Rapid High-accuracy Modeling Simulation Method for Full Trajectory of the Ballistic Missile

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Abstract— In view of the existing problems of trajectory model for ballistic missile, a method for full trajectory model of a ballistic missile over a rotating, ellipsoid earth has been developed. The method is based on the theory of minimal energy. The error of flight range is calculated with the method of Bessel's geodetic inverse solution and fourth-order Runge-Kutta, and then corrected by dichotomy iteration. The process is repeated until the ballistic missile impacts the target location within a predefined miss distance tolerance. Simulation results show that both the speed of ballistic generation and confidence get a noticeable rise with the established trajectory model in this paper.

1. INTRODUCTION

A ballistic trajectory is the path of an object that is dropped, thrown, served, launched. In the extraction technology for ballistic targets, establish ballistic trajectory model is the first step. One of the earliest studies of trajectory prediction and determination was conducted by Johann Heinrich Lambert. A method for the determination of the ballistic trajectory was developed with only giving the launch position and impact point over a spherical model [1]. A practical analytical approximation of projectile trajectories in 2D and 3D roughly based on linear drag model was introduced [2]. The ballistic missile trajectory based on elliptic trajectory theory and 4D external trajectory model was predicted [3]. B. Xiao [4] researched a method of energy ballistic orbit, based on the theory of two-body orbit trajectory, but without considering the characteristics of different flight phases for missile, F. Zhang [5] firstly obtained terminal ballistic parameters without considering the earth rotation, then corrected the terminal velocity by iteration to reduce the affection of the earth rotation. In many existing methods, the characteristics of different flight phases, the affection of the earth rotation and the low speed of ballistic generation are often ignored, and other contributions rely on a prior information. According to these researches, an iterative optimization method is proposed to establish a full ballistic trajectory model in this paper. Firstly, ballistic parameters at burnout point are initially estimated according to the theories of minimal energy trajectory and elliptic trajectory. Secondly, dynamic differential equations and missile airframe state vector are established. Finally, the error of flight range is calculated with the method of Bessel's geodetic inverse solution and fourth-order Runge-Kutta, and corrected by dichotomy iteration until the ballistic missile impacts the target location within a predefined miss distance tolerance. It turns out that the method can markedly decrease the iterations, speed up ballistic generation and improve the accuracy of the result. Moreover, the trajectory model established in this paper has been used in a signature extraction system for missile herd targets, and has shown good performance.

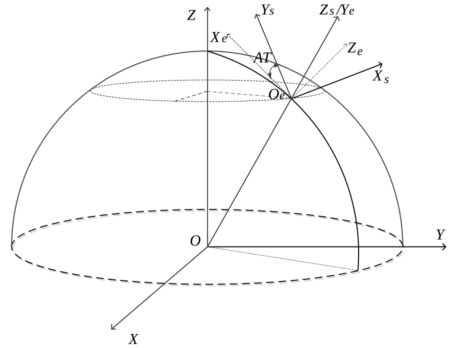

Figure 1: Coordinate system definition.

2. COORDINATE TRANSFORMATION

According to the flight dynamics, the mathematics model which describes targets motion principles would be highly simplified if choosing suitable reference system. The coordinate systems used in establishing kinematic equations for missile are Launch frame (X_e, Y_e, Z_e), East-North-Up (ENU) frame (X_s, Y_s, Z_s) and Earth-Centered-Inertial (ECI) frame (X, Y, Z), A_T is azimuth. Figure 1 depicts the three coordinate systems. As depicted in Figure 1, the transformation from an ENU position vector to an ECI position vector is given by the following vector-matrix operation

$$\mathbf{r_{eci}} = C_e^s \mathbf{r_{enu}} \tag{1}$$

the elements of the transformation matrix C_e^s are given by

$$C_e^s = \begin{bmatrix} -\sin j_0 & \cos j_0 & 0 \\ -\sin w_0 \cos j_0 & \sin w_0 \sin j_0 & \cos w_0 \\ \cos w_0 \cos j_0 & \cos w_0 \sin j_0 & \sin w_0 \end{bmatrix} \tag{2}$$

Here w_0 is the longitude, and j_0 is the latitude.

