Efficient Analysis of EM Scattering from Rotating Structures Using a Fast Iterative Physical Optics Method

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Abstract— Analysis of the electromagnetic fields scattering from rotating structures such as aerogenerators, aircraft propellers, turbojet and turbofan, helicopter rotors, etc. has an increased interest due to the influence in civil and military radars efficiency and in radiation characteristics of on-board antennas. In this paper an Iterative Physical Optics method accelerated by a fast Far Field Approximation is presented. It has been developed in such a way that the computation time is reduced by means of a multilevel Far Field Approximation, a preprocessing of the parameters that are invariant with the rotations, a fast calculation of the rotation transformations of the parameters affected by the movement and a parallelization schema based on Fortran Coarrays.

1. INTRODUCTION

The interaction of EM fields with rotating structures such as wind turbines, aircraft propellers, turbojet and turbofan,helicopter rotors, etc. can produce Doppler effects [1] and spatial modulations [2] degrading the performances of radars and antennas. To predict accurately the scattering is necessary to compute the interaction of EM fields with the rotating and fixed parts and the interaction of scattered fields from one part with the others for each relative position between them taking into account also the different electromagnetic material properties. In the case of complex rotating structures such as turbofan or turbojet with hundreds of blades rotating at different speeds and directions, it is necessary to analyze efficiently thousands of relative positions in order to obtain useful results. Previous works use Method of Moments [2], Physical Optics [3] or Finite Elements [4] to perform this analysis. This paper presents an Iterative Physical Optics (IPO) method [5, 6] accelerated by a fast Far Field Approximation (FFA) [7] in order to analyze accurately and efficiently this problem. It has been developed in such a way that the computation time is reduced by means of a multilevel FFA (MLFFA), a preprocessing of the parameters that are invariant with the rotations, a fast calculation of the rotation transformations of the parameters affected by the movement [8] and a parallelization schema based on FORTRAN co-arrays [9]. Cross comparison with Multilevel Fast Multipole Method (MLFMM) results of several test cases shows an excellent agreement at a much lower computational cost. Complex cases such as time-variant radar cross section of a large aircraft with four rotating propellers have also been computed in affordable time.

2. IPO AND MLFFA

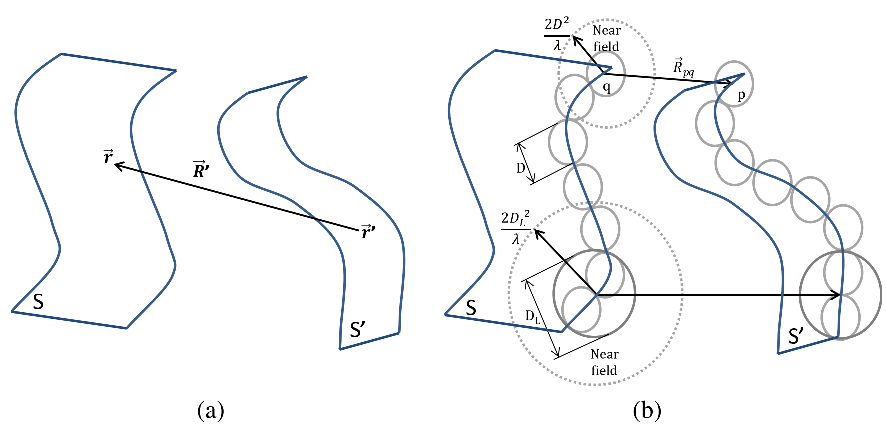


Figure 1: Geometry of the problem: (a) The integral equation is discretized using point sampling, the surfaces are assumed to be locally flat at the sample points and two orthogonal current directions are defined at each point. (b) FFA and MLFFA grouping.

