Efficient Numerical Solution for Time-domain Volume Integral Equations

J. Zhang and M. S. Tong
Department of Electronic Science and Technology
Tongji University, 4800 Cao'an Road, Shanghai 201804, China

Abstract— Electromagnetic (EM) analysis for inhomogeneous penetrable structures relies on the formulation of volume integral equations (VIEs) in integral equation approach and time-domain VIEs (TDVIEs) are required for transient interaction with the structures. The TDVIEs are usually solved by combining the method of moments (MoM) with well-designed basis function in spatial domain and a march-on-in-time (MOT) scheme in temporal domain. We propose a different approach in which the Nyström method is used in the spatial domain and the MoM with Laguerre basis and testing functions is employed in the temporal domain. The proposed approach can simplify the numerical implementation and fully solve the stability problem as in the traditional approach. A numerical example is presented to illustrate the approach and good results have been observed.

1. INTRODUCTION

Integral equation approach is widely used for solving electromagnetic (EM) problems. For inhomogeneous penetrable structures, the volume integral equations (VIEs) are indispensable, and furthermore the time-domain version (TDVIEs) should be applied when a transient interaction with the structures exists [1]. The TDVIEs are usually solved by combining the method of moments (MoM) with a well-designed basis function like the Schaubert-Wilton-Glisson (SWG) basis function [2] in spatial domain and a march-on-in-time (MOT) scheme in temporal domain [3]. The MoM requires conforming meshes in geometric discretization and also needs to perform two-fold integrations in evaluating matrix elements, resulting in certain inconvenience. On the other hand, the MOT has a well-known stability problem which has not been fully solved yet although various improvements were proposed [4].

In this work, we propose a different scheme to solve the TDVIEs for transient EM problems with penetrable objects. Unlike the conventional approach, the Nyström method is used in the spatial domain while the MoM with Laguerre basis and testing functions is employed in the temporal domain. The Nyström method uses a point-matching procedure to discretize integral equations and does not use any basis and testing functions so that the implementation can be simplified. Also, the method does not require conforming meshes and can endure low-quality meshes, leading to much convenience in geometric discretization [5]. In temporal domain, the expansion of Laguerre basis function can naturally enforce the causality and also the Galerkin's testing procedure can fully solve the numerical stability problem in a march-on-in-degree (MOD) manner [6]. Although the MOD may not be more efficient than the MOT, it is very suitable to solve transient problems [7]. Numerical examples for transient EM scattering by a dielectric cylinder are presented and the merits of proposed approach has been demonstrated.

2. TIME-DOMAIN VOLUME INTEGRAL EQUATIONS (TDVIES)

Consider the scattering of transient EM wave by a dielectric object with a volume V in the free space with a permittivity ϵ_0 and permeability μ_0. The problem can be formulated by the TDVIEs which is reduced to a single electric-field TDVIE (EF-TDVIE) when the object is nonmagnetic. The EF-TDVIE states that the total electric field is equal to the summation of incident electric field and scattered electric field. The scattered electric field $\mathbf{E}^{sca}(\mathbf{r},t)$ is related to the magnetic vector potential $\mathbf{A}(\mathbf{r},t)$ and electric scalar potential $\Phi(\mathbf{r},t)$ by

$$\mathbf{E}^{sca}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t) - \nabla\Phi(\mathbf{r},t) = -\frac{\mu_0}{4\pi}\int_V \frac{1}{R}\frac{\partial}{\partial t}\mathbf{J}(\mathbf{r}',\tau)dS' - \frac{1}{4\pi\epsilon_0}\nabla\int_V \frac{1}{R}q(\mathbf{r}',\tau). \quad (1)$$

where $\mathbf{J}(\mathbf{r},t)$ and $q(\mathbf{r}',t)$ are the current density and charge density induced in the object, respectively, $R = |\mathbf{r}-\mathbf{r}'|$ is the distance between an observation point $\mathbf{r}$ and a source point $\mathbf{r}'$, and

