Electromagnetic Force on Charged Objects with the Angular Velocity

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Abstract— The paper studies the influence of angular velocity on the electromagnetic force for one rotational charged object. In the complex octonion space, the radius vector and the integrating function of field potential can be combined together to become one compounding radius vector. The latter may be considered as the 'radius vector' in the complex-octonion compounding space (one function space). The complex-octonion compounding space will be regarded as the extension of one complex-octonion compounding field. In the complex-octonion compounding field, the angular velocity will impact the compounding field strength, field source and linear momentum and so on. The study reveals that the angular velocity has a direct influence on the compounding angular momentum, compounding torque, and compounding force and so forth, including the compounding force term exerting on the rotational charged object.

1. INTRODUCTION

The quaternion was applied by J. C. Maxwell to research the electromagnetic theory. Nowadays some scholars studied the electromagnetic and gravitational fields by means of the complex quaternion. The complex quaternion space for gravitational fields and that for electromagnetic fields can be combined together to become one complex octonion space. As a result the physics feature of two fields can be described simultaneously with the complex octonion [1], including the force in the presence of the gravitational field, electromagnetic field, and angular velocity.

In the complex octonion space, the radius vector and the integrating function of field potential are able to be combined together to become the compounding radius vector, which may be considered as the 'radius vector' in the compounding space (or one function space). In the physics, the space is the extension of the field [2]. Similarly the compounding space will be regarded as the extension of the compounding field. By analogy the field and space should be generalized to the compounding field and compounding space respectively. Further it is fit for studying the impact of the angular velocity on the field strength and field source and so on, from the compounding field and compounding space.

Making use of the complex octonion, the paper claims that there are some kinds of equilibrium states in these two fields. The field potential and velocity ratio both will impact the field strength, field source, and equilibrium state. So the variation of field potential may alter the force or torque. This inference may figure out the physical phenomenon relevant to the over-speed movement of stars on the fringe of a galaxy [3], although this uncertainty remains as puzzling as ever.

2. GRAVITATIONAL SOURCE

In the complex quaternion space for the gravitational field, the basis vector is $\mathbb{H}_g = (\mathbf{i}_0, \mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3)$, and the radius vector is $\mathbb{R}_g = ir_0\mathbf{i}_0 + \Sigma r_j\mathbf{i}_j$. The radius vector can be combined with the physical quantity, $\mathbb{X}_g = ix_0\mathbf{i}_0 + \Sigma x_j\mathbf{i}_j$, to become the compounding radius vector,

$$\bar{\mathbb{R}}_g = \mathbb{R}_g + k_{rx}\mathbb{X}_g, \tag{1}$$

where $\bar{\mathbb{R}}_g = i\bar{r}_0 + \bar{\mathbf{r}} = i\bar{r}_0\mathbf{i}_0 + \Sigma\bar{r}_j\mathbf{i}_j$. $\mathbb{X}_g$ is the integrating function of complex-quaternion gravitational potential, $\mathbb{A}_g = ia_0\mathbf{i}_0 + \Sigma a_j\mathbf{i}_j$. i is the imaginary unit. $r_0 = v_0 t$; v_0 is the speed of light; t denotes the time. $\bar{r}_k$, r_k, a_k, and x_k are all real. $k_{rx} = 1/v_0$. $\mathbf{i}_0 = 1$. $j = 1, 2, 3$; $k = 0, 1, 2, 3$.

In the mathematics, $\bar{\mathbb{R}}_g$ may be considered as one 'radius vector' in the compounding space of complex quaternions, which is one kind of function space with the quaternion basis vector $\mathbb{H}_g$. Further the $\mathbb{A}_g$ and complex-quaternion velocity ratio, $\mathbb{T}_g = it_0\mathbf{i}_0 + \Sigma t_j\mathbf{i}_j$, will combine together to become the complex-quaternion compounding velocity ratio,

$$\bar{\mathbb{T}}_g = \mathbb{T}_g + k_{rx}\mathbb{A}_g, \tag{2}$$

where $\mathbb{T}_g = i\Box^\times \circ \mathbb{R}_g$; $\mathbb{A}_g = i\Box^\times \circ \mathbb{X}_g$; $\bar{\mathbb{T}}_g = i\Box^\times \circ \bar{\mathbb{R}}_g = i\bar{t}_0\mathbf{i}_0 + \Sigma\bar{t}_j\mathbf{i}_j$. $\Box = i\mathbf{i}_0\partial_0 + \Sigma\mathbf{i}_j\partial_j$, with $\nabla = \Sigma\mathbf{i}_j\partial_j$, and $\partial_k = \partial/\partial r_k$. The symbol $\circ$ denotes the octonion multiplication. $\times$ denotes the complex conjugate. $\bar{t}_k$, a_k, and t_k are all real.

