A Study of Periodic Multilayered Structure in Fractional Dimension Space and Euclidian Space

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Abstract— In this paper, Transfer Matrix Method and modified Maxwell equations are used to find the general expressions for reflection and transmission coefficients of periodic multilayered structure. The structure is placed in D dimension space where D is integer for Euclidean space and non-integer for Fractional space. Characteristics of these structure are studied when an electromagnetic wave strikes on it at both normal and oblique incident angle. The final expressions are the function of frequency, dimension and incident angle. Numerical results for multilayered periodic strong chiral — strong chiral structure are presented. This study provides motivation to investigate the electromagnetic waves propagation in multilayered structures at fractional boundaries.

1. INTRODUCTION

Advances in the metamaterials (MTM) has significantly attracted the attention of researchers. Multilayered structures composed of composite materials are being widely used in applications such as antenna designing, lens designing, frequency selective surfaces etc. [1–3]. Since, metallic structures have losses associated with it, metamaterial structures are used to achieve desirable outcomes. Until recently, frequency, incident angle and constitutive parameters behaviour were studied but with the introduction of fractional calculus, effect of dimension can also be studied. Irregular surfaces, porous media, and complex structures can be modelled with fractional calculus [4]. In 1982, Mandelbrot introduced the concept of fractal for the first time [5]. Fractional calculus can be used to study the characteristics of homogenous models at fractal boundaries [6, 7]. It provides a way to introduce solutions of electromagnetic problems in fractional dimension space [8, 9]. In 1996, study of scaler wave equation was carried out using fractional integration and later analysis of source distributions which are equivalent to fractional dimension Dirac delta function [10, 11]. Therefore, a solution in D dimension space is required to understand the behaviour of the structures. Lately, solutions to plane wave, differential electromagnetic (EM) wave, cylindrical wave and spherical wave in D-dimension fractional space are developed in [4].

There are several types of metamaterials being used by researchers; single negative materials are those whose either permittivity or permeability is negative, double negative materials have both permittivity and permeability negative, materials whose image is non-superposable are called chiral materials. Chiral materials are further divided into chiral nihility and strong chiral materials; chiral nihility materials have permittivity ($\epsilon = 0$) and permeability ($\mu = 0$) equal to zero for certain range of frequencies and chirality is non-zero ($\kappa \neq 0$), when chirality to refractive index ratio is greater than unity the material is called strong chiral material (SC). In this paper SC–SC multilayered structure is placed in fractional space and euclidian space for study and analysis. In Section 2, model and formulation for n-layered periodic structure are discussed and expressions for reflected and transmitted fields are computed. In Section 3, numerical results for both the cases are presented and discussed followed by conclusion. Please note time dependency in this paper is taken as e^{jwt} and is kept suppressed.

2. MODEL AND FORMULATION

A homogenous, non-dispersive periodic SC-SC multilayered structure is placed in D dimension space as shown in Figure 1. The width, refractive index, chirality and optical width of slab i are d_i, n_i, κ_i and $\mid n_i \mid d_i$, respectively where ($i = A, B$). The general incident, reflected and transmitted

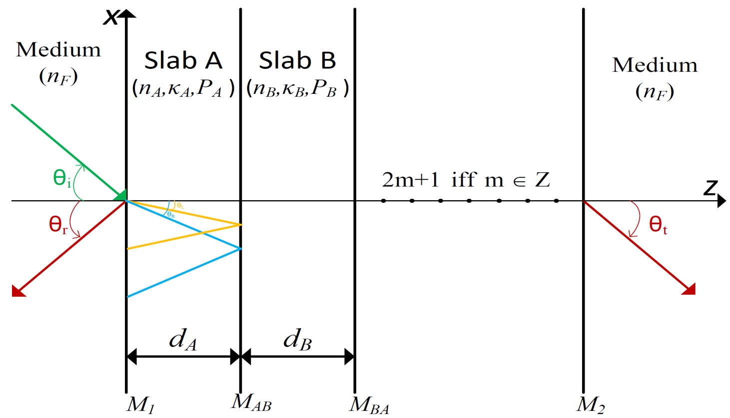

Figure 1: Five layered metmaterial structure placed in D dimension space.

