Phase Behaviour of an Azimuthally Magnetized Two-layered Ferrite-dielectric Circular Waveguide

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Abstract— The areas of propagation in the $\bar{r}_0 - \bar{\beta}$ — phase plane, the $\bar{\beta}(\bar{r}_0)$ — phase characteristics and the lower and upper cut-off frequencies (the lower and upper normalized critical guide radii $\bar{r}_{0cr}$ and $\bar{r}_{0en\pm}$, resp.) of the circular waveguide, containing a thick latching ferrite cylinder, magnetized azimuthally to remanence, surrounded by a thin dielectric toroid, are studied for the normal TE_{01} mode, assuming equal values of the relative permittivities of both strata, ($\bar{r}_0$ and $\bar{\beta}$ — the guide radius and phase constant, normalized with respect to the frequency and relative permittivity of anisotropic filling, "+" and "−" — subscripts, labeling quantities, relevant to positive, resp. negative direction of its magnetization). The principle outcome of the investigation is that the removal even of an insignificant part from the ferrite in the vicinity of the wall and its replacement by an isotropic medium changes substantially the phase behaviour of the configuration, compared to the case in which it fills in the whole transmission line, inspected earlier. This is visible for both signs of ferrite magnetization, but especially noticeable if its sign is positive (counter-clockwise). Besides, the alteration of the size of outer layer, of its permittivity, keeping that of the inner one the same and of the direction of magnetization of the ferrite load, are the most important factors that determine the position, boundaries, shape and dimensions of the region in the phase plane in which the wave might be sustained, as well as the distribution of its two kinds of cut-off frequencies (the critical radii).

1. INTRODUCTION

The circular waveguides, entirely or partially filled with ferrite of azimuthal magnetization are apt for the development of nonreciprocal digital phase shifters, operating in the normal TE_{01} mode, since they can provide differential phase shift when switching the remanent magnetization of their anisotropic load between its two stable states [1–8]. For certain combinations of parameters the multilayered ferrite-dielectric structures could afford an increased value of this important quantity [1–4, 8]. Due to the complexity of the tasks, however, still the phase behaviour of these configurations is not sufficiently clarified.

The purpose of this paper is the investigation of the phase behaviour of a two-layered geometry of the above type in which the anisotropic medium takes up its inner and the isotropic one — its outer region, using results of the recent solution of the propagation problem for the normal TE_{0n} modes in it [4]. To simplify the study, the discussion is confined to the TE_{01} mode solely. As earlier [4], the relative permittivities of both media are assumed equal. In addition, the relative size of the area, occupied by dielectric is small, compared to the one of the transmission line and the lower part of frequency band in which the wave could be sustained, is considered only.

2. FORMULATION OF THE PROBLEM

An infinitely long, uniform, lossless perfectly conducting circular waveguide of radius r_0, loaded with a coaxially positioned latching ferrite cylinder of radius r_1, magnetized azimuthally to remanence by an infinitely thin axial wire and a dielectric toroid of inner and outer radii r_1 and r_0, resp., is considered. The ferrite has a Polder permeability tensor $\vec{\mu} = \mu_0[\mu_{ij}]$, $i, j = 1, 2, 3$, with nonzero components $\mu_{ii} = 1$ and $\mu_{13} = \mu_{31} = -j\alpha$, $\alpha = \gamma M_r/\omega(-1 < \alpha < 1)$, γ — gyromagnetic ratio, M_r — ferrite remanent magnetization, ω — angular frequency of the wave and a scalar permittivity $\varepsilon = \varepsilon_0\varepsilon_r$. The dielectric possesses a scalar permittivity and permeability $\varepsilon^d = \varepsilon_0\varepsilon_d$ and $\mu^d = \mu_0\mu_d$, resp., $\mu_d = 1$. It is accepted that $\varepsilon_r = \varepsilon_d$. The propagation of normal TE_{0n} modes with phase constant β is explored. Normalized quantities are introduced by the relations: $\bar{\beta} = \beta/(\beta_0\sqrt{\varepsilon_r})$, $\bar{r}_0 = \beta_0 r_0\sqrt{\varepsilon_r}$ and $\bar{r}_1 = \beta_0 r_1\sqrt{\varepsilon_r}$ with $\beta_0 = \omega\sqrt{\varepsilon_0\mu_0}$ [4–6].