3. MINIMAL ENERGY TRAJECTORY MODEL

A method to determine the trajectory of a ballistic missile over a rotating, ellipsoid earth is developed with only giving the launch position, impact position, range angle in free-flight phase and geocentric distance at burnout point. Initial ballistic parameters at burnout point are estimated according to the theories of minimal energy trajectory and elliptic trajectory. In order to simplify the complexity of model, particle dynamic equations in flight and missile airframe state vector are established. Finally, The error of flight range is calculated with the method of Bessel's geodetic inverse solution and fourth-order Runge-Kutta, and corrected by dichotomy iteration. The process is repeated until the ballistic missile impacts the target location within a predefined miss distance tolerance.

3.1. Ballistic Parameters

The exact mathematical relationship among range angle in free-flight phase β_e, energy parameter γ_k and optimum ballistic angle θ_k are given by the following equation

$$\sin\frac{\beta_e}{2} = \frac{\gamma_k}{2e}\sin 2\theta_k \tag{3}$$

$$e = \sqrt{1 + \gamma_k(\gamma_k - 2)\cos^2\theta_k} \tag{4}$$

where e is ellipse eccentricity. The relationship between γ_k and θ_k is deduced and the optimum θ_k is obtained if β_e is given by the method of maximum-minimum. γ_k is determined by r_k and V_k, the equation is as follow

$$\gamma_k = \frac{r_k V_k^2}{u} \tag{5}$$

r_k is the geocentric distance from burnout point to the earth's center, and V_k is the missile speed in this point. If r_k is given, the argument of γ_k is determined by V_k, and when θ_k is optimal, γ_k is minimal, V_k would also be minimal, similarly, the energy cost in full missile flight would also be minimum.

3.2. Boost Phase Trajectory Model

Dynamic and kinematic model for missile in boost flight phase is established in ECI coordinate system after analyzing the forces of missile in the phase, and the roles of missile are decided by propulsion, gravity, aerodynamic resistance, Coriolis and centripetal. A standard ellipsoid model of the earth is established and corrected by the second harmonic, dynamic equations in this flight phase are as follows

$$\dot{\mathbf{p}} = \mathbf{v} \quad \dot{\mathbf{v}} = \mathbf{a_P} + \mathbf{a_D} + \mathbf{a_G} + \mathbf{a_{cor}} + \mathbf{a_{cen}} \tag{6}$$

Here $\mathbf{a_P}$ is propulsion acceleration, the direction is linear in velocity. $a_D = \frac{C_D(v(t))S\rho(h(t))v^2(t)}{2m(t)}$ is aerodynamic resistance acceleration, the direction is opposite to velocity, $C_D(v)$ is resistance coefficient, $\rho(h)$ is air density. $\mathbf{a_G}$ is gravity acceleration, $\mathbf{a_{cor}}$ is Coriolis acceleration, and kinematic $\mathbf{a_{cen}}$is centripetal acceleration. Kinematic equations for missile are as follows:

$$\begin{cases} \dot{\mathbf{v}}_\mathbf{x} = (a_p - a_D - a_{cen})\frac{\mathbf{v_x}}{v_0} - \frac{u}{r_0^3}\left(\mathbf{p_x} + \frac{C_e}{r_0^2}\left(1 - 5\left(\frac{p_z}{r_0}\right)^2\right)\mathbf{p_x}\right) - a_{cor}\frac{\mathbf{p_x}}{r_0} \\ \dot{\mathbf{v}}_\mathbf{y} = (a_p - a_D - a_{cen})\frac{\mathbf{v_y}}{v_0} - \frac{u}{r_0^3}\left(\mathbf{p_y} + \frac{C_e}{r_0^2}\left(1 - 5\left(\frac{p_z}{r_0}\right)^2\right)\mathbf{p_y}\right) - a_{cor}\frac{\mathbf{p_y}}{r_0} \\ \dot{\mathbf{v}}_\mathbf{z} = (a_p - a_D - a_{cen})\frac{\mathbf{v_z}}{v_0} - \frac{u}{r_0^3}\left(\mathbf{p_z} + \frac{C_e}{r_0^2}\left(1 - 5\left(\frac{p_z}{r_0}\right)^2\right)\mathbf{p_z}\right) - a_{cor}\frac{\mathbf{p_z}}{r_0} \end{cases} \tag{7}$$

where, $r_0 = \sqrt{p_x^2 + p_y^2 + p_z^2)}$ is the scalar distance from missile to the center of the earth, $v_0 = \sqrt{v_x^2 + v_y^2 + v_z^2}$ is the scalar velocity or speed of the missile, $C_e = \frac{3J_2R^2}{2}$ and J_2 is the second harmonic.

