Electromagnetic Force in the Complex Quaternion Space

Zi-Hua Weng
School of Physics and Mechanical & Electrical Engineering
Xiamen University, Xiamen 361005, China

Abstract— J. C. Maxwell applied simultaneously the vector terminology and the quaternion analysis to study the electromagnetic features. Nowadays the spaces of electromagnetic and gravitational fields can be chosen as the quaternion spaces, and the coordinates of quaternion spaces are able to be the complex numbers. The quaternion space of the electromagnetic field is independent to that of the gravitational field. These two quaternion spaces can combine together to become one octonion space. Contrarily the octonion space can be separated into two subspaces, the quaternion space and the S-quaternion space. In the quaternion space, it is able to deduce the field strength, field source, angular momentum, torque, force, and mass continuity equation etc. in the gravitational field. In the S-quaternion space, it is able to infer the field strength, field source, and current continuity equation etc. in the electromagnetic field. The results reveal that the quaternion space is suitable to depict the gravitational features, including the gravity and the mass continuity equation etc.. Meanwhile the S-quaternion space, it is suitable to describe the electromagnetic features, including the electromagnetic force and the current continuity equation etc..

1. INTRODUCTION

In the works about the electromagnetic theory, J. C. Maxwell mingled the quaternion analysis and vector terminology to depict the electromagnetic feature. Similarly the scholars begin to study the physics feature of gravitational field with the quaternion and octonion. A. Singh [1] applied the complex quaternion to deduce directly the Maxwell's equations in the classical electromagnetic theory. K. Morita [2] developed the study of the quaternion field theory. Similarly J. Edmonds [3] utilized the quaternion to depict the wave equation and gravitational theory in the curved space-time. F. A. Doria [4] adopted the quaternion to research the gravitational theory. A. S. Rawat etc. [5] discussed the gravitational field equation with the quaternion treatment. Moreover several of the scientists apply the octonion analysis to study the electromagnetic theory. V. L. Mironov etc. [6] described the electromagnetic equations and related features with the algebra of octonions. M. Gogberashvili [7] used the octonion to discuss the electromagnetic theory. O. P. S. Negi etc. [8] depicted the Maxwell's equations by means of the octonion.

The ordered couple of quaternions compose the octonion. On the contrary, the octonion is able to be separated into two parts, the quaternion and the S-quaternion (short for the second quaternion), and their coordinates can be chosen as the complex numbers. In the paper, it is able to find that the quaternion space is suitable to describe the gravitational features, while the S-quaternion space is proper to depict the electromagnetic features.

In the paper, the quaternion space and the S-quaternion space can be introduced into the field theory, in order to describe the physical feature of gravitational and electromagnetic fields. In the electromagnetic theory described with the complex quaternion, it is able to deduce directly the Maxwell's equations in the classical electromagnetic theory, without the help of the current continuity equation [9]. In this approach, substituting the S-quaternion for the quaternion, one can obtain the Maxwell's equations still. On the basis of this approach, the paper is able to deduce directly the force and the current continuity equation etc. further. Similarly the paper can apply the quaternion to research the gravitational theory, deducing directly the force, torque, energy, and the mass continuity equation etc..

