Evaluation of the Angular Spectrum of Scattered High Frequency Radio Waves in the Anisotropic Collision Magnetized Ionospheric Plasma

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Abstract— Second-order statistical moments of scattered radiation in turbulent collision magnetized plasma. General dispersion equation is obtained at arbitrary angles of inclination of both external magnetic field and wave vector of an incident electromagnetic wave. Statistical characteristics of the phase fluctuations of scattered high frequency radio waves in the collision magnetized plasma caused by the electron density and external magnetic field fluctuations taking into account polarization coefficients for both ordinary and extraordinary waves are considered. The influence of collision frequency on the broadening and displacement of maximum of the angular spectrum is analyzed. Numerical calculations are carried out for the F-region of the ionosphere using experimental data.

1. INTRODUCTION

Investigation of statistical characteristics of small-amplitude electromagnetic waves propagating through a turbulent plasma slab is very important in many practical applications associated with both natural and laboratory plasmas [1]. It has been established that the energy loss due to the collisions between plasma particles can lead to a decrease in the amplitude of the electromagnetic waves and also to an appreciable distortion of the angular spectrum of radiation in the events of multiple scattering by random smooth inhomogeneities of the medium. In [2] it was shown that for a fixed collision frequency between plasma particles, the degree to which the absorption influences the angular spectrum of the scattered waves depends strongly on the propagation direction of the original incident wave with respect to the plasma boundary and also on the strength of an external magnetic field using the complex geometrical optics approximation. Broadening of the angular power spectrum of scattered electromagnetic waves in turbulent collision magnetized plasma for both power-law and anisotropic Gaussian correlation function of electron density fluctuations has been considered in [3] in complex ray (-optics) approximation.

In the present work, using the perturbation method, we calculate statistical characteristics of scattered electromagnetic waves in turbulent collision magnetized ionospheric plasma taking into account both electron density and external magnetic field fluctuations; and also polarization coefficients for both ordinary and extraordinary waves. General dispersion equation is obtained at arbitrary angles of inclination of both external magnetic field and wave vector of an incident electromagnetic wave. The influence of collision frequency on the broadening of the angular spectrum is analyzed. Numerical calculations were carried out for the F-region of the ionosphere using experimental data.

2. FORMULATION OF THE PROBLEM

At high frequency the effect of ions can be neglected. Maxwell's equations and equation of motion of the electron in collision plasma $ig\omega\mathbf{w} = (e/m)(\mathbf{E} + [\mathbf{w}\cdot\mathbf{H_0}]/c)$ lead to the wave equation for the electric field

$$\mathrm{grad}\,\mathrm{div}\,\mathbf{E} - \Delta\mathbf{E} - k_0^2\,\mathbf{E} = -\frac{k_0^2 vg}{g^2-u}\left\{\mathbf{E} - \frac{i\sqrt{u}}{g}[\mathbf{E}\cdot\mathbf{m}] - \frac{u}{g^2}(\mathbf{E}\cdot\mathbf{m})\,\mathbf{m}\right\}, \tag{1}$$

where $\mathbf{w}$ is the velocity of electron, $g = 1 - is$, $s = \nu_{eff}/\omega$, $\nu_{eff} = \nu_{ei} + \nu_{en}$ is the effective collision frequency of electron with other plasma particles, $k_0 = \omega/c$, $u = \omega_H^2/\omega^2$ and $v = \omega_p^2/\omega^2$ are non-dimensional magneto-ionic parameters, $\omega_H = eH_0/mc$ and $\omega_p = 4\pi N\,e^2/m\omega^2$ are the electron gyrofrequency and plasma frequencies, respectively, N-electron density, e and m are the charge and mass of an electron, $\mathbf{m} = \mathbf{H_0}/H_0$ is the unite vector along the direction of an external magnetic field locating in the yoz plane. Complex refractive index of the collisional magnetized plasma is

$$N^2 \equiv (n - i\,\kappa)^2 = 1 - \frac{2v(g-v)}{2g(g-v) - u_T \pm \sqrt{u_T^2 + 4u_L(g-v)^2}}, \tag{2}$$

