# A Fast Gaussian Beam Tracing Method for Quasi-optical System Analysis Based on Gabor Frame Expansion

**Hai Wang**[1], **Zejian Lu**[1], **Fangyuan Cheng**[1], **Junsheng Yu**[1], **Xiaodong Chen**[2], **Xiaoming Liu**[1], **and Yuan Yao**[1]

[1]School of Electronic Engineering
Beijing University of Posts and Telecommunications, Beijing 100876, China
[2]School of Electronic Engineering and Computer Science
Queen Marry University of London, E1 4NS, UK

**Abstract**— Quasi-optical systems and techniques have been the topic of research in the past decades. The field of feed is expanded into a set of sub-GBs based on Gabor frame expansion method and field tracing is through astigmatic Gaussian beam (GB) techniques. Single astigmatic GB reflection under arbitrary curved surface is successfully analyzed. Near-field scanning result of quasi-optical systems having two parabolic mirrors matching up to $-50$ dB below peak value has been verified. Numerical and experimental comparison reveal a good trade-off between the accuracy and the computational effort.

## 1. INTRODUCTION

With the increase of frequency, GB analysis method is more accurate and practical especially in analyzing unsymmetrical and large electrically reflector antenna, with respect to Physical Optics (PO) [1] or Geometrical Optics (GO). Uniform geometrical theory of diffraction (UTD) is suitable to study curved edges illuminated by electromagnetic. In engineering applications, there is an urgent hope that quickly analyzing and obtaining emergent field of reflection mirror of millimeter wave on the premise of guaranteeing certain accuracy requirement.

The conventional modeling approaches in multi-reflector quasi-optical system analysis [2] are constituted by two aspects: expansion techniques and tracking techniques [3–5]. In that theory, electromagnetic field is first expanded into Gaussian beams (GBs), with different direction and energy, then they are traced into a reflector where reflection or diffraction occur and emergent beam field can be obtained via superimposing all GBs [6]. However, there is no doubt that GBs are often considered as paraxial transmission wave equation, which can be regarded as focused pure GB, and the energy of electromagnetic emergent from the feed is mainly up to the number of GBs in the center of optical axis, which also means that around the mirror the number and energy of GBs are very small.

Based on this conclusion, a fast quasi-optical analysis method is proposed on the assumption that the diffraction on the reflector edge is rare thus can be neglected if the reflector size is four times larger than the beam width on the surface. In this method, the field pattern of feed is expanded into sets of sub-GBs using Gabor frame expansion method [7–9], and near area of specular reflection point is approximated as parabolic model. Furthermore, introducing the astigmatic GB [10] as elementary beam wave equation to find out every emergent beam field so multi-reflection mirrors system can be continuous analyzed.

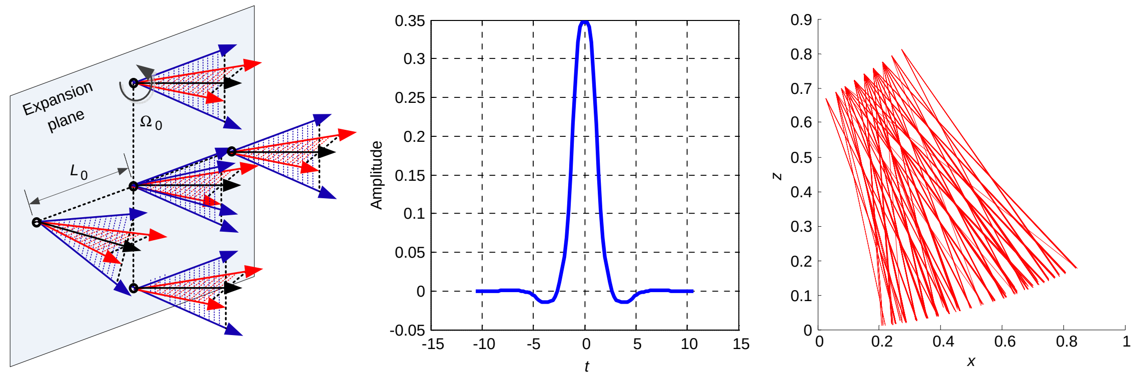
Feed is expanded into a series of sub-GBs based on Gabor frame expansion technique and the characters of astigmatic GB are viewed in Section 2. A quasi-optical system having two parabolic mirrors is discussed in Section 3, and in Section 3, we summarize all of the work.