2. FIELD EQUATIONS

In the quaternion space $\mathbb{E}_g$ for the gravitational field, the basis vector is $\mathbb{E}_g = (\mathbf{i}_0, \mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3)$, the radius vector is $\mathbb{R}_g = ir_0\mathbf{i}_0 + \Sigma r_k\mathbf{i}_k \equiv ir_0 + \mathbf{r}$, and the velocity is $\mathbb{V}_g = iv_0\mathbf{i}_0 + \Sigma v_k\mathbf{i}_k \equiv iv_0 + \mathbf{v}$. The gravitational potential is $\mathbb{A}_g = ia_0\mathbf{i}_0 + \Sigma a_k\mathbf{i}_k \equiv ia_0 + \mathbf{a}$, the gravitational strength is $\mathbb{B}_g = h_0\mathbf{i}_0 + \Sigma h_k\mathbf{i}_k \equiv h_0 + \mathbf{h}$, and the gravitational source is $\mathbb{S}_g = is_0\mathbf{i}_0 + \Sigma s_k\mathbf{i}_k \equiv is_0 + \mathbf{s}$. In the S-quaternion space $\mathbb{E}_e$ for the electromagnetic field, the basis vector is $\mathbb{E}_e = (\mathbf{I}_0, \mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3)$, the radius vector is $\mathbb{R}_e = iR_0\mathbf{I}_0 + \Sigma R_k\mathbf{I}_k \equiv i\mathbf{R}_0 + \mathbf{R}$, and the velocity is $\mathbb{V}_e = iV_0\mathbf{I}_0 + \Sigma V_k\mathbf{I}_k \equiv i\mathbf{V}_0 + \mathbf{V}$. The electromagnetic potential is $\mathbb{A}_e = iA_0\mathbf{I}_0 + \Sigma A_k\mathbf{I}_k \equiv i\mathbf{A}_0 + \mathbf{A}$, and the electromagnetic source is

$\mathbb{S}_e = iS_0\mathbf{I}_0 + \Sigma S_k\mathbf{I}_k \equiv i\mathbf{S}_0 + \mathbf{S}$. The electromagnetic strength is $\mathbb{B}_e = H_0\mathbf{I}_0 + \Sigma H_k\mathbf{I}_k \equiv \mathbf{H}_0 + \mathbf{H}$, with $\mathbf{H}_0 = H_0\mathbf{I}_0$. Herein $\mathbb{E}_e = \mathbb{E}_g \circ \mathbf{I}_0$. The symbol $\circ$ denotes the octonion multiplication. r_j, v_j, a_j, s_j, R_j, V_j, A_j, S_j, h_0, and H_0 are all real. h_k and H_k are all complex numbers. i is the imaginary unit. $\mathbf{i}_0 = 1$. $j = 0, 1, 2, 3$. $k = 1, 2, 3$.

Two quaternion spaces, $\mathbb{E}_g$ and $\mathbb{E}_e$, may compose one octonion space, $\mathbb{E} = \mathbb{E}_g + \mathbb{E}_e$. In the octonion space $\mathbb{E}$ for the electromagnetic and gravitational fields, the octonion radius vector is $\mathbb{R} = \mathbb{R}_g + k_{eg}\mathbb{R}_e$, the octonion velocity is $\mathbb{V} = \mathbb{V}_g + k_{eg}\mathbb{V}_e$, with k_{eg} being the coefficient. The octonion field potential is $\mathbb{A} = \mathbb{A}_g + k_{eg}\mathbb{A}_e$, the octonion field strength is $\mathbb{B} = \mathbb{B}_g + k_{eg}\mathbb{B}_e$. From here on out, some physics quantities are extended to the octonion functions, according to the characteristics of octonion. Apparently $\mathbb{V}$ and $\mathbb{A}$ etc. are all octonion functions of $\mathbb{R}$.

The octonion field source $\mathbb{S}$ of the electromagnetic and gravitational fields can be defined as,

$$\mu\mathbb{S} = -(i\mathbb{B}/v_0 + \Box)^* \circ \mathbb{B} = \mu_g\mathbb{S}_g + k_{eg}\mu_e\mathbb{S}_e - (i\mathbb{B}/v_0)^* \circ \mathbb{B}, \tag{1}$$

where the field strength is $\mathbb{B} = \Box \circ \mathbb{A}$. $\mathbb{B}_g = \Box \circ \mathbb{A}_g$, $\mathbb{B}_e = \Box \circ \mathbb{A}_e$. The quaternion operator is $\Box = i\mathbf{i}_0\partial_0 + \Sigma\mathbf{i}_k\partial_k$. $\nabla = \Sigma\mathbf{i}_k\partial_k$, $\partial_j = \partial/\partial r_j$. μ, μ_g, and μ_e are coefficients. $\mu_g < 0$, and $\mu_e > 0$. $*$ denotes the conjugation of octonion. $v_0 = \partial r_0/\partial t$. v_0 is the speed of light, and t is the time. In the case for single one particle, a comparison with the classical field theory reveals that, $\mathbb{S}_g = m\mathbb{V}_g$, and $\mathbb{S}_e = q\mathbb{V}_e$. m is the mass density, while q is the density of electric charge.