where: $u_T = u\sin^2\alpha$, $u_L = u\cos^2\alpha$, α is the angle between vectors $\mathbf{k}$ and $\mathbf{H_0}$, upper sign corresponds to the ordinary wave, lower sign to the extraordinary wave; n is refractive index, κ is absorption index. Using perturbation method each parameter in the Equation (1) we submit as a sum of the mean value and small fluctuating terms: $\mathbf{E} = \langle\mathbf{E}\rangle + \mathbf{e}$, $\mathbf{H_0} = \langle\mathbf{H_0}\rangle + \mathbf{h_0}$, $N = \langle N\rangle + n$. The angular brackets indicate the statistical average. We assume that the mean values of electron density and magnetic field do not depend on coordinates. Fluctuating fields of scattered radio waves are random functions of the spatial coordinates and satisfy set of stochastic differential equation taking into account both electron density and external magnetic field fluctuations:

$$\left(\frac{\partial^2}{\partial x_i \partial x_j} - \Delta\delta_{ij} - k_0^2\varepsilon_{ij}\right) e_j = j_i, \tag{3}$$

where the current density is $\mathbf{j}(\mathbf{r}) = -\frac{k_0^2 v_0 g}{g^2-u_0}\{-i\frac{\sqrt{u_0}}{g}[\langle\mathbf{E}\rangle\mathbf{h'_0}] - \frac{u_0}{g}(\langle\mathbf{E}\rangle\mathbf{m})\mathbf{h'_0} - \frac{u_0}{g^2}(\langle\mathbf{E}\rangle\mathbf{h'}_0)\mathbf{m}\} - \frac{k_0^2\,v_0\,g}{g^2-u_0}\{n' + \frac{2u_0}{g^2-u_0}(\mathbf{m}\,\mathbf{h'}_0)\}\{\langle\mathbf{E}\rangle - i\frac{\sqrt{u_0}}{g}[\langle\mathbf{E}\rangle\mathbf{m}] - \frac{u_0}{g^2}(\langle\mathbf{E}\rangle\mathbf{m})\mathbf{m}\}$; where $\mathbf{h'_0} = \mathbf{h_0}/|\langle\mathbf{H_0}\rangle|$ and $n' = n/\langle N\rangle$ are external magnetic field and electron density fluctuations which are random functions of the spatial coordinates. Components of the dielectric permittivity for the collision plasma at $(1 \pm u_0) \gg s^2$, on the pole (vertical magnetic field $\alpha = 0°$) are [1]:

$$\begin{aligned}&\varepsilon_{xx} = \varepsilon_{yy} = \eta' - i\,s\,\tilde{\eta}'', \quad \varepsilon_{zz} = \varepsilon' - is\tilde{\varepsilon}'', \quad \varepsilon_{xy} = s\tilde{\mu}' - i\mu'', \quad \eta' = 1 - \frac{v_0}{1-u_0}, \quad \tilde{\eta}'' = \frac{v_0(1+u_0)}{(1-u_0)^2},\\ &\tilde{\mu}' = \frac{2\,v_0\sqrt{u_0}}{(1-u_0)^2}, \quad \mu'' = \frac{v_0\sqrt{u_0}}{1-u_0}, \quad \varepsilon' = 1 - v_0, \quad \tilde{\varepsilon}'' = v_0.\end{aligned} \tag{4}$$

Index "zero" indicates the mean values of non-dimensional plasma parameters. Components of the wave vector in homogeneous medium are determined as:

$$k_x = k_0 N\sin\theta\sin\varphi \equiv k_0\tau_1, \quad k_y = k_0 N\sin\theta\cos\varphi \equiv k_0\tau_2, \quad k_z = k_0 N\cos\theta \equiv k_0 q, \quad \tau_1^2 + \tau_2^2 + q^2 = N^2,$$

where: θ is the angle between wave vector $\mathbf{k}$ and z axis, φ is the angle between projection of vector $\mathbf{k}$ on the xoy plane and x axis.

The dispersion relation in general case, at $\theta \neq 0$, $\varphi \neq 0$ and $\alpha \neq 0$ has the following form:

$$\Delta \equiv a_4\,q^4 + a_3\,q^3 + a_2\,q^2 + a_1\,q + a_0 = 0, \tag{5}$$

here: $a_4 = \varepsilon_{zz}$, $a_3 = 2\,\tau_2\,\varepsilon_{yz}$, $a_2 = \varepsilon_{yz}^2 - \varepsilon_{xz}^2 - \varepsilon_{zz}(\varepsilon_{xx} + \varepsilon_{yy}) + \tau_1^2(\varepsilon_{xx} + \varepsilon_{zz}) + \tau_2^2(\varepsilon_{yy} + \varepsilon_{zz})$, $a_1 = 2\,\tau_2[\varepsilon_{yz}(\tau_1^2 + \tau_2^2) - \varepsilon_{xx}\varepsilon_{yz} - \varepsilon_{xy}\varepsilon_{xz}]$, $a_0 = (\tau_1^2 + \tau_2^2 - \varepsilon_{zz})(\tau_1^2\varepsilon_{xx} + \tau_2^2\varepsilon_{yy} - \varepsilon_{xx}\,\varepsilon_{yy} - \varepsilon_{xy}^2) + \varepsilon_{yz}^2(\tau_2^2 - \varepsilon_{xx}) - 2\,\varepsilon_{xy}\,\varepsilon_{xz}\varepsilon_{yz} + \varepsilon_{xz}^2(\varepsilon_{yy} - \tau_1^2)$.

We seek the solution set of Equation (3) by expansion of the Fourier integral over x and y coordinates using the boundary conditions: fluctuations are negligible below and above plasma layer. If the plane xoy coincides with the lower boundary of slab, at $z \geq L$ the waves propagating in the negative direction must be absent, and at $z \leq 0$ — in the positive direction. Snell's law gives: $\tau_1^2 + \tau_2^2 = N^2\sin^2\theta_0$, where θ_0 is an incident angle. Spectral components of the current density on the pole ($\alpha = 0°$) substantially simplified:

$$\begin{aligned}&\tilde{g}_y(æ, z) = \mp ik_0^2\langle E_x\rangle\left[(D_1 + i\,s\,D_3)\,n' - (D_2 + i\,s\,D_4)h'_{0z}\right], \quad \tilde{g}_y(æ, z) = \pm i\tilde{g}_x(æ, z),\\ &\tilde{g}_z(æ, z) = k_0^2\langle E_x\rangle(F_1 + i\,sF_2)\left[h'_{0x}(æ, z) \pm ih'_{0y}(æ, z)\right],\end{aligned} \tag{6}$$

where: $D_1 = \frac{v_0}{1-u_0}(1 \pm \sqrt{u_0})$, $D_2 = -\frac{v_0}{1-u_0}[\pm\sqrt{u_0} + \frac{2u_0(1\pm\sqrt{u_0})}{1-u_0}]$, $D_3 = \frac{v_0}{(1-u_0)^2}(1 \pm \sqrt{u_0})^2$, $D_4 = -\frac{2v_0}{(1-u_0)^2}[\pm\sqrt{u_0} + \frac{u_0(3+u_0\pm4\sqrt{u_0})}{1-u_0}]$, $F_1 = \frac{v_0}{1-u_0}(u_0 \pm \sqrt{u_0})$, $F_2 = \frac{v_0}{(1-u_0)^2}[u_0(3-u_0) \pm 2\sqrt{u_0}]$.

Hence, current density fluctuations in the xoy plane contain both electron density and external magnetic field fluctuations, while the z component — only magnetic field fluctuations.

We investigate statistical characteristics of scattered high frequency radio waves in the turbulent collisional magnetized plasma slab caused by electron density fluctuations if wave propagates along the external magnetic field z-axis ($\theta = \varphi = 0$). Taking into account that phase fluctuations φ_1 are determined by the expression $\varphi_1 = \mathrm{Im}(e/\langle E_x\rangle)$, applying the residue theory for the poles of the Equation (5), for the y component of scattered radiation and arbitrary correlation function of the phased fluctuations $W_\varphi(\mathbf{r_1}, \mathbf{r_2}) = \langle \varphi_1(\mathbf{r_1})\varphi_1^*(\mathbf{r_2})\rangle$ at the observation points $\mathbf{r_1}$ and $\mathbf{r_2}$ caused by electron density fluctuations has the following form:

$$\begin{aligned}
&\langle \varphi_1(x+\rho_x, y+\rho_y, L)\varphi_1^*(x,y,L)\rangle_{yD}\\
&= -D_1^2 k_0^2 L \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} d\rho_z \exp(ik_x\rho_x + ik_y\rho_y) W_n\left(k_x, k_y, \rho_z\right)\\
&\left(\frac{2}{\delta_2}\left(E_1 - s\cdot E_2\right)\left[\cos\left(k_{02}\rho_z\right) - \frac{\sin(2k_{02}L)}{2k_{02}L} ch(s\cdot\alpha_2 k_{02}\rho_z)\right] - \frac{1}{\delta_1}(E_3 + s\cdot E_4)\right.\\
&\cdot\left\{\frac{\sin(p_2k_0L)}{p_2k_0L}\cos\left(\frac{p_1}{2}k_0\rho_z\right) ch\left(s\cdot\frac{q_1}{2}k_0\rho_z\right) - (J_1 + s\cdot J_2)\cos\left(\frac{p_2}{2}k_0\rho_z\right) ch\left(s\cdot\frac{q_2}{2}k_0\rho_z\right)\right.\\
&\left.\left.+ (J_3 - s\cdot J_4)\sin\left(\frac{p_2}{2}k_0\rho_z\right)\ sh\left(s\cdot\frac{q_2}{2}k_0\rho_z\right)\right\}\right), \qquad (7)
\end{aligned}$$

where: $E_1 = \varepsilon'\mu'' - \frac{1}{k_0^2}[(\eta' - \mu'')k_x^2 + \varepsilon' k_y^2]$, $E_2 = \frac{G_2}{k_0^2}k_xk_y$, $E_4 = \frac{G_1}{k_0^2}k_xk_y$, $E_3 = \frac{1}{k_0^2}[(\eta' + \mu'')k_x^2 + \varepsilon' k_y^2]$, $G_1 = \tilde{\varepsilon}'' + \tilde{D}_3(\eta' + \mu'' - \varepsilon') - \frac{1}{2\varepsilon'^2}(\tilde{\eta}'' - \tilde{\mu}')$, $G_2 = \tilde{\varepsilon}'' + \tilde{D}_3(\eta' - \mu'' - \varepsilon') - \frac{1}{2\varepsilon'^2}(\tilde{\eta}'' + \tilde{\mu}')$, $\delta_1 = 4\mu''\sqrt{\eta' + \mu''}$, $\delta_2 = 4\mu''\sqrt{\eta' - \mu''}$, $p_1 = \sqrt{a} + \sqrt{c}$, $q_1 = \sqrt{c}\alpha_2 - \sqrt{a}\alpha_1$, $p_2 = \sqrt{a} - \sqrt{c}$, $q_2 = \sqrt{c}\alpha_2 + \sqrt{a}\alpha_1$, $\alpha_1 = b/2a$, $\alpha_2 = d/2c$, $k_{02} = k_0\sqrt{c}$, $\tilde{D}_3 = \frac{1\pm\sqrt{u_0}}{1-u_0}$, $J_1 = \frac{\sin(p_1k_0L)}{p_1k_0L}ch(sq_1k_0L)$, $J_2 = \frac{q_1}{p_1}\frac{\cos(p_1k_0L)}{p_1k_0L}sh(sq_1k_0L)$, $J_3 = \frac{\cos(p_1k_0L)}{p_1k_0L}sh(sq_1k_0L)$, $J_4 = \frac{q_1}{p_1}\frac{\sin(p_1k_0L)}{p_1k_0L}ch(sq_1k_0L)$, ρ_x and ρ_y are distances between observation points spaced apart at small distances. Broadening of the angular power spectrum (APS) is determined as [4]: $\langle k_{x,y}^2\rangle = -\partial^2 W_n/\partial\rho_{x,y}^2$.

3. NUMERICAL CALCULATIONS

Analytical and numerical calculations will be carried out for anisotropic Gaussian correlation function of electron density fluctuation [3].

$$W_n\left(k_x, k_y, \rho_z\right) = \frac{\langle N_1^2\rangle}{4\pi}\frac{l_\parallel^2}{\chi\Gamma_0}\exp\left(-\frac{m^2}{l_\parallel^2}\rho_z^2 + ink_x\rho_z\right)\exp\left(-\frac{k_x^2\, l_\parallel^2}{4\,\Gamma_0^2} - \frac{k_y^2\, l_\parallel^2}{4\,\chi^2}\right), \qquad (8)$$