3.3. Post-boost Phase Trajectory Model

3.3.1. Dynamic and Kinematic Model in Post-boost Phase

Dynamic and kinematic model for missile in post-boost flight phase is established in ECI coordinate system, comparing to the forces of missile in boost phase, there is no propulsion, dynamic equations are as follows

$$\dot{\mathbf{p}} = \mathbf{v} \quad \dot{\mathbf{v}} = \mathbf{a_D} + \mathbf{a_G} + \mathbf{a_{cor}} + \mathbf{a_{cen}} \tag{8}$$

Kinematic equations for missile are as follows:

$$\begin{cases} \dot{\mathbf{v}}_\mathbf{x} = (-a_D - a_{cen})\frac{\mathbf{v_x}}{v_0} - \frac{u}{r_0^3}\left(\mathbf{p_x} + \frac{C_e}{r_0^2}\left(1 - 5\left(\frac{p_z}{r_0}\right)^2\right)\mathbf{p_x}\right) - a_{cor}\frac{\mathbf{p_x}}{r_0} \\ \dot{\mathbf{v}}_\mathbf{y} = (-a_D - a_{cen})\frac{\mathbf{v_y}}{v_0} - \frac{u}{r_0^3}\left(\mathbf{p_y} + \frac{C_e}{r_0^2}\left(1 - 5\left(\frac{p_z}{r_0}\right)^2\right)\mathbf{p_y}\right) - a_{cor}\frac{\mathbf{p_y}}{r_0} \\ \dot{\mathbf{v}}_\mathbf{z} = (-a_D - a_{cen})\frac{\mathbf{v_z}}{v_0} - \frac{u}{r_0^3}\left(\mathbf{p_z} + \frac{C_e}{r_0^2}\left(1 - 5\left(\frac{p_z}{r_0}\right)^2\right)\mathbf{p_z}\right) - a_{cor}\frac{\mathbf{p_z}}{r_0} \end{cases} \tag{9}$$

The missile airframe state vector is comprised of 6 elements that define the missile position and velocity. This state vector is written as

$$\mathbf{X} = (\mathbf{p_x}, \mathbf{p_y}, \mathbf{p_z}, \mathbf{v_x}, \mathbf{v_y}, \mathbf{v_z}) \tag{10}$$

Aiming to the problem of no solution to these nonlinear differental equations, the method of fourth-order Runge-Kutta is applied to numerical integration in this paper, fourth-order Runge-Kutta equations is written as

$$\begin{gathered} \mathbf{X_{i+1}} = \mathbf{X_i} + \frac{h}{6}\left(X_{1i} + 2X_{2i} + 2X_{3i} + X_{4i}\right) \\ \begin{cases} X_{1i} = f_i(t, \mathbf{X_i}) \\ X_{2i} = f_i(t + \frac{h}{2}, \mathbf{X_i} + \frac{h}{2}X_{1i}) \\ X_{3i} = f_i(t + \frac{h}{2}, \mathbf{X_i} + \frac{h}{2}X_{2i}) \\ X_{4i} = f_i(t + \frac{h}{2}, \mathbf{X_i} + hX_{3i}) \end{cases} \end{gathered} \tag{11}$$

where h is the integration step. Simulations show that fourth-order Runge-Kutta is highly accurate than higher-order Runge-Kutta.

3.3.2. Dichotomy Iteration

The following expression is to determine whether the simulation results within a predefined miss distance tolerance

$$\Delta s = |S_T - S_P| < \varepsilon \tag{12}$$

S_T is geodetic distance, called theoretical flight range, which is estimated by the method of Bessel's geodetic inverse solution. The main idea of the method is that two points on the ellipsoid are

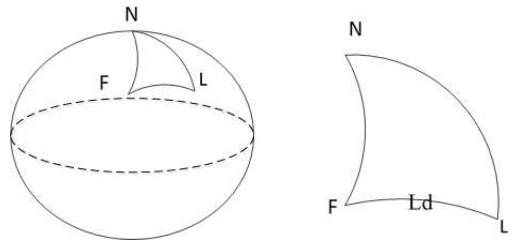

Figure 2: Bessel projection operation.