The paper adopts the gauge condition, $h_0 = -\partial_0 a_0 + \nabla \cdot \mathbf{a} = 0$. And then the above can be written as $\mathbf{h} = i\mathbf{g}/v_0 + \mathbf{b}$, with $\mathbf{h} = \Sigma h_k\mathbf{i}_k$. One component of gravitational strength is the gravitational acceleration, $\mathbf{g}/v_0 = \partial_0\mathbf{a} + \nabla a_0$. The other is $\mathbf{b} = \nabla \times \mathbf{a}$, which is similar to the magnetic flux density. Similarly the paper adopts the gauge condition, $\mathbf{H}_0 = -\partial_0\mathbf{A}_0 + \nabla \cdot \mathbf{A} = 0$. While the electromagnetic strength can be written as $\mathbf{H} = i\mathbf{E}/v_0 + \mathbf{B}$, $\mathbf{H} = \Sigma H_k\mathbf{I}_k$. The electric field intensity is $\mathbf{E}/v_0 = \partial_0\mathbf{A} + \nabla \circ \mathbf{A}_0$, and the magnetic flux density is $\mathbf{B} = \nabla \times \mathbf{A}$.

Expanding of Eq. (1) is able to deduce the Maxwell's equations in the classical electromagnetic theory, and the Newton's law of universal gravitation in the classical gravitational theory when $\mathbf{b} = 0$ and $\mathbf{a} = 0$.

Table 1: The multiplication table of octonion.

	1	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{I}_0$	$\mathbf{I}_1$	$\mathbf{I}_2$	$\mathbf{I}_3$
1	1	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{I}_0$	$\mathbf{I}_1$	$\mathbf{I}_2$	$\mathbf{I}_3$
$\mathbf{i}_1$	$\mathbf{i}_1$	-1	$\mathbf{i}_3$	$-\mathbf{i}_2$	$\mathbf{I}_1$	$-\mathbf{I}_0$	$-\mathbf{I}_3$	$\mathbf{I}_2$
$\mathbf{i}_2$	$\mathbf{i}_2$	$-\mathbf{i}_3$	-1	$\mathbf{i}_1$	$\mathbf{I}_2$	$\mathbf{I}_3$	$-\mathbf{I}_0$	$-\mathbf{I}_1$
$\mathbf{i}_3$	$\mathbf{i}_3$	$\mathbf{i}_2$	$-\mathbf{i}_1$	-1	$\mathbf{I}_3$	$-\mathbf{I}_2$	$\mathbf{I}_1$	$-\mathbf{I}_0$
$\mathbf{I}_0$	$\mathbf{I}_0$	$-\mathbf{I}_1$	$-\mathbf{I}_2$	$-\mathbf{I}_3$	-1	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$
$\mathbf{I}_1$	$\mathbf{I}_1$	$\mathbf{I}_0$	$-\mathbf{I}_3$	$\mathbf{I}_2$	$-\mathbf{i}_1$	-1	$-\mathbf{i}_3$	$\mathbf{i}_2$
$\mathbf{I}_2$	$\mathbf{I}_2$	$\mathbf{I}_3$	$\mathbf{I}_0$	$-\mathbf{I}_1$	$-\mathbf{i}_2$	$\mathbf{i}_3$	-1	$-\mathbf{i}_1$
$\mathbf{I}_3$	$\mathbf{I}_3$	$-\mathbf{I}_2$	$\mathbf{I}_1$	$\mathbf{I}_0$	$-\mathbf{i}_3$	$-\mathbf{i}_2$	$\mathbf{i}_1$	-1

