Analyzing Five-layer Planar Optical Waveguides with Kerr-type Nonlinear Metamaterial Guiding Films

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Abstract— In this paper, we proposed a method to analyze the electric field distributions of the five-layer planar optical waveguide structure with nonlinear metamaterial guiding films. The proposed nonlinear metamaterial planar optical waveguide is composed of two different kinds of media. One is the Kerr-type nonlinear metamaterial in the guiding films, and the other is the linear double-positive medium (DPS) in the interaction layer, in the cladding and in the substrate. We have derived the transcendental equations of the proposed five-layer planar optical nonlinear metamaterial waveguide structure. The propagation characteristics of the transverse electric (TE) waves were investigated analytically and numerically in the this structure. The analytical and numerical results show excellent agreement. The simulation results are helpful to figure out the propagation situation of the TE polarized waves in the five-layer nonlinear metamaterial waveguide structure. The transverse magnetic (TM) polarized waves can also be predicted by the similar process. It can also be very useful for designing a planar nonlinear metamaterial waveguide devices.

1. INTRODUCTION

Recently, a great number of papers are presented and investigated in the metamaterial. The metamaterial is a new type of artificial materials as well as so-called the left-handed medium (LHM). The metamaterial exhibits simultaneously negative dielectric permittivity and magnetic permeability [1–3]. In the past, most of papers have been concerned with linear metamaterial in wave propagation, applications, and components [4–12]. The kerr-type nonlinear medium has a phenomenon of optical solitons. From the theoretical investigation of spatial optical solitons, the applications of the kerr-type nonlinear medium have been proposed [13, 14]. Zharov et al. [15] have analyzed the nonlinear properties of left-handed metamaterials in two-dimensional periodic structure created by arrays of wires and split-ring resonators. Shadrivov et al. [16] have studied both linear and nonlinear surface waves at the interface between a left-handed medium and a conventional medium. Here we investigate the electric field distributions of the five-layer planar optical waveguide structure with the kerr-type nonlinear metamaterial guiding films. The propagation characteristics of the TE waves were investigated analyticallyand numerically. The analytical and numerical results show excellent agreement.

2. ANALYSES AND NUMERICAL RESULTS

In this section, the modal theory was used to drive the TE-polarized field for the five-layer planar optical waveguide with the kerr-type nonlinear metamaterial guiding films, as shown in Fig. 1. We consider the TE waves propagating along the z-direction. The wave equation can be reduced to as follows.

$$\frac{\partial^2 E_y}{\partial z^2}+\frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial y^2}=\frac{n_j}{c^2}\frac{\partial^2 E_y}{\partial t^2}\ , \quad j=f,c,s,i \tag{1}$$

In Eq. (1) the electric field in each layer can be expressed as:

$$E_y(x,z,t)=E_j(x)\exp[j(\omega t-\beta k_o z)], \qquad j=f,c,s,i \tag{2}$$

where β is the effective refractive index, ω is the angular frequency, and k_0 is the wave number in the free space. For the Kerr-type nonlinear metamaterial, the square of the refractive index n_f^2 can be expressed as:

$$n_f^2=\mu_f\varepsilon_f+\alpha\left|E(x)\right|^2 \tag{3}$$

where $\mu_f\varepsilon_f$ is the square of the linear refractive index of the nonlinear metamaterial and α is the nonlinear coefficient. By substituting Eq. (2) into the wave equation Eq. (1), considering the case

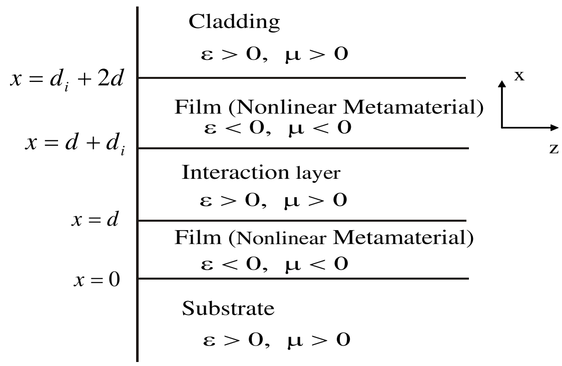

Figure 1: The structure of five-layer planar optical waveguide with nonlinear metamaterial film.

