Channel Equilibration in Wideband Digital Array Radar Test-bed

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Abstract— In actual digital array radar (DAR) systems, analog components will cause frequency-dependent amplitude and phase mismatch among receive channels. This paper demonstrates the design and implement of the channel equilibration subsystem of an 8-element DAR test-bed. An ultra-wideband waveform generator is designed for generating various calibration signals and the pre-distortion algorithm is presented to compensate for the reconstruction errors. In the receive channels, an improved inverse Fourier Transform algorithm for filter design is investigated in detail based on analysis of real receive channels. The processing results of measured data show the high performance of channel equilibration algorithms.

1. INTRODUCTION

Wideband Digital array radar (DAR) is widely used in space surveillance, for it meets the requirements for multi-functions and multi-beams simultaneously while providing large dynamic range, resistance to interferes and flexible control [1–3]. In actual systems, analog components such as feed lines, amplifiers, band-pass filters and ADCs will cause the distortion of frequency responses of each channel as well as amplitude and phase mismatch between receive channels, which will severely degrades the performances of the array.

Active calibration algorithms are widely researched and applied to compensate for channel mismatch. Lars Pettersson in Swedish Defense Research Agency designed a 15 tap equalizing FIR filter in a 12 channel S-band digital beam-forming antennas, and the cancellation ration was improved from -29 dB to -75 dB [4]. In the 96-element, L-band DAR prototype established by the Office of Naval Research, receive data were fed into a 20-tap, 16 bit complex FIR for equalization [5] . Wang Fang proposed a modified equilibration algorithm making use of Fourier Transform, which had excellent real-time performance [6]. Zhang Yue improved the filter-design algorithm by linear-fitting and designed the equilibration filter for a DAR test-bed with the bandwidth of 200 MHz [7]. As the simultaneous bandwidth of DAR test-bed has reached up to 500 MHz nowadays, the fundamental challenge in channel equilibration is the design of high quality wideband waveform generator as well as the realization of equilibration algorithms on FPGA-based processors [8, 9].

This paper focuses on channel equilibration of an 8-element L/S band DAR test-bed. In Section 2, a flexible wideband waveform generator is designed. Pre-distortion method according to working mode of DAC is implemented to improve the quality of waveforms. In Section 3, the inverse Fourier Transform algorithm is studied based on analysis of measured data, and then the equilibration filter is realized by FFT. In Section 4, experiments are carried out in the anechoic chamber, and some results are demonstrated in detail. Conclusions are drawn in Section 5.

2. WIDEBAND WAVEFORM GENERATION AND CALIBRATION

2.1. Design of Wideband Waveform Generator

Channel equilibration module in the DAR test-bed is illustrated in Figure 1. The equilibration algorithms are carried out in three steps. Firstly, wideband calibration signals are generated by the waveform generator according to the command of control computer. Then the signals are injected into each digital module through calibration network that is composed of a power divider and a coupler. Secondly, the calibration data are received by digital modules and transmitted to the control computer through optical fiber links, and the frequency responses are obtained. Thirdly, design the equilibration filters and send the coefficients to digital module for measured data equilibration.

The wideband waveform generator is composed of a FPGA carrier board and digital-to-analog convertor (DAC) card. The carrier board is a VPX product equipped with two Xilinx Virtex chips XC6VSX315T. The high-speed wideband DAC is integrated on a FPGA Mezzanine Card (FMC), which can be plugged into the FPGA carrier board. It is a single-channel, 12 bits DAC with the highest sampling rate of 3 Gsps. By setting the chip to work in different modes, it is capable of generating analog waveforms from 0 GHz to 6 GHz. When the DAR test-bed works in calibration mode, FPGA receives commands from the controller, and produces desired waveforms by direct digital synthesis (DDS). The low-speed baseband digital data are sent to the DAC to generate high-frequency analog waveforms.