3. OCTONION FORCE

In the octonion space, the octonion linear momentum density $\mathbb{P}$ is defined from the octonion field source $\mathbb{S}$, and can be written as follows,

$$\mathbb{P} = \mu\mathbb{S}/\mu_g, \tag{2}$$

where $\mathbb{P} = \mathbb{P}_g + k_{eg}\mathbb{P}_e$. $\mathbb{P}_g = \{\mu_g\mathbb{S}_g - (i\mathbb{B}/v_0)^* \circ \mathbb{B}\}/\mu_g$, and $\mathbb{P}_e = \mu_e\mathbb{S}_e/\mu_g$.

From the above, one can define the octonion angular momentum, $\mathbb{L} = (\mathbb{R} + k_{rx}\mathbb{X})^\times \circ \mathbb{P}$, and the octonion torque density, $\mathbb{W} = -v_0(i\mathbb{B}/v_0 + \Box) \circ \mathbb{L}$. Herein $\mathbb{X}$ is the integral function of field potential $\mathbb{A}$, and $\times$ denotes the conjugation of complex number, with k_{rx} being the coefficient. If we neglect the contribution of some tiny terms, the octonion torque density $\mathbb{W}$ will be reduced to the term $(2\mathbb{P}v_0)$ approximately, when $\mathbf{r}$ is one three-dimensional vector. And then the octonion force density $\mathbb{N}$ can be defined as,

$$\mathbb{N}/2 = -v_0(i\mathbb{B}/v_0 + \Box) \circ \mathbb{P}, \tag{3}$$

and the above can be expressed as,

$$\begin{aligned}\mathbb{N}/2 &= -v_0(i\!\!\mathbb{B}_g \circ \mathbb{P}_g/v_0 + ik_{eg}^2 \mathbb{B}_e \circ \mathbb{P}_e/v_0 + \Box \circ \mathbb{P}_g) \\ &= -k_{eg} v_0(i\!\!\mathbb{B}_g \circ \mathbb{P}_e/v_0 + i\!\!\mathbb{B}_e \circ \mathbb{P}_g/v_0 + \Box \circ \mathbb{P}_e),\end{aligned} \tag{4}$$

where $\mathbb{N} = \mathbb{N}_g + k_{eg}\mathbb{N}_e$. $\mathbb{N}_g/2 = -v_0(i\!\!\mathbb{B}_g \circ \mathbb{P}_g/v_0 + ik_{eg}^2 \mathbb{B}_e \circ \mathbb{P}_e/v_0 + \Box \circ \mathbb{P}_g)$ situates on the quaternion space $\mathbb{E}_g$, including the inertial force, gravity, and electromagnetic force etc.. $\mathbb{N}_e/2 = -v_0(i\!\!\mathbb{B}_g \circ \mathbb{P}_e/v_0 + i\!\!\mathbb{B}_e \circ \mathbb{P}_g/v_0 + \Box \circ \mathbb{P}_e)$ stays on the S-quaternion space $\mathbb{E}_e$, and is the interacting term between the gravitational and electromagnetic fields.

Table 2: The multiplication of the operator and octonion physics quantity.

definition	expression meaning
$\nabla \cdot \mathbf{a}$	$-(\partial_1 a_1 + \partial_2 a_2 + \partial_3 a_3)$
$\nabla \times \mathbf{a}$	$\mathbf{i}_1(\partial_2 a_3 - \partial_3 a_2) + \mathbf{i}_2(\partial_3 a_1 - \partial_1 a_3) + \mathbf{i}_3(\partial_1 a_2 - \partial_2 a_1)$
∇a_0	$\mathbf{i}_1 \partial_1 a_0 + \mathbf{i}_2 \partial_2 a_0 + \mathbf{i}_3 \partial_3 a_0$
$\partial_0 \mathbf{a}$	$\mathbf{i}_1 \partial_0 a_1 + \mathbf{i}_2 \partial_0 a_2 + \mathbf{i}_3 \partial_0 a_3$
$\nabla \cdot \mathbf{A}$	$-(\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3)\mathbf{I}_0$
$\nabla \times \mathbf{A}$	$-\mathbf{I}_1(\partial_2 A_3 - \partial_3 A_2) - \mathbf{I}_2(\partial_3 A_1 - \partial_1 A_3) - \mathbf{I}_3(\partial_1 A_2 - \partial_2 A_1)$
$\nabla \circ \mathbf{A}_0$	$\mathbf{I}_1 \partial_1 A_0 + \mathbf{I}_2 \partial_2 A_0 + \mathbf{I}_3 \partial_3 A_0$
$\partial_0 \mathbf{A}$	$\mathbf{I}_1 \partial_0 A_1 + \mathbf{I}_2 \partial_0 A_2 + \mathbf{I}_3 \partial_0 A_3$