$\beta < \varepsilon_f \mu_f$ and matching the boundary conditions, the transverse electric fields in each layer can be written as:

$$\begin{aligned}
E_y(x) &= E_c \exp\{-P_c[x-(2d+w)]\} \quad 2d+w<x && (4)\\
E_y(x) &= b\cdot cn\{A[x-(d+w)+x_0]\,|B\} \quad d+w<x<2d+w && (5)\\
E_y(x) &= E_i\left\{\exp[-P_i(x-d)]+\exp\{P_i[x-(d+w)]\}\right\} \quad d<x<d+w && (6)\\
E_y(x) &= b\cdot cn\{A[x+x_0]\,|B\} \quad 0<x<d && (7)\\
E_y(x) &= E_s \exp[P_s x] \quad x<0 && (8)
\end{aligned}$$

where

$$A=\left[\left(a^2+b^2\right)\left(\frac{\alpha_2 k_0^2}{2}\right)\right]^{1/2},\quad B=\frac{b^2}{a^2+b^2},\quad a^2=\frac{\sqrt{H^4+2\alpha_2 k_0^2 K}+H^2}{\alpha_2 k_0^2}$$
$$b^2=\frac{\sqrt{H^4+2\alpha_2 k_0^2 K}-H^2}{\alpha_2 k_0^2},\quad H^2=k_0^2\left(n_2^2-\beta^2\right),\quad P^2=k_0^2\left(\beta^2-n_1^2\right)\qquad j=c,i,s \tag{9}$$

The parameters E_s an E_d are the values of the electric field at the lower (at $x=0$) and upper (at $x=w+2d$) boundaries of the film, respectively. K and x_0 ars the first and the second constant of integration. The cn is a specific Jacobian elliptic function. For simplicity, th modulus B is omitted in the following discussions. On the other hand, the first integration can be expressed as follows:

$$\begin{aligned}
K &= k_0^2 E_s^2\left(\left(\frac{\mu_f}{\mu_s}\right)^2\left(\beta^2-\varepsilon_s\mu_s\right)-\left(\varepsilon_f\mu_f-\beta^2\right)+\frac{\alpha E_s^2}{2}\right)\\
&= k_0^2 E_c^2\left(\left(\frac{\mu_f}{\mu_c}\right)^2\left(\beta^2-\varepsilon_c\mu_c\right)-\left(\varepsilon_f\mu_f-\beta^2\right)+\frac{\alpha E_c^2}{2}\right) && (10)
\end{aligned}$$

By the matching boundary condition, the transcendental equation can be expressed as:

$$\begin{aligned}
\frac{E_d P \mu_f}{\mu_i} &= \left\{A\cdot[E_{i1}\exp(-Rw)+E_{i2}]\cdot sn[Ad]dn[Ad]\cdot\left\{1-B\cdot\left[1-\left(\frac{E_{i1}\exp(-Rw)+E_{i2}}{b}\right)^2\right]\right\}\right.\\
&\quad -A\cdot[E_{i1}\exp(-Rw)+E_{i2}]\cdot B_4\cdot sn[A_4 d]cn^2[Ad]dn[Ad]\left[1-\left(\frac{E_{i1}\exp(-Rw)+E_{i2}}{b}\right)^2\right]\\
&\quad +\left\{cn[Ad]dn^2[Ad]-\left[\frac{E_{i1}\exp(-Rw)+E_{i2}}{b}\right]^2\cdot B_4\cdot sn^2[Ad]cn[Ad]\right\}\\
&\quad \left.\cdot R\cdot[E_{i1}\exp(-Rw)-E_{i2}]\right\}\frac{\mu_f}{\mu_i}/P\cdot\left\{1-B\cdot sn^2[Ad]\cdot\left[1-\left(\frac{E_{i1}\exp(-Rw)+E_{i2}}{b}\right)^2\right]\right\}^2 && (11)
\end{aligned}$$

where sn and dn are the Jacobian elliptic functions. Eq. (11) can be solved numerically. on the other hand, for the case $\varepsilon_f \mu_f < \beta$, we only have to replace A, B, a, b, and x_0 in Eq. (9) with A', B', a', b', and x'_0, and get these equations with similar representations. When constants β and E_0 are determined, all the other constants , K, A, a, b, B, and E_d are also determined.