$\tau = t - R/c$ is the retarded time with c being the speed of light in free space. By using the continuity equation and introducing a new source vector $\mathbf{e}(\mathbf{r}, t)$ which is related to the charge density by $q(\mathbf{r}, t) = -\nabla \cdot \mathbf{e}(\mathbf{r}, t)$, we can write the EF-TDVIE as

$$\frac{\mu_0}{4\pi}\int_V \frac{1}{R}\frac{\partial^2 \mathbf{e}(\mathbf{r}', \tau)}{\partial t^2} dV' + \frac{1}{4\pi\epsilon_0}\int_V \nabla\nabla\left(\frac{1}{R}\right)\cdot \mathbf{e}(\mathbf{r}', \tau) dV' = \frac{\mathbf{e}(\mathbf{r}, t)}{[\epsilon_0 - \epsilon(\mathbf{r})]} + \mathbf{E}^{inc}(\mathbf{r}, t), \quad \mathbf{r} \in V. \quad (2)$$

3. NYSTRÖM-BASED SCHEME FOR SOLVING THE TDVIES

The dielectric object is discretized into N small tetrahedral elements and ΔV_n is the volume of the nth tetrahedron. The unknown source vector $\mathbf{e}(\mathbf{r}', t)$ can then be expanded as

$$\mathbf{e}(\mathbf{r}', t) = \sum_{n=1}^{N} e_n(t)\mathbf{e}_n(\mathbf{r}'). \quad (3)$$

If we apply the Nyström method in spatial domain, i.e., the integration over a small tetrahedron is replaced with a summation under a quadrature rule provided that the integrand is regular, then we have

$$\frac{\mathbf{e}(\mathbf{r}, t)}{[\epsilon_0 - \epsilon(\mathbf{r})]} + \mathbf{E}^{inc}(\mathbf{r}, t) = \frac{\mu_0}{4\pi}\sum_{n=1}^{N}\sum_{q=1}^{Q}\frac{w_{nq}}{R}\frac{\partial^2 e_n(\tau)}{\partial t^2}\mathbf{e}_n(\mathbf{r}'_{nq}) + \frac{1}{4\pi\epsilon_0}\sum_{n=1}^{N}\sum_{q=1}^{Q} w_{nq}\nabla\nabla\left(\frac{1}{R}\right)\cdot \mathbf{e}_n(\mathbf{r}'_{nq}) e_n(\tau) \quad (4)$$

where $\mathbf{e}_n(\mathbf{r}'_{nq}) = \mathbf{e}_{nq}$ is the value of unknown source vector at the qth quadrature point in the nth tetrahedron. If we choose the pth quadrature point in the mth tetrahedron as an observation point to perform a collocation procedure, then we have the following space-domain matrix equation

$$\frac{\mathbf{e}_{mp} e_m(t)}{[\epsilon(\mathbf{r}_{mp}) - \epsilon_0]} + \sum_{n=1}^{N}\sum_{q=1}^{Q}\left[\frac{\mu_0}{4\pi}\frac{\partial^2 e_n(\tau)}{\partial t^2}\mathbf{a}_{mpnq} + \frac{e_n(\tau)}{4\pi\epsilon_0}\mathbf{b}_{mpnq}\right] = \mathbf{E}^{inc}(\mathbf{r}_{mp}, t), \; m=1, \ldots, N; \; p=1, \ldots, Q \quad (5)$$

where

$$\mathbf{a}_{mpnq} = \frac{w_{nq}\mu_0}{4\pi R_{mpnq}}\mathbf{e}_{nq}, \quad \mathbf{b}_{mpnq} = \frac{w_{nq}}{4\pi\epsilon_0}\nabla\nabla\left(\frac{1}{R}\right)\cdot \mathbf{e}_{nq}. \quad (6)$$

The above procedure could have a singularity problem which occurs when $m = n$ but we have developed a robust treatment technique [8].

In the temporal domain, we use the Laguerre function $\phi_j(t) = e^{-t/2}L_j(t)$ as a basis function to expand $e_n(t)$ [6]

$$e_n(t) = \sum_{j=0}^{\infty} e_{nj}\phi_j(st) \quad (7)$$

where s is the scaling factor to adjust the support of expansion. Substituting this expansion to Eq. (5) and using $\phi_i(st)$ as a testing function to test the equation, we can obtain

$$\begin{aligned}
&\sum_{n=1}^{N}\sum_{q=1}^{Q}\left(0.25 s^2 \mathbf{a}_{mpnq} + \mathbf{b}_{mpnq}\right) e_{nj} I_{ii}(sR_{mpnq}/c) \\
&-\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left(0.25 s^2 \mathbf{a}_{mpnq} + \mathbf{b}_{mpnq}\right) e_{nj} I_{ij}(sR_{mpnq}/c) \\
&-\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{j=0}^{i} s^2 \mathbf{a}_{mpnq}\sum_{k=0}^{j-1}(j-k) e_{nk} I_{ij}(sR_{mpnq}/c) + \frac{\mathbf{e}_{mp} e_{mi}}{[\epsilon_0 - \epsilon(\mathbf{r}_{mp})]} + \mathbf{V}_{mpi} \quad (8)
\end{aligned}$$

where $I_{ij}(sR_{mpnq}/c)$ and $\mathbf{V}_{mpi}$ can be determined according to the appendix in [6]. Combining the space-domain and time-domain expansion coefficients together, we can solve the matrix equation with a march-on-in-degree (MOD) procedure.

