Chaos Generation Utilizing Optically Square-wave-injected Semiconductor Lasers

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Abstract— Benefiting from a non-zero value of the linewidth enhancement factor which leads to a variety of dynamics different from any other lasers, the broad bandwidth of chaos can be easily generated. There are several applications in chaotic laser, such as broadband random-bit generation, ranging, and secured communications. In the past two decays, most studies of the chaos generations focused on the nonlinear dynamics of the traditional continuous wave (cw) injection and dynamical pulse and sine wave injection. In this work, we study chaos generation utilizing an optically square-wave-injected semiconductor laser with tunable duty cycle. By injecting a slave laser with periodic optical square-wave at specific parameters, chaos oscillation and chaos pulsing can be generated individually. Compared with continuous wave injection with the same injected conditions, the chaos oscillation generated by square-wave injection shows larger bandwidth and more complex behaviors. Furthermore, if the operating variables are adjusted to the center of the chaos area in the dynamical mapping, the waveforms of power spectra of the generated chaos are much smoother and broader. In this paper, the distribution map of chaos oscillation and pulsing oscillation are investigated. When the duty cycle is operated at high level, chaos oscillation is observed. On the other hand, if we operate the condition at high repetition rate and strong injection strength, nonlinear dynamics is driven into the chaos pulsing states and the bandwidth of chaos pulsing is two times larger than chaos oscillation which is operated in week injection strength. The three parameters, duty cycle, repetition rate, and injection strength, all play important roles in chaos generation utilizing optical square-wave injection system.

1. INTRODUCTION

Nonlinear dynamics of semiconductor lasers have been widely investigated in recent years. Subject to different types of perturbations, such as optical feedback [1], optical injection [2], optoelectronic feedback [3], and dual-beam injection [4], rich dynamical behaviors can be easily generated. The main reasons which lead to a variety of dynamics in semiconductor lasers are the low reflectivity of the internal mirrors in the laser cavity and a non-zero value of the linewidth enhancement factor. The variety of dynamics are including stable locking, periodic oscillation, chaos pulsing, and chaos oscillation [5, 6]. Chaos pulsing and chaos oscillation are the phenomena of irregular variations of system outputs. Potential applications utilizing chaotic states have been extensively explored, such as secured communications [7], ranging [8, 9], and broadband random-bit generation [10]. In the past two decays, most studies utilized traditional continuous wave injection and dynamical pulse and sine wave injection to generate chaos, in this work, we focused on the chaos which is generated by an optically square-wave-injected semiconductor laser with tunable duty cycle. Compared with continuous wave injection with the same injected conditions, the chaos oscillation generated by square-wave injection shows larger bandwidth and more complex behaviors. Additionally, when the conditions are operated at high repetition rate and strong injection strength, the nonlinear dynamics is driven into the chaos pulsing states and the bandwidth is larger than that is operated at low repetition rate and week injection strength.

2. SCHEMATIC SETUP

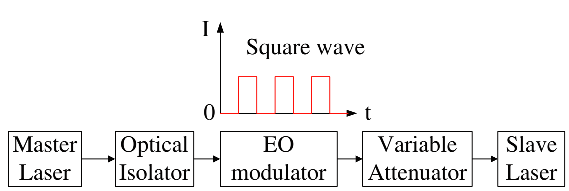

Figure 1: Schematic setup of a semiconductor laser under repetitive optical square wave injection.

Figure 1 is the schematic setup of the semiconductor laser under repetitive optical square-wave injection. In Fig. 1, EO modulator is an optical device in which a signal-controlled element exhibiting the electro-optic effect is used to make continuous wave change into square-wave and variable attenuator is to control the injection strength of square-wave input. The dynamics of the slave laser is controlled by adjusting the operational parameters. The nonlinear behavior of the optically injected slave laser can be derived by normalizing three coupled rate equations [11]:

$$\frac{da}{dt} = \frac{1}{2}\left[\frac{\gamma_c\gamma_n}{\gamma_s\tilde{J}}\tilde{n} - \gamma_p(2a+a^2)\right](1+a) + \xi_i(t)\gamma_c\cos(\Omega t+\phi) \tag{1}$$