3.1. Gravitational and Electromagnetic Forces

In the quaternion space $\mathbb{E}_g$, the component $\mathbb{N}_g$ of octonion force $\mathbb{N}$ can be written as follows

$$\mathbb{N}_g = iN_{10}^i + N_{10} + i\mathbf{N}_1^i + \mathbf{N}_1, \tag{5}$$

where $\mathbb{P}_g = ip_0 + \mathbf{p}$. $\mathbf{p} = \Sigma p_k \mathbf{i}_k$. $\mathbb{P}_e = i\mathbf{P}_0 + \mathbf{P}$. $\mathbf{P} = \Sigma P_k \mathbf{I}_k$. $\mathbf{P}_0 = P_0 \mathbf{I}_0$. $\mathbf{N}_1 = \Sigma N_{1k} \mathbf{i}_k$, $\mathbf{N}_1^i = \Sigma N_{1k}^i \mathbf{i}_k$. P_j, p_j, N_{1j}, and N_{1j}^i are all real.

When there are the gravitational and electromagnetic fields, the above can be expressed as

$$N_{10}^i/2 = -\mathbf{b} \cdot \mathbf{p} - k_{eg}^2(\mathbf{B} \cdot \mathbf{P}), \tag{6}$$

$$N_{10}/2 = v_0 \partial_0 p_0 - v_0 \nabla \cdot \mathbf{p} + \mathbf{g} \cdot \mathbf{p}/v_0 + k_{eg}^2(\mathbf{E} \cdot \mathbf{P}/v_0), \tag{7}$$

$$\mathbf{N}_1^i/2 = -v_0 \partial_0 \mathbf{p} + p_0 \mathbf{g}/v_0 - v_0 \nabla p_0 - \mathbf{b} \times \mathbf{p} + k_{eg}^2(\mathbf{E} \circ \mathbf{P}_0/v_0 - \mathbf{B} \times \mathbf{P}), \tag{8}$$

$$\mathbf{N}_1/2 = -v_0 \nabla \times \mathbf{p} + \mathbf{g} \times \mathbf{p}/v_0 + p_0 \mathbf{b} + k_{eg}^2(\mathbf{E} \times \mathbf{P}/v_0 + \mathbf{B} \circ \mathbf{P}_0), \tag{9}$$

where N_{10} is the power density, including the term capable of translating into the Joule heat in the electromagnetic field. N_{10}^i contains the current helicity of electromagnetic field. $\mathbf{N}_1$ is the torque derivative. $\mathbf{N}_1^i$ is the force density, including that of the inertial force, gravity, Lorentz force, and energy gradient etc.. $\nabla(\mathbb{B}^* \circ \mathbb{B}/\mu_g)$ is only dealt with the gradient of the norm of field strength, but is independent of the mass as well as the quantity of electric charge.

