Parabolic Strip Telescope

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Abstract— We present a proposal of a new telescope type using a rotating parabolic strip as the primary mirror. It is a principal modification of the design of telescopes from the times of Galileo and Newton. In order to demonstrate the basic idea, the image of an artificial constellation observed by this kind of telescope was reconstructed using the techniques described in this article. We present a working model of this new telescope, we have used an assembly of the primary mirror — a strip of acrylic glass parabolic mirror 30 cm long and 10 cm wide shaped as a parabolic cylinder of a focal length of 1 m — and an artificial constellation, a set of LED diodes in a distance of 15 m. In order to reconstruct the image, we made a series of snaps, each taken after the constellation had been rotated by 5 degrees. Using three different algorithms, we reconstructed the image of this artificial constellation. This contribution is based on [1] with new telescope designs and new experimental tests.

1. INTRODUCTION

The current telescopic systems are based on two basic principles: the Galileo-Kepler system uses optical lenses as the primary optic element (refractors) and the Newton-Cassegrain system has primary refracting surfaces shaped like a circle or n-gon. Their angular resolution is therefore identical in all directions. Images are acquired by means of photographic cameras, astronomic cameras or light-emission spectrometers. Images from CCDs and spectrometers are stored in a computer for further processing. In all the cases, images are taken in the same plane. The primary optical elements contained in both the reflectors and the refractors concentrate parallel light beams to a focal point. The other optical elements make use of different geometric configurations. For more details go to [2].

As a rule, the telescope is mounted on a support frame allowing to exactly adjust the position of the telescope and to carry out fine rectification of its movement. For this purpose, the azimuthal, parallactic or four-axis assemblies are used. In the case of azimuthal assembly, one axis is horizontal and the other vertical. A telescope rotates along the vertical axis in relation to the cardinal points; the height above the horizon is set in the horizontal axis. When observing celestial bodies, the positioning of a telescope around both axes must be changed smoothly; it is better-known as so called Dobson platform. The parallactic assembly has one axis parallel with the earth's axis pointing to the pole. The other axis is perpendicular to the first axis. When one object is tracked, a telescope rotates around the polar axis in a steady speed of 15 arc degrees per hour. The four-axis assembly is intended especially for the tracking of man-made satellites orbiting the Earth. Three of the axes are adjusted so that a telescope could track an object only by rotating around one axis.

An important parameter of the telescope is its angular resolution. It is determined by the smallest angle that enables us to distinguish two points. The smaller the angle, the better the angular resolution. If the diameter of an objective is D, then its plan area P is given by the following relation

$$P = \pi \frac{D^2}{4}, \quad \delta \approx 1.22 \frac{\lambda}{D}$$

for the monochromatic light of a wavelength of λ [3].

The main reason for the construction of telescopes of large diameters is that astronomers require a better angular resolution. As it follows from what was specified above, the disadvantage of these known solutions is that to achieve a better angular resolution, especially a large size of the primary optical element is required. The weight of these massive objectives is heavy, they are demanding in terms of their construction, and the technology and poduction is expensive.

1.1. Teleskop with a Rotating Objective Element

The principle of our new system was inspired by the fact that the angular resolution is inversely proportional to the length of an antenna, and by the technology of X-ray computer tomography (CT). An X-ray source is located on one side and an X-ray camera on the opposite side of an x-rayed sample. The integral absorptions of X-rays in different angles are measured step by step during the rotation of the sample. The total absorption of all photons coming along different lines perpendicular to the camera are registered as points of a one-dimensional picture. Finally, the inverse Radon transform is used to reconstruct the image of absorption of X-rays in different points of the media.

Modern mathematical methods and software developed for CT involve the gathering of projection data from multiple directions and the feeding of the data into a tomographic reconstruction software algorithm processed by a computer [5]. Mathematical filters are used to improve the reconstructed image [6]. The idea and the image processing as performed by means of the technology of Single-Photon Emission Computer Tomography [7] is the same as in the case of our parabolic strip telescope. This technology is used in nuclear medicine where a patient is injected with a radiopharmaceutical which emits gamma rays. The emitted gamma rays are collected by a gamma camera and the emitted image is reconstructed [8].