We used the formulae derived in the preceding section to calculate the transverse electric field functions in the five-layer planar optical waveguides with the kerr-type nonlinear metamaterial guiding films. We use numerical examples to presented some simulation results. The numerical results are shown from the Figs. 2–6. Figure 2 shows the curve of the transcendental equation in nonlinear metamaterial waveguide with constant: $\mu_f = -2$, $d_f = 3\,\mu\text{m}$, $d_i = 3\,\mu\text{m}$, $\varepsilon_f \mu_f = 1.57$, $\varepsilon_s \mu_s = \varepsilon_c \mu_c = 1.55$, $\alpha = 6.3786\,\mu\text{m}^2/\text{V}^2$, and $\lambda = 1.3\,\mu\text{m}$. As the results shown in Figs. 2–6, the electric field distributions for several points as shown in the Fig. 2.

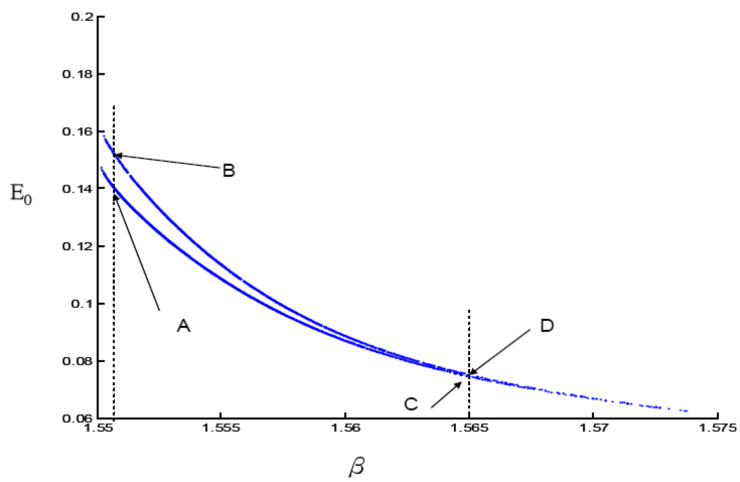

Figure 2: Dispersion curve of the five-layer waveguide with kerr-type nonlinear metamaterial guiding film.

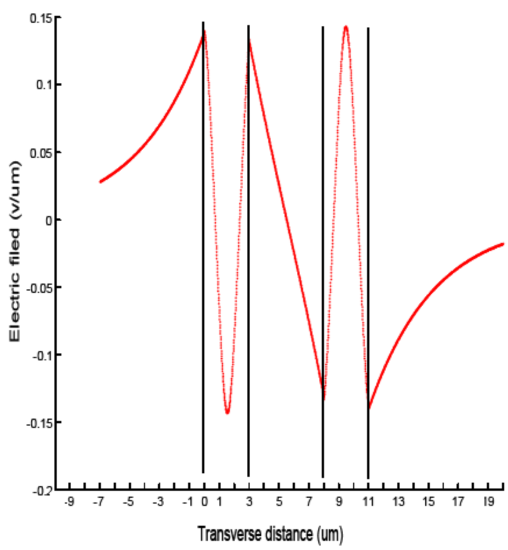

Figure 3: The electric filed distributions with respect to A point in the Fig. 2.