Using the approximate Magnetic Field Integral Equation (MFIE), the electric current at point $\vec{r}$ on a target surface S due to an incident magnetic field $\vec{H}^i$ in presence of other surface S' (Fig. 1(a)) can be calculated by

$$\vec{J}(\vec{r}) = 2\hat{n} \times \vec{H}^i(\vec{r}) + 2\hat{n} \times \int_{S'} \vec{J}(\vec{r}\,') \times \vec{R}' \frac{e^{-jkR'}}{4\pi R'}(jk + \frac{1}{R'})dS' \quad (1)$$

being $\hat{n}$ the unitary normal vector at point $\vec{r}$, $\vec{r}\,'$ a point on surface S', $\vec{R}' = \vec{r} - \vec{r}\,'$ and $R' = |\vec{R}'|$.

The next iterative formula can be used to find J for the n iteration

$$\vec{J}^{(n)}(\vec{r}) = 2\hat{n} \times \vec{H}^i(\vec{r}) + 2\hat{n} \times \int_{S'} \vec{J}^{(n-1)}(\vec{r}\,') \times \vec{R}' \frac{e^{-jkR'}}{4\pi R'}(jk + \frac{1}{R'})dS' \quad (2)$$

with an initial guess of

$$\vec{J}^{(0)}(\vec{r}) = 2\hat{n} \times \vec{H}^i(\vec{r}) \quad (3)$$

Each iteration step adds another reflection between surfaces. The iteration halts after a pre-specified number of steps based on the number of expected reflections or when the relative change in the currents becomes small enough to neglect subsequent iterations. An iterative schema is presented in [5] in order to converge to a stable solution by defining a residual error for each iteration and minimizing this residual error using an iterative algorithm. The complexity of such schema is driven by the computation of operator $\bar{Z}$ with a cost of $O(N^2)$.

$$\bar{Z}\vec{J}^{(n)}(\vec{r}) = \vec{J}^{(n)}(\vec{r}) - 2\hat{n} \times \int_{S'} \vec{J}^{(n-1)}(\vec{r}\,') \times \vec{R}' \frac{e^{-jkR'}}{4\pi R'} \left(jk + \frac{1}{R'}\right) dS' \quad (4)$$

Far Field Approximation is used to reduce the computational cost. FFA is similar to Fast Multipole Method (FMM) in that the unknown current elements are spatially grouped and the radiation integral is evaluated one pair of groups at a time. For groups which are separated by less than the far field distance $\frac{2D^2}{\lambda}$, being D the size of the groups and λ the wavelength, the integral is computed using direct numerical integration. For groups separated by more than the far field distance (Fig. 1(b)), the FFA is used

$$\vec{H}_{pq}(\vec{r}) \approx \vec{H}_{pq}(\vec{r}_q) e^{-jk(\vec{r}-\vec{r}_q)\cdot\hat{R}_{pq}} \quad (5)$$

where $\vec{H}_{pq}$ is the magnetic field radiated from source group p to far field group q, $\vec{r}_p$ and $\vec{r}_q$ are the center of groups p and q respectively, $\vec{R}_{pq} = \vec{r}_q - \vec{r}_p$ and $\hat{R}_{pq} = \frac{\vec{R}_{pq}}{|\vec{R}_{pq}|}$. In this way, $\vec{H}_{pq}(\vec{r}_q)$ is calculated first by integrating over the elements of group p, them FFA is used to find $\vec{H}_{pq}(\vec{r})$ for all the sample points within group q. Using FFA the complexity of IPO is reduced to $O(N^{\frac{3}{2}})$. Considering groups with at least a sampling point inside, these groups can be spatially grouped again in higher level groups with a size $D_L > D$ (Fig. 1(b)), being able to apply the FFA between these more populated groups if they are separated at least by a far field distance $\frac{2D_L{}^2}{\lambda}$. This grouping process can be continued until a level with only one group, obtaining a multilevel FFA. Applying the FFA for each couple of groups at the adequate level, it is possible to reduce the complexity of the operator $\bar{Z}$ to $O(N \log(N))$. Several techniques can be used to perform the space partition, in particular it has being used an octree partition as in most of MLFMM implementations.