Similarly, in this complex-quaternion compounding space, the complex-quaternion compounding gravitational potential is defined as,

$$\bar{\mathbb{A}}_g = \mathbb{A}_g + K_{rx}\mathbb{T}_g, \tag{3}$$

where $\bar{\mathbb{A}}_g = i\Box^\times \circ \bar{\mathbb{X}}_g = i\bar{a}_0\mathbf{i}_0 + \Sigma\bar{a}_j\mathbf{i}_j$. $\bar{\mathbb{X}}_g = \mathbb{X}_g + K_{rx}\mathbb{R}_g$. $K_{rx} = 1/k_{rx}$. $\bar{a}_k$ is real.

Further the complex-quaternion compounding gravitational strength, $\bar{\mathbb{F}}_g = \bar{f}_0\mathbf{i}_0 + \Sigma\bar{f}_j\mathbf{i}_j$, is defined from the above,

$$\bar{\mathbb{F}}_g = \Box \circ \bar{\mathbb{A}}_g = \mathbb{F}_g + K_{rx}\mathbb{Y}_g, \tag{4}$$

where the curl of velocity ratio is, $\mathbb{Y}_g = \Box \circ \mathbb{T}_g = iy_0\mathbf{i}_0 + \Sigma y_j\mathbf{i}_j$. The gravitational strength is, $\mathbb{F}_g = \Box \circ \mathbb{A}_g = if_0\mathbf{i}_0 + \Sigma f_j\mathbf{i}_j$. In the gravitational field, φ is the scalar potential, while $\mathbf{a}$ is the vectorial potential. The gauge equation is, $-\bar{f}_0 = \partial_0\bar{a}_0 - \nabla \cdot \bar{\mathbf{a}} = 0$. $a_0 = \varphi/v_0$ is the scalar. $\mathbf{a} = \Sigma(a_j\mathbf{i}_j)$, $\bar{\mathbf{a}} = \Sigma(\bar{a}_j\mathbf{i}_j)$. $\bar{f}_k$, f_k, and y_k are all real.

In the complex-quaternion compounding space, the complex-quaternion compounding field source $\bar{\mathbb{S}}$ is defined from the above,

$$\left(i\bar{\mathbb{F}}_g/v_0 + \Box\right)^* \circ \bar{\mathbb{F}}_g = -\mu\bar{\mathbb{S}} = -\mu_g\bar{\mathbb{S}}_g + i\bar{\mathbb{F}}_g^* \circ \bar{\mathbb{F}}_g/v_0, \tag{5}$$

where μ and μ_g are the constants. The complex-quaternion compounding gravitational source is, $\bar{\mathbb{S}}_g = i\bar{s}_0\mathbf{i}_0 + \Sigma\bar{s}_j\mathbf{i}_j$. $\bar{\mathbb{S}}_g = \mathbb{S}_g + K_{rx}\mathbb{Z}_g$. $\mu_g\mathbb{S}_g = -\Box^* \circ \mathbb{F}_g$, and $\mu_g\mathbb{Z}_g = -\Box^* \circ \mathbb{Y}_g$. $\bar{\mathbb{F}}_g^* \circ \bar{\mathbb{F}}_g/(2\mu_g)$ is one energy term in the gravitational field. $*$ denotes the octonion conjugate. For one single mass particle, there is, $\bar{\mathbb{S}}_g = m\bar{\mathbb{V}}_g$, with m being the density of inertial mass. $\bar{\mathbb{V}}_g = \partial\bar{\mathbb{R}}_g/\partial t = i\bar{v}_0\mathbf{i}_0 + \Sigma\bar{v}_j\mathbf{i}_j$. $\bar{s}_k$ and $\bar{v}_k$ are all real. $\mu_g < 0$.

3. ELECTROMAGNETIC SOURCE

The complex octonion space $\mathbb{E}$ is able to be separated into two orthogonal subspaces, the complex quaternion space $\mathbb{H}_g$ and the complex S-quaternion space, $\mathbb{H}_e = (\mathbf{I}_0, \mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3)$. The $\mathbb{H}_g$ is propitious to describe the gravitational field, while the $\mathbb{H}_e$ is fit for depicting the electromagnetic field, with $\mathbb{H}_e = \mathbb{H}_g \circ \mathbf{I}_0$. In the complex octonion space, the basis vector $\mathbb{O} = (\mathbf{i}_0, \mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3, \mathbf{I}_0, \mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3)$.

In the complex S-quaternion space for electromagnetic fields, the radius vector is $\mathbb{R}_e = iR_0\mathbf{I}_0 + \Sigma R_j\mathbf{I}_j$, with the basis vector being $\mathbb{H}_e$. And the $\mathbb{R}_e$ can be combined with the physical quantity, $\mathbb{X}_e = iX_0\mathbf{I}_0 + \Sigma X_j\mathbf{I}_j$, to become the compounding radius vector,

$$\bar{\mathbb{R}}_e = \mathbb{R}_e + k_{rx}\mathbb{X}_e, \tag{6}$$

where $\bar{\mathbb{R}}_e = i\bar{R}_0 + \bar{\mathbf{R}} = i\bar{R}_0\mathbf{I}_0 + \Sigma\bar{R}_j\mathbf{I}_j$. $\mathbb{X}_e$ is the integrating function of complex-quaternion electromagnetic potential, $\mathbb{A}_e = iA_0\mathbf{I}_0 + \Sigma A_j\mathbf{I}_j$. R_k, $\bar{R}_k$, A_k, and X_k are all real.