3. PHASE BEHAVIOUR

Figures 1(a), (b) and 2(a), (b) depict the domain of transmission of the normal TE_{01} wave in the structure under study (portrayed by light green), magnetized in positive ($\alpha_+ > 0$) and negative ($\alpha_- < 0$) direction, resp., assuming anisotropic cylinder to waveguide radius ratio $\rho = 0.9$ and 0.95, resp. ($\rho = \bar{r}_1/\bar{r}_0$, $0 < \rho < 1$). The corresponding normalized $\bar{\beta}(\bar{r}_0)$ — phase characteristics with α as parameter are pictured here by solid and dashed lines, as well. The $\alpha = 0$ — solid curve (the lower border of the area in question for $\alpha_+ > 0$) dovetails to a geometry, wholly filled with dielectric of relative permittivity ε_r. In case of negative magnetization this boundary is composed of parts of the phase lines for small $\alpha_- < 0$. For upper limits of the areas inspected serve the En_{1-}- and En_{1+}-dotted envelopes (the sets of termination points of the $\bar{\beta}(\bar{r}_0)$ — characteristics for higher frequencies) at which the propagation ceases. The appearance of the latter may be interpreted as

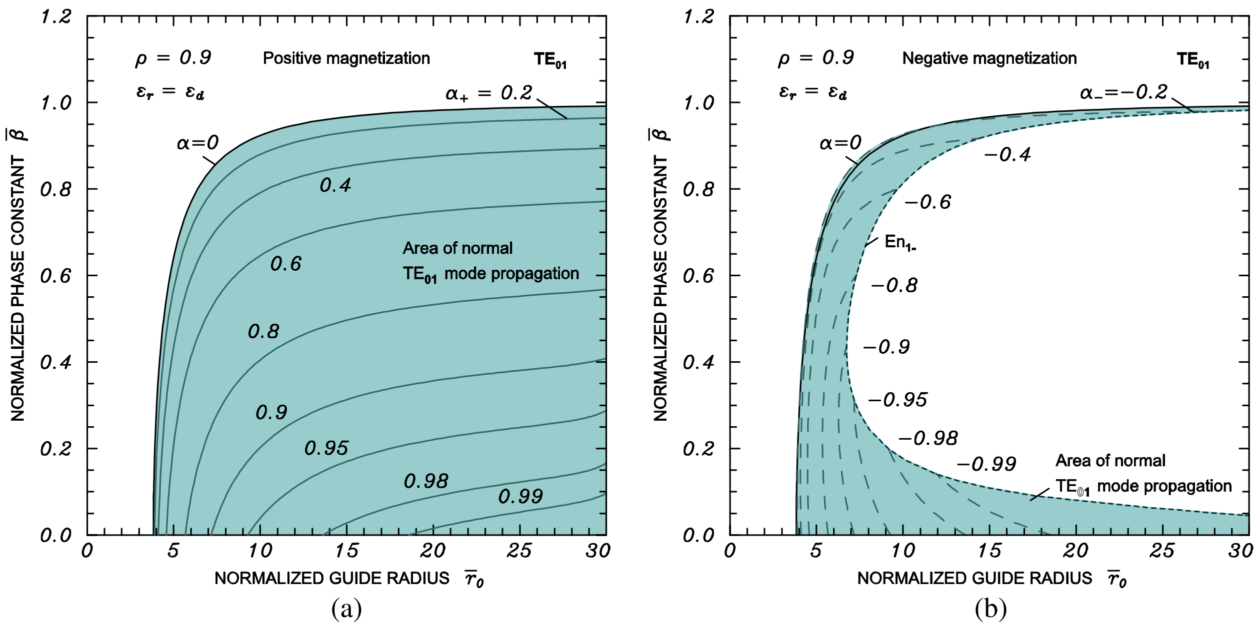


Figure 1: Area of transmission and phase characteristics of the ferrite-dielectric waveguide, magnetized azimuthally in: (a) positive and (b) negative direction for the normal TE_{01} mode in case $\rho = 0.9$.