When $\mathbb{N}_g = 0$, the equation for force equilibrium is yielded from $\mathbf{N}_1^i = 0$. The mass continuity equation is derived from $N_{10} = 0$, and it will be reduced to that in the classical field theory when there is no field strength. The part current helicity of charged particle can be inferred from $N_{10}^i = 0$. While $\mathbf{N}_1 = 0$ can deduce the curl of toque component $\mathbf{p}v_0$. And this equation can infer the angular velocity of Larmor precession with respect to the orbital angular momentum of charged particle. The equation $\mathbf{N}_1^i = 0$ means that the gravitational acceleration $\mathbf{g}$ is the counterpart of the linear acceleration, $v_0 \partial_0 \mathbf{v}$. The equation $\mathbf{N}_1 = 0$ states that the component $\mathbf{b}$ is the counterpart of the velocity curl, $\nabla \times \mathbf{v}$, which is the double of the precessional angular velocity when the dimension of vector $\mathbf{r}$ is 2.

Comparing with the force density in the classical field theory states that $k_{eg}^2 = \mu_g/\mu_e < 0$. The force density $\mathbf{N}_1^i$ in the above requires that the gravitational field must situate on the quaternion space $\mathbb{E}_g$, while the electromagnetic field can only stay on the quaternion space $\mathbb{E}_e$, but not vice versa. The inertial force term $(-v_0 \partial_0 \mathbf{p})$ plays a crucial role in the space discrimination.

3.2. Interacting Force

In the S-quaternion space $\mathbb{E}_e$, the component $\mathbb{N}_e$ of octonion force $\mathbb{N}$ can be written as follows

$$\mathbb{N}_e = i\mathbf{N}_{20}^i + \mathbf{N}_{20} + i\mathbf{N}_2^i + \mathbf{N}_2, \tag{10}$$

where $\mathbf{N}_{20} = \Sigma N_{20}\mathbf{I}_0$, $\mathbf{N}_{20}^i = \Sigma N_{20}^i\mathbf{I}_0$. $\mathbf{N}_2 = \Sigma N_{2k}\mathbf{I}_k$, $\mathbf{N}_2^i = \Sigma N_{2k}^i\mathbf{I}_k$. N_{2j} and N_{2j}^i are all real.

When there are the gravitational and electromagnetic fields, the above can be expressed as

$$\mathbf{N}_{20}^i/2 = -\mathbf{b}\cdot\mathbf{P} - \mathbf{B}\cdot\mathbf{p}, \tag{11}$$

$$\mathbf{N}_{20}/2 = v_0\partial_0\mathbf{P}_0 - v_0\nabla\cdot\mathbf{P} + \mathbf{g}\cdot\mathbf{P}/v_0 + \mathbf{E}\cdot\mathbf{p}/v_0, \tag{12}$$

$$\mathbf{N}_2^i/2 = -v_0\partial_0\mathbf{P} - v_0\nabla\circ\mathbf{P}_0 + \mathbf{g}\circ\mathbf{P}_0/v_0 - \mathbf{b}\times\mathbf{P} + p_0\mathbf{E}/v_0 - \mathbf{B}\times\mathbf{p}, \tag{13}$$

$$\mathbf{N}_2/2 = -v_0\nabla\times\mathbf{P} + \mathbf{g}\times\mathbf{P}/v_0 + \mathbf{b}\circ\mathbf{P}_0 + \mathbf{E}\times\mathbf{p}/v_0 + p_0\mathbf{B}, \tag{14}$$

where $\mathbf{N}_{20}$ covers the current continuity equation.

When $\mathbb{N}_e = 0$, the equilibrium equation of force-like is yielded from $\mathbf{N}_2^i = 0$. The current continuity equation is derived from $\mathbf{N}_{20} = 0$, and it will be reduced to that in the classical field theory when there is no field strength. Meanwhile $\mathbf{N}_{20}^i = 0$ deduces the helicity-like of the charged particle. $\mathbf{N}_2 = 0$ states that the strength components $\mathbf{b}$ and $\mathbf{B}$ will impact the curl of the torque component $(\mathbf{P}v_0)$.