In the octonion space for the gravitational and electromagnetic fields, the $\bar{\mathbb{R}}_e$ and $\bar{\mathbb{R}}_g$ can be combined together to become the compounding radius vector,

$$\bar{\mathbb{R}} = \bar{\mathbb{R}}_g + k_{eg}\bar{\mathbb{R}}_e = \mathbb{R} + k_{rx}\mathbb{X}, \tag{7}$$

where $\mathbb{X} = \mathbb{X}_g + k_{eg}\mathbb{X}_e$, and $\mathbb{R} = \mathbb{R}_g + k_{eg}\mathbb{R}_e$. k_{eg} is the coefficient, with $k_{eg}^2 = \mu_g/\mu_e$.

In the mathematics, $\bar{\mathbb{R}}$ can be considered as the 'radius vector' in the complex-octonion compounding space (or function space) with the basis vector $\mathbb{O}$. In the complex-octonion compounding space, the complex-octonion velocity, $\mathbb{T} = \mathbb{T}_g + k_{eg}\mathbb{T}_e$, is combined with the complex-octonion potential, $\mathbb{A} = \mathbb{A}_g + k_{eg}\mathbb{A}_e$, to become the complex-octonion compounding velocity,

$$\bar{\mathbb{T}} = \bar{\mathbb{T}}_g + k_{eg}\bar{\mathbb{T}}_e = \mathbb{T} + k_{rx}\mathbb{A}, \tag{8}$$

where $\mathbb{T} = i\Box^\times \circ \mathbb{R}$. $\bar{\mathbb{T}} = i\Box^\times \circ \bar{\mathbb{R}}$. $\mathbb{A} = i\Box^\times \circ \mathbb{X}$. $\mathbb{T}_e = i\Box^\times \circ \mathbb{R}_e = iT_0\mathbf{I}_0 + \Sigma T_j\mathbf{I}_j$. $\bar{\mathbb{T}}_e = \mathbb{T}_e + k_{rx}\mathbb{A}_e$. $\bar{\mathbb{T}}_e = i\Box^\times \circ \bar{\mathbb{R}}_e = i\bar{T}_0\mathbf{I}_0 + \Sigma\bar{T}_j\mathbf{I}_j$. $\mathbb{A}_e = i\Box^\times \circ \mathbb{X}_e = iA_0\mathbf{I}_0 + \Sigma A_j\mathbf{I}_j$. T_k, $\bar{T}_k$, and A_k are all real.

In the complex-octonion compounding space, the complex-octonion compounding electromagnetic potential, $\bar{\mathbb{A}}$, is defined as,

$$\bar{\mathbb{A}} = \bar{\mathbb{A}}_g + k_{eg}\bar{\mathbb{A}}_e = \mathbb{A} + K_{rx}\mathbb{T}, \tag{9}$$

where $\bar{\mathbb{A}}_e = i\Box^\times \circ \bar{\mathbb{X}}_e = i\bar{A}_0\mathbf{I}_0 + \Sigma\bar{A}_j\mathbf{I}_j$. $\bar{\mathbb{A}} = i\Box^\times \circ \bar{\mathbb{X}}$. $\bar{\mathbb{X}} = \bar{\mathbb{X}}_g + k_{eg}\bar{\mathbb{X}}_e$. $\bar{A}_k$ is real.

Similarly the complex-octonion compounding electromagnetic strength, $\bar{\mathbb{F}} = \bar{\mathbb{F}}_g + k_{eg}\bar{\mathbb{F}}_e$, is defined from the compounding potential $\bar{\mathbb{A}}$,

$$\bar{\mathbb{F}} = \Box \circ \bar{\mathbb{A}} = \mathbb{F} + K_{rx}\mathbb{Y}, \tag{10}$$