The same approach can be used when a parabolic strip is the primary mirror [9, 10] of a telescope, following the scheme shown in Figure 2. The images of tracked objects are comprised of lines. Each line represents the integral intensity of light incoming from an object or objects perpendicular to the strip (parallel to the focal line) located inside the field of view, which is guaranteed by the geometry. When making a series of photos while rotating the telescope around its optical axis, the inverse Radon transformation can be used to reconstruct the image with the above mentioned angular resolution. It is also possible to use other tools, for example Matlab [8]. Secondary optical elements can be used to focus the lines from the focal plane into points.

The technological construction of the parabolic strip is simpler than the planar or paraboloid surface, because the stress of material helps to maintain the geometry. A precise parabolic bracket is required to support the parabolic strip made of elastic mirrors. It is also simpler to use adaptive optics for the correction of optical defects. One can control the surface by a laser pointer located on one side of the strip. By the detection of its reflected light on the other side of the strip during the scan, the whole surface can be controlled and corrected by corrective elements. Of course, secondary mirrors can be added, then we have a Newton — like or Cassegrain — like telescope. The area and the angular resolution of a parabolic strip telescope are

$$P = LW, \quad \delta_L = \frac{\lambda}{L},$$

where λ is the wavelength of light, L is the length of the projection of the strip on the tangent plane at the vertex line — in Figure 2 it is the length of the x-axis projection of the strip — and W is the width of the strip.

The proposed telescopic system involves preferably parallactic mounting and an instrument for image digitalization connected to a computer.

The mounting, however, has to perform one more rotation than the usual mounting: rotation of the primary element or of the whole system around the optical axis is necessary in order to reconstruct details of an object. The instrument for the digitalization of images has to be located in the image plane of a telescope.

2. PROCESSING IMAGES FROM A PARABOLIC STRIP TELESCOP

The basic mode of tracking by means of a rotational telescope will display various intensities for the parallel line segment corresponding to various tracked points. Line segments may correspond to more than one observed points on straight lines.

It is apparent that all the points correspond to the given line segments. On the edge, one line segment corresponds to five points while the middle three line segments correspond to two points. Changes in the intensities on the line segments correspond to the number of LEDs in the image. For the studied methods, it is necessary to pre-process every image and to use the set of images originating during the rotation of a telescope around the optical axis. For this processing, images of objects (line segments of various thickness values and various intensities) must be the same everywhere because they pertain to the same objects. Every image of an object will be displayed as a system of parallel line segments. The scanning device can be oriented so that all line segments would be parallel with one axis of an image.

To measure objects from the field of vision of a telescope, which is determined by the angle between the centre of an image and the edge of the parabolic strip, it is necessary to "trim" the line segments so that they would be reduced to the surroundings of the centre and then expand to the whole image. First, a number of images is scanned. Each of them is turned by an angle of θ. Then the values from every image matrix are added up by columns, symmetrically around the centre $\pm n$ (multiplication of a matrix by vector $\vec{e}$) so that vector $\vec{x}_k$ originates. Tensor multiplication of this vector by the all-ones matrix is executed so that we would obtain the final image $\mathbf{Y_k}$ in the $m \times m$ form, where we have already achieved the desired shape. Image $\mathbf{Q}$ results from the turning by an angle corresponding to the angle by which a telescope was turned. It is assigned a number and stored. The algorithm continues in this way until it reaches $\theta/360$ number of images, i.e., we have the images evenly spread.

