Anisotropy and Non-reciprocity in Boundary Conditions: Generalized PEMC Surface

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Abstract— Boundary conditions play an important role in electromagnetics, and not the least in connection with metamaterials and metasurfaces. In this paper we present a generalization of the PEMC (perfect electromagnetic conductor) boundary. The generalization extends the isotropic and non-reciprocal character of the PEMC boundary into anisotropic and reciprocal dimensions. The effect of the generalized boundary on the polarization of incident waves is analyzed.

1. INTRODUCTION

The perfect electromagnetic conductor (PEMC) boundary [1] is a lossless boundary defined with the following conditions for the tangential components of the electric and magnetic fields:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = -\begin{pmatrix} 1/M & 0 \\ 0 & 1/M \end{pmatrix}\begin{pmatrix} H_x \\ H_y \end{pmatrix} \tag{1}$$

where it is assumed that the boundary is locally perpendicular to the z-axis ($\mathbf{u}_z$ pointing into the domain), and M is the admittance-type PEMC parameter. These boundary conditions are non-reciprocal, and appear on the surface of a so-called axion medium [2]. As is known, the PEMC (axion) medium, defined by the conditions $\mathbf{D} = M\mathbf{B}$, $\mathbf{H} = -M\mathbf{E}$, contains both the perfect electric conductor (PEC) and perfect magnetic conductor (PMC) as special cases ($1/M = 0$ and $M = 0$, respectively).

The particular character of the condition (1) becomes clear when it is paralleled with the (isotropic) impedance boundary condition (IBC) [3]

$$\mathbf{n} \times \mathbf{E} = Z_s \mathbf{n} \times (\mathbf{n} \times \mathbf{H}) \tag{2}$$

where $\mathbf{n}$ is the unit normal, in the present case $\mathbf{n} = \mathbf{u}_z$. Writing this in matrix form, the non-diagonality and antisymmetry of (2) contrasts strongly with (1):

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 0 & Z_s \\ -Z_s & 0 \end{pmatrix}\begin{pmatrix} H_x \\ H_y \end{pmatrix} \tag{3}$$

Let us set the PEMC condition in a broader setting of boundary conditions. In terms of the classification of boundary conditions with the four elementary dyadics $\overline{\overline{\mathrm{I}}}$, $\overline{\overline{\mathrm{J}}}$, $\overline{\overline{\mathrm{K}}}$, $\overline{\overline{\mathrm{L}}}$ [4], the general surface impedance dyadic $\overline{\overline{\mathrm{Z}}}_s$ in

$$\mathbf{n} \times \mathbf{E} = \overline{\overline{\mathrm{Z}}}_s \cdot \mathbf{n} \times (\mathbf{n} \times \mathbf{H}) \tag{4}$$

reads

$$\overline{\overline{\mathrm{Z}}}_s = Z_I \begin{pmatrix} +1 & 0 \\ 0 & +1 \end{pmatrix} + Z_J \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix} + Z_K \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} + Z_L \begin{pmatrix} 0 & +1 \\ +1 & 0 \end{pmatrix} \tag{5}$$

where Z_I equals Z_s in (3) and $Z_J = 1/M$ in (1). For the boundary to be lossless, the following needs to apply [4]: $\overline{\overline{\mathrm{Z}}}_s^T = -\overline{\overline{\mathrm{Z}}}^*$, the star denoting the complex conjugate. This means that for lossless boundary conditions the following is required:

$$\mathrm{Re}\{Z_I\} = 0, \quad \mathrm{Re}\{Z_K\} = 0, \quad \mathrm{Re}\{Z_L\} = 0, \quad \mathrm{Im}\{Z_J\} = 0 \tag{6}$$

Furthermore, the reciprocity condition means symmetry: $\overline{\overline{\mathrm{Z}}}_s^T = -\overline{\overline{\mathrm{Z}}}_s$, in other words $Z_J = 0$ with no restriction on Z_I, Z_K, Z_L.

2. GENERALIZED PEMC BOUNDARY

One possibility to generalize the PEMC boundary condition (1) is to allow the two diagonal "admittances" to be different, while still keeping it complementary to the IBC. This means allowing the parameter Z_L to be nonzero. For a lossless boundary, (6) requires that Z_L has to be purely imaginary.

This leads to the following boundary condition:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = -\begin{pmatrix} P & 0 \\ 0 & P^* \end{pmatrix} \begin{pmatrix} \eta H_x \\ \eta H_y \end{pmatrix} \tag{7}$$

where the characteristic impedance η has been included to reduce dimensions from the complex boundary parameter P. Note that if P is real, the boundary distills down to the classical PEMC (1) with $\eta MP = 1$.

Of course other generalizations of (1) can be envisioned. However, (7) is the only one which retains the diagonal character of the relation between the tangential fields. This choice extends the PEMC condition into anisotropic domain [5] and also adds a reciprocal part into the purely non-reciprocal condition (1).