3. PRE-PROCESSING OF INVARIANT DATA

All those geometrical data that are invariant with the rotation transformation can be computed only once before the rotation loop. These data include the values of vector $\vec{R}'$ and its module in (1), (2) and (4) when $\vec{r}$ and $\vec{r}\,'$ are both in the fixed part or in the same rotating part of the target and $\hat{R}_{pq}$ in (5) when group p and q are also in those parts. Other parameters related with PO, as the curvature of surfaces to calculate its reflection coefficients or the relationship between surfaces of the same fixed or rotating part to determine the shadow zones, are also pre-computed previously to the rotation loop.

4. FAST COMPUTATION OF ROTATIONS

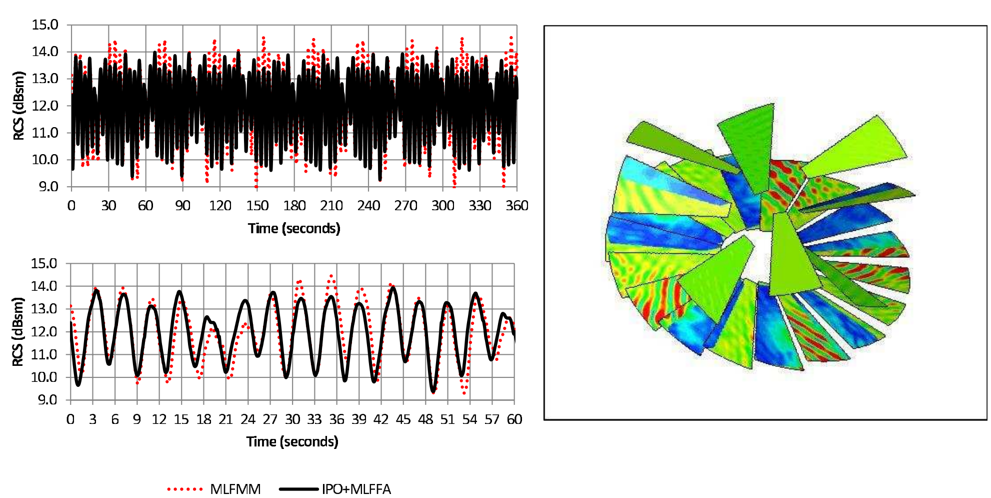
Many other parameters (sampling points, normals, MLFFA groups geometry, ...) are affected by rotations and must be computed efficiently. A rotation around an arbitrary axis can be defined by an origin point (a, b, c) plus a unitary direction vector $\langle u, v, w\rangle$ to build the axis and an angular speed to determine the angle θ rotated at each time step. The rotation can be decomposed in the following transformations of the affected structure: T_P to move the axis origin point to $(0, 0, 0)$, T_{xz} to move the axis to xz plane, T_z to move the axis to z axis, $R_z(\theta)$ to rotate around the z axis, T_z^{-1}, T_{xz}^{-1} and T_P^{-1}. The total transformation matrix $T_{tot} = T_P^{-1}T_{xz}^{-1}T_z^{-1}R_z(\theta)T_zT_{xz}T_P$ is presented in [8], multiplying it by vector $\langle x, y, z, 1\rangle$ yields the result of rotating the point (x, y, z). When the time steps are uniforms, the angle θ is a constant and the matrix T_{tot} is computed only once. For each time step, the affected parameters are rotated using T_{tot} by means of a very efficient vector-matrix product routine.

5. PARALLELIZATION USING COARRAYS

Fortran programming language includes since its 2008 standard version a new parallelization schema named Coarray Fortran. It uses the model of single-program multiple-data (SPMD) where a single program runs on multiple machines with different local data that is occasionally shared across machines. Each execution context is called an "image". In Coarray Fortran, variables can have the CODIMENSION attribute, which is similar to an array dimension. This means that such a variable not only exists on all the images, but also that the content of this variable on any given image can be accessed by all other images. This feature can be used to parallelize at a high level the IPO, MLFFA and rotation algorithms by distributing the different surfaces of the structure between images, so that each one computes all data relative to their own surfaces and accessing the data of the rest of surfaces to calculate the interaction between them. As point sampling is a function of the area of surfaces, work and memory load is balanced distributing surfaces among images in a way that total area is similar for all of them.