where: $m^2 = \chi^2/\Gamma_0^2$, $\Gamma_0^2 = \sin^2\gamma_0 + \chi^2\cos^2\gamma_0$, $n = (\chi^2 - 1)\sin\gamma_0\cos\gamma_0/\Gamma_0^2$. The average shape of electron density irregularities has the form of elongate ellipsoid of rotation. The rotation axis is located in the plane of geomagnetic meridian. The ellipsoid is characterized with two parameters: the anisotropy factor for the irregularities equaling to the ratio of ellipsoid axes $\chi = l_\parallel/l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities with respect to the external magnetic field) and orientation equaling to the inclination angle γ_0 of the prolate irregularities with respect to the external magnetic field (sometimes with respect to horizon). Anisotropy of the shape of irregularities is connected with the difference of diffusion coefficients in the field align and field perpendicular directions. In the case $k_0L < 1$ for nonmagnetic isotropic ($\chi = 1$) collisionless plasma ($s = 0$), at $\rho_x = \rho_y = 0$ we obtain the well-known formula for the variance of the phase fluctuations $W_{\varphi n} = \sqrt{\pi}\sigma_n^2 v_0^2 k_0^2 L\, l/4$ [1].

The results of numerical calculations of the APS are illustrated in graphical form. The plots are constructed for an incident wave 3 MHz ($k_0 = 6.28\cdot 10^{-2}$ m) using the following dimensionless parameters: $v_0 = 0.28$, $u_0 = 0.22$.

Figure 1 illustrates curves of the spectral width of scattered radio waves versus angle γ_0 for irregularities closely aligned along the magnetic field lines of the F-region in the principle plane. At fixed anisotropic factor $\chi = 13$, increasing characteristic linear scale of irregularities in the interval $l_{||} = 1.1 \div 2.4$ km, angle of inclination is $\gamma_0 \approx 3°$, and the width of the spectrum not

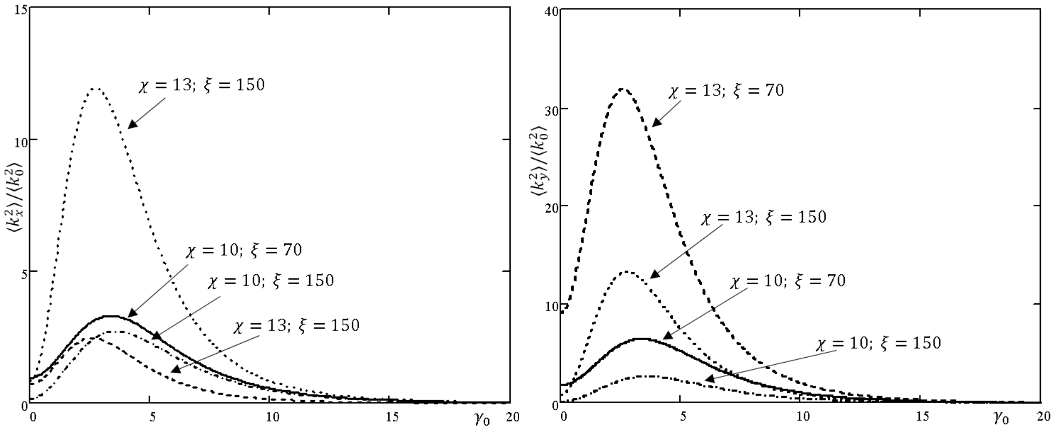

Figure 1: Plots of the spectral width of the APS in the principle and perpendicular planes as function of inclination angle of prolate irregularities γ_0 at different anisotropic parameter χ and ξ.