where $\mathbb{F}_e = \Box \circ \mathbb{A}_e = iF_0\mathbf{I}_0 + \Sigma F_j\mathbf{I}_j$; $\bar{\mathbb{F}}_e = \Box \circ \bar{\mathbb{A}}_e = i\bar{F}_0\mathbf{I}_0 + \Sigma\bar{F}_j\mathbf{I}_j$; $\mathbb{F} = \Box \circ \mathbb{A} = \mathbb{F}_g + k_{eg}\mathbb{F}_e$. $\mathbb{Y} = \Box \circ \mathbb{T}$. $\mathbb{Y}_e = \Box \circ \mathbb{T}_e = iY_0\mathbf{I}_0 + \Sigma Y_j\mathbf{I}_j$. In the electromagnetic field, ϕ is the scalar potential, $\mathbf{A} = \Sigma A_j\mathbf{I}_j$ is the vectorial potential. $\mathbf{A}_0 = A_0\mathbf{I}_0$, with $A_0 = \phi/v_0$. $\bar{\mathbf{A}}_0 = \bar{A}_0\mathbf{I}_0$, $\bar{\mathbf{A}} = \Sigma\bar{A}_j\mathbf{I}_j$. The gauge equation is $-\bar{\mathbf{F}}_0 = \partial_0\bar{\mathbf{A}}_0 - \nabla \cdot \bar{\mathbf{A}} = 0$. Y_k, F_k, and $\bar{F}_k$ are all real.

In the complex-octonion compounding space, the complex-octonion compounding field source $\bar{\mathbb{S}}$ is defined from the $\bar{\mathbb{F}}$,

$$\left(i\bar{\mathbb{F}}/v_0 + \Box\right)^* \circ \bar{\mathbb{F}} = -\mu\bar{\mathbb{S}} = -\left(\mu_g\bar{\mathbb{S}}_g + k_{eg}\mu_e\bar{\mathbb{S}}_e\right) + i\bar{\mathbb{F}}^* \circ \bar{\mathbb{F}}/v_0, \tag{11}$$

where μ_e is the constant, with $\mu_e > 0$. $\bar{\mathbb{S}}_e = i\bar{S}_0\mathbf{I}_0 + \Sigma\bar{S}_j\mathbf{I}_j$. $\bar{\mathbb{S}}_e = \mathbb{S}_e + K_{rx}\mathbb{Z}_e$. $\mu_e\mathbb{S}_e = -\Box^* \circ \mathbb{F}_e$, and $\mu_e\mathbb{Z}_e = -\Box^* \circ \mathbb{Y}_e$. $\bar{\mathbb{F}}^* \circ \bar{\mathbb{F}}/\mu_g = \bar{\mathbb{F}}_g^* \circ \bar{\mathbb{F}}_g/\mu_g + \bar{\mathbb{F}}_e^* \circ \bar{\mathbb{F}}_e/\mu_e$. $\bar{\mathbb{F}}_e^* \circ \bar{\mathbb{F}}_e/(2\mu_e)$ is one energy term in the electromagnetic field, and is different to the electromagnetic field energy. For a single charged particle, $\bar{\mathbb{S}}_e = q\bar{\mathbb{V}}_e$. q is the density of electric charge. $\bar{\mathbb{V}}_e = \partial\bar{\mathbb{R}}_e/\partial t = i\bar{V}_0\mathbf{I}_0 + \Sigma\bar{V}_j\mathbf{I}_j$. $\bar{V}_k$ and $\bar{S}_k$ are all real.

4. ANGULAR MOMENTUM

In the complex-octonion compounding space, the linear momentum density, $\bar{\mathbb{P}} = \bar{\mathbb{P}}_g + k_{eg}\bar{\mathbb{P}}_e$, is defined as

$$\bar{\mathbb{P}} = \mu\bar{\mathbb{S}}/\mu_g, \tag{12}$$

where $\bar{\mathbb{P}}_g = i\bar{p}_0 + \bar{\mathbf{p}} = i\bar{p}_0\mathbf{i}_0 + \Sigma\bar{p}_j\mathbf{i}_j$. $\bar{p}_0 = \hat{m}\bar{v}_0$, $\bar{p}_j = m\bar{v}_j$; $\hat{m} = m + \triangle m$, with $\hat{m}$ being the density of gravitational mass. $\bar{\mathbb{P}}_e = i\bar{P}_0 + \bar{\mathbf{P}} = i\bar{P}_0\mathbf{I}_0 + \Sigma\bar{P}_j\mathbf{I}_j$. $\bar{P}_0 = M\bar{V}_0$, and $\bar{P}_j = M\bar{V}_j$; $M = q\mu_e/\mu_g$. $\triangle m = -\bar{\mathbb{F}}^* \circ \bar{\mathbb{F}}/(\bar{v}_0 v_0\mu_g)$. $\bar{P}_k$ and $\bar{p}_k$ are all real.

In the above the linear momentum $\bar{\mathbb{P}}$ consists of a few terms, including the linear momentum term $\bar{\mathbb{S}}_g$ of one mass object, the term $k_{eg}\mu_e\bar{\mathbb{S}}_e/\mu_g$ of one charged particle, the term $\bar{\mathbb{F}}_g^* \circ \bar{\mathbb{F}}_g/(\mu_g v_0)$ of gravitational fields, and the term $\bar{\mathbb{F}}_e^* \circ \bar{\mathbb{F}}_e/(\mu_e v_0)$ of electromagnetic fields.