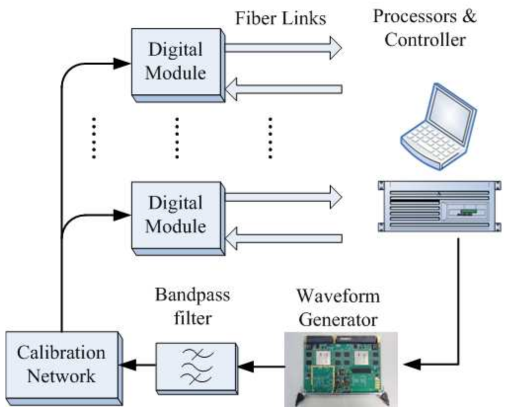


Figure 1: Block of channel equilibration subsystem.

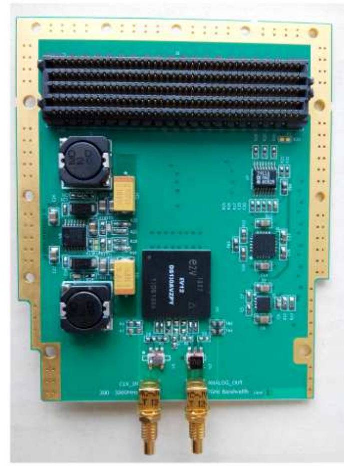


Figure 2: High-speed wideband DAC card.

2.2. Pre-distortion Algorithm for Waveform Generation

The DAC integrates a sample-and-hold to reconstruct waveforms from baseband data. However, it is impossible to realize an impulse function in actual components. Therefore reconstruction errors are introduced, leading to deterioration of analog waveforms.

Suppose the desired signal is $s(t)$, the sampling rate is T_s and the baseband data can be expressed as $s(nT_s)$, So the reconstruction function is $s_r(t) = \sum\limits_{n=-\infty}^{\infty} s(nT_s)q(t-nT_s)$, where $q(t)$ is the reconstruction factor.The Fourier Transform of $s_r(t)$ is as follows:

$$S_r(\omega) = \sum_{n=-\infty}^{\infty} s(nT_s)\int_{-\infty}^{+\infty} q(t-nT_s)e^{-j\omega t}dt = Q(\omega)(\sum_{n=-\infty}^{\infty}\int_{-\infty}^{+\infty} s(t)e^{-jnT_s\omega}dt = Q(\omega)\tilde{S}(\omega) \tag{1}$$

where $\tilde{S}(\omega) = \int\limits_{-\infty}^{+\infty} s(t)\cdot(\sum\limits_{n=-\infty}^{\infty}\delta(t-nT_s))e^{-j\omega t}dt = \frac{1}{T_s}\sum\limits_{n=-\infty}^{\infty} S(\omega - n\frac{2\pi}{T_s})$ and $S(\omega)$ is Fourier Transform of the desired signal. It is clear that $\tilde{S}(\omega)$ is the extension of $S(\omega)$ by the period of $\frac{2\pi}{T_s}$.

It can be concluded from Equation (1) that the output waveform is the combination of extension of the desired signal and the modulation function $Q(\omega)$. The distortion resulting from the modulation can be calibrated by pre-distortion method. That means imposing the inverse filter of $Q(\omega)$ on baseband data to compensate for the distortion.

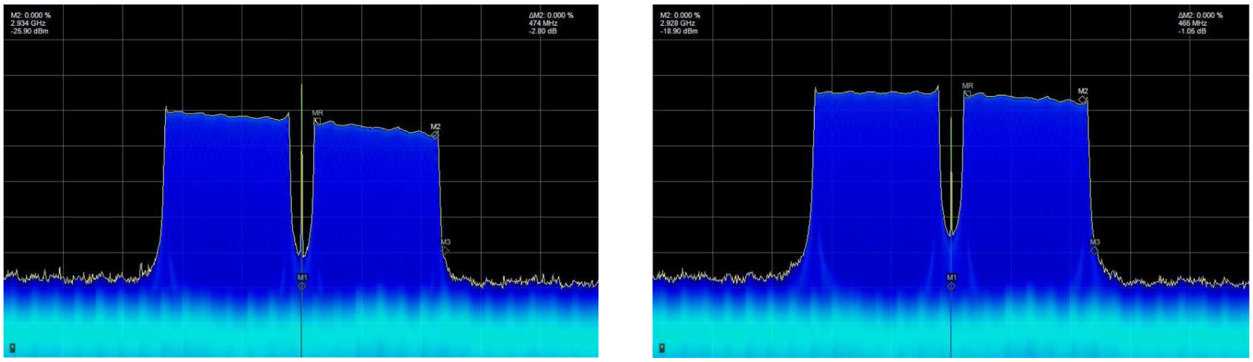
Taking the DAC in Figure 2 as an example, The Fourier Transform of reconstruction factor is:

$$Q(\omega) = T_{\mathrm{s}} \cdot e^{-j\frac{T_{\mathrm{s}}}{2}\omega} \cdot \sin\left(\frac{T_{\mathrm{s}}}{4}\omega\right) \cdot \mathrm{sinc}\left(\frac{T_{\mathrm{s}}}{4}\omega\right) \tag{2}$$

Then the pre-distortion function is obtained:

$$P(\omega) = \frac{1}{T_s} \cdot e^{-j\frac{T_{\mathrm{s}}}{2}\omega} \cdot \frac{1}{\sin(\frac{T_{\mathrm{s}}}{4}\omega)\cdot\mathrm{sinc}(\frac{T_{\mathrm{s}}}{4}\omega)} \qquad \frac{2\pi}{T_s} < |\omega| < \frac{3\pi}{T_s} \tag{3}$$

In this paper, the baseband data of the LFM signal (the bandwidth is 500 MHz, and the center frequency is 2.7 GHz) is generated by the FPGA. After pre-distortion processing, the calibrated data is send to DAC. The analog waveforms are shown in Figure 3. After calibration, the ripple is reduced and the quality is improved significantly.



(a) (b)

Figure 3: Spectrum of 2.45-2.95 GHz LFM signal. (a) before calibration. (b) after calibration.

3. FREQUENCY DOMAIN EQUILIBRATION ALGORITHM

3.1. Improved Inverse Fourier Transform Algorithm

In actual DAR system, equilibration filters are usually added into the receive channels to compensate for channel mismatch. Reference [7] has proposed an improved inverse Fourier Transform algorithm to design the equilibration filters and applied it in a DAR prototype with the bandwidth of 200 MHz successfully. Since the instantaneous bandwidth of the DAR test-bed has increased to 500 MHz, this paper makes further study on the algorithm with the help of measured data.

Suppose there are channels in the system, the frequency response of reference channel and i th channel is $S_{ref}(k)$ and $S_i(k)$ respectively. The desired response of i th equilibration filter is:

$$H_{di}\left(k\right)=\frac{S_{ref}\left(k\right)}{S_i\left(k\right)}\qquad i=1,2,\cdots,M\quad k=0,1,\ \cdots,K-1 \tag{4}$$

Suppose the complex coefficients of the ith equilibration filter is $h_i(n)\quad n=0,1,\cdots,N-1$, then the discrete frequency response is:

$$H_i\left(k\right)=\sum_{n=0}^{N-1}h_i\left(n\right)\mathrm{e}^{-j\omega n}|_{\omega=\frac{2\pi}{M}k} \tag{5}$$

According to the Least-Square criterion, when the mean square error between $H_{di}(k)$ and $H_i(k)$ reaches the minimum value, the optimum $h_i(n)$ can be obtained taking account of the Parsval theorem as follows:

$$\min_h E_i=\min_h\sum_{k=0}^{K-1}|H_{id}(k)-H_i(k)|^2=\min_h\sum_{n=0}^{K-1}|h_{id}(n)-h_i(n)|^2 \tag{6}$$

In Equation (6), $h_i(n)$ is the coefficients of equilibration filter, and it only has N values, so it can be expressed as:

$$\min_h E_i=\min_h\sum_{n=0}^{P}|h_{id}(n)|^2+\sum_{n=P+1}^{P+N}|h_{id}(n)-h_i(n)|^2+\sum_{n=P+N+1}^{K-1}|h_{id}(n)|^2 \tag{7}$$

The optimum values of $h_i(n)$ can be obtained that by choosing the P maximum values of $h_{id}(n)$.