fields in D $(2 \le D \le 3)$ dimension are [4],

$$\mathbf{E}_i = \left[E_{i\|}\left(\hat{x}\cos\theta_i + \hat{z}\sin\theta_i\right) + E_{i\perp}\hat{y}\right] e^{-jk_F(-x\sin\theta_i)} \left(k_F z\cos\theta_i\right)^n \left[H_n^{(2)}\left(k_F z\cos\theta_i\right)\right], \tag{1}$$

$$\mathbf{H}_i = \frac{1}{\eta}\left[E_{i\|}\hat{y} - E_{i\perp}\left(\hat{x}\cos\theta_i + \hat{z}\sin\theta_i\right)\right] e^{-jk_F(-x\sin\theta_i)} \left(k_F z\cos\theta_i\right)^{nh} \left[H_{nh}^{(2)}\left(k_F z\cos\theta_i\right)\right], \tag{2}$$

$$\mathbf{E}_r = \left[E_{r\|}\left(\hat{x}\cos\theta_r - \hat{z}\sin\theta_r\right) + E_{r\perp}\hat{y}\right] e^{-jk_F(-x\sin\theta_r)} \left(k_F z\cos\theta_r\right)^n \left[H_n^{(1)}\left(k_F z\cos\theta_r\right)\right], \tag{3}$$

$$\mathbf{H}_r = \frac{1}{\eta}\left[-E_{r\|}\hat{y} + E_{r\perp}\left(\hat{x}\cos\theta_r - \hat{z}\sin\theta_r\right)\right] e^{-jk_F(-x\sin\theta_r)} \left(k_F z\cos\theta_r\right)^{nh} \left[H_{nh}^{(1)}\left(k_F z\cos\theta_r\right)\right], \tag{4}$$

$$\mathbf{E}_t = \left[E_{t\|}\left(\hat{x}\cos\theta_t + \hat{z}\sin\theta_t\right) + E_{t\perp}\hat{y}\right] e^{-jk_F(-x\sin\theta_t)} \left(k_F z\cos\theta_t\right)^n \left[H_n^{(2)}\left(k_F z\cos\theta_t\right)\right], \tag{5}$$

$$\mathbf{H}_t = \frac{1}{\eta}\left[E_{t\|}\hat{y} - E_{t\perp}\left(\hat{x}\cos\theta_t + \hat{z}\sin\theta_t\right)\right] e^{-jk_F(-x\sin\theta_t)} \left(k_F z\cos\theta_t\right)^{nh} \left[H_{nh}^{(2)}\left(k_F z\cos\theta_t\right)\right] \tag{6}$$

In the above equations subscript i, r, and t represent the incident, reflected and transmitted wave. Perpendicular and parallel component of the waves are represented by $\perp$ and $\|$, respectively. Hankel function of the second kind and Hankel function of the first kind represents the forward travelling wave and backward travelling wave respectively and $n = \frac{|3-D|}{2}$ and $nh = \frac{|D-1|}{2}$ where D represents the dimension. Transfer Matrix Method (TMM) is applied on Equations (1)–(6) and electric and magnetic field expressions for strong chiral medium in [12], to compute field expressions on either side of the structure. The final expression is of the following form,

$$\begin{bmatrix} E_{i\|} \\ E_{i\perp} \\ E_{r\|} \\ E_{r\perp} \end{bmatrix} = [T]\begin{bmatrix} E_{t\|} \\ E_{t\perp} \end{bmatrix} \tag{7}$$

$$[T] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} = [M_1][P_A][T_1]^m[M_2] \tag{8}$$

$$[T_1] = [M_{AB}][P_B][M_{BA}][P_A] \tag{9}$$

In Equations (8) and (9), $[P_A]$ and $[P_B]$ represents the propagation matrix, it includes the phase component of the wave. $[M_1]$, $[M_2]$, $[M_{AB}]$, and $[M_{BA}]$ represents matching matrix, it relates the fields on either side of the slab. All the propagation matrices and matching matrices are 4×4 except, $[M_2]$ whose size is 4×2. Transfer matrix, $[T]$, relates the field on either side of the structure. The derived transfer matrix is for odd numbers of slabs. All the expressions computed are for D dimension (inserting integer value of D recovers classical results).