3. REFLECTION WITH NORMAL INCIDENCE

Assume that an incident plane wave $\mathbf{E}^i$ with polarization $\mathbf{E} = \mathbf{u}_x E_x + \mathbf{u}_y E_y$ hits the generalized PEMC boundary (7) with normal incidence. Then the reflected field $\mathbf{E}^r = \overline{\overline{\mathrm{R}}} \cdot \mathbf{E}^i$ can be calculated from the reflection matrix

$$\overline{\overline{\mathrm{R}}} = \frac{1}{|P|^2+1} \begin{pmatrix} |P|^2-1 & -2P \\ 2P^* & |P|^2-1 \end{pmatrix} \tag{8}$$

The eigenvectors in reflection (those polarizations that keep their polarization in the reflection) are

$$P\,\mathbf{u}_x \pm \mathrm{j}|P|\mathbf{u}_y \tag{9}$$

with eigenvalues

$$\frac{|P|^2-1 \mp 2\mathrm{j}|P|}{|P|^2+1} = \frac{(|P| \mp \mathrm{j})^2}{|P|^2+1} \tag{10}$$

From this it can be seen that the reflection is always 100% in amplitude (both eigenvalues have unity absolute value). Also, the eigenpolarizations are circular if P is real (classical PEMC), and otherwise elliptical. The other limiting case is purely imaginary P, which leads to linear eigenpolarizations.

4. RENORMALIZATION OF PARAMETERS

Let us express the generalized PEMC surface impedance P with two angular parameters in the following way

$$P = \tan\beta\, \exp(\mathrm{j}\varphi) \tag{11}$$

with $\varphi = 0$ for PEMC, $\beta = 0$ for PEC, and $\beta = \pi/2$ for PMC.

Then the reflection matrix looks like

$$\overline{\overline{\mathrm{R}}} = \begin{pmatrix} -\cos(2\beta) & -\sin(2\beta)\exp(\mathrm{j}\varphi) \\ \sin(2\beta)\exp(-\mathrm{j}\varphi) & -\cos(2\beta) \end{pmatrix} \tag{12}$$

with eigenvectors as

$$\exp(\mathrm{j}\varphi)\mathbf{u}_x \pm \mathrm{j}\mathbf{u}_y \tag{13}$$

and the eigenvalues are

$$-\exp(\pm \mathrm{j}2\beta) \tag{14}$$

5. BEHAVIOR OF THE POLARIZATION

The polarization vector $\mathbf{p}$ describes the polarization state (character of a complex vector) of a complex vector $\mathbf{a}$. It is defined as [4]

$$\mathbf{p}(\mathbf{a}) = \frac{\mathbf{a} \times \mathbf{a}^*}{\mathrm{j}\mathbf{a} \cdot \mathbf{a}^*} \tag{15}$$

The handedness of the vector is positive when looked into the direction of the vector. The magnitude $|\mathbf{p}| = 2b/(1+b^2)$ gives information about the ellipticity of the vector (b is the axis ratio of the ellipse).

The polarization vector for the eigenpolarizations can be seen to be simple:

$$\mathbf{p} = \mp \frac{\Re\{P\}}{|P|} \mathbf{u}_z = \mp \cos\varphi\, \mathbf{u}_z \tag{16}$$

In other words, the polarizations are of opposite handedness but have the same elliptical shape. In addition, the polarization does not depend at all on the parameter β. Figure 1 shows the axis ratio of the eigenpolarization depending on φ. We can see the monotonous change from the circularly polarized PEMC case to the linear polarization for the fully anisotropic surface.

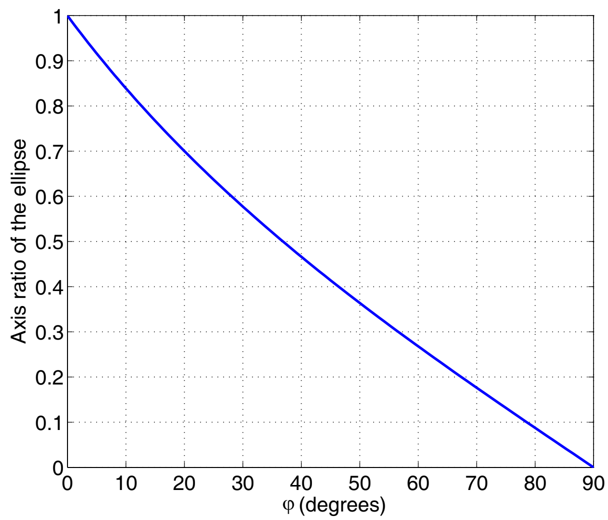

Figure 1: The axis ratio of the eigenpolarizations in normal-incidence reflection from the generalized PEMC boundary, as function of the φ parameter. The case $\varphi = 0$ corresponds to the pure PEMC surface.

6. CONCLUSION

A generalization of the fully non-reciprocal PEMC boundary condition in the form (7) retains the non-diagonal character of the PEMC impedance dyadic (5) but extends the isotropy of a pure PEMC case into anisotropic boundaries. In addition, the modification introduces a reciprocal component into the boundary. The boundary is lossless, in other words all incident fields are reflected with reflection coefficient $|R| = 1$. As is known, a PEMC boundary rotates the polarization plane of a linearly polarized incident field in reflection, and hence the additional parameteric dimension that the generalized PEMC boundary offers gives interesting potential for polarization control of electromagnetic waves.

Promising progress toward practical realizations of PEMC-metasurfaces has been documented [6]. The materialization of generalized PEMC surfaces of the type that was outlined above is certainly conceivable due to the fact that the additional parameters correspond to anisotropic but reciprocal effects.

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