However, in practical circumstance, the traditional inverse Fourier Transform method is sensitive to the out-band noise. A more practical method is to poly-fit the out-band frequency response, that is, replace the desired response in Equation (6) by Equation (8):

$$H_{di}\left(k\right)=\begin{cases}\frac{S_{ref}(k)}{S_i(k)} & \frac{K}{2}(1-\frac{2B}{f_s})\le k\le\frac{K}{2}(1+\frac{2B}{f_s})\\ a+b\cdot k & \qquad others\end{cases} \tag{8}$$

where a and b are the coefficients which are determined by poly-fit of the in-band data.

Taking the first channel as an example, the measured frequency response is shown in Figure 4(a). The in-band response changes slowly while the out-band response changes dramatically and discontinuously. Figure 4(b) demonstrates the modified desired response and the frequency responses of equilibration filters with 40, 80, 160 taps. It can be learned that a 160-tap complex filter is needed to approach the frequency response of the channel with the bandwidth of 500 MHz.

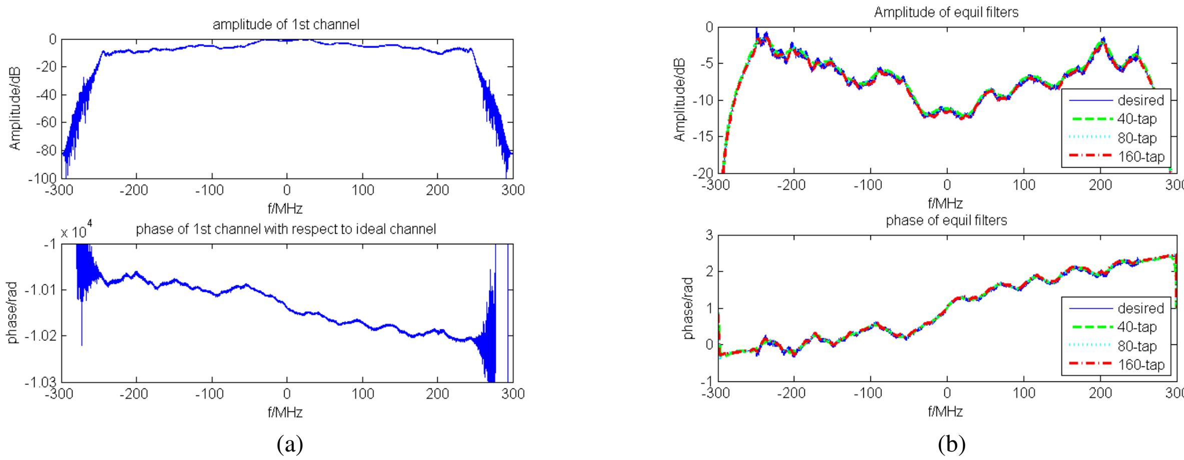


Figure 4: Measured channel responses and equilibration filter. (a) Response of the first channel. (b) Responses of equilibration filters with different taps.

3.2. Realization of Channel Equilibration in FPGA

In conventional method, channel equilibration is realized by complex FIR filter. However, as the bandwidth increases, the taps of equilibration filters becomes so high that it is unaffordable for FPGA. In order to realize equilibration in actual systems, FFT based on Overlap addition method is applied to simplify the implementation and the requirements for hardware.

