Effects of Focusing on Scintillations of Higher Order Laser Modes in Non-Kolmogorov Turbulence

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Abstract— The scintillation index of focused higher order laser beam propagating in non-Kolmogorov atmospheric turbulence is formulated by employing the Rytov method and the equivalence of the structure constant. Our evaluations are performed for even modes. The equivalence formula for the structure constant is extracted from our earlier work in which the equivalence is obtained by equating the scintillation indices found in the Kolmogorov and the non-Kolmogorov turbulence. If not specified otherwise, the focused beam is defined when the focal length is equal to the link length. For the focused higher order laser beams, as the power law exponent of the non-Kolmogorov spectrum decreases, the scintillations decrease. At any power law exponent, the scintillations tend to become larger when the mode order of the focused beam becomes larger, i.e., the focused Gaussian beam is advantageous over the focused higher order laser beams for any realization of the non-Kolmogorov turbulence. Again being valid for any power law exponent, increase in the source size is found to decrease the intensity fluctuations of all the focused higher order mode scintillations. Especially for the larger order beams, focusing the higher order beam at a distance smaller than the link length results in a change in the behaviour of the scintillation index versus the power law exponent. In such cases, the scintillations are observed to increase. Comparison of the focused higher order beam scintillations with the previously obtained collimated higher order beam scintillations yields that the focused higher order beam scintillations are lower. Collimated higher order beams exhibit lower scintillations than the collimated Gaussian beams whereas this is reversed in the focused case. Another observation in such comparison shows that the difference of the intensity fluctuations between the Gaussian and the higher order beams are much larger in the focused case, especially at larger power law exponent values.

1. INTRODUCTION

The intensity fluctuations occurring in atmospheric turbulence are studied for many different incidences. Among these incidences, higher order mode operation of laser, forms one set. As known, the radiation originating from a laser resonator together with external configurations can yield single mode field distributions other than the widely used TEM_{00} (Gaussian) field which are referred as the higher order modes. These types of modes can be operated in collimated or focused forms. The scintillations of collimated higher order modes are evaluated in both Kolmogorov [1, 2] and non-Kolmogorov turbulence [3]. Focusing of a beam in atmospheric turbulence results in completely different intensity fluctuations [4–6]. Atmospheric turbulence deviating from the Kolmogorov spectrum also varies the scintillation behaviour of the beams when compared to the scintillation behaviour in Kolmogorov turbulence [7–10].

In this paper, the scintillation index in non-Kolmogorov turbulence is evaluated when the excitation is a focused higher order laser beam.

2. FORMULATION

In Rytov method, the scintillation index is given by $m^2 = 4B_\chi(L)$ where $B_\chi(L)$ is the correlation of the log-amplitude which is previously reported in [11] for a general type optical beam and L is the propagation distance. In the $B_\chi(L)$ solution provided in [11], we employ the focused higher order mode field as the incident beam, which is expressed by $u_s(s_x, s_y) = H_n(s_x/\alpha_s)H_m(s_y/\alpha_s)\exp\{-[0.5k\,\beta(s_x^2 + s_y^2)]\}$ where $k = 2\pi/\lambda$, λ is the wavelength, H_n is the Hermite polynomial of order n, (s_x, s_y) is the transverse source coordinate, α_s is the source size, $\beta = 1/(k\alpha_s^2)+(i/F)$, F is the focal length. This will yield the scintillations as

$$m^2 = \frac{4\pi}{[H_n(0)\,H_m(0)]^2}\mathrm{Re}\left\{\int_0^L d\eta\int_0^\infty \kappa d\kappa\int_0^{2\pi} d\theta\left[E(\eta,\kappa,\theta,L)\,E(\eta,-\kappa,\theta,L)+|E(\eta,\kappa,\theta,L)|^2\right]\Phi_n(\kappa)\right\}, \tag{1}$$

where $\kappa \exp(i\theta)$ is the two-dimensional spatial frequency in polar coordinates, η is the distance along the propagation axis,

$$E(\eta, \kappa, \theta, L) = ik \exp\left[-\frac{0.5i(L-\eta)(1+\beta i\eta)\kappa^2}{k(1+\beta iL)}\right] H_n\left(\frac{\eta - L}{k}\varsigma\kappa\cos\theta\right) H_m\left(\frac{\eta - L}{k}\varsigma\kappa\sin\theta\right),$$

$\varsigma = \{\alpha_s(1-i\beta L)^{1/2}[1-iL(\frac{2}{k\alpha_s^2}-\beta)]^{1/2}\}^{-1}$, and the spectrum in non-Kolmogorov turbulence is $\Phi_n(\kappa) = h(\alpha)\kappa^{-\alpha}$, $0 \le \kappa < \infty$, $3 < \alpha < 4$, with the equivalence of the structure functions involved [12], $h(\alpha) = -\Gamma(\alpha)(k/L)^{\alpha/2-11/6} C_n^2/\{8\pi^2\Gamma(1-0.5\alpha)[\Gamma(0.5\alpha)]^2 \sin(0.25\pi\alpha)\}$, $3 < \alpha < 4$, $\Gamma(.)$ is the gamma function, α is the power law exponent of the non-Kolmogorov spectrum, $i = (-1)^{1/2}$. Note that Eq. (1) is valid for even order modes.