3. RESULTS

The expressions for reflected and transmitted fields are function of frequency (f), width (d), incident angle (θ_i), chirality (κ), and dimension (D). Figure 2 shows the reflectance and transmittance at oblique incidences for five layered structured composed of SC-SC alternating layers. It can be seen

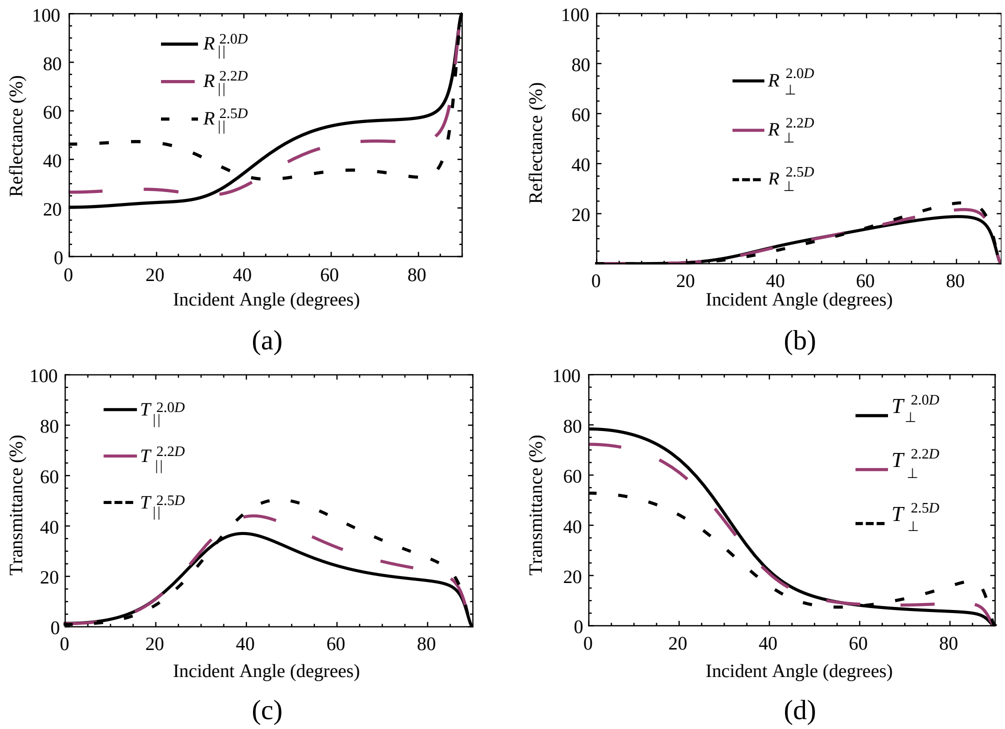

Figure 2: Reflectance and transmittance for five layered SC-SC structure. $n_A = 0.8$, $n_B = 0.6$, $\kappa_A = 1.2$, $\kappa_B = 1.0$, $| n_A | d_A =| n_B | d_B = \lambda_0/4$, $f/f_0 = 0.5$.