2.1. Simulation Algorithm

To check the function, we first simulated images from a telescope corresponding to real pictures. We proceeded as follows: first, we produced a series of pictures of an object turned by multiples of angle θ and then we added up the intensities of the points of the images that are perpendicularly projected to the same point on individual straight lines turned by the angles passing through the centre. Thus, we obtained the vectors and by means of the tensor product of the vectors and all-ones matrix we will obtain a number of pictures corresponding to the simulated observations where points are displayed as segment lines. The model is first turned by the required angle of θ by means of interpolation in the MATLAB software. Then the image matrix is added up by columns so that vector $\vec{x}_k$ results. Tensor multiplication of this vector by the all-ones matrix is executed. In this way, we obtain the final image $\mathbf{Q}$ in the $m \times m$ form. The image is assigned a number and stored. The algorithm continues in this way until it reaches $\theta/360$ number of images.

$$\mathbf{X}_k \in \mathbf{R}^{m \times m} \tag{1}$$
$$\theta_1, \theta_2, \ldots, \theta_N \in \langle 0, 2\pi \rangle \tag{2}$$
$$\mathbf{Y}_k = \mathrm{rot}(\mathbf{X}_k, \theta_k) \tag{3}$$
$$\vec{x}_k = \vec{e}\mathbf{Y}_k, \quad \vec{e} = (1, \ldots, 1) \in \mathbf{R}^m \tag{4}$$
$$\mathbf{Q} = \mathbf{Y}_k \tag{5}$$

2.2. Summation Algorithm

The summation algorithm is based on the principle of the reciprocal adding up of resulting projections. Intensities of the points that are covered by segment lines of individual images are accentuated by the summation of multiple images. The other points in lines can be removed by subtracting the matrix with constants in all the places. The value of the constants can be changed so that an image would become apparent.

$$\mathbf{X}_k \in \mathbf{R}^{m \times m} \tag{6}$$
$$\theta_1, \theta_2, \ldots, \theta_N \in \langle 0, 2\pi \rangle \tag{7}$$
$$\vec{x}_k = \vec{e}\mathbf{X}_k, \quad \vec{e} = (0, \ldots, 0, 1, \ldots, 1, 0, \ldots, 0) \in \mathbf{R}^m \tag{8}$$
$$\mathbf{Y}_k = f\vec{x}_k, \quad f = (1, \ldots, 1)^T \in \mathbf{R}^m \tag{9}$$
$$\mathbf{Z}_k = \mathrm{rot}(\mathbf{Y}_k, \theta_k) \tag{10}$$
$$\mathbf{Q} = \sum_{k=1}^{N} \mathbf{Z}_k \tag{11}$$

2.3. Radon Inverse Transform

The basic principle of a telescope is based on the Radon transform, however it is not possible to use the Radon inverse transform directly, especially because this algorithm does not always have a solution. Therefore its approximations are used. The process of reconstruction is hence based on the filtered back projection. This algorithm was originally designed for the reconstruction of an image obtained from computed tomography

$$\mathbf{X}_k \in \mathbf{R}^{m\times m} \tag{12}$$
$$\theta_1, \theta_2, \ldots, \theta_N \in \langle 0, 2\pi\rangle \tag{13}$$
$$\vec{x}_k = \vec{e}\mathbf{X}_k, \quad \vec{e} = (0, \ldots, 0, 1, \ldots, 1, 0, \ldots, 0) \in \mathbf{R}^m \tag{14}$$
$$\mathbf{Y}_k = f\vec{x}_k, \quad f = (1, \ldots, 1)^T \in \mathbf{R}^m \tag{15}$$
$$\mathbf{S}_k = \text{Ramplfilter}\mathbf{Y}_k \tag{16}$$
$$\mathbf{Z}_k = rot(\mathbf{S}_k, \theta_k) \tag{17}$$
$$\mathbf{Q} = \sum_{k=1}^{N} \mathbf{Z}_k \tag{18}$$

2.4. Multiplication Algorithm

The multiplication algorithm is an enhanced analogy to the summation algorithm. It is based on reciprocal multiplication of individual pre-processed matrices. The advantage is that if we have a zero (i.e., a dark point) in one matrix, the zero will remain in the resulting matrix when we execute the multiplication. High values in matrices and more complicated weighting of an image during the multiplication.