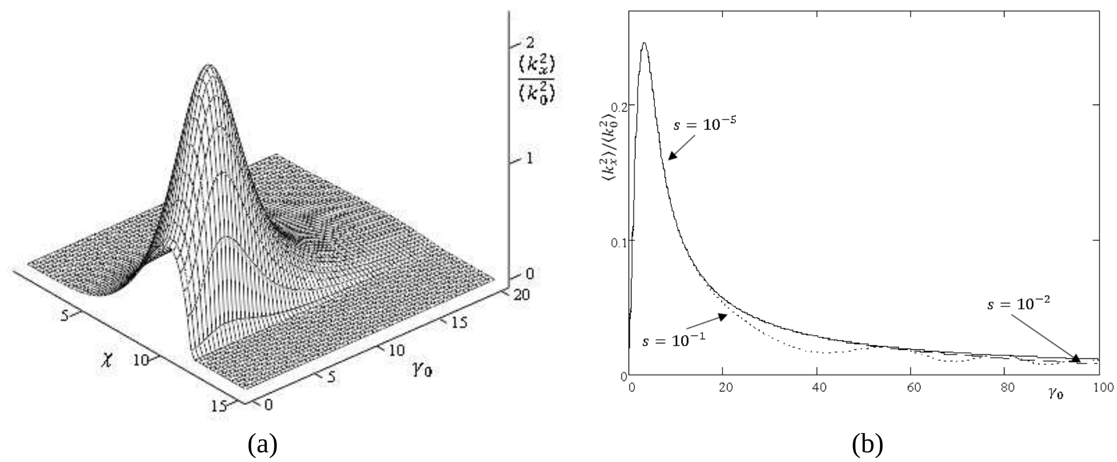

Figure 2: (a) 3-D picture of the spectral width of the APS in the principle plane versus anisotropy factor χ and angle of inclination γ_0. (b) Dependence of the broadening of the APS as function of the parameter ξ in isotropic case $\chi = 1$ for different s.

varies. Decreasing parameter $\chi = 10$ maximum of the spectrum is at $l_{||} \approx 1\,\text{km}$ and $\gamma_0 \approx 3.5°$; for irregularities with $l_{||} \approx 2.4\,\text{km}$ the width of the APS increases twice and the maximum displaced slightly, $\gamma_0 \approx 3°$.

Figure 2(a) depicts 3D Figure of the APS of scattered electromagnetic waves in the principle plane having Gaussian form for anisotropic plasma irregularities; Figure 2(b) illustrates curves of broadening of the APS in isotropic case $\chi = 1$. Maximum of the spectrum is at $\xi = 3.4$ not depending on the collision frequency between plasma particles and corresponds to the characteristic spatial scale of irregularities of electron density fluctuations $l_{||} = 50\,\text{m}$.

Increasing parameter s up to $s = 10^{-1}$, tail of the spectrum oscillated and three minimums appear: $\xi_{\min}^{(1)} = 40.3$, $\xi_{\min}^{(2)} = 68.5$ and $\xi_{\min}^{(3)} = 89.4$ corresponding to the: $l_{||}^{(1)} \approx 600\,\text{m}$, $l_{||}^{(2)} \approx 1.1\,\text{km}$ and $l_{||}^{(3)} \approx 1.4\,\text{km}$. The obtained results are is in agreement with the observation data from earth satellites at an average height of 362 km [5].

4. CONCLUSIONS

Statistical characteristics of scattered radio waves are calculated in turbulent collision magnetized ionospheric plasma using the perturbation method. General dispersion equation is obtained in zero approximation at arbitrary angles of inclination of both external magnetic field and wave vector of an incident electromagnetic wave. Correlation function of the phase fluctuations of scattered electromagnetic waves is obtained for arbitrary correlation function of electron density fluctuations taking into account polarization coefficients for both ordinary and extraordinary waves. The width of the APS substantially depends of the anisotropic factors characterizing field aligned irregularities, while location of maximum of the APS not substantially displaced. For isotropic irregularities three minimum appear in the APS allowing estimate characteristic linear scale of plasma irregularities. Numerical calculations for 3 MHz electromagnetic waves are in agreement with satellite observation data between heights of 153 and 617 km.

REFERENCES

1. Gershman, B. N., L. M. Erukhimov, and Y. Y. Yashin, *Wave Phenomena in the Ionosphere and Space Plasma*, Nauka, Moscow, 1979.
2. Jandieri, G. V., V. G. Gavrilenko, and A. A. Semerikov, "On the effect of absorption on multiple wave-scattering in a magnetized turbulent plasma," *Wave Random Media*, Vol. 9, 427–440, 1999.
3. Jandieri, G. V., A. Ishimaru, V. G. Jandieri, A. G. Khantadze, and Z. M. Diasamidze, "Model computations of angular power spectra for anisotropic absorptive turbulent magnetized plasma," *Progress In Electromagnetics Research*, Vol. 70, 307–328, 2007.
4. Jandieri, G. V., V. G. Gavrilenko, V. G. Jandieri, and A. V. Sorokin, "Some properties of the angular power distribution of electromagnetic waves multiply scattered in a collisional magnetized turbulent plasma," *Plasma Physics Report*, Vol. 31, No. 7, 604–615, 2005.
5. Chen, A. A. and G. S. Kent, "Determination of the orientation of ionospheric irregularities causing scintillation of signals from earth satellites," *J. Atm., Terr. Physics*, Vol. 34, 1411–1414, 1972.