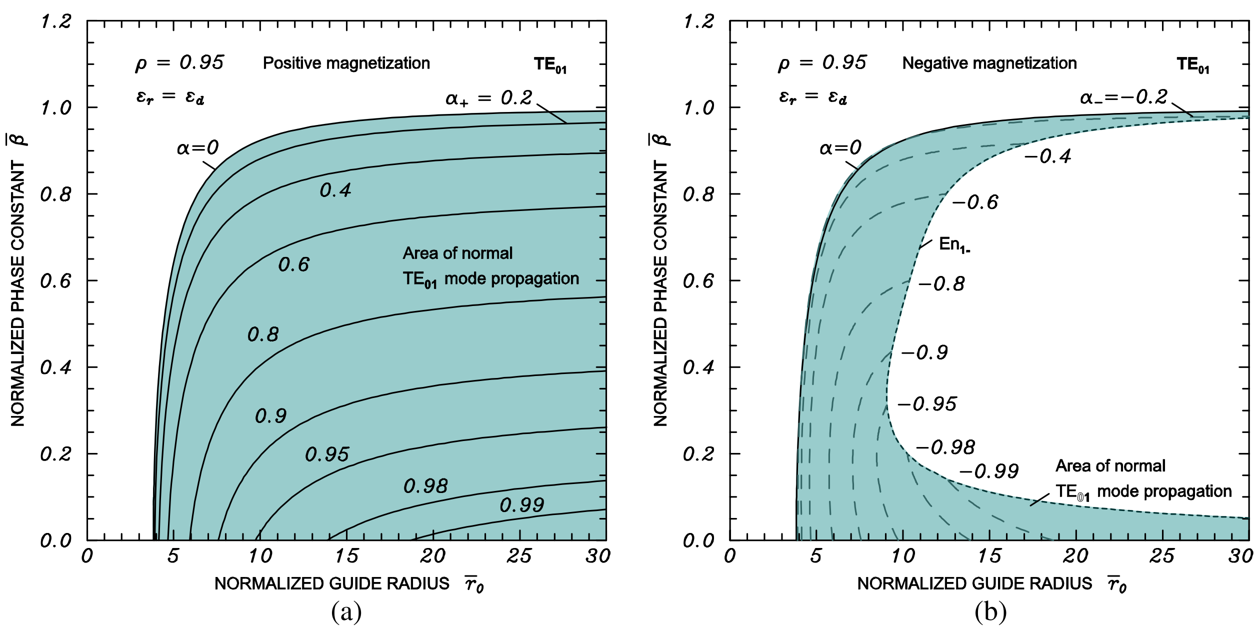


Figure 2: Area of transmission and phase characteristics of the ferrite-dielectric waveguide, magnetized azimuthally in: (a) positive and (b) negative direction for the normal TE_{01} mode in case $\rho = 0.95$.

a springing up of upper cut-off frequencies of the configuration. Accordingly, the conventional ones may be called lower cut-off frequencies.

The phase diagrams are plotted, employing the procedure, worked out earlier [4]. It uses the roots of the characteristic equation of the structure $\xi^{(c)}_{k_\pm,n}(\bar{\varepsilon},\rho,\alpha_\pm)$ [4], derived in terms of complex Kummer confluent hypergeometric and the real Bessel and Neumann functions [9]. ($k_\pm$ — imaginary part of the complex first parameter of Kummer function, $k_\pm = \alpha_\pm\bar{\beta}/(2\bar{\beta}_2)$, $\bar{\beta}_2 = \beta_2/(\beta_0\sqrt{\varepsilon_r})$, $\beta_2 = [\omega^2\varepsilon_0\mu_0\varepsilon_r(1-\alpha^2)-\beta^2]^{1/2}$ — radial wavenumbers in the anisotropic medium [4]).