Table 3: Some definitions of the physics quantity in the gravitational and electromagnetic fields described with the complex quaternion/S-quaternion spaces.

physics quantity	definition	note
radius vector	$\mathbb{R} = \mathbb{R}_g + k_{eg}\mathbb{R}_e$	$k_{eg}^2 = \mu_g/\mu_e < 0$
integral function	$\mathbb{X} = \mathbb{X}_g + k_{eg}\mathbb{X}_e$	
field potential	$\mathbb{A} = i\Box^\times \circ \mathbb{X}$	
field strength	$\mathbb{B} = \Box \circ \mathbb{A}$	gauge condition
field source	$\mu\mathbb{S} = -(i\mathbb{B}/v_0 + \Box)^* \circ \mathbb{B}$	field equations
linear momentum	$\mathbb{P} = \mu\mathbb{S}/\mu_g$	
angular momentum	$\mathbb{L} = (\mathbb{R} + k_{rx}\mathbb{X})^\times \circ \mathbb{P}$	$k_{rx} = 1/v_0$
octonion torque	$\mathbb{W} = -v_0(i\mathbb{B}/v_0 + \Box) \circ \mathbb{L}$	torque, work, energy
octonion force	$\mathbb{N} = -(i\mathbb{B}/v_0 + \Box) \circ \mathbb{W}$	power, force

4. CONCLUSIONS

The paper introduced the quaternion/S-quaternion space into the field theory, to describe the physics features of gravitational and electromagnetic fields. The space extended from the electromagnetic field is independent of that from the gravitational field. Meanwhile the space of gravitational field and of electromagnetic field can be chosen as the quaternion space and S-quaternion space respectively. Furthermore the components of those two spaces and of relevant physics quantities may be complex numbers.

The above means that the vector analysis and quaternion analysis are suitable to describe the physics feature of gravitational and electromagnetic fields within two different confines. To a certain extent, the vector analysis may describe the vast majority of physics features of those two fields. Within a wider range, the quaternion analysis is able to depict more physics features of those two fields, including the force and continuity equation.

It should be noted that the paper discussed only some simple cases about the angular momentum, torque, force, and continuity equations etc. in the field theory described with the quaternion/S-quaternion. However it clearly states that the complex quaternion and S-quaternion spaces are able to availably describe the physics features of electromagnetic and gravitational fields. This will afford the theoretical basis for further analysis, and is helpful to research the property of the force and of continuity equations in the following researches.

ACKNOWLEDGMENT

This project was supported partially by the National Natural Science Foundation of China under grant number 60677039.

REFERENCES

1. Singh, A., "Quaternionic form of the electromagnetic-current equations with magnetic monopoles," *Lettere al Nuovo Cimento*, Vol. 31, No. 5, 145–148, 1981.
2. Morita, K., "Quaternions, lorentz group and the dirac theory," *Progress of Theoretical Physics*, Vol. 117, No. 3, 501–532, 2007.
3. Edmonds, J. D., "Quaternion wave equations in curved space-time," *International Journal of Theoretical Physics*, Vol. 10, No. 2, 115–122, 1974.
4. Doria, F. A., "A Weyl-like equation for the gravitational field," *Lettere al Nuovo Cimento*, Vol. 14, No. 13, 480–482, 1975.
5. Rawat, A. S. and O. P. S. Negi, "Quaternion gravi-electromagnetism," *International Journal of Theoretical Physics*, Vol. 51, No. 3, 738–745, 2012.
6. Mironov, V. L. and S. V. Mironov, "Octonic representation of electromagnetic field equations," *Journal of Mathematical Physics*, Vol. 50, No. 1, 012901, 10 pages, 2009.
7. Gogberashvili, M., "Octonionic electrodynamics," *Journal of Physics A*, Vol. 39, No. 22, 7099–7104, 2006.
8. Chanyal, B. C., P. S. Bisht, and O. P. S. Negi, "Generalized octonion electrodynamics," *International Journal of Theoretical Physics*, Vol. 49, No. 6, 1333–1343, 2010.
9. Honig, W. M., "Quaternionic electromagnetic wave equation and a dual charge-filled space," *Lettere al Nuovo Cimento*, Vol. 19, No. 4, 137–140, 1977.