## 2. METHOD

### 2.1. Gabor Frame Expansion Technique

In digital signal processing, according to the sampling theorem, the sampling values can restore the original signal. The field that incident upon the input plane in quasi-optical system may not possess an exact GB profile, which mean that decomposing into a sum of GBs. Gabor frame expansion is based on the fact that any form of field distribution can be expanded into a set of GBs, which have own direction and coefficients that are frequency-dependent. Field incident from feed are expanded into sub-GBs are showed in Fig. 1(a) and the expansion formula is as follow

$$f(t) = \sum_{m,n} A_{mn}\omega_{mn}(t) = \sum_{m,n} A_{mn}\omega(t - mL_0)\exp(jn\Omega_0 t) \tag{1}$$




(a) (b) (c)

Figure 1: (a) Incident field is decomposed into a set of sub-GBs in the expansion plane. (b) Gabor frame window function. (c) The actual feed is expanded into sub-GBs projection in the $xoz$ plane.

where $A_{mn}$ is to be solved out coefficient of the GBs, $\omega(t) = \pi^{-1/4}\exp(-t^2/2)$ is the Gabor frame window function, which usually is the function of Gaussian shape and showed in Fig. 1(b). $L_0$ and $\Omega_0$ is space spectrum and angle spectrum sampling interval respectively. The coefficient of formula is

$$A_{mn\mu v} = \iint f(x,y)\exp\ [-jK_0(nx+vy)]\tilde{\omega}(x-mL_0)\tilde{\omega}(y-\mu L_0)dxdy \tag{2}$$

where $|m|$, $|\mu| \leq M/2$, $|n|$, $|\nu| \leq N/2$, $M$ and $N$ is the number of space spectrum and angle spectrum sampling respectively.

### 2.2. Astigmatic GB Tracking Technique

Feed is expanded into a set of sub-GBs. It is necessary to find out the characters of a single astigmatic GB reflection under arbitrary curved surface. An astigmatic GB equation is as follows:

$$\vec{H}_i(x_i, y_i, z_i) = \left(\vec{x}_i H_1^i + \vec{y}_i H_2^i\right)\sqrt{\frac{\det(Q^i(z_i))}{\det(Q^i(0))}}\exp\left(-jkS^i\right) \tag{3}$$

where $S^i = z_i + \frac{1}{2}[x_i \quad y_i]\, Q^i(z_i)\,[x_i \quad y_i]'$, $Q^i(z_i) = \begin{bmatrix} \frac{1}{z_i+\rho_1^i} & 0 \\ 0 & \frac{1}{z_i+\rho_2^i} \end{bmatrix}$, $\rho_1^i$ and $\rho_2^i$ have relationship with distance from the position of beam waist in $x_i$ and $y_i$ directions respectively to reflection point of mirror and the waist size of the input beam. $Q$ matrix is wavefront curvature matrix and represents the actual propagation process of light path by function $[Q_i(z_i)]^{-1} = [Q_i(0)]^{-1} + z_i \cdot I$. Emergent field of astigmatic GB equation has

$$\vec{H}_r(x_r, y_r, z_r) = \left(\vec{x}_r H_1^r + \vec{y}_r H_2^r\right)\sqrt{\frac{\det(Q^r(z_r))}{\det(Q^r(0))}}\exp\left(-jkS^r\right) \tag{4}$$

where

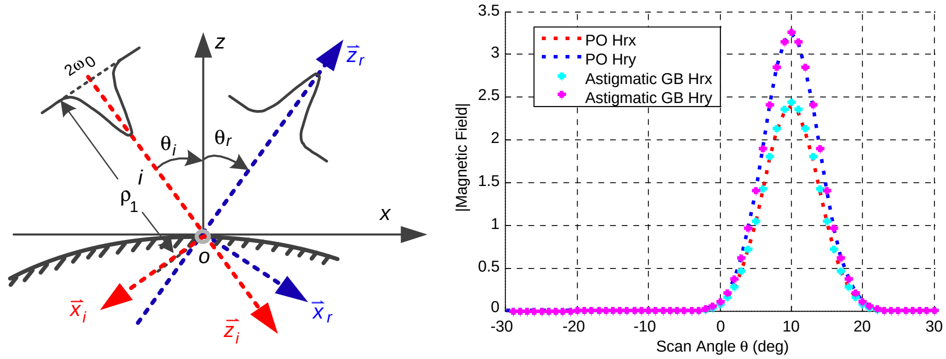
$$Q^r(z_r) = \frac{1}{z_r+\rho_1^r}\frac{1}{z_r+\rho_2^r}\begin{bmatrix} \rho_1^r\rho_2^r Q_{11}^r + z_r & \rho_1^r\rho_2^r Q_{12}^r \\ \rho_1^r\rho_2^r Q_{21}^r & \rho_1^r\rho_2^r Q_{22}^r + z_r \end{bmatrix} \tag{5}$$

Tracking techniques need to solve out elementary GB intersection location in mirror on the basis of expansion start location and propagation direction. On the specular reflection points, three coordinate system, which are input, principal axes and output coordinate system as showed in Fig. 2(a), are established so that to obtain GB reflection field formula. A single GB reflection model of obliquely incidence and its reflection result comparing with PO having same surface are showed in Fig. 2(b).