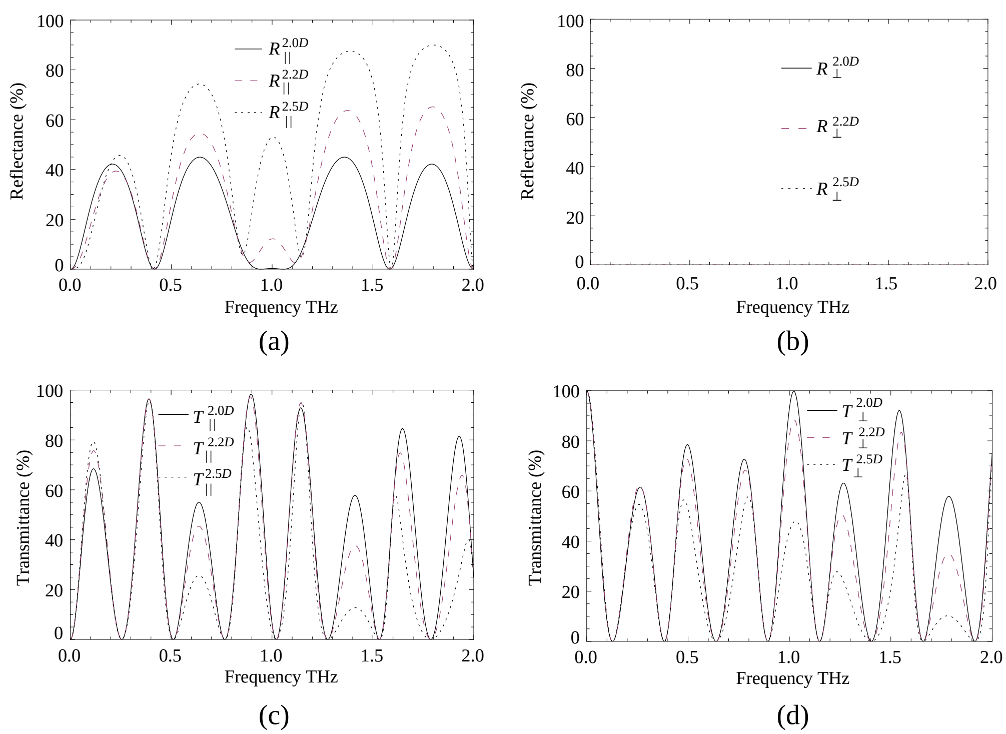

Figure 3: Reflectance and transmittance for five layered SC-SC structure. $n_A = 0.8$, $n_B = 0.6$, $\kappa_A = 1.2$, $\kappa_B = 1.0$, $| n_A | d_A =| n_B | d_B = \lambda_0/4$, $\theta_i = 0^\circ$.

in Figure 2(a), there is no significant power change in regions (0°–30°) and (55°–80°). Initially reflected power for $2.5D$ substrate is greater than $2.3D$ and $2.0D$ but after 40°, reflected power for $2.5D$ decreases and becomes lower than $2.3D$ and $2.0D$. Figure 2(b), shows that there is no significant difference in reflected power profile for perpendicular component of the wave. Figure 2(c), shows the transmitted power for parallel component of the wave. It can be seen that there is marginal difference of transmitted powers in (35°–85°). The maximum transmitted power for $2.5D$ substrate structure is approximately 50% and it is 35% for $2.0D$ substrate. Figure 2(d), shows the transmitted power for the perpendicular component of the wave. At 0° transmitted power is approximately 78°, 72° and 53° for $2.0D$, $2.3D$ and $2.5D$ substrate, respectively. It means that polarization is rotated for the wave passing through all three types of substrate. Figure 3, shows the effect of frequency and dimension on the behavior of the five layered structure when a wave strike at normal incidence. It is seen in Figure 3(a) that the parallel component of the reflected power has bell-shaped windows for all substrates. At 1.0 THz, reflected power for $2.0D$, $2.3D$ and $2.5D$ are 0%, 10% and 55%, respectively. There is no perpendicular component of the reflected wave at 0° as shown in Figure 3(b), since only parallel component of the wave is incident. Figures 3(c) and 3(d), shows the behavior of transmitted component. Narrowbands with high transmitted power exists at regular intervals with power approximately equal to unity at 1.0 THz.

4. CONCLUSION

In the presented paper, the electromagnetic behavior of periodic multilayered structure when placed in fractional dimension space and integer dimension space is studied and discussed. Transfer Matrix Method is used to find the expressions for reflected and transmitted wave in D dimension space. These expressions are then numerically presented for analysis. All the numerical results satisfy the law of conservation of power. Inserting the integer value of D recovers the classical results. This study provides motivation to investigate the electromagnetic waves propagation in multilayered structures at fractional boundaries and waveguides filled with fractal media.

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