$$\frac{d\phi}{dt} = -\frac{b}{2}\left[\frac{\gamma_c\gamma_n}{\gamma_s\tilde{J}}\tilde{n} - \gamma_p(2a+a^2)\right] - \frac{\xi_i(t)\gamma_c}{1+a}\sin(\Omega t+\phi) \tag{2}$$

$$\frac{d\tilde{n}}{dt} = -\gamma_s\tilde{n} - \gamma_n(1+a)^2\tilde{n} - \gamma_s\tilde{J}(2a+a^2) + \frac{\gamma_s\gamma_p}{\gamma_c}\tilde{J}(2a+a^2)(1+a)^2 \tag{3}$$

where ϕ, a, and $\tilde{n}$ representing the optical phase difference, normalized optical field and normalized carrier density, respectively. γ_c is the cavity decay rate, γ_n is the differential carrier relaxation rate, γ_p is the nonlinear carrier relaxation rate, γ_s is the spontaneous carrier relaxation rate, b is the linewidth enhancement factor, $\tilde{J}$ is the normalized dimensionless injection current parameter, Ω is the detuning frequency, and n is the effective refractive index. The dimensionless injection parameter $\xi_i(t)$ is the normalized strength of the injection field received by the injected laser. The injection profile of each Square wave is described as:

$$\xi_i(t) = \begin{cases} \xi_{sq}, & \text{if } T \leq D \times \tau \\ 0, & \text{otherwise} \end{cases}$$

Here ξ_{sq} is the normalized injection strength of the square wave. T is the period of function, D is the duty cycle of the square wave, and τ is the duration that the function are active. The relaxation resonance frequency of the laser used in this simulation is $f_r = 2.47$ GHz. By adjusting the controllable operational parameters including the normalized injection strength of square wave ξ_{sq}, the detuning frequency Ω between the master laser and the slaver laser, the duty cycle D of square wave, and repetition frequency f_{rep} of the repetitive square waves, various dynamical states including frequency-locked state, periodic oscillation, and chaos states can be observed.

3. RESULTS

Figure 2 shows the chaos oscillation generated by continuous wave injection and square-wave injection. The red and black curves indicate the input and output of slave laser, respectively. Figs. 2(a)–(b) show the time series and the corresponding power spectra under traditional continuous wave injection which is operated at the boundary between chaos oscillation and period-doubling oscillation area. The dynamics in Fig. 2(a) shows a period-doubling oscillation and the power spectra in Fig. 2(b) displays high intensity signal at specific frequency which gives rise to a narrow bandwidth. The bandwidth at this condition is about 4.36 GHz. It is obvious that this waveform is not a good choice to be applied to the application in chaotic lidar. However, instead of traditional continuous wave injection, we used square-wave injection with duty cycle $D = 50\%$ under the same conditions of detuning frequency and injection strength. The chaos oscillation became more complex, as shown in Figs. 2(c)–(d). Figs. 2(c)–(d) show time series and power spectra at the repetition rate $f_{rep} = 200$ MHz, respectively. Compared with continuous wave injection, square-wave injection at repetition rate $f_{rep} = 200$ MHz demonstrate a smoother power spectra, this chaos oscillation still displays a distinguish peak at 8 GHz in the power spectra caused by the slow repetition rate. High quality chaos waveform can be achieved if the repetition rate is operated at the center of the chaos area, and nonlinear dynamics could be more complicated than the fixed repetition rate at 200 MHz.

Moreover, the distribution map of chaos oscillation and pulsing oscillation is investigated. Fig. 3 shows the chaos map and corresponding bandwidth with various injection strength and repetition rate at fixed duty cycle of $D = 50\%$. The red dots in Fig. 3(a) represent chaos oscillation and the black dots represent chaos pulsing. As shown in Fig. 3(a), chaos pulsing occurs at high injection strength and high repetition rate area and almost all of chaos pulsing demonstrate better bandwidth than chaos oscillation, as can be seen in Fig. 3(b). The definition of bandwidth here is the 80% of total energy from the DC in power spectra. When the repetition rate is fixed at 4.8 GHz, operating

