Solitary Wave Induced in a Sinusoidal Water Surface Wave Field of Hydrodynamics

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Abstract— Monitoring for nonlinear processes of water surface waves is introduced in brief. It was found a kind of nonlinear water surface waves which had a single crest wave with an infinitely long wave in a water tank. This wave was named as solitary wave on water surface in an experiment first. Then, a solitary wave had been led in a mathematical formulation. Another nonlinear wave known as a cnoidal wave was found in a form of cn function mathematically. An extensive experiment in an water tank has led to see a process in which an input function of linear sinusoidal wave induced a nonlinear wave quite similar to the solitary wave.

1. INTRODUCTION

This work is concerning to a problem on monitoring of solitary wave on water surface. For this purpose, a historical review is introduced about an experiment of a single crested wave in brief. It was found a kind of nonlinear water surface waves which had a single crest wave with an infinitely long wave in a water tank. This wave was named as solitary wave on the water surface in an experiment first. Then, a solitary wave had been led in a mathematical formulating procedure. Another nonolinear wave known as a cnoidal wave was found in a form of cn function mathematically. An extensive experiment had been led to see a process in which an input function of linear sinusoidal wave induced a nonolinear wave quite similar to the solitary wave.

2. LINEAR MODEL OF WATER SURFACE WAVE

At starting to introduce some specific features of nonlinear water surface wave, a brief note is given some note to water waves for a convenience in an hydrodynamic understanding of the present interesting nonlinear waves.

First, assume that water is isotropic for a linear water surface wave model.

This assumption has been introduced even in the publication by Lamb in 1879 [1].

What is essential is the problems on a sinusoidal water surface wave under assuming of a small amplitude as a linear problem of water waves in hydrodynamics.

3. NONLINEAR PROCESS OF SOLITARY WAVE

On the other hand, there are various kinds of aperiodic waves.

For example, Lamb in 1879 had introduced about a solitary wave which was called by Scott Russel as seen in "Report on Waves" appeared in British Association Report in 1844 [1]. Following what noted Lamb had noted in 1879, the author could Introduce the work undertaken by Scott Russel as follow.

Scott Russell, in his interesting experimental investigations, was led to pay great attention to a particular type which he called the 'so1itary wave'. This is a wave consisting of a single elevation, of height not necessarily small compared with the depth of the fluid, which, if properly started, may travel for a considerable distance along a uniform canal, with little or no change of type. Waves of depression, of similar relative amplitude, were found not to possess the same character of performance, but to break up into series of shorter waves.

What noted above was given independently by Bousinesquq [2] and Rayleigh (as noted in his publication [3]).

The above note can specify the solitary wave. Then, some note in relation to one of the water waves reduced by a mathematical procedure as what is noted by Lamb in 1879 can be seen as follow.

Russel's solitary type which may be regarded as an extreme case of Russel's solitary type may be regarded as an extreme case of Stokes' oscillatry waves of permanent type, the wave length being great compared with the depth of the canal, so that the widely separated elevations are particularly independent of one another. Now, the methods of approximation employed by Stokes become, however, unsuitable when the wave-length much exceeds the depth; and subsequent investigations of solitary waves of permanent type have proceeded on different lines.

3.1. What Lamb Noted

Lamb notes that Rayleigh, treating the problem as one of steady motion, starts virtually the formula

$$\phi + i\psi = F(x + iy) = \exp[iy(d/dx)]F(x), \tag{1}$$

where $F(x)$ is real. The notations ϕ and ψ are for stream function and potential velocity.

Lamb's solution of the above differential equation can be written as following, i.e.,

$$\eta = a\,\mathrm{sech}^2[(1/2)(x/b)], \tag{2}$$

where,

$$y - h = \eta, \text{ and, } b^2 = \left[h^2(h + a)/(3a)\right], \tag{3}$$

if the origin of x be taken beneath the summit. The approximations consist in neglecting the fourth power of the ratio $(h + a)/(2b)$. The theory of the solitary waves has been treated by Weinstein in 1926 [1]. Then, it can be seen that

$$c^2 = g(h + a), \tag{4}$$

in the field of the earth's gravity field.

The motion at the outskirts of the solitary wave can be represented by a very simple formula. Lamb in 1987 had communicated from Stokes that McCowan investigated to reduce,

$$c^2 = (g/m)\tan(mh), \tag{5}$$

$$\text{where, } m\alpha = (2/3)\sin^2[m(h + (2/3)a)], \text{ and, } a = \alpha\tan[(1/2)m(h + a)], \tag{6}$$

and, the notations a and α are the maximum elevation above the mean level and a subsidiary constant.