6. RESULTS

IPO, MLFFA and fast computation of rotations have been implemented in an existing PO solver parallelized using coarrays. A first case was analyzed consisting in the monostatic radar cross section (RCS) computation of a time-varying target comprised of several coaxial wheels of planar blades rotating around Z axis. The test-case definition was originally submitted by MBDA France as part of the 2014 ISAE EM Workshop [10]. Fig. 2 shows the results of time varying RCS computed at 10 GHz illuminating the structure with a plane wave from $Z = +\infty$ polarized with the electric field along X axis. Lower wheel is fixed, middle wheel is rotating clockwise with an angular speed of 1 degree/second and upper wheel is rotating counterclockwise at 2 degree/second. Time step is 0.1 seconds with a total time of 360 seconds and 3601 different relative positions. For this test-case with most of the surfaces rotating, the computational cost of IPO+MLFFA is three times lower than MLFFM one and the difference between both methods is lower than 1 dBsm. For cases where the amount of rotating surfaces is much smaller than the fixed ones the pre-processing of invariant



(a) (b)

Figure 2: Rotating blades test-case: (a) Comparison of IPO+MLFFA and MLFMM results. (b) Current distribution at time zero.

parts made IPO+MLFFA even more efficient.

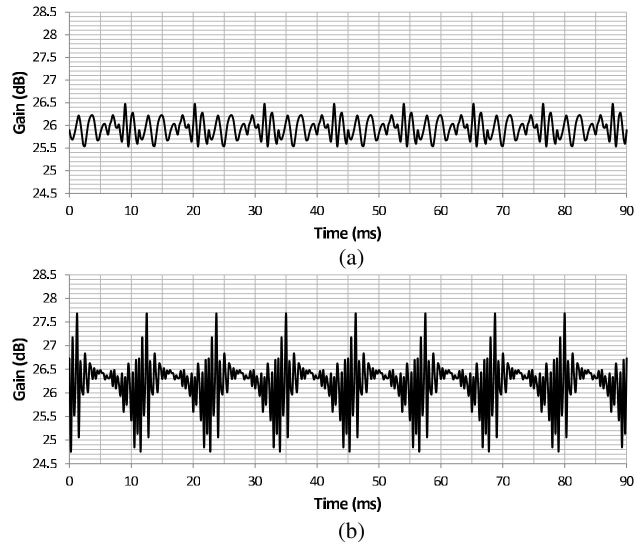


Figure 3: Time variation of GPS antenna gain: (a) Direction with minimum effect. (b) Direction with maximum effect.

Rotating structures were demonstrated to introduce a periodic modulation onto the Global Positioning System (GPS) satellite signals [11]. Fig. 3 shows the time variation of the gain of a GPS antenna on-board a large transport turboprop aircraft due to the rotation of their four propellers. The antenna is located in the upper fuselage between the propellers, the frequency is 1575 MHz corresponding to an aircraft electrical length of 225 wavelengths. The propellers are rotating at 655 revolutions per minute, the considered time step is 0.25 milliseconds with a total time of 90 milliseconds and 360 different relative positions of the propellers. A gain variation of about 1 dB is observed in the direction where the effect of the propellers rotation is minimum (Fig. 3(a)) while in the direction where the effect is maximum, the gain variation is about 3 dB (Fig. 3(b)).

7. CONCLUSIONS

An efficient method to analyze the EM scattering from rotating structures based on a fast iterative PO is presented. The traditional iterative PO schema is accelerated by mean of a multilevel far field approximation reducing the complexity of the iteration computation. Pre-processing of data not varying with the rotations and the use of a matrix algorithm to compute efficiently the rotations reduce even more the computational cost. Finally a parallel schema based on Coarray Fortran has been used to implement the code in an easy way with a very good scalability. The solver including all these features has been tested through the computation of simple and complex cases comparing with MLFMM results obtaining an excellent agreement with a calculation cost several times lower.

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