Table 1 lists the roots $\xi^{(c)}_{0,n}(\bar{\varepsilon},\rho,\alpha_\pm)$ ($k_\pm = 0$) of the equation mentioned and the lower cut-off frequencies of the configuration (the lower normalized critical guide radius) $\bar{r}_{0cr\pm} = \xi^{(c)}_{0,n}(\bar{\varepsilon},\rho,\alpha_\pm)/(2\sqrt{1-\alpha^2_\pm})$ at which no propagation takes place ($\bar{\beta}_{cr\pm} = 0$) [10]. Table 2 presents the $L_1(c,n)$ and $\bar{L}_{3\pm}(c,\bar{\varepsilon},\rho,\alpha_\pm,n)$ numbers and the upper cut-off frequencies of the waveguide (the upper normalized critical guide radii) $\bar{r}_{0en-} = L_1(c,n)/(|\alpha_{en-}|\sqrt{1-\alpha^2_{en-}})$, resp. $\bar{r}_{0en\pm} = L_{3\pm}(c,\bar{\varepsilon},\rho,\alpha_{en\pm},n)/(|\alpha_{en\pm}|\sqrt{1-\alpha^2_{en\pm}})$, pertinent to the case $\bar{\beta}_{en\pm} = \sqrt{1-\alpha^2_{en\pm}}$, ($c = 3$, $\bar{\varepsilon} = \varepsilon_d/\varepsilon_r = 1$, $0 < \rho < 1$, $-1 < \alpha_- < 0$, $0 < \alpha_+ < 1$, $n = 1,2,3,\ldots$, $|\alpha_-| \equiv |\alpha_+| \equiv |\alpha|$, $\bar{r}_{0cr-} \equiv \bar{r}_{0cr+} \equiv \bar{r}_{0cr}$, $\bar{r}_{0en-} \neq \bar{r}_{0en+}$) [4,9]. All data correspond to normal TE_{01} mode ($n = 1$). The calculations are performed for $|\alpha| = 0.05(0.05)0.95$, on condition that $\rho = 1$, 0.95 and 0.9. (For details on the definition and modeling of the various L numbers see Refs. [4,7,8,10]. In some of them the notation $L(c,n)$ is advanced instead of $L_1(c,n)$.) The distribution of $\bar{r}_{0cr}$ and $\bar{r}_{0en\pm}$ vs. $|\alpha|$ is presented in Figs. 3(a), (b) and 4(a), (b), resp. for the aforesaid numerical equivalents of ρ. The subscripts "*cr*" and "*en*±" mark off quantities, relative to the lower and upper cut-off frequencies (to the En_{1-}- and En_{1+}-lines.) Note that the above expressions allow to write the equation of the envelopes $\bar{\beta}_{en_-} = \bar{\beta}_{en_-}(\bar{r}_{0en-})$, resp. $\bar{\beta}_{en\pm} = \bar{\beta}_{en_\pm}(\bar{r}_{0en\pm})$ in parametric form [4]. The case $\rho = 1$ answers to waveguide, completely filled with ferrite in which there is only one envelope, relevant to negative ferrite magnetization — the En_{1-}-line of equation $\bar{\beta}_{en_-} = \bar{\beta}_{en_-}(\bar{r}_{0en-})$, see Ref. [3]).

Table 1: Values of the roots of the characteristic equation of the ferrite-dielectric structure $\xi^{(c)}_{0,n}(\rho)$ and of the lower cut-off frequencies $\bar{r}_{0cr}$ for the TE_{01} mode in case $\varepsilon_d = \varepsilon_r$, $\rho = 1$, 0.95 and 0.9 as a function of $|\alpha|$.