Then, Lamb notes what are specific in the solitary wave. That is to say, the extreme form of the wave when the crest has sharp angle of 120 degree was examined. Adding to that, the limiting value of the ratio a/h was found to be 0.78, in which case the wave-velocity is given by $c^2 = 1.56gh$.

Lamb states as that by a slight modification the investigation of Rayleigh and Boussinesq can be made to give the theory of a system of oscillatory waves of finite height in a canal of limited depth [4].

3.2. McCowan and Solitary Wave

McCowan [5] must be a reference in the early age of solitary wave's description.

Assuming a reference water depth h, then, a solitary wave η can be described in a mathematical form, i.e., as following to Lamb,

$$\eta = a\,\mathrm{sech}^2\left[(1/2)\left(3a/\left(h^3\right)\right)^{1/2}x\right], \tag{7}$$

where, the notation x is the horizontal axis and, the notation a is found in trocoidal wave,

$$x = x_0 a\ \exp(kz_0))\sin(kx_0), \text{ and, } z = z_0 + a\ \exp(kz_0)\cos(kx_0), \tag{8}$$

which is called as Gerstner wave in a consideration of historical background of its finding, as Gerstner reduced theoretically in 1802 though Rankin independently found it in 1863.

As for the small amplitude wave, formulation can be in a linear expression though this solitary wave has to be solved on the basis of nonlinear equation referring Lagrangean technique ia consideration.

4. RESONANT COUPLING OF SOLITARY WAVE

Nakamura [6] has found a specific condition can produce a solitary wave in a simple water basin experimentally though a cnoidal wave with cn function was in his interest.

In Nakamura's work [6], he expected a uni-nodal lateral oscillation can be seen first.

The two solitary waves translated to meet at the center of the barrier to increase the maximum height of the water level was about twice of those at the two side comers.

At the center of the barrier, a couple of the solitary wave meets to make a collision each other. After that, a couple of the solitary wave continue the cross motion as if the two waves have never distorted at the collision.

The collision cycle was to coincize to the cycle of to the incident sinusoidal wave as an input function of energy.

So that, a center slit of the barrier must be acted as an energy supply for the two solitary waves to maintain their cross motion between the two side walls during the input function of the sinusoidal wave is repeatedly generated.

When the input function acts only for one cycle of the sinusoidal wave, the couple of the solitary wave becomes to be faint and repeat reflection on the wall of both side up to decay out and disappear the couple of the wave.

This process was seen at the frequency of the input function is satisfying a resonant mode under a certain combination when the boundary condition, bathymetric condition, and energy supply at the center of the barrier are satisfied the resonant mode of the couple of solitary wave in the water basin.

An illustration for various waves found on the water surface is shown by a dot as in Figure 1 [6]. Then, the couple of the solitary wave observed in the water basin fit to the McCowan limit extending an asymptotic line to the case of solitary wave of the water wave theory (a dot in Figure 1).

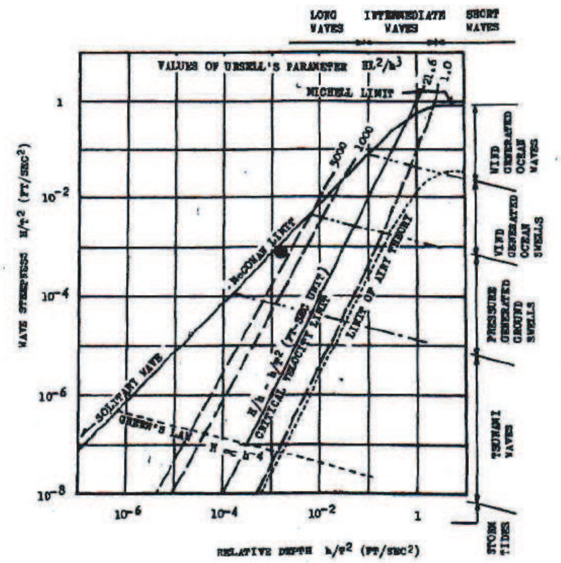

Figure 1: Characteristics of the couple of solitary waves. (1) A dot is for the experimental result. (2) McCowan's limit of breaking wave is demonstrated. (3) McCowan's limit extonding to solitary ware in noticed. [courtesy of ASCE].

This result supports that the cross motion of the couple of solitary wave along the bathymetric line of the specific constant waterdepth crossing to the direction of the input function of sinusoidal wave.

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