From the definition of complex-quaternion velocity ratio $\mathbb{T}_g$, one can find its relation with the velocity, $\mathbb{V}_g = \partial\mathbb{R}_g/\partial t$. Sometimes the physics quantity $(v_0\mathbb{T}_g)$ possesses the same vectorial part with $\mathbb{V}_g$, but their scalar parts are different. In the case, there are, $v_0 t_j = v_j$, and $v_0 t_0 \neq v_0$. Herein $\mathbb{V}_g = iv_0\mathbf{i}_0 + \Sigma v_j\mathbf{i}_j$. v_k is real.

In the complex-octonion compounding space for the electromagnetic and gravitational fields, the complex-octonion angular momentum, $\bar{\mathbb{L}} = \bar{\mathbb{L}}_g + k_{eg}\bar{\mathbb{L}}_e$, can be defined from the complex-octonion compounding radius vector $\bar{\mathbb{R}}$ and linear momentum $\bar{\mathbb{P}}$. That is,

$$\bar{\mathbb{L}} = \bar{\mathbb{R}}^\times \circ \bar{\mathbb{P}}, \tag{13}$$

where $\bar{\mathbb{L}}_g = \bar{L}_{10} + i\bar{\mathbf{L}}_1^i + \bar{\mathbf{L}}_1$; $\bar{\mathbf{L}}_1^i = \Sigma\bar{L}_{1j}^i\mathbf{i}_j$, $\bar{\mathbf{L}}_1 = \Sigma\bar{L}_{1j}\mathbf{i}_j$. $\bar{\mathbb{L}}_e = \bar{\mathbf{L}}_{20} + i\bar{\mathbf{L}}_2^i + \bar{\mathbf{L}}_2$. $\bar{\mathbf{L}}_2^i = \Sigma\bar{L}_{2j}^i\mathbf{I}_j$, $\bar{\mathbf{L}}_2 = \Sigma\bar{L}_{2j}\mathbf{I}_j$, $\bar{\mathbf{L}}_{20} = \bar{L}_{20}\mathbf{I}_0$. $\bar{\mathbf{L}}_1$ covers the angular momentum. $\bar{\mathbf{L}}_2$ consists of the magnetic dipole momentum. $\bar{\mathbf{L}}_2^i$ includes the electric dipole momentum. And $\bar{\mathbf{L}}_{20}$ may contains the spin magnetic momentum. $\bar{L}_{1k}$, $\bar{L}_{1j}^i$, $\bar{L}_{2k}$, and $\bar{L}_{2j}^i$ are all real.

5. OCTONION TORQUE

In the complex-octonion compounding space for the electromagnetic and gravitational fields, the complex-octonion torque, $\bar{\mathbb{W}} = \bar{\mathbb{W}}_g + k_{eg}\bar{\mathbb{W}}_e$, is defined from $\bar{\mathbb{L}}$ and $\bar{\mathbb{F}}$,

$$\bar{\mathbb{W}} = -v_0(i\bar{\mathbb{F}}/v_0 + \Box) \circ \bar{\mathbb{L}}, \tag{14}$$

where $\bar{\mathbb{W}}_g = i\bar{W}_{10}^i + \bar{W}_{10} + i\bar{\mathbf{W}}_1^i + \bar{\mathbf{W}}_1$. $\bar{\mathbf{W}}_1^i = \Sigma\bar{W}_{1j}^i\mathbf{i}_j$, $\bar{\mathbf{W}}_1 = \Sigma\bar{W}_{1j}\mathbf{i}_j$. $\bar{W}_{10}^i$ is the compounding energy density, and $\bar{\mathbf{W}}_1^i$ is the compounding torque density. $\bar{\mathbb{W}}_e = i\bar{W}_{20}^i + \bar{W}_{20} + i\bar{\mathbf{W}}_2^i + \bar{\mathbf{W}}_2$. $\bar{\mathbf{W}}_2^i = \Sigma\bar{W}_{2j}^i\mathbf{I}_j$, $\bar{\mathbf{W}}_2 = \Sigma\bar{W}_{2j}\mathbf{I}_j$. $\bar{W}_{1k}$, $\bar{W}_{1k}^i$, $\bar{W}_{2k}$, and $\bar{W}_{2k}^i$ are all real.

The compounding energy includes the proper energy, kinetic energy, work, gravitational potential energy, electromagnetic potential energy, gravitational field energy, and electromagnetic field energy and so on. The compounding energy density can be expressed as,