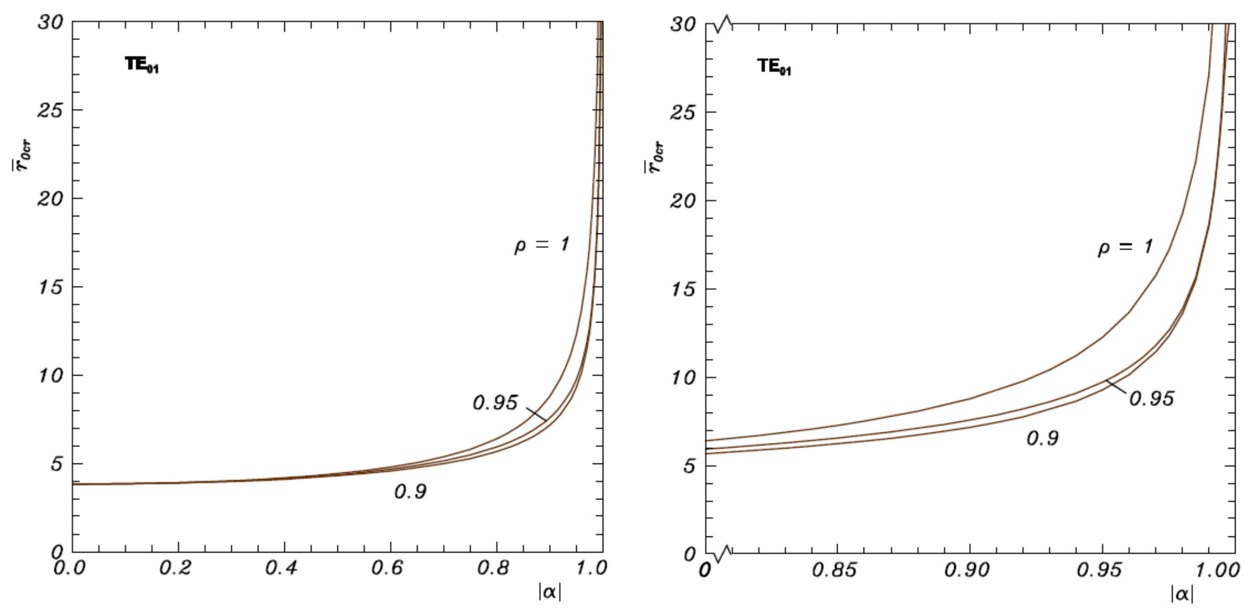
ρ	$\|\alpha\|$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
1	$\xi^{(c)}_{0,n}(\rho)$	7.66341									
	$\bar{r}_{0cr}$	3.83650	3.85101	3.87555	3.91072	3.95737	4.01672	4.09043	4.18073	4.29069	4.42447
0.95	$\xi^{(c)}_{0,n}(\rho)$	7.66249	7.65969	7.65494	7.64810	7.63896	7.62725	7.61257	7.59439	7.57196	7.54427
	$\bar{r}_{0cr}$	3.83604	3.84914	3.87127	3.90290	3.94474	3.99777	4.06329	4.14308	4.23948	4.35569
0.9	$\xi^{(c)}_{0,n}(\rho)$	7.66167	7.65642	7.64753	7.63478	7.61785	7.59629	7.56951	7.53670	7.49681	7.44842
	$\bar{r}_{0cr}$	3.83563	3.84750	3.86752	3.89611	3.93384	3.98154	4.04031	4.11161	4.19741	4.30035
ρ	$\|\alpha\|$	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
1	$\xi^{(c)}_{0,n}(\rho)$	7.66341									
	$\bar{r}_{0cr}$	4.58796	4.78963	5.04215	5.36546	5.79299	6.38618	7.27379	8.79054	12.27128	
0.95	$\xi^{(c)}_{0,n}(\rho)$	7.50991	7.46681	7.41192	7.34051	7.24492	7.11197	6.91715	6.61022	6.07361	
	$\bar{r}_{0cr}$	4.49607	4.66676	4.87669	5.13939	5.47665	5.92664	6.56547	7.58244	9.72556	
0.9	$\xi^{(c)}_{0,n}(\rho)$	7.38965	7.31793	7.22970	7.11992	6.98130	6.80295	6.56801	6.24916	5.79836	
	$\bar{r}_{0cr}$	4.42407	4.57371	4.75679	4.98494	5.27737	5.66913	6.23408	7.16828	9.28481	

Confronting the results obtained with the previous ones shows that:

i) The presence even of a slight non-uniformity in the anisotropic medium (of a thin dielectric toroid) in contact with the structure wall changes totally the phase behaviour of the waveguide compared to the case in which the ferrite occupies it entirely [3,6]. For negative magnetization this is clearly visible juxtaposing Figs. 1(b) and 2(b) with Fig. 3 [6]. For positive magnetization it follows from Table 2, indicating the appearance of upper cut-off frequencies in case $\rho = 0.95$ and 0.9, not observed when $\rho = 1$ (ferrite-filled waveguide). The upper limit of the propagation area (the envelope limit En_{1+}) however appears for values of $\bar{r}_0$, lying out of the scale chosen (larger than 30).