## 3. DOUBLE REFLECTORS QUASI-OPTICAL SYSTEM

### 3.1. System Design

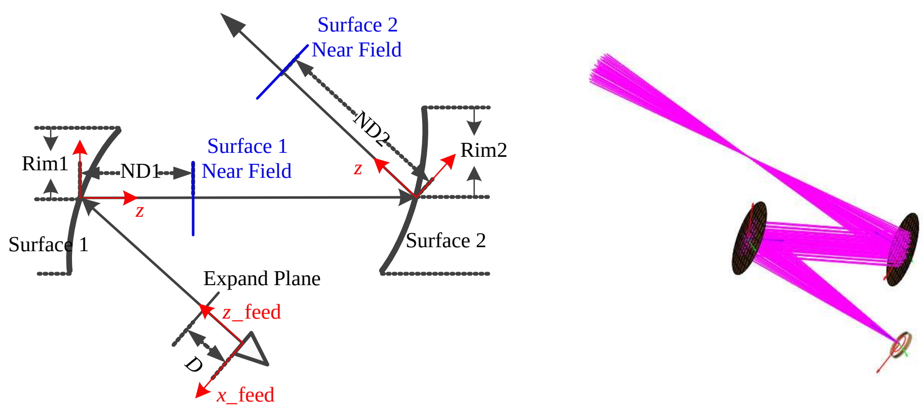
We set up a set of quasi-optical reflector system made up of two parabolic surfaces, working at the frequency of 30 GHz. The geometry model of the double reflectors system is showed in Fig. 3.




(a) (b)

Figure 2: (a) The model of GB reflection from a curved surface, which will decide $Q$ matrix. (b) A single astigmatic GB obliquely incident angle equal to 10 degree on the reflector point, and reflected field is scanned comparing between PO and astigmatic GB method.

The feed is placed on the focus of the first reflection mirror. The size of two parabolic mirrors is equal to 2 times of surface parameters $Rim_1$ and $Rim_2$ respectively. The field from feed is defined through the near-field pattern and is expanded at a plane with the distance D from phase center of feed. Two origins of output-field observation coordinate system are placed at specular reflection point where center beam intersects with the mirror respectively. In this coordinate system, the positive $z$-direction is created along the propagation direction of the reflected beam and the near-field scanning distance of two surfaces is $ND_1$ and $ND_2$ respectively.




(a) (b)

Figure 3: (a) Geometry of the double quasi-optical reflectors system. (b) 3D view of geometric model of double quasi-optical reflectors system is set up in the GRASP simulation software.

### 3.2. Simulation Validation

The Gaussian feed is radiating from the focus of first parabolic mirror, which is same of two being quite equal to 1 meters, working at 30 GHz along the feed positive $z$-axis. The expanded GB field consecutively goes through two parabolic mirrors. Location and mirror parameters are as follows:

A. The rotation angle of feed around the $y$ axis of global coordinate system is 150 degree and the distance from expand plane to phase center of feed is $D = 0.29$ m. Near-field pattern of feed being expanded into sets of sub-GBs using Gabor frame expansion method and its recovered field pattern after propagation distance 0.5 m are showed in Fig. 4(a). There are 3897 GBs in total counted in expanded plane.

B. The radius of reflector surface 1 is $Rim_1 = 4 * \omega = 0.338$ m and near-field scanning distance $ND_1 = 0.5$ m. This $\omega$ is waist size of center beam on the mirror reflection point of surface 1, and the initial GB waist size is $\omega_0 = 0.05$ m. The near-field scanning result of surface 1 is showed in Fig. 4(b). There are 1137 GBs counted on reflector 1.