$$\begin{aligned}\bar{W}_{10}^i &= \left(\bar{\mathbf{g}}\cdot\bar{\mathbf{L}}_1^i/v_0-\bar{\mathbf{b}}\cdot\bar{\mathbf{L}}_1\right)-v_0\left(\partial_0\bar{L}_{10}+\nabla\cdot\bar{\mathbf{L}}_1^i\right)+k_{eg}^2\left(\bar{\mathbf{E}}\cdot\bar{\mathbf{L}}_2^i/v_0-\bar{\mathbf{B}}\cdot\bar{\mathbf{L}}_2\right)\\ &\approx -\left\{v_0\bar{p}_0\left(\bar{v}_0/v_0\right)+\bar{v}_0\bar{p}_0\left(\nabla\cdot\bar{\mathbf{r}}\right)\right\}-\left\{v_0\left(\partial_0\bar{\mathbf{r}}\right)\cdot\bar{\mathbf{p}}+v_0\bar{\mathbf{r}}\cdot\partial_0\bar{\mathbf{p}}\right\}\\ &\quad-\left\{\left(\bar{p}_0\bar{a}_0+\bar{\mathbf{a}}\cdot\bar{\mathbf{p}}\right)+k_{eg}^2\left(\bar{\mathbf{A}}_0\circ\bar{\mathbf{P}}_0+\bar{\mathbf{A}}\cdot\bar{\mathbf{P}}\right)\right\}\\ &\quad+k_{eg}^2\left\{\left(\bar{\mathbf{E}}/v_0\right)\cdot\left(\bar{\mathbf{r}}\circ\bar{\mathbf{P}}_0\right)-\bar{\mathbf{B}}\cdot\left(\bar{\mathbf{r}}\times\bar{\mathbf{P}}\right)\right\}+\left\{\left(\bar{\mathbf{g}}/v_0\right)\cdot\left(\bar{p}_0\bar{\mathbf{r}}\right)-\bar{\mathbf{b}}\cdot\bar{\mathbf{L}}_1\right\},\end{aligned}\tag{15}$$

where the gauge conditions for the compounding field potential are chosen as, $\nabla\times\bar{\mathbf{x}}=0$, and $\nabla\times\bar{\mathbf{X}}=0$. The gravitational potential is, $\bar{\mathbb{A}}_g=i\bar{a}_0+\bar{\mathbf{a}}$, with $\bar{a}_0=\partial_0\bar{x}_0+\nabla\cdot\bar{\mathbf{x}}$, and $\bar{\mathbf{a}}=\partial_0\bar{\mathbf{x}}-\nabla\bar{x}_0$. The electromagnetic potential is, $\bar{\mathbb{A}}_e=i\bar{\mathbf{A}}_0+\bar{\mathbf{A}}$, with $\bar{\mathbf{A}}_0=\partial_0\bar{\mathbf{X}}_0+\nabla\cdot\bar{\mathbf{X}}$, and $\bar{\mathbf{A}}=\partial_0\bar{\mathbf{X}}-\nabla\circ\bar{\mathbf{X}}_0$. The gravitational strength is, $\bar{\mathbf{f}}=i\bar{\mathbf{g}}/v_0+\bar{\mathbf{b}}$, with $\bar{f}_0=0$. The gravitational acceleration is, $\bar{\mathbf{g}}/v_0=\partial_0\bar{\mathbf{a}}+\nabla\bar{a}_0$. The other component is, $\bar{\mathbf{b}}=\nabla\times\bar{\mathbf{a}}$. The electromagnetic strength is, $\bar{\mathbf{F}}=i\bar{\mathbf{E}}/v_0+\bar{\mathbf{B}}$, with $\bar{\mathbf{F}}_0=0$. The electric field intensity is, $\bar{\mathbf{E}}/v_0=\partial_0\bar{\mathbf{A}}+\nabla\circ\bar{\mathbf{A}}_0$, and the magnetic flux density is, $\bar{\mathbf{B}}=\nabla\times\bar{\mathbf{A}}$. $\bar{\mathbf{F}}=\Sigma(\bar{F}_j\mathbf{I}_j)$. $\bar{\mathbf{B}}=\Sigma(\bar{B}_j\mathbf{I}_j)$. $\bar{\mathbf{E}}=\Sigma(\bar{E}_j\mathbf{I}_j)$. $\bar{\mathbf{f}}=\Sigma(\bar{f}_j\mathbf{i}_j)$. $\bar{\mathbf{g}}=\Sigma(\bar{g}_j\mathbf{i}_j)$. $\bar{\mathbf{b}}=\Sigma(\bar{b}_j\mathbf{i}_j)$. $\bar{B}_j$, $\bar{E}_j$, $\bar{g}_j$, and $\bar{b}_j$ are all real. $\bar{F}_j$ and $\bar{f}_j$ are all complex.