Table 2: Values of the $L_1(c, n)$, $\bar{L}_{3-}(c, \bar{\varepsilon}, \rho, \alpha_-, n)$ and $\bar{L}_{3+}(c, \bar{\varepsilon}, \rho, \alpha_+, n)$ numbers and of the upper cut-off frequencies $\bar{r}_{0cr}$ for the TE_{01} mode in case $\varepsilon_d = \varepsilon_r$, $\rho = 1$, 0.95 and 0.9 as a function of $|\alpha|$.

ρ	$\lvert\alpha\rvert$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
1	L_1	6.59365									
	$\bar{r}_{0en-}$	132.03823	66.26872	44.46072	33.64810	27.23959	23.04009	20.11104	17.98567	16.40772	15.22739
0.95	L_{3-}	6.42086	6.41569	6.40685	6.39399	6.37658	6.35383	6.32463	6.28743	6.23994	6.17892
	$\bar{r}_{0en-}$	128.57799	64.48008	43.20110	32.62919	26.34280	22.20207	19.29052	17.15036	15.52755	14.26959
	L_{3+}	48.51598	48.37313	48.13345	47.79447	47.35258	46.80286	46.13881	45.35198	44.43147	43.36313
	$\bar{r}_{0en+}$	971.53477	486.16823	324.56174	243.90013	195.62215	163.54244	140.72614	123.70766	110.56376	100.14286
0.9	L_{3-}	5.64657	5.63076	5.60398	5.56557	5.51458	5.44973	5.36943	5.27170	5.15415	5.01401
	$\bar{r}_{0en-}$	114.36997	56.59123	37.78738	28.40167	22.78172	19.04291	16.37710	14.37973	12.82564	11.57937
	L_{3+}	24.58168	24.50957	24.38857	24.21741	23.99422	23.71647	23.38081	22.98291	22.51715	21.97626
	$\bar{r}_{0en+}$	492.24937	246.33049	164.45108	123.58393	99.12448	82.87204	71.31289	62.69103	56.03193	50.75199
ρ	$\lvert\alpha\rvert$	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
1	L_1	6.59365									
	$\bar{r}_{0en-}$	14.35461	13.73678	13.34863	13.18995	13.29156	13.73678	14.72569	16.80764	22.22799	
0.95	L_{3-}	6.09947	5.99424	5.85188	5.65482	5.37652	4.97996	4.42183	3.66507	2.69066	
	$\bar{r}_{0en-}$	13.27875	12.48801	11.84695	11.31191	10.83804	10.37491	9.87533	9.34249	9.07055	
	L_{3+}	42.12844	40.70273	39.05234	37.12992	34.86577	32.15101	28.79961	24.44509	18.13731	
	$\bar{r}_{0en+}$	91.71505	84.79735	79.06017	74.27469	70.28279	66.98127	64.31854	62.31210	61.14304	
0.9	L_{3-}	4.84813	4.65297	4.42473	4.15939	3.85280	3.50075	3.09894	2.64290	2.12768	
	$\bar{r}_{0en-}$	10.55454	9.69369	8.95772	8.32044	7.76652	7.29322	6.92089	6.73692	7.17266	
	L_{3+}	21.35071	20.62784	19.79038	18.81409	17.66332	16.28255	14.57715	12.36132	9.15513	
	$\bar{r}_{0en+}$	46.48122	42.97466	40.06497	37.63570	35.60591	33.92198	32.55533	31.50978	30.86302	

