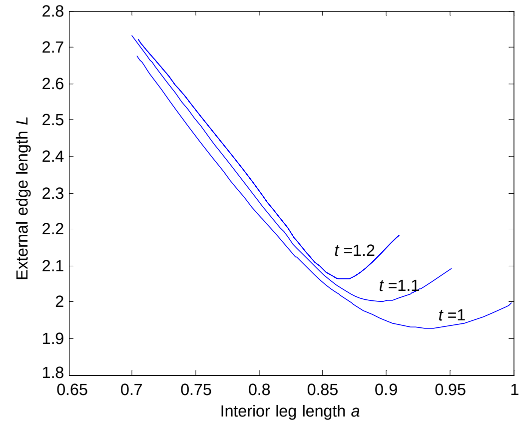

Figure 7: The relation between edge length with interior edge length l at different ratio t.

Table 1: Comparison of the edge lengths of different panel geometries.

Types of CR / Properties	straight line: square	straight line: pentagon	straight line: hexagon	circular arc	ellipse edge
Panel area	S	S	S	S	S
Interior leg length S	$\sqrt{S}$	$0.866\sqrt{S}$	$0.9469\sqrt{S}$	$0.932\sqrt{S}$	$0.932\sqrt{S}$
Minimum of external edge length	$2\sqrt{S}$	$2.1\sqrt{S}$	$1.9441\sqrt{S}$	$1.9282\sqrt{S}$	$1.9282\sqrt{S}$

The parameter equation for the circular arc is:

$$\begin{cases} y = f(\tau) = y_0 + r\cos\tau \\ z = g(\tau) = (l - x) + r\sin\tau \end{cases}, \quad \tau \in \left[\frac{\pi}{2} + \arctan\left(\frac{y_0}{x}\right), \frac{\pi}{2}\right] \tag{13}$$

Using (7), (8) and (11)∼(13), and through a numerical approximation approach [8], the relationship between edge length L with l has been derived and shown in Figure 5. When $l = 0.932\sqrt{S}$, the minimal edge length $L_{\min} = 1.9282\sqrt{S}$, which is smaller than that of the hexagon panel geometry.

3.3. Curve $\mathbf{C}_1$: Ellipse Arc

Now consider the more general case of the ellipse edge. Assume that the curve $\mathbf{C}_1$ is part of an ellipse with the center at $(y_0,\, l - x)$, semi-major axis $b = ta$ (t is the ratio between semi-major and short axis) and semi-minor axis a as shown in Figure 6. From Figure 6, the line segment

$$x = \frac{-y_0^2/t^2 - 4y_0^2 - l^2 + 4ly_0}{2l - 4y_0} - (2y_0 - l) \tag{14}$$

The parameter equation for the ellipse arc is:

$$\begin{cases} y = f(\tau) = y_0 + b\cos\tau \\ z = g(\tau) = (l - x) + a\sin\tau \end{cases}, \quad \left[\frac{\pi}{2} + \arctan\left(\frac{y_0}{x}\right), \frac{\pi}{2}\right] \tag{15}$$

The relation between edge lengths with interior edge length l at different ratio t are shown in Figure 7. We can see that when $t = 1$ (i.e., the circular arc case), the minimal edge length $L_{\min} = 1.9282\sqrt{S}$ when $l = 0.932\sqrt{S}$. The results showed that the minimal external edge length is obtained when the semi-major axis of the ellipse equals to semi-minor axis, which means that the panel geometry with circular arc has the minimal external edge length.

4. CONCLUSION

This paper analyzed the edge lengths of different types of self-illustration corner reflector. The simplest panel geometry of the self-illustration corner reflector is the polygon, but it would not have the minimal external edge length according to the isoperimetric theorem. This paper extended the type of panel geometry to the more general case (elliptical arc) for analysis. The results showed that the minimal external edge length is obtained when the semi-major axis of the ellipse equals to semi-minor axis, which means that the panel geometry with circular arc has the minimal external edge length. Its value was smaller than that of the polygon, which verified the assumption derived from the isoperimetric theorem. In the future the RCS accuracy will be validated by simulation and RCS testing the prototype of the circular arc self-illustration corner reflector.

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