M^2-factor for the Partially Coherent Elegant Laguerre-Gaussian Beam Propagating through the Turbulent Ocean

B. Wang, Y. S. Yuan, Z. F. Cui, and J. Qu
Department of Physics, Anhui Normal University, Wuhu 241000, China

Abstract— On account of the extended Huygens-Fresnel principle and the second order moments of the Wigner distribution function, the analytical expression of the beam propagation factor (M^2-factor) for partially coherent elegant Laguerre-Gaussian (PCELG) beam has been theoretically derived. The corresponding analysis and discussions reveal that the M^2-factor of the PCELG beam in oceanic turbulence vary with changes in parameters of the beam and the oceanic turbulence. The acquired results may have some certain reference value for laser tracking, remote sensing and optical communication under the water.

1. INTRODUCTION

In the last decades, the propagation properties of beams in free space, an arbitrary medium and atmosphere have been theoretically and experimentally studied by many scholars [1–5]. As an extension of the standard Laguerre-Gaussian beam, the elegant Laguerre-Gaussian beam with a more symmetrical form is proposed [6]. Since then, some researchers have paid their attention to the propagation properties of the PCELG beam through the turbulent atmosphere and obtained a lot of achievements [7, 8].

Recently, the new power spectrum for the turbulent ocean, considering the temperature and salinity fluctuations, has been used to investigate the propagation properties of beams in oceanic turbulence. N. Farwell et al. use the new spectrum to deal with the intensity and coherence properties of a Gaussian beam propagating in the clear water turbulent ocean [9]. Chen et al. obtain the evolution of propagation quality factor for partially coherence Hermite-Gaussian beams traveling through the turbulent ocean [10]. Zhao et al. focus on the polarization degree of an anisotropic electromagnetic Gaussian Schell-model beam propagating in the clear-water turbulent ocean [11]. Besides, Fu et al. have revealed how the average intensity, polarization degree and the coherence degree for the partially coherent radially polarized doughnut beam in oceanic turbulence are affected by the parameters of the light source and turbulent ocean [12].

This paper aims to specifically investigate the M^2-factor of the PCELG beam through the oceanic turbulence. By applying the deduced analytical expressions, corresponding numerical calculations and discussions have been carried out.

2. THEORETICAL MODEL

For dealing with the M^2-factor of the PCELG beam through the turbulent ocean, we take the power spectrum of the refractive index fluctuations in ocean water, specifically for isotropic and homogeneous oceanic turbulence, nearly introduced in Ref. [11]. Here, ignoring the scattering and absorption by some possible suspended particles and supposing the eddy thermal diffusivity and the diffusion of the salt equal in value, the power spectrum takes the following form

$$\phi_n(\kappa) = 0.388 \times 10^{-8} \varepsilon^{-1/3} \kappa^{-11/3} \left[1 + 2.35 (\kappa\eta)^{2/3}\right] f(\kappa, \zeta, \chi_T) \tag{1}$$

where ε denotes the rate of dissipation of turbulent kinetic energy per unit mass of fluid, ranging from $10^{-4}\,\text{m}^2/\text{s}^3$ to $10^{-10}\,\text{m}^2/\text{s}^3$, $\eta = 10^3\,\text{m}$ describing the Kolmogorov micro scale (inner scale), with [11]:

$$f(\kappa, \zeta, \chi_T) = \frac{\chi_T}{v^2} \left[\zeta^2 \exp(-A_T\delta) + \exp(-A_S\delta) - 2\zeta \exp(-A_{TS}\delta)\right]$$

With x_T indicating the rate of dissipation of mean-square temperature, varying in the range from $10^{-4}\,\text{K}^2/\text{s}$ to $10^{-10}\,\text{K}^2/\text{s}$, other parameters are set as $A_T = 1.863 \times 10^{-2}$, $A_S = 1.9 \times 10^{-4}$, $A_{TS} = 9.41 \times 10^{-3}$, and $\delta = 8.284(\kappa\eta)^{(4/3)} + 12.978(\kappa\eta)^2$, ζ showing the relative strength of temperature and salinity fluctuations, which in the ocean water can vary in the interval $[-5; 0]$, combining with the maximum value 0 corresponding to the case under the domination of temperature-driven turbulence, the minimum value -5 corresponding to the situation that salinity-driven turbulence gains the upper hand. It is necessary to note here, if $\phi_n(\kappa) = 0$, the condition is namely for propagation in free space.

Assuming that a PCELG beam propagates along z axis in the positive direction, into the half-space of $z \geq 0$, the electric field at the source plane in the rectangular coordinate system can be expressed as

$$E^{(0)}(\boldsymbol{\rho}, 0) = R\frac{(-1)^n}{2^{2n+m} \cdot n!}\sum_{t=0}^{n}\sum_{s=0}^{m} i^s \binom{n}{t}\binom{m}{s} H_{2t+m-s}\left(\frac{x}{w_0}\right) H_{2n-2t+s}\left(\frac{y}{w_0}\right) \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \tag{2}$$

where $\boldsymbol{\rho} = (x, y)$ representing the transverse coordinates in