An Efficient Algorithm for the Calculation of Quantum Radar Cross Section of Flat Objects

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Abstract— In this paper, an efficient algorithm for simulating the quantum radar cross section of arbitrary shaped flat objects is proposed. Compared with previous studies on the simulation of quantum radar cross section the proposed algorithm used an alternative formula. Thus the proposed algorithm is more computationally efficient, and more favorable for large quantum system simulations.

1. INTRODUCTION

Quantum information science experienced dramatic developments in recent years. Those progresses have led to the advances in the field of quantum sensing and detection. The idea of using entangled photon pairs in a radar system to harness detection can be considered as the beginning of the quantum radar [1]. The quantum radar has been proposed as a candidate of new generation detection technologies [2]. Compared with classical radar, quantum radar features with anti-jamming capability [3], high resolution [4], low power consumption, and low delectability.

Similar as the radar cross section [5], the quantum radar cross section is a measurement of the ability of an object scattering the quantized electromagnetic waves in given directions. It is a crucial input parameter for designing a quantum radar system. However, only a few studies have been done on how to obtain quantum radar cross section [6–8]. Marco Lanzagorta defined the quantum radar cross section quantitatively [6, 7], and he proposed an algorithm for numerical simulating the quantum radar cross section. Marco gave the simulated quantum radar cross section of rectangular plates with different sizes. Unfortunately, the simulation is very time consuming. As presented in [6], even for a simple quantum system with one incident photon and 10,000 atoms, the simulation time is around 4 hours on a Intel 1.6 GHz desktop computer. In multiple incident photons cases, the problem becomes even worse, because the computational complexity increases exponentially with the increase of the number of incident photons. It is not feasible to simulate large quantum systems using Marco's method. In this work, we derived a different formulation, and implemented a more efficient algorithm based on the new formulation for quantum radar cross section simulation. Furthermore, the propose algorithm is able to deal with arbitrary shaped flat objects. Several numerical examples are given to demonstrate the efficiency of the proposed method. The proposed algorithm is an excellent candidate for quantum radar cross section simulation.

This paper is organized as follows. In the Section 2, an alternative formulation of quantum radar cross section are presented. In the Section 3, numerical examples of quantum radar cross section of flat objects are given with discussions, followed by the conclusions in the Section 4.

2. THEORY AND FORMULATION

Analog to classical radar cross section, the quantum radar cross section is defined as,

$$\sigma_Q = \lim_{R\to\infty} 4\pi R^2 \frac{\langle I_s(\mathbf{r}_s, \mathbf{r}_d, t)\rangle}{\langle I_i(\mathbf{r}_s, \mathbf{r}_d, t)\rangle} \tag{1}$$

where the R is the distance between radar and object. $\mathbf{r}_s$, $\mathbf{r}_d$ denote the location of the scatters and location of detector. $\langle I_i\rangle$, $\langle I_s\rangle$ are the incident and scattered field intensity, respectively [6]. Assuming the incident wave is homogeneous distributed on the target which is consist by N atoms, the incident wave on the target surface can be expressed as,

$$\langle I_i(\mathbf{r}_s, \mathbf{r}_d, t)\rangle = \frac{\frac{1}{N}\sum_{i=1}^{N}\left|\Psi_\gamma^{(i),\mathrm{INC}}(\Delta r_{si}, t)\right|^2}{S} \tag{2}$$

where $\Psi_\gamma^{(i),\mathrm{INC}}$ denotes the incident photon wave function, which can be represented by,

$$\Psi_\gamma^{(i),\mathrm{INC}}(\Delta r_{si}, t) = \frac{E_0}{r_{si}}\Theta(t - r_{si}/c)e^{-(i\omega+\Gamma/2)(t-r/c)} \tag{3}$$

where $\Theta(t - r_{si}/c)$ denotes a step function. If the absorption effect is ignored, all the incident energy on the target is scattered away, the intensity of the scattered field can be expressed as,

$$\langle I_s(\mathbf{r}_s, \mathbf{r}_t, t)\rangle = \frac{S_\perp}{r_{si}^2}\frac{1}{N}\left|\sum_{i=1}^{N}\Psi_\gamma^{(i),\mathrm{SCT}}(\Delta r, t)\right|^2 \tag{4}$$

where $\Psi_\gamma^{(i),\mathrm{SCT}}(\Delta r, t)$ denotes the incident photon wave function, which can be represented by,

$$\Psi_\gamma^{(i),\mathrm{SCT}}(\Delta r_{id}, t) = \frac{E_0}{r_{id}}\Theta(t - r_{si}/c)e^{-(i\omega+\Gamma/2)(t-\Delta R/c)} \tag{5}$$