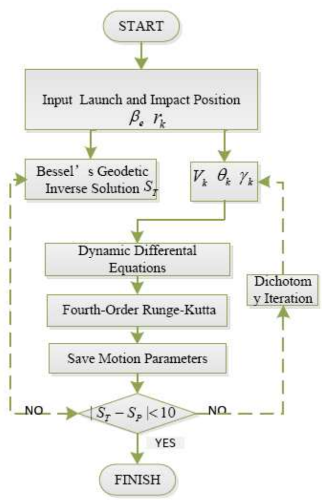

Figure 3: Simulation order of operation.

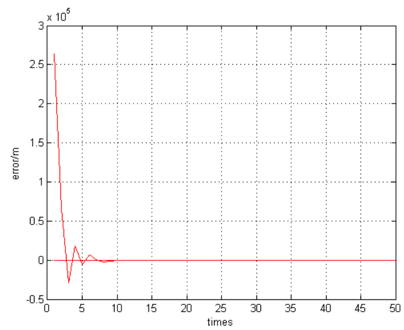

Figure 4: The range error vs. iteration time.

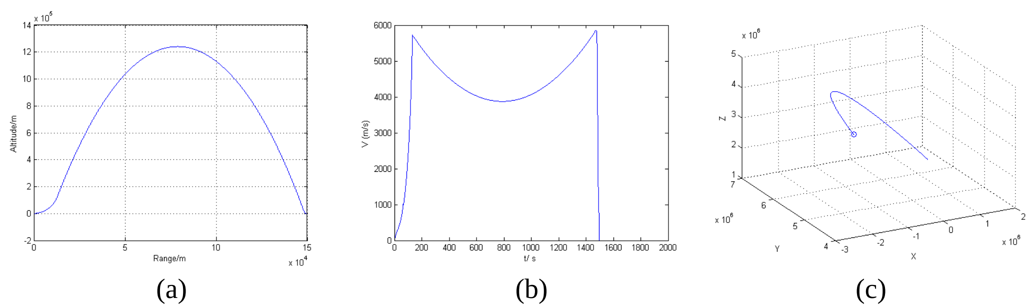

Figure 5: Ballistic missile trajectory. (a) Altitude vs. range. (b) velocity vs. time. (c) Minimal energy trajectory.

projected onto a secondary sphere based on the conditions of Bessel ellipsoid projection, then calculated by solving a spherical triangle. Figure 2 shows the procedure of Bessel projection. S_T is written as

$$S = A\sigma + \left(B'' + C''y\right)\sin\sigma \tag{13}$$

σ and y are calculated by the geodetic longitude and geodetic latitude at launch point and impact point, all the coefficients in Eq. (13) are calculated with Krasophuskii ellipsoid elements. S_P is practical flight range which is integrated by the method of fourth-order Runge-Kutta in the procedure of missile trajectory simulation.

The steps of dichotomy iteration and error correction can be summarized as follows:

1) Calculate S_T by Eq. (13) without considering the earth rotation in first iteration.

2) Generate the ballistic trajectory according to the dynamic and kinematic model established in this paper, save S_P and the flight time T in trajectory generation, calculate Δs.

3) Repeat first step, calculate S_T with taking into account the angle caused by the affection of the earth rotation in step 2). Then repeat second step, using the method of dichotomy iteration to correct Δs by adjusting the terminal velocity scalar, the whole process is repeated until the ballistic

missile impacts the target location within a predefined miss distance tolerance ε, $\varepsilon = 10$.

Figure 3 shows the algorithm of the whole trajectory simulation program.

4. SIMULATION

The simulation results in the paper show that the flight range error is 0.03684 m with 15 iteration times by the method of dichotomy iteration, Figure 4 shows the change of range error with iteration times.

Figure 5 shows the altitude and velocity of missile in whole flight and the full ballistic trajectory in 3D.

5. CONCLUSION

Large quantities of missile flight data is demanded in the technology of signature extraction for missile herd targets, rapid and highly accurate trajectory generation has important practical significance. In the paper, a method for full ballistic trajectory model of a ballistic missile over a rotating, ellipsoid earth is developed, and the ballistic trajectory generation is highly accelerated by iterative optimization. Simulation results show that the method has a high degree of confidence. A logical upgrade to the simulation is that improving the method of iteration to correct the error exactly from two points (range and azimuth). The idea will decrease the iteration times and improve simulation confidence, finally contribute to a more accurate trajectory, especially for long flight time missile.

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