$$\mathbf{X}_k \in \mathbf{R}^{m\times m} \tag{19}$$
$$\theta_1, \theta_2, \ldots, \theta_N \in \langle 0, 2\pi\rangle \tag{20}$$
$$\vec{x}_k = \vec{e}\mathbf{X}_k, \quad \vec{e} = (0, \ldots, 0, 1, \ldots, 1, 0, \ldots, 0) \in \mathbf{R}^m \tag{21}$$
$$\mathbf{Y}_k = f\vec{x}_k, \quad f = (1, \ldots, 1)^T \in \mathbf{R}^m \tag{22}$$
$$\mathbf{Z}_k = \mathrm{rot}(\mathbf{Y}_k, \theta_k) \tag{23}$$
$$\mathbf{Q} = \prod_{k=1}^{N} \mathbf{Z}_k \tag{24}$$

2.5. Iteration Algorithm

The iteration algorithm is based on mutual multiplication of the final projections that proceeds in multiple steps. Images are decomposed to subsets. The subsets are multiplied and extracted until one image remains.

$$\mathbf{X}_k \in \mathbf{R}^{m\times m} \tag{25}$$
$$\theta_1, \theta_2, \ldots, \theta_N \in < 0, 2\pi > \tag{26}$$
$$\vec{x}_k = \vec{e}\mathbf{X}_k, \quad \vec{e} = (0, \ldots, 0, 1, \ldots, 1, 0, \ldots, 0) \in \mathbf{R}^m \tag{27}$$
$$\mathbf{Y}_k = f\vec{x}_k, \quad f = (1, \ldots, 1)^T \in \mathbf{R}^m \tag{28}$$
$$\mathbf{Z}_k = \mathrm{rot}(\mathbf{Y}_k, \theta_k) \tag{29}$$
$$\mathbf{Q} = \prod_{k=1}^{N} \mathbf{Z}_k \tag{30}$$

2.6. Non Standard Observation Method

The telescope can be successfully used for the tracking that requires a good resolving ability in one direction. In this case it is not necessary to rotate the mirror around an axis and it is possible to use a standard parallactic assembly with the adjusting of an angle so that it would be possible to set the directions of the required resolving abilities. This type of tracking can be successfully used for example for some special observations executed to give accuracy to the movement of objects in one direction. The method is based on one of the properties of a mirror, i.e., that every point of light in the field of vision displays as a line segment the length of which equals to the width of a parabolic mirror. The centre of the line segment is shifted against the centre of an image by an angle which is the same as the angle by which a tracked object is shifted against the axis of a telescope. The line segments are perpendicular to the reflector plane. Ideally, each of the line segments has the same intensity in any point. If line segments of multiple points partially overlap or connect one to another, the position and luminance of individual points can be identified on the basis of an analysis of proportion of intensities of lines.

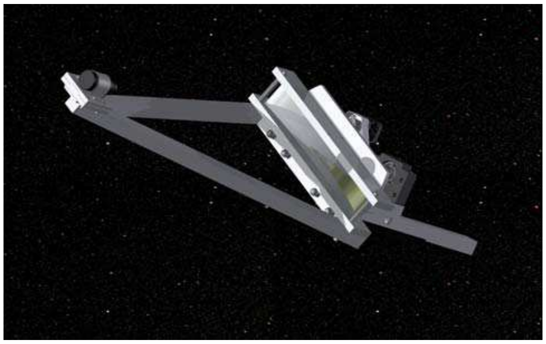

Figure 1: Parabolic strip telescope consists of a parabolic strip mirror, CCD camera in the image plane, supported by mounting.

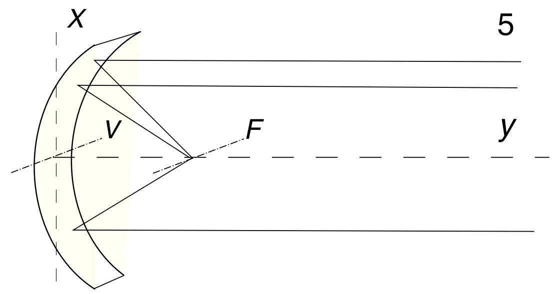

Figure 2: The paraxial beams 5 are reflected by strip on the focus line F, where V is the vertex line of the strip.

The intensities of the point sources along a line will jump grow at the beginning of a line segment and jump decrease at its end. Thanks to this, it is possible to reconstruct an image from the measurements of intensities. The ideal angular resolving ability in a point is lower in the direction of a line than in the direction perpendicular to it.