Then the quantum radar cross section can be expressed as,

$$\begin{aligned}\sigma_Q &= \lim_{R\to\infty} 4\pi R^2 \frac{\langle I_s(\mathbf{r}_s, \mathbf{r}_d, t)\rangle}{\langle I_i(\mathbf{r}_s, \mathbf{r}_d, t)\rangle} = \lim_{R\to\infty} 4\pi R^2 \frac{\frac{S_\perp}{N}\left|\Sigma_{i=1}^{N}\Psi_\gamma^{(i),\mathrm{SCT}}(\Delta r, t)\right|^2}{\frac{1}{NS}\Sigma_{i=1}^{N}\left|\Psi_\gamma^{(i),\mathrm{INC}}(\Delta r_{si}, t)\right|^2}\\ &= \lim_{R\to\infty} 4\pi R^2 \frac{SS_\perp \frac{1}{r_{si}^2}\left|\Sigma_{i=1}^{N}\frac{E_0}{r_{id}}e^{(-i\omega-\Gamma/2)(t-\Delta R_i/c)}\right|^2}{\Sigma_{i=1}^{N}\left|\frac{E_0}{r_{si}}e^{(-i\omega+\Gamma/2)(t-r_i/c)}\right|^2}\end{aligned} \tag{6}$$

we use far field approximation $r_{si} \approx r_{id} \approx R$, Equation (6) can be simplified as,

$$\sigma_Q \approx 4\pi \lim_{R\to\infty} \frac{SS_\perp\left|\Sigma_{i=1}^{N}e^{(-i\omega-\Gamma/2)(t-\Delta R_i/c)}\right|^2}{\Sigma_{i=1}^{N}\left|e^{(-i\omega+\Gamma/2)(t-r_i/c)}\right|^2} \tag{7}$$

Then we further assume $\Gamma = 0$, Equation (7) can be simplified as,

$$\sigma_Q \approx 4\pi \lim_{R\to\infty} \frac{SS_\perp\left|\Sigma_{i=1}^{N}e^{-i\omega(\Delta R_i/c)}\right|^2}{\Sigma_{i=1}^{N}\left|e^{-i\omega(r_i/c)}\right|^2} \tag{8}$$

Compared with the formulation proposed in [6], as shown in Equation (9),

$$\sigma_Q \approx 2\pi S_\perp \lim_{R\to\infty} \frac{\left|\Sigma_{i=1}^{N}e^{-i\omega(\Delta R_i/c)}\right|^2}{\int_0^{2\pi}\int_0^{\pi}\left|\Sigma_{i=1}^{N}e^{-i\omega(r_i'/c)}\right|^2 \sin\theta' d\theta' d\phi'} \tag{9}$$

the new Equation (8) avoids the evaluation of the integration of a rapid changing pattern on a spherical surface as shown in the denominator of Equation (9), which requires a large number of integration points to be evaluated accurately. Therefore the proposed algorithm is much more efficient than the one based on Equation (9). For the multiple incident photons circumstances, this work follow the exact procedure used in [6]. In the next section, numerical examples are given to demonstrate the advantages of the proposed algorithm.

3. NUMERICAL EXAMPLES

In this section, we will use serval numerical examples to demonstrate the correctness and efficiency of the proposed algorithm.

3.1. Rectangular Perfect Electrical Conductor (PEC) Plate

The first example is a rectangular PEC plate which can be found in [6]. The PEC plate is 2.5λ (λ is the wave length) in length and 4λ in width. The plate is placed in XOY plane. The mono-static quantum radar cross section in the XOZ plane ($\phi = 0°$) is calculated. The simulated mono-static quantum radar cross section is shown in the Figure 1.