Suppose the receive data is $x(n)$, the equilibration filter is $h(n)$, and $x(n)$ can be divided into a series of segments:

$$x_i(n) = \begin{cases} x(n) & iN_2 \le n \le (i+1)N_2 - 1 \\ 0 & others \end{cases} \tag{9}$$

Then the equilibration output is:

$$y(n) = x(n) * h(n) = \sum_{i=-\infty}^{\infty} x_i(n) * h(n) \tag{10}$$

The length of $x_i(n)$ and $h(n)$ is N_2 and N respectively, so convolution can be realized by FFT:

$$y(n) = \sum_{i=-\infty}^{\infty} IFFT(FFT(x_i(n), L) \cdot FFT(h(n), L)) \tag{11}$$

Where $L = N + N_2 - 1 = 2^M$.

4. EXPERIMENTAL RESULTS

To demonstrate the performance and effectiveness of the channel equilibration algorithms, experiments are carried out in an anechoic chamber by the DAR test-bed. When power on, the test-bed works in calibration mode. The high-quality LFM signal with the bandwidth of 500 MHz and center frequency of 2.7 GHz is generated by the wideband waveform generator.

The equilibration filters are designed using improved inverse Fourier Transform algorithm and their coefficients are sent to digital module. The measured data are captured and equilibrated by the filters, and the results are demonstrated in Figure 5(a). After equilibration, the amplitude ripple is decreased significantly, and the phase linearity is improved effectively.

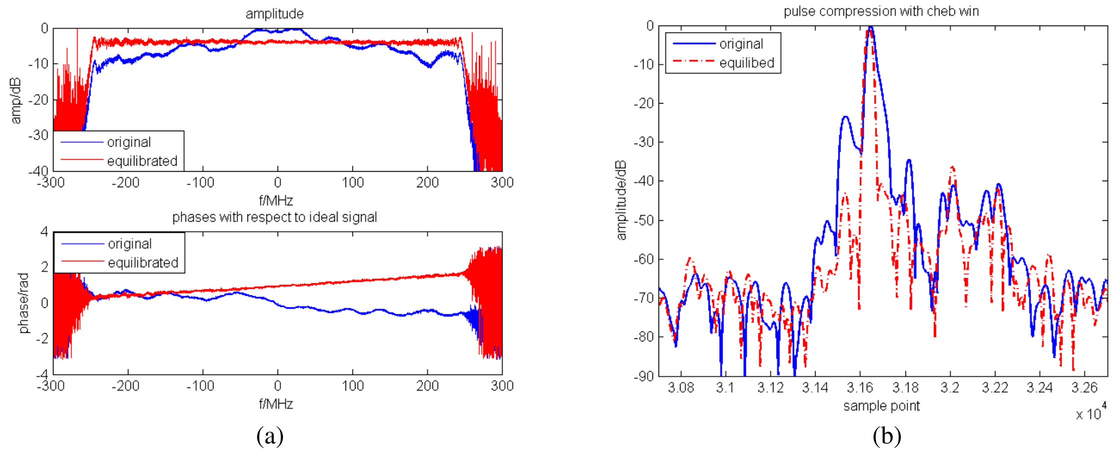


Figure 5: Processing results of measured data.Solid: before equilibration. Dash: after equilibration. (a) Frequency domain responsesl. (b) Pulse compression.

The compare of pulse compression before and after channel equilibration are illustrated in Figure 5(b). After equilibration, the main lobe becomes narrower and the side lobes are suppressed, leading to an increase in system resolution and dynamic range.

The convolution of equilibration filters and measured data are realized by FFT based on overlap addition. Taking a 160-tap equilibration filter as an example, the FFT method only need only 58 DSP slices in FPGA, while the FIR filter method need 480. So the FFT method is feasible to realize in wideband DAR systems.

5. CONCLUSION

This paper demonstrates the design and implement of channel equilibration subsystem in wideband DAR test-bed. The wideband waveform generator is capable of generating various wideband waveforms by DDS. An effective structure of improved inverse Fourier Transform algorithm is presented, which is feasible to realize in FPGA. Experiments are carried out by an 8-element wideband DAR test-bed. The expected results have shown the effectiveness and high performance of the channel calibration module.

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