3. THE PROOF-OF-PRINCIPLE EXPERIMENT

In order to show that the principle works we have prepared a very basic experiment.

For the sake of simplification, the telescope was stationary. Figure 3 shows a parabolic strip telescope of a length of 30 cm. The artificial constellation was represented by means of a series of

Figure 3: The parabolic strips of length 30 cm, for the proof-of-principle experiment was used.

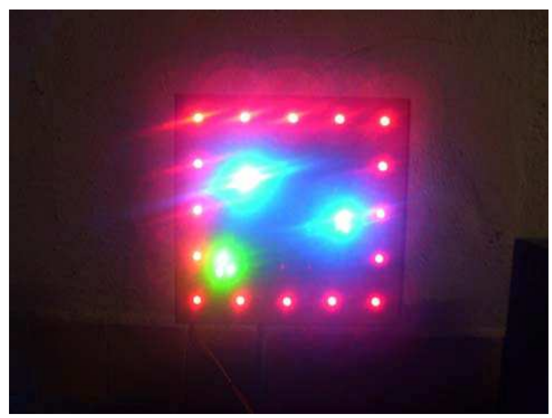

Figure 4: Two artificial constellations were used to demonstrate the principle.

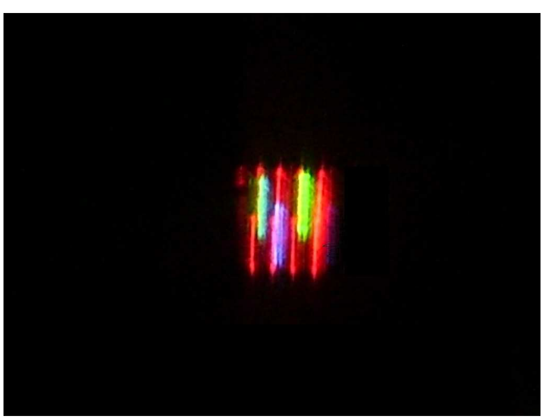

Figure 5: One of series of images from a camera.

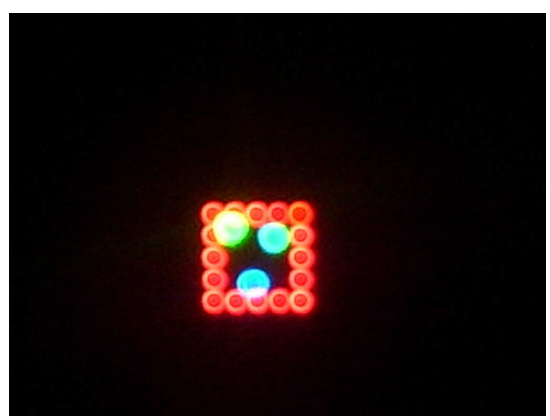

Figure 6: The image reconstructed from a series of pictures made with angular step of 5 degrees.

LED diodes. The artificial constellations used are shown in Figure 4.

The constellation was then rotated in steps by 5 degrees and the images were photographed by an ordinary digital camera. Figure 5 shows one of the photos. The reconstructed image is shown in Figure 6. For the image processing, Matlab was used.

The principle of the telescope was successfully tested. The resulting image can be compared with the reconstructed images in [8]. With higher quality components and more measurements at finer angle steps, the reconstructed image would be of better quality.

4. ADVANTAGES

- high mounting not being necessary,
- low wind influence,
- low gravitational influence.

5. CONCLUSIONS

The rotational telescope is a promising technology. It has a potential to supplement the existing types of telescopes used to observe the Universe. Although rotational telescopes are more complex in terms of their assembly and the processing of a final image, but contrary to standard reflectors, the mirror of this type of a telescope is much lighter and cheaper. This enables to construct telescopes with the primary optical element of a size of tens of meters, while the construction remains to be simple and production costs low. The ideal deployment of the telescope will be on an orbit where the size of mirrors could run to hundreds of meters and their resolving abilities will be sufficient for direct tracking of extra solar objects.

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