This example is simulated on a laptop computer with Intel 2.4 GHz CPU. The total simulation time for 181 incident angles is 7.5 seconds, i.e., 0.04 second per incident angle. Simulated results agree well with results presented in [7] as shown in the Figure 1. As discussed in Section 2, since a different formulation is used, the simulation time is reduced from around 4 hours [7] to less than 10 second on a laptop computer.

Next, the same rectangular PEC plate is used to study the effect of the atom lattice structure on the simulated quantum radar cross section results. Two different atom lattices are used to represent the same rectangular PEC plate. The first lattice is a uniform square lattice, and the second lattice is a triangular lattice. Furthermore, the location of the atom in the triangular lattice can have a small random perturbation in the XOY plane. The maximum offset of the atom is five percent of the rectangular PEC plate length. The number of the atoms keeps the same for the two different lattices. The calculated quantum radar cross section of the rectangular PEC plate using different lattice structures are shown in Figure 2. In this figure, it is shown the type of the lattice structure does not have a noticeable effect on the simulation quantum radar cross section results of a rectangular PEC plate.

The same rectangular PEC plate is used to study the effect of the number of atoms on the simulation of the quantum radar cross section. The quantum radar cross section obtained by using a lattice with a large number of atoms is used as the reference results to calculate the relative error of the simulated quantum radar cross section using different number of atoms, as shown in the Figure 3. From the Figure 3, we conclude that if a relative accuracy less than 1% is required, an atom number larger than 30 per wavelength is needed.

3.2. Elliptical PEC Plate

The second example is an elliptical PEC disk, which is used to demonstrate the capability of the algorithm to deal with arbitrary shaped flat objects. The length of the major semi-axes of the disk

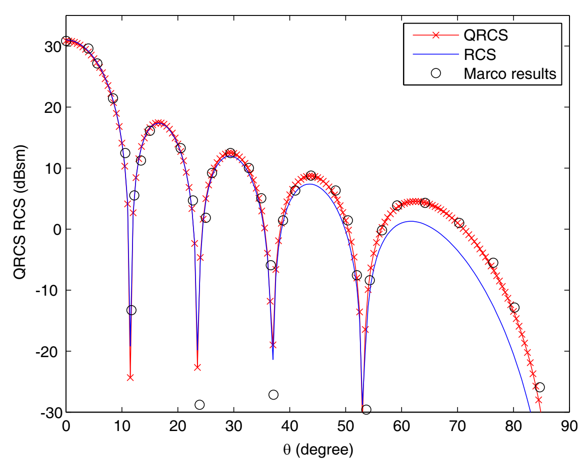

Figure 1: The simulated quantum radar cross section of a rectangular PEC plate.

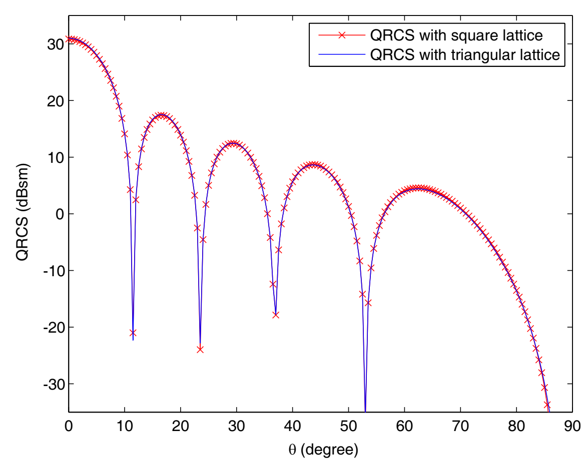

Figure 2: The simulated quantum radar cross section simulation using different atom structures.