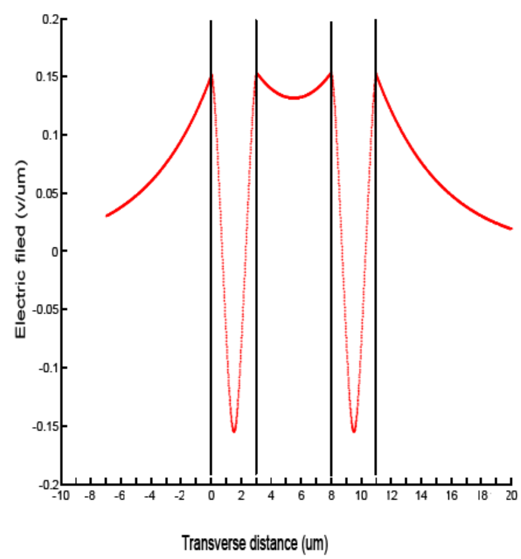

Figure 4: The electric filed distributions with respect to B point in the Fig. 2

3. CONCLUSION

In this paper, we have analyzed the transverse electric field of each layer in five-layer planar optical waveguide with the kerr-type nonlinear metamaterial guiding films. The transcendental equation can be depicted by the value of the boundary electric filed and the effective refraction index. We also show the evolutions of the electric filed with different effective refraction index. The similar process can be used to predict the propagation characteristics of TM waves in this structure. The simulation result is also helpful to figure out the propagation situation of the TE polarized waves.

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REFERENCES

1. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Sov. Phys. Usp.*, Vol. 10, 509–514, 1968.
2. Shelby, R. A., D. R. Smith, and S. Schultz, *Science*, Vol. 292, 77–79, 2001.
3. Ziolkowski, R. W. and E. Heyman, "Wave propagation in media having negative permittivity and permeability," *Phys. Rev. E*, Vol. 64, 056625, 2001.
4. Manapati, M. B. and R. S. Kshetrimayum, "SAR reduction in human head from mobile phone radiation using single negative metamaterials," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 10, 1385–1395, 2009.
5. Hwang, R.-B., H.-W. Liu, and C.-Y. Chin, "A metamaterial-based E-plane horn antenna," *Progress In Electromagnetics Research*, Vol. 93, 275–289, 2009.
6. Huang, M. D. and S. Y. Tan, "Efficient electrically small prolate spheroidal antennas coated with a shell of double-negative metamaterials," *Progress In Electromagnetics Research*, Vol. 82, 241–255, 2008.
7. Si, L.-M. and X. Lv, "CPW-FED multi-band omni-directional planar microstrip antenna using composite metamaterial resonators for wireless communications," *Progress In Electromagnetics Research*, Vol. 83, 133–146, 2008.
8. Al-Naib, I. A. I., C. Jansen, and M. Koch, "Single metal layer CPW metamaterial band-pass flter," *Progress In Electromagnetics Research Letters*, Vol. 17, 153–161, 2010.
9. Ziolkowski, R. W. and E. Heyman, "Wave propagation in media having negative permittivity and permeability," *Phys. Rev. E*, Vol. 64, 056625, 2001.
10. Milonni, P. W. and G. J. Maclay, "Quantized-field description of light negative-index media," *Opt. Commun.*, Vol. 228, 161–165, 2003.
11. Mirza, I. O., J. N. Sabas, S. Shi, and D. W. Prather, "Experimental demonstration of metamaterial based phase modulation," *Progress In Electromagnetics Research*, Vol. 93, 1–12, 2009.
12. Sabah, C. and S. Uckun, "Multilayer system of Lorentz/drude type metamaterials with dielectric slabs and its application to electromagnetic lters," *Progress In Electromagnetics Research*, Vol. 91, 349–364, 2009.
13. Sukhorukov, A. A., Y. S. Kivshar, H. S. Eisenberg, and Y. Silberberg, "Spatial Optical Solitons in Waveguide Ar-rays," *IEEE Journal of Quantum Electron*, Vol. 39, No. 1, 31–50, 2003.
14. Johansson, M., A. A. Sukhorukov, and Y. S. Kivshar, "Dis-crete reduced-symmetry solitons and second band vor-tices in two dimensional nonlinear waveguide arrays," *Physical Review: E*, Vol. 80, No. 4, 046604–046619, 2009.
15. Zharov, A. A., I. V. Shadrivov, and Y. S. Kivshar, "Nonlinear properties of left-handed metamaterials," *Phys. Rev. Lett.*, Vol. 91, 037401-1, 2003.
16. Shadrivov, I. V., A. A. Sukhorukov, Y. S. Kivshar, A. A. Zharov, A. D. Boardman, and P. Egan, "Nonlinear surface waves in left-handed materials," *Phys. Rev. E*, Vol. 69, 016617, 2004.