In a similar way, the compounding torque density can be expressed as

$$\begin{aligned}\bar{\mathbf{W}}_1^i &= \left(\bar{\mathbf{g}}\times\bar{\mathbf{L}}_1^i/v_0-\bar{L}_{10}\bar{\mathbf{b}}-\bar{\mathbf{b}}\times\bar{\mathbf{L}}_1\right)-v_0\left(\partial_0\bar{\mathbf{L}}_1+\nabla\times\bar{\mathbf{L}}_1^i\right)+k_{eg}^2\left(\bar{\mathbf{E}}\times\bar{\mathbf{L}}_2^i/v_0-\bar{\mathbf{B}}\circ\bar{\mathbf{L}}_{20}-\bar{\mathbf{B}}\times\bar{\mathbf{L}}_2\right)\\ &\approx \bar{p}_0\bar{\mathbf{g}}\times\bar{\mathbf{r}}/v_0+k_{eg}^2\left\{\bar{\mathbf{E}}\times\left(\bar{\mathbf{r}}\circ\bar{\mathbf{P}}_0\right)/v_0-\bar{\mathbf{B}}\times\left(\bar{\mathbf{r}}\times\bar{\mathbf{P}}\right)\right\}-v_0\bar{\mathbf{r}}\times\partial_0\bar{\mathbf{p}}-\left(\bar{\mathbf{r}}\cdot\bar{\mathbf{p}}\right)\bar{\mathbf{b}}-\bar{\mathbf{b}}\times\left(\bar{\mathbf{r}}\times\bar{\mathbf{p}}\right)+\bar{\mathbf{a}}\times\bar{\mathbf{p}},\end{aligned}\tag{16}$$

where the compounding torque consists of a few terms, including the torque terms caused by the gravity, inertial force, Lorentz force, and other force terms and so forth.

6. OCTONION FORCE

In the complex-octonion compounding space for the gravitational and electromagnetic fields, the compounding force is able to encompass various force terms, including the inertial force, the gravity, the electromagnetic force, the gradient of energy, and the interacting force between the dipole moment with the magnetic strength and so on.

In the complex-octonion compounding space, the octonion force, $\bar{\mathbb{N}}=\bar{\mathbb{N}}_g+k_{eg}\bar{\mathbb{N}}_e$, is defined from $\bar{\mathbb{W}}$ and $\bar{\mathbb{F}}$,

$$\bar{\mathbb{N}}=-(i\bar{\mathbb{F}}/v_0+\Box)\circ\bar{\mathbb{W}},\tag{17}$$

where $\bar{\mathbb{N}}_g=i\bar{N}_{10}^i+\bar{N}_{10}+i\bar{\mathbf{N}}_1^i+\bar{\mathbf{N}}_1$. $\bar{\mathbf{N}}_1^i=\Sigma\bar{N}_{1j}^i\mathbf{i}_j$, $\bar{\mathbf{N}}_1=\Sigma\bar{N}_{1j}\mathbf{i}_j$. The scalar part $\bar{N}_{10}$ is the compounding power, and the vectorial part $\bar{\mathbf{N}}_1^i$ is the compounding force. $\bar{\mathbb{N}}_e=i\bar{N}_{20}^i+\bar{N}_{20}+i\bar{\mathbf{N}}_2^i+\bar{\mathbf{N}}_2$. $\bar{\mathbf{N}}_2^i=\Sigma\bar{N}_{2j}^i\mathbf{I}_j$, $\bar{\mathbf{N}}_2=\Sigma\bar{N}_{2j}\mathbf{I}_j$. $\bar{N}_{1k}$, $\bar{N}_{1k}^i$, $\bar{N}_{2k}$, and $\bar{N}_{2k}^i$ are all real.

In case the compounding field strength is relatively weak, the compounding power density

$$\bar{N}_{10}=\left(\partial_0\bar{W}_{10}^i-\nabla\cdot\bar{\mathbf{W}}_1\right)+\left(\bar{\mathbf{g}}\cdot\bar{\mathbf{W}}_1/v_0+\bar{\mathbf{b}}\cdot\bar{\mathbf{W}}_1^i\right)/v_0+k_{eg}^2\left(\bar{\mathbf{E}}\cdot\bar{\mathbf{W}}_2/v_0+\bar{\mathbf{B}}\cdot\bar{\mathbf{W}}_2^i\right)/v_0,\tag{18}$$

can be written approximately as,

$$\begin{aligned}\bar{N}_{10}/k_p &\approx \partial_0\left(\bar{p}_0\bar{v}_0\right)-\nabla\cdot\left(\bar{v}_0\bar{\mathbf{p}}\right)+L_{10}\left(\bar{\mathbf{b}}\cdot\bar{\mathbf{b}}-\bar{\mathbf{g}}\cdot\bar{\mathbf{g}}/v_0^2\right)/\left(v_0k_p\right)\\ &\quad+k_{eg}^2\bar{L}_{10}\left(\bar{\mathbf{B}}\cdot\bar{\mathbf{B}}-\bar{\mathbf{E}}\cdot\bar{\mathbf{E}}/v_0^2\right)/\left(v_0k_p\right)+k_{eg}^2\bar{\mathbf{E}}\cdot\bar{\mathbf{P}}/v_0+\bar{\mathbf{g}}\cdot\bar{\mathbf{p}}/v_0,\end{aligned}\tag{19}$$

where $k_p=(k-1)$ is the coefficient, with k being the dimension of compounding vector $\bar{\mathbf{r}}$.