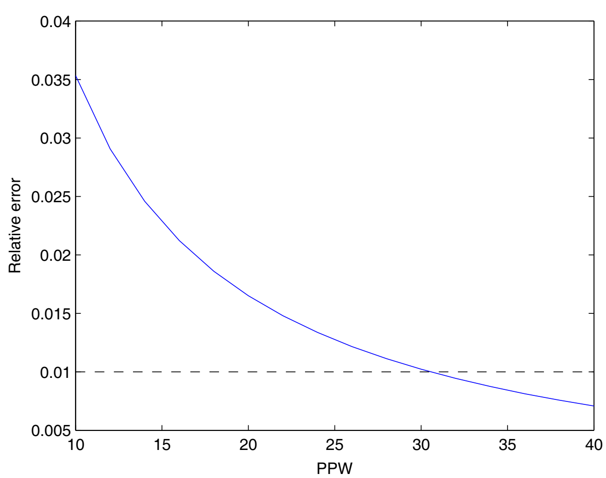

Figure 3: The relative error of simulated quantum radar cross section using different number of atoms.

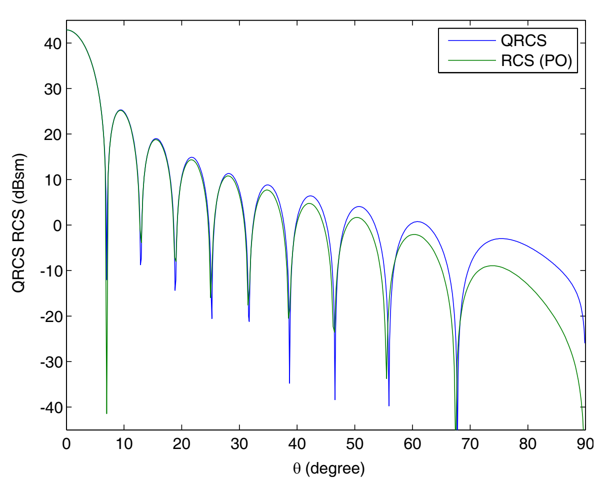

Figure 4: The simulated quantum radar cross section of a PEC elliptical disk.

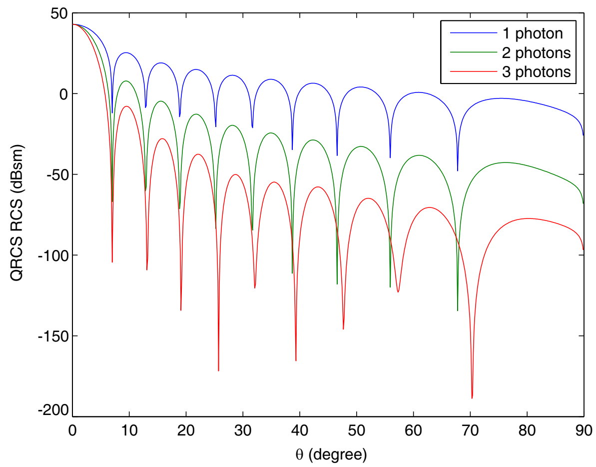

Figure 5: The simulated quantum radar cross section of a PEC elliptical disk with multiple incident photons.

is 5λ , and the length of the minor semi-axes of the disk is 2.5λ. The elliptical PEC disk is placed in XOY plane, the mono-static quantum radar cross section in XOZ plane ($\phi = 0°$) is calculated. The simulated quantum radar cross section is shown in the Figure 4, with physical optics results as reference.

Using the same object, the effect of multiple incident photons is studied. The simulated quantum radar cross section results for multiple incident photons are shown in Figure 5. From the simulated results in Figure 5, we notice the quantum radar cross section in multiple incident photons case is very different from single incident photon case. The main lobe in multiple incident case is narrower than single incident case, which implies using multiple photons can increase the spatial resolution of the quantum radar [9]. Another interesting phenomenon can be observed in Figure 5 is that the peak value of the main lobe keeps unchanged for all cases.

4. CONCLUSIONS

In summary, an efficient algorithm based on an alternative formula for the quantum radar cross section simulation is proposed. The new algorithm is able to simulate the quantum radar cross section of arbitrary shaped flat objects. The accuracy and efficiency of the proposed method is demonstrated by numerical examples. The effects of the atoms lattice structure and the number of the atoms on the simulated quantum radar cross section are studied. The efficiency of the algorithm can be further improved by parallelization, which can be implemented straight forward. The proposed algorithm is very suitable for simulating large photon-quantum systems.

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