3. RESULTS

All the figures show the variation of the scintillation index versus the power law exponent of the non-Kolmogorov spectrum, α for various source and medium parameters. If not indicated, the higher order beam is focused at the propagation distance, i.e., $F = L$. From Fig. 1, it is seen that higher α values yield higher scintillations for all higher order modes and at the same, the scintillations become large as the mode order increases. It is not presented here but found that the increase in the source size decreases the scintillations of the focused higher order beams in non-Kolmogorov medium. Fig. 2 is plotted for the same modes as in Fig. 1 but with different source size, structure constant and wavelength. Change in the parameters naturally changes the scintillation index values, however the trend versus α remains the same. From Figs. 1 and 2, it is concluded that the scintillation index of the focused Gaussian beam is smaller than the scintillation index of the focused higher order laser beams for any α.

Figure 3 reflects the effect of the focal length on the scintillations of the given higher order modes if the focal length is different than $F = L$. The observation is that when the higher orders beams are focused at a distance before the receiver, the scintillations of all the even higher order modes attain closer values at all power law exponent. We additionally report, without presenting the results in a figure, that for the larger order beams, focusing the higher order beam at a distance

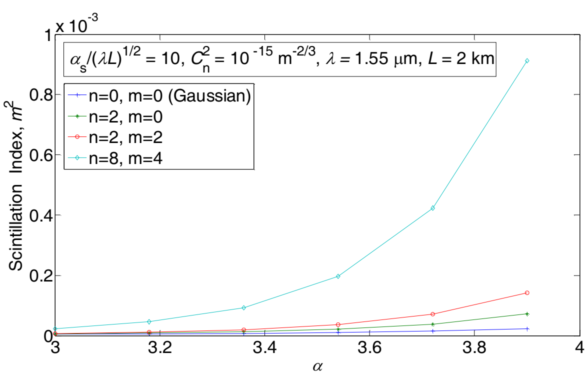

Figure 1: m^2 versus α for the indicated modes and parameters.

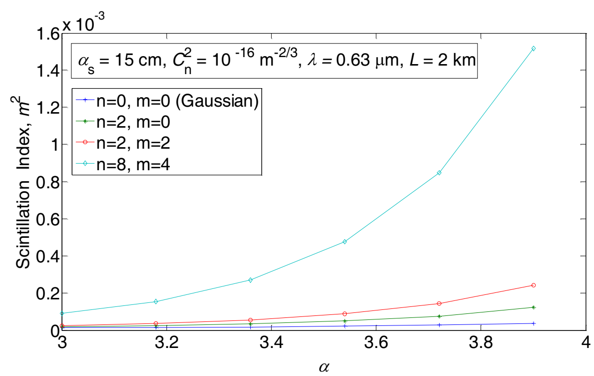

Figure 2: m^2 versus α for the same modes as in Fig. 1 but at different parameters.

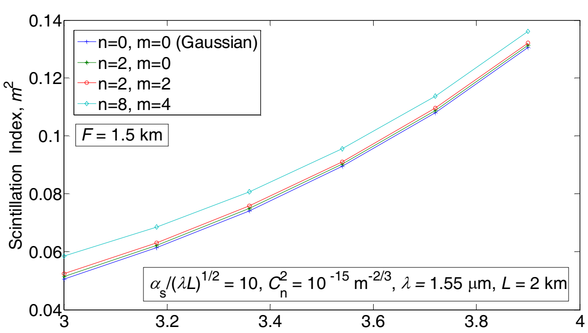

Figure 3: m^2 versus α for the same modes and parameters as in Fig. 1 but at $F = 1.5$ km.

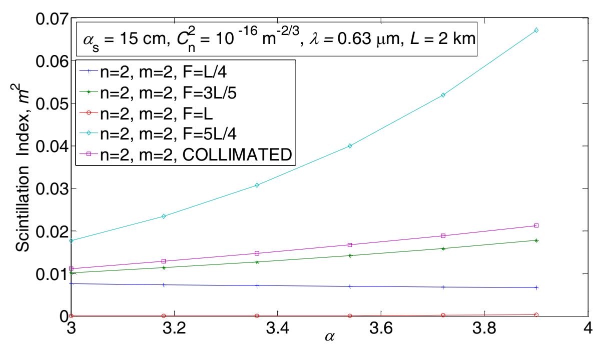

Figure 4: Comparison of m^2 at different focal lengths.

smaller than the link length will vary the behaviour of the scintillations versus α. In Fig. 4, the effect of the focal length on the scintillation index of the same higher order mode is examined as the power law exponent is varied. It is observed that depending on the chosen mode order and the source and medium parameters, the scintillations will show different variations, but the focused beam ($F = L$) will exhibit the smallest intensity fluctuations for all the power law exponent values of the non-Kolmogorov turbulence. This trend was found to be the opposite for collimated Gaussian beams in non-Kolmogorov turbulence [3].

4. CONCLUSION

Focused higher order mode scintillations are investigated in non-Kolmogorov atmospheric turbulence. Being valid for all the focused higher order modes, the scintillations become larger as the power law exponent and the mode order increase. Change in the source and medium parameters vary the scintillations, however, the trend versus α remains unchanged. When the higher orders modes are focused at a distance different than the propagation distance, the behaviour of the intensity fluctuations varies based on the chosen mode order, source and medium parameters where the minimum scintillations occurs at $F = L$ for any power law exponent of the non-Kolmogorov spectrum.

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