When the compounding field strength is comparative weak, the compounding force

$$\begin{aligned}\bar{\mathbf{N}}_1^i &= \left(\bar{W}_{10}^i\bar{\mathbf{g}}/v_0+\bar{\mathbf{g}}\times\bar{\mathbf{W}}_1^i/v_0-\bar{W}_{10}\bar{\mathbf{b}}-\bar{\mathbf{b}}\times\bar{\mathbf{W}}_1\right)/v_0-\left(\partial_0\bar{\mathbf{W}}_1+\nabla\bar{W}_{10}^i+\nabla\times\bar{\mathbf{W}}_1^i\right)\\ &\quad+k_{eg}^2\left(\bar{\mathbf{E}}\circ\bar{\mathbf{W}}_{20}^i/v_0+\bar{\mathbf{E}}\times\bar{\mathbf{W}}_2^i/v_0-\bar{\mathbf{B}}\circ\bar{\mathbf{W}}_{20}-\bar{\mathbf{B}}\times\bar{\mathbf{W}}_2\right)/v_0,\end{aligned}\tag{20}$$

can be written approximately as,

$$\begin{aligned}\bar{\mathbf{N}}_1^i/k_p \approx\ & -\partial_0(\bar{\mathbf{p}}\bar{v}_0) + \bar{p}_0\bar{\mathbf{g}}/v_0 + \bar{L}_{10}\left(\bar{\mathbf{g}}\times\bar{\mathbf{b}} + k_{eg}^2\bar{\mathbf{E}}\times\bar{\mathbf{B}}\right)/\left(v_0^2 k_p\right)\\ & -\bar{\mathbf{b}}\times\bar{\mathbf{p}} - \nabla\left(\bar{p}_0\bar{v}_0\right) + k_{eg}^2\left(\bar{\mathbf{E}}\circ\bar{\mathbf{P}}_0/v_0 - \bar{\mathbf{B}}\times\bar{\mathbf{P}}\right),\end{aligned} \tag{21}$$

where $(\bar{p}_0\bar{\mathbf{g}}/v_0)$ is the gravity. $\partial_0(-\bar{v}_0\bar{\mathbf{p}})$ is the inertial force. $\{k_{eg}^2(\bar{\mathbf{E}}\circ\bar{\mathbf{P}}_0/v_0 - \bar{\mathbf{B}}\times\bar{\mathbf{P}})\}$ covers the electromagnetic force. $\nabla(\bar{p}_0\bar{v}_0)$ is the energy gradient.

7. CONCLUSIONS AND DISCUSSIONS

According to the viewpoint of function space, the radius vector and velocity ratio can be combined together to become one 'radius vector' in the compounding space. And the compounding space is able to be considered as the extension of one compounding field, according to the relationship between the field with the space. In the compounding space, there are several kinds of equilibrium states, although the torque and force both may not be equal to zero. This means that the velocity ratio will impact the angular momentum, torque, and force and so forth. Therefore the physics feature of the rotational charged object should depart from the deduction deriving from the Newton's law of gravitation and the classical Coulomb's law.

In the complex-octonion space for electromagnetic and gravitational fields, when the torque $\mathbf{W}_1^i = 0$ and the force $\mathbf{N}_1^i = 0$, the neutral/charged object stays on one equilibrium state. Similarly in the complex-octonion compounding space, when $\bar{\mathbf{W}}_1^i = 0$ and $\bar{\mathbf{N}}_1^i = 0$, the rotational neutral/charged object stays on one kind of 'equilibrium state', although there may be $\mathbf{W}_1^i \neq 0$ and $\mathbf{N}_1^i \neq 0$. And it is correspondent with $\bar{\mathbf{f}} = 0$ (that is, the weightlessness state), while $\mathbf{f} \neq 0$. It may be able to be applied to figure out the stable over-speed movement of the rotational stars on the fringe of a galaxy. Also it may be used to explain the stable over-speed movement of the rotational charged objects.

In some special situations, the above equations in the complex-octonion compounding space will be reduced to the simply cases in the complex-octonion space. When $\mathbb{T}$, $\mathbb{Y}$, and $\mathbb{Z}$ are all approximate to zero, $\bar{\mathbb{A}}$, $\bar{\mathbb{F}}$, and $\bar{\mathbb{S}}$ will be degenerated into $\mathbb{A}$, $\mathbb{F}$, and $\mathbb{S}$ respectively, further the physics quantities $\bar{\mathbb{L}}$, $\bar{\mathbb{W}}$, and $\bar{\mathbb{N}}$ will be degenerated into $\mathbb{L}$, $\mathbb{W}$, and $\mathbb{N}$ respectively.

It should be noted that the study for the influence of angular velocity on the electromagnetic force of rotational charged objects examined only some simple cases. Despite its preliminary characteristics, this study can clearly indicate that the angular velocity should impact the equilibrium state and electromagnetic force and so forth. For the future studies, the research will focus on the investigation about the equilibrium state of rotational charged objects.

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