4. NUMERICAL EXAMPLE

We consider the transient EM scattering by a dielectric cylinder to demonstrate the approach. The cylinder, which is centered at the origin of a rectangular coordinate system as shown in Fig. 1, has a height $h = 1.0\,\mathrm{m}$ and a radius $a = 0.5\,\mathrm{m}$ at its cross section. The relative permittivity of the cylinder is $\epsilon_r = 2.0$ and it is discretized into 886 tetrahedrons. A Gaussian plane wave is illuminating the scatterer along the $-z$ direction and the electric field is defined by

$$\mathbf{E}^{inc}(\mathbf{r}, t) = \hat{x}\frac{4e^{-\gamma^2}}{\sqrt{\pi}T}, \quad \gamma = \frac{4}{T}(ct - ct_0 + \mathbf{r}\cdot\hat{z}) \tag{9}$$

where T is the width of Gaussian impulse, and t_0 is the time delay denoting the time when the pulse peaks at the origin. We choose a Gaussian pulse with $T = 4.0\,\mathrm{lm}$ (light meter) and $ct_0 = 6.0\,\mathrm{lm}$ as an incident wave. Fig. 2 plots the solution of the θ-component of normalized far-zone scattered electric field observed along the backward direction. The result agrees well with the solution obtained from the inverse discrete Fourier transform (IDFT) of frequency-domain solution.

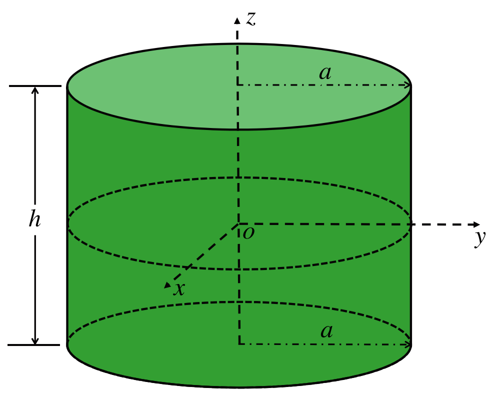

Figure 1: Geometry of a dielectric cylinder for transient scattering.

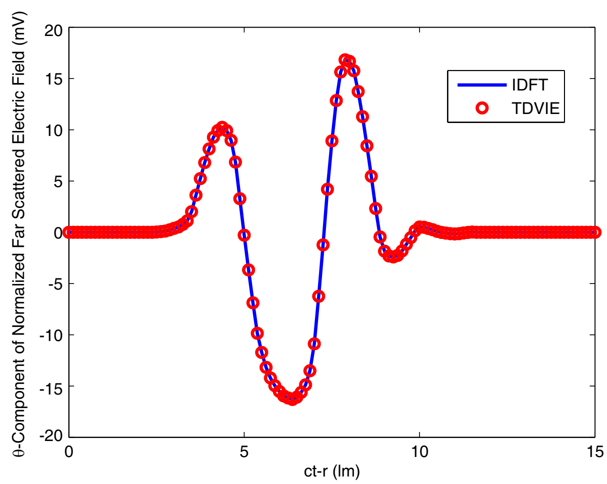

Figure 2: Solution of θ-component of normalized far-zone back-scattered electric field for transient scattering by a dielectric cylinder.

5. CONCLUSION

The TDVIEs are used to formulate the transient EM problems with penetrable objects. We propose a different scheme to solve the TDVIEs by combining the Nyström method in spatial domain and the MoM with the Laguerre basis and testing functions in temporal domain. The benefits of the scheme include the simple mechanism of implementation with allowance of using nonconforming and low-quality meshes in spatial domain, and also the natural satisfaction of causality with a full elimination of instability in temporal domain. A numerical example for transient EM scattering by a dielectric cylinder is presented and good results have been observed.

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