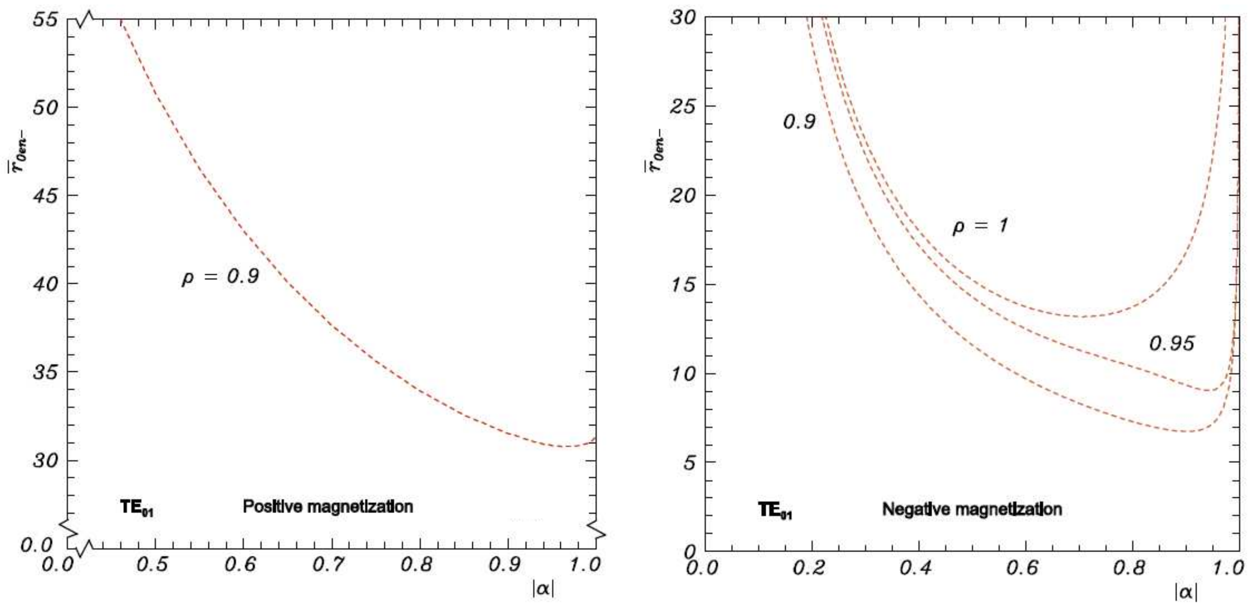


(a) (b)

Figure 3: Dependence of the lower cut-off frequencies $\bar{r}_{0cr}$ of the ferrite-dielectric waveguide for the normal TE_{01} mode on $|\alpha|$ with ρ as parameter, assuming: (a) $|\alpha| = \langle 0 \div 1 \rangle$; (b) $|\alpha| = \langle 0.8 \div 1 \rangle$.

ii) In view of the substantial influence of the value of $\varepsilon_d/\varepsilon_r$ — ratio on the phase portrait of a geometry, containing dielectric cylinder and ferrite toroid, established before [8], it might be expected that a similar dependence might be observed in the behavour of the structure under study.

iii) An insignificant variation of the parameter ρ (of the size of the area, taken by dielectric), keeping the $\varepsilon_d/\varepsilon_r$ — ratio constant, leads to a drastic deformation of phase curves, resp. of the



(a) (b)

Figure 4: Dependence of the upper cut-off frequencies $\bar{r}_{0en+}$ and $\bar{r}_{0en-}$ of the ferrite-dielectric waveguide, magnetized azimuthally in: (a) positive and (b) negative direction for the normal TE_{01} mode on $|\alpha|$ with ρ as parameter.

region of transmission (cf. Figs. 1(a), (b) with Figs. 2(a), (b) for $\rho = 0.95$ and 0.9).

iv) A reversal of M_r, preserving ρ, the $\varepsilon_d/\varepsilon_r$ — ratio and $|\alpha|$ the same, entails an entire alteration of the position, limits, form and size of the domain of propagation.

v) The lower and upper cut-off frequencies for all ρ and $\varepsilon_d/\varepsilon_r$ — ratios change their places, if $|\alpha|$ is large (cf. Figs. 1(b) and 2(b) for negative magnetization and Tables 1 and 2 for both signs of M_r).

vi) The area in which the structure provides differential phase shift (may operate as a phase shifter) strongly depends on the dimension of the part of waveguide cross-section, taken by dielectric. In case it enlarges, the phase shift is afforded in a narrower frequency band.

4. CONCLUSION

The replacement of the outer part of the circular ferrite waveguide of azimuthal magnetization, propagating normal TE_n modes even by a thin dielectric toroid effects considerably its phase portrait. Envelope curves (resp. additional kind of cut-off frequencies) spring up at which the phase characteristics for both signs of magnetization terminate (the propagation stops). The dimensions of the area of wave propagation, of the frequency band in which differential phase shift is produced, are substantially influenced by the magnitude of dielectric insert.

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