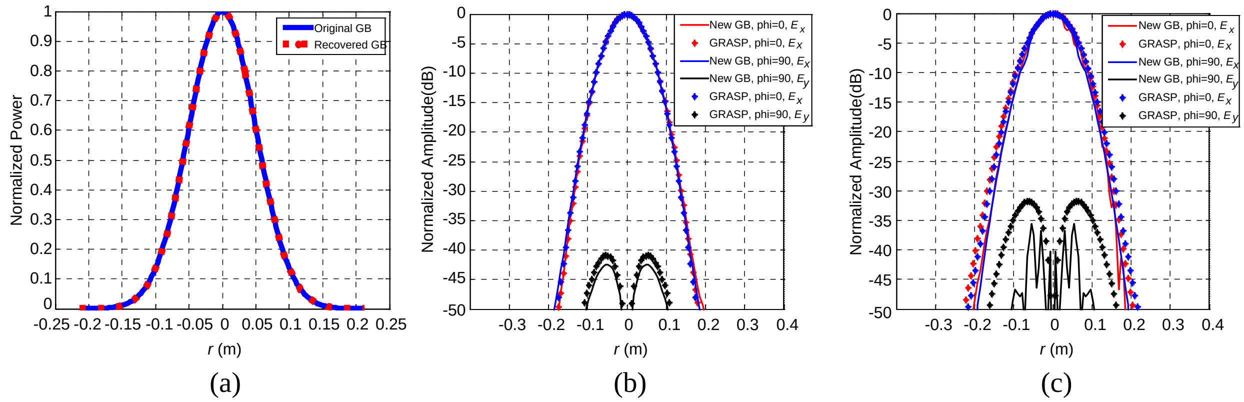



Figure 4: (a) Near-field pattern of feed (solid line) and its recovered field pattern (dotted line) after propagation distance 0.5 m. (b) Near-field scanning reflected after surface 1. (c) Near-field scanning reflected after surface 2.

C. The radius of reflector surface 2 is $Rim_2 = 0.338$ m and near-field scanning distance $ND_2 = 1.5$ m. The distance from center of surface 1 to surface 2 is 1.874 m. The near-field scanning result of surface 2 is showed in Fig. 4(c). There are 839 GBs counted on reflector 2.

## 4. CONCLUSION

A fast GB tracing method for quasi-optical system of two parabolic surfaces based on Gabor frame expansion technique at the frequency of 30 GHz has been presented.

Two reflection mirrors having different number of GBs respectively, meanwhile contrasting near-field scanning, reveal that GBs are paraxial transmission wave equation and energy of emergent is mainly up to the number of GBs in the center of optical axis. Gabor frame expansion coefficients are time-dependent, which result in calculating separately for each frequency. Astigmatic GB equation can characterize the transformation of the GB reflection, and obtaining reflection field formula only calculates $Q$ matrix according to the incident field.

Near-field scanning results of main polarization and cross polarization in phi = 0 degree and phi = 90 degree are also matched up to $-50$ dB below peak value, which demonstrate the correctness of the algorithm, including Gabor frame expansion and tracking by astigmatic GB techniques. The accuracy of the method is connected with the distance of expanded plane and the number of GBs. Selecting parameters reasonably can achieve good trade-off between the accuracy and the computational effort.

## ACKNOWLEDGMENT

This work was supported by the CAEP THz Science and Technology Foundation under the Grant No. CAEPTHZ201210.

## REFERENCES

1. Hillairet, J., J. Sokoloff, et al., "Analytical physical optics scattering from a PEC finite plate illuminated by a vector gaussian beam," *Electromagnetics in Advanced Applications*, 283–286, Sep. 2007.
2. Goldsmith, P. F., "Quasi-optical techniques," *IEE Proc.* Vol. 80, No. 11, 1729–1747, Nov. 1992.
3. Hristos, T. A. and H. P. Prabhakar, "Closed form solution for three-dimensional reflection of an arbitrary Gaussian beam by a smooth surface," *Radio Science*, Vol. 37, No. 2, Apr. 2002.
4. Casaletti, M., S. Skokic, S. Maci, and S. Sørensen, "Beam expansion in multi-reflector quasi-optical systems," *Antennas and Propagation of Fourth European Conference*, 1–4, Apr. 2010.
5. Deschamps, G. A., "Ray techniques in electromagnetics," *IEE Proc.*, Vol. 60, No. 9, 1022–1035, Sep. 1972.
6. Chou, H.-T. and H. P. Prabhakar, "Uniform asymptotic solution for electromagnetic reflection and diffraction of an arbitrary Gaussian beam by a smooth surface," *Radio Science*, Vol. 32, No. 4, 1319–1336, Jul. 1997.
7. Lugara, D., C. Letrou, A. Shlivinski, E. Heyman, and A. Boag, "Frame-based Gaussian beam summation method: Theory and applications," *Radio Science*, Vol. 38, No. 2, Apr. 2003.

8. Bastiaans, M. J., "Gabor's expansion of a signal into Gaussian elementary signals," *IEE Proc.*, Vol. 68, No. 4, 538–539, Apr. 1980.
9. Feichtinger, H. G. and T. Strohmer, *Gabor Analysis and Algorithms: Theory and Applications,* Springer-Verlag, 1997.
10. Evgenia, K., W. Gudrun, S. Dennis, et al., "Modeling of the general astigmatic Gaussian beam and its propagation through 3D optical systems," *Applied Optics*, Vol. 52, No. 24, 6030–6040, 2013.