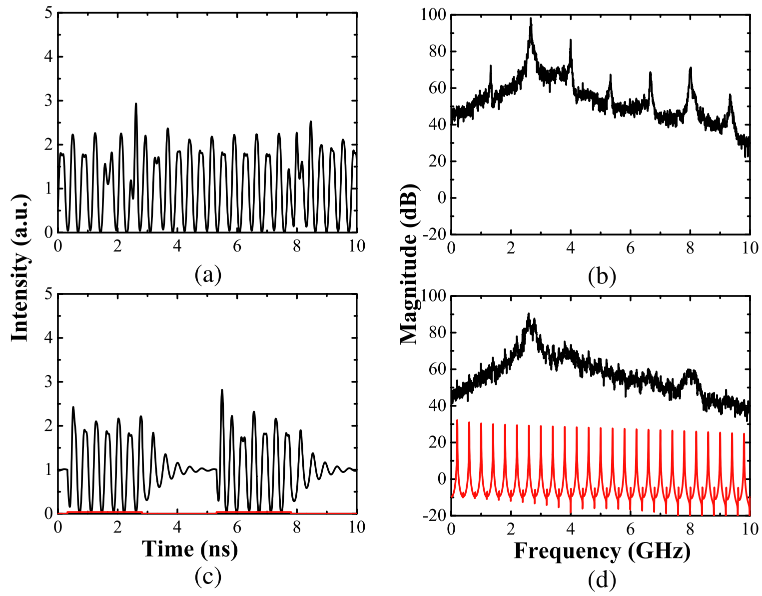

Figure 2: Comparison between continuous wave injection and square-wave injection. (a)–(b) Chaos oscillation generated by continuous wave injection operated at the boundary between period-2 and chaos with injection strength $\xi_i = 0.04$ and detuning frequency $\Omega = -0.7$ GHz. (c)–(d) chaos oscillation generated by square-wave injection with duty cycle $D = 50\%$ and repetition rate $f_{rep} = 200$ MHz. The red curves are the input of slave laser and the black curves are the output of slave laser.

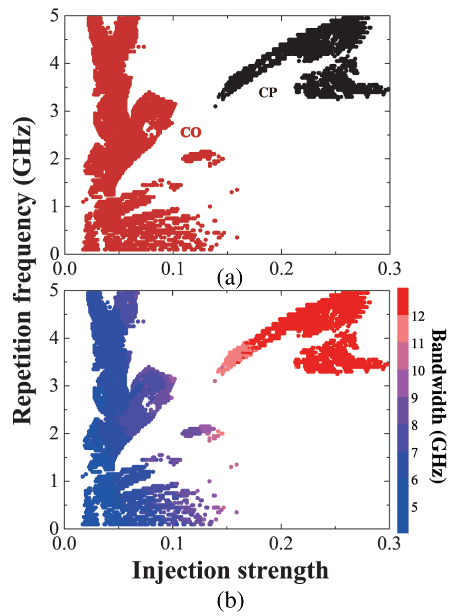

Figure 3: (a) The map of chaos and (b) the map of bandwidth with square-wave injection operated at duty cycle $D = 50\%$ and detuning frequency $\Omega = -0.7$ GHz. The red dots and the black dots in Fig. 3(a) represent chaos oscillation and chaos pulsing, respectively.

the injection strength $\xi_{sq} = 0.05$ and $\xi_{sq} = 0.273$ to generate chaos oscillation and chaos pulsing with the bandwidths of 7.1 GHz and 14.5 GHz are demonstrated, respectively. As the result, the bandwidth of chaos pulsing is two times larger than chaos oscillation. Except the repetition rate and injection strength, duty cycle is also an key parameter for chaos generation. Chaos oscillation is usually found at high duty cycle condition and chaos pulsing is easily observed when operating at low duty cycle.

4. CONCLUSION

In this work, we have numerically investigated chaos oscillation and chaos pulsation generated by square-wave injection. Under the same conditions of detuning frequency and injection strength, the waveforms of chaos oscillation generated by square-wave injection are more complex than traditional continuous wave injection. Furthermore, when increasing the repetition rate of square-wave injection, the chaos waveforms show more smoother than operating at low repetition rate. In this paper, the distribution map of chaos oscillation and chaos pulsing is also obtained and studied. Chaos oscillation is usually observed at the conditions of low repetition rate and low injection strength. In addition, due to the bandwidth enhancement effect, chaos with larger bandwidth is easily found under strong injection strength.

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