On Frequency Optimization of Assymetric Resonant Inductive Coupling Wireless Power Transfer Links

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Abstract— Resonant Inductive Coupling Wireless Power Transfer (RIC-WPT) is a leading field of research due to the growing number of applications that can benefit from this technology: from biomedical implants to consumer electronics, fractionated spacecraft and electric vehicles amongst others. However, current applications are limited to symetric point-to-point-links. New challenges and applications of RIC-WPT emphasize the necessity to explore, predict and optimize the behavior of these links for different configurations: multi-point RIC-WPT networks and assymetrical systems. In this work a design methodology oriented towards the optimization of assymetric RIC-WPT links is presented, resulting in a closed analytical formulation of the optimal frequency at which an assymetrical RIC-WPT link should operate. Finally, the resulting efficiency-optimized link si explored and compared to previous results obtained in RIC-WPT symetric configurations [1, 2].

1. INTRODUCTION

Power transfer efficiency is key in wireless power transfer systems. In particular, in RIC-WPT links, the efficiency of the physical layer strongly depends upon a) the frequency of operation (resonant frequency of the link), b) the losses of the transmitter and receiver coils and c) the mutual inductance between them. Since both the losses and the mutual inductance are frequency-dependant, it is of interest to analyze the optimal frequency at which a given link should operate to maximize its efficiency. This has been previously studied for symetric point-to-point RIC-WPT links [2], but it is still unexplored for assymetrical configurations (different transmitter and receiver sizes), in which the difference between transmitter and receiver minimum-loss frequencies emphasize the need for an optimal system co-designed frequency of operation. In this work, the power transfer efficiency of impedance-matched Assymetric RIC-WPT links is studied in terms of frequency (Section 2) and a closed analytical expression of the optimal frequency of operation is provided (Section 3). Finally, the System Efficiency of Assymetric Frequency-Optimized RIC-WPT links is analyzed and compared to the Symetric Frequency-Optimized Configuration (Section 4).

2. EFFICIENCY IN ASSYMETRIC RIC-WPT

The power transfer efficiency in RIC-WPT links, defined as the ratio between the power delivered to the load and the total input power, can be expressed as a function of the input frequency ω, the load R_L, transmitter and receiver losses (R_1, R_2), and the mutual inductance between them M_{12} [5]

$$\eta = \frac{R_L(\omega M_{12})^2}{(\omega M_{12})^2(R_2 + R_L) + R_1(R_2 + R_L)^2} \tag{1}$$

If impedance matching conditions are fulfilled ($R_L = \sqrt{R_2^2 + (\omega M_{12})^2 R_2/R_1}$ [3]), the efficiency of the system only depends upon the equivalent resistance of the coils (radiative, ohmic and dielectric losses), the mutual impedance between them and the frequency of operation, resulting in:

$$\eta_o = \frac{\sqrt{1 + \left(\frac{\omega M_{12}}{\sqrt{R_1 R_2}}\right)^2} - 1}{1 + \left(\frac{\omega M_{12}}{\sqrt{R_1 R_2}}\right)^2 + 1} = \frac{\sqrt{1 + S_a^2} - 1}{\sqrt{1 + S_a^2} + 1} \tag{2}$$

It can be observed in 2 that, to maximize efficiency, the relationational factor S_a — which is equivalent to the Coupled mode Theory K/Γ [6] — has to be maximized.

$$S_a = \frac{\omega M_{12}}{\sqrt{R_1 R_2}} \tag{3}$$

To accomplish this, it is necessary to 1) Maximize the frequency of the resonators (ω), 2) maximize the mutual inductance between coils (M_{12}) and 3) minimize the transmitter (R_1) and receiver (R_2) losses, which depend upon the technological parameters of the coils and the separation between them as follows:

$$\begin{aligned} M_{12} &= f(N_1, a_1, N_2, a_2, D_{12}) \\ R_1 &= f(\omega, N_1, a_1, b_1, c_2, \sigma_1) \\ R_2 &= f(\omega, N_2, a_2, b_2, c_2, \sigma_2) \end{aligned} \tag{4}$$

where two circular loop antennas with $N_{1,2}$ turns, coil diameters $a_{1,2}$, wire radius $b_{1,2}$, inter-turn separation $c_{1,2}$, conductivity $\sigma_{1,2}$ and a distance D_{12} between them, have been assumed.

To maximize the efficiency, the frequency should be chosen so that S_a is maximized (highest frequency, maximum mutual inductance and minumum coil's losses). In the forecoming sections, the mutual inductance as well as the coil's losses are derived for N-turn circular loop antennas.

2.1. Mutual Inductance

In the quasi-static limit, at large distances ($D_{12} \gg a_1$) the magnetic flux density at the receiver coil as a result of the transmitter coil has the form of a dipole [4].

$$B_{12} \simeq \frac{\mu_0}{2} \frac{N_1 i a_1^2}{D_{12}^3} \tag{5}$$

where coaxial orientation between coils has been assumed. The mutual inductance is then found froum the flux through the N_2 linkages in the receiver coil:

$$M_{12} = N_2 \frac{\partial \Psi_{12}}{\partial i} \simeq \frac{\pi}{2} N_1 N_2 \mu_0 \frac{a_1^2 a_2^2}{D_{12}^3} \tag{6}$$

2.2. Evaluation of Losses

The losses of the resonators depend upon their constituent materials ($\sigma_{1,2}, \delta_{1,2}$) and geometry ($a_{1,2}, b_{1,2}, c_{1,2}$) and can be divided into Radiative Losses (R_r), Ohmic Losses (R_o) and Dielectric Losses (R_d):

$$R_1 = R_r^1 + R_o^1 + R_d^1 \tag{7}$$

In this work, the losses of two electrically small ($a \ll \lambda$) circular loop antennas (chosen for their low radiation resistance [42]) are considered.

The radiation losses of a circular N_1-turn loop antenna with loop radius a_1 can be found by [4]:

$$R_r^1 = 20\pi^2 N_1^2 a_1^4 \frac{\omega^4}{c_o^4} \tag{8}$$

The ohmic resistance, which is in general much larger than the radiation resistance, depends upon the proximity effect (if the spacing between the turns in the loop antenna is small) and the skin effect. The total ohmic resistance for an N_1-turn circular loop antenna with loop radius a_1, wire radius b_1 and loop separation $2c_1$ is given by [4]:

$$R_o^1 = \frac{N_1 a_1}{b_1} R_s^1 \left(\frac{R_p^1}{R_0^1} + 1 \right) \tag{9}$$

where $R_s^1 = \sqrt{\omega\mu_0/2\sigma_1}$ is the surface impedance of the conductor and R_p^1 is the ohmic resistance per unit length due to proximity effect.

Finally, if a dielectric loop antenna is considered, the dielectric losses are given by:

$$R_d^1 = \frac{\tan\delta_1}{\omega C_1} = \omega L_1 \tan\delta_1 \simeq \omega 4\mu_0 a_1 N_1^2 \tan\delta_1 \tag{10}$$

where $\tan\delta_1$ is the loss tangent of the coil and L_1 has been approximated to $L_1 = 4\mu_0 a_1 N_1^2$. Defining the ratios of assymetry in number of turns (u_N), antenna diameter (u_a), wire radius (u_b), inter-turn distance (u_c), conductivity (u_σ) and loss tangent (u_δ) as follows:

$$\begin{aligned} u_N &= \frac{N_2}{N_1}; \quad u_a = \frac{a_2}{a_1}; \quad u_b = \frac{b_2}{b_1} \\ u_\sigma &= \frac{\sigma_2}{\sigma_1}; \quad u_c = \frac{c_2}{c_1}; \quad u_\delta = \frac{\delta_2}{\delta_1} \end{aligned} \tag{11}$$

the losses at the receiver (assuming $u_\delta = u_c = 1$) can be expressed as:

$$R_2 = R_r^2 + R_o^2 + R_d^2 = u_N^2 u_r^4 R_r^1 + \frac{u_N u_a}{u_b\sqrt{u_\sigma}} R_o^1 + u_a u_N^1 R_d^1 \tag{12}$$

3. FREQUENCY OPTIMIZATION OF ASSYMETRIC RIC-WPT

3.1. Optimal Frequency

To find the optimal frequency at which the Assymetric RICWPT link should operate, it is necessary to take the derivative of S_a with respect to ω. To do this, the losses in the transmitter coil are expressed as:

$$\begin{aligned} R_r^1 &= C_r^1 \omega^4; \quad C_r^1 = \frac{20\pi^2 N_1^2 a_1^4}{c_o^4} \\ R_o^1 &= C_o^1 \sqrt{\omega}; \quad C_o^1 \simeq \frac{N_1 a_1}{b_1}\sqrt{\frac{\mu_0}{2\sigma_1}} \\ R_d^1 &= C_d^1 \omega; \quad C_d \simeq 4\mu_0 a_1 N_1^2 \tan\delta_1 \end{aligned} \tag{13}$$

where C_r^1, C_o^1 and C_d^1 are the frequency-independent cofficients corresponding to the radiation, ohmic and dielectric losses respectively. Similarly, the losses at the receiver coil can be defined as:

$$\begin{aligned} R_r^2 &= C_r^2 \omega^4; \quad C_r^2 = K_r C_r^1; \quad K_r = u_N^2 u_a^4 \\ R_o^2 &= C_o^2 \sqrt{\omega}; \quad C_o^2 = K_o C_o^1; \quad K_o = \frac{u_N u_a}{u_b\sqrt{u_\sigma}} \\ R_d^2 &= C_d^2 \omega; \quad C_d^2 = K_d C_d^1; \quad K_d = u_a u_N^2 \end{aligned} \tag{14}$$

where K_r, K_o and K_d model the effect of the assimetries between transmitter and receiver upon the receiver's losses. Finally, the mutual inductance can be expressed as a function of the equivalent mutual inductance obtained in a symetric link ($u_{N,a,b,c,\delta} = 1$) as:

$$M_{12}^a = K_m M_{12}; \quad K_m = u_N u_a^2; \tag{15}$$

Once this is known, both the mutual inductance (15) and the transmitter and receiver losses (13), (14) can be substituted in Equation (3) resulting in:

$$S_a = \frac{\omega K_m M_{12}}{\sqrt{(C_r^1\omega^4 + C_o^1\sqrt{\omega} + C_d^1\omega)(K_r C_r^1\omega^4 + K_o C_o^1\sqrt{\omega} + K_d C_d^1\omega)}} \tag{16}$$

which is then be derived with respect to ω to obtain the optimal frequency at which the assymetric link should operate. The resulting ω_o^a is the solution of:

$$6\omega^7 C_R^2 K_r + 6\omega^4 C_d C_r \left(\frac{K_d + K_r}{2}\right) + 5\omega^{\frac{7}{2}} C_o C_r \left(\frac{K_o + K_r}{2}\right) - \omega^{\frac{1}{2}} C_d C_o \left(\frac{K_o + K_d}{2}\right) - K_o C_o^2 = 0 \tag{17}$$

For illustration purposes and in order to achieve a closed analytical formulation, dielectric-less transmitter and receiver coils are assumed ($C_d^1 = C_d^2 = 0$), obtaining:

$$\omega_{opt}^a = K_\omega \omega_{opt} \tag{18}$$

where:

$$K_\omega = \left(\frac{(K_o + K_r)}{K_r}\left[\sqrt{\frac{25}{16} + \frac{6K_o K_r}{(K_o + K_r)^2}} - \frac{5}{4}\right]\right)^{\frac{2}{7}} \tag{19}$$

and ω_{opt} is the optimal frequency corresponding to a symmetric link ($u_{N,a,b,c,\sigma,\delta} = 1$):

$$\omega_{opt} = \left(\frac{C_o^1}{6C_r^1}\right)^{\frac{2}{7}} \tag{20}$$

3.2. Maximum Efficiency

Once the optimal frequency at which the link should operate is obtained, the maximum relational factor S_a and the resulting maximum efficiency $\eta^a_{\max}$ can be found by substituting ω by ω_{opt} in (3):

$$S_a^{\max} = S^{\max} \frac{7K_m\sqrt{K_w}}{\sqrt{K_r K_w^7 + 6K_w^{\frac{7}{2}}(K_o + k_r) + 36K_o}} \tag{21}$$

where $S^{\max}$ is the maximum relational factor S for a symetric link:

$$S^{\max} = \frac{6M}{7C_o^1}\left(\frac{C_o^1}{6C_r^1}\right)^{\frac{1}{7}} \tag{22}$$

and the maximum power transfer efficiency is then obtained by:

$$\eta_a^{\max} = \frac{\sqrt{1 + (S_a^{\max})^2} - 1}{\sqrt{1 + (S_a^{\max})^2} + 1} \tag{23}$$

4. RESULTS

The results obtained above regarding the optimal frequency of the assymetric link, the maximum relational factor $S_{a,\max}$ and the corresponding maximum efficiency $\eta_{s,\max}$ are illustrated below. First, the normalized frequency deviation due to the link assymetry is shown in Figure 1:

$$\Delta w = \frac{w_o^a - w_o^s}{w_o^s} = k_w - 1 \tag{24}$$

where $k_w = f(K_o, K_r)$ as per Equation (19). It can be observed that, when the assymetry between transmitter and receiver represents around a 40% of difference either in the radiation or the ohmic

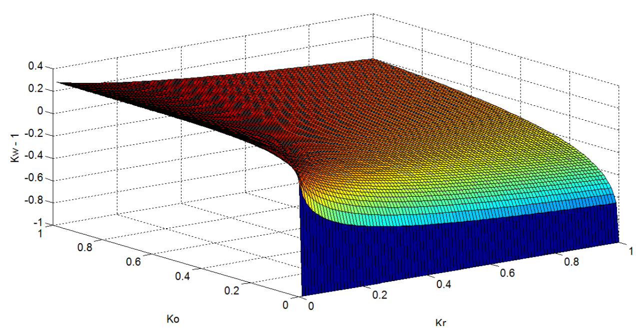

Figure 1: Normalized Frequency Deviation (Δw) with $C_o = C_r = 0.05$.

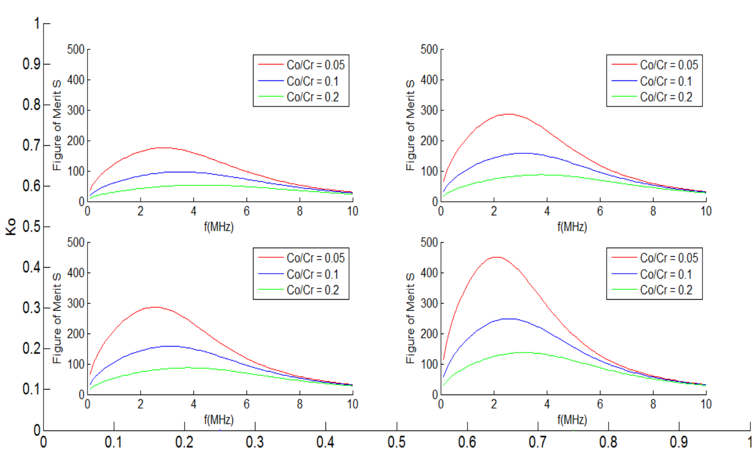

Figure 2: $S^a_{\max}$ study for different K_o, K_r configurations.

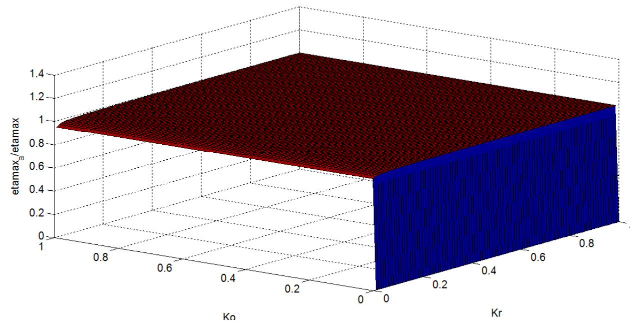

Figure 3: Normalized Maximum Efficiency for different K_o, K_r configurations. $C_o = C_r = 0.05$.

losses, the optimal frequency of operation for the assymetric link experiments a 20% deviation from the optimal frequency in the symetric link, showcasing the impact of this study. When assessing the effect of the assymetry in RICWPT links, it is also of interest to study how this assymetry (described in a compressed manner by the coefficients K_r, K_o and K_m) affects $S_{\max}$ and the maximum achievable efficiency. To do this, Figure 4 illustrates the obtained $S_{\max}$ for different K_o/K_r and $C_o = C_r$ coefficients as a function of frequency, where the frequency deviation explained in Figure 1 can be observed. Finally, Figure 4 shows the resulting maximum efficiency ($\eta^a_{\max} = \eta^a|_{w=w^a_{opt}}$) normalized with respect to the symetric link maximum efficiency ($\eta^a_{\max} = \eta^a|_{w=w^s_{opt}}$).

5. CONCLUSIONS

The efficiency of Assymetric RIC-WPT systems has been studied and presented in this work, together with a closed analytical formulation for optimal frequency in RIC-WPT Assymetric links. Finally, the scalability of Assymetric Frequency- Optimized RIC-WPT Links has been assessed and benchmarked with Symetric Frequency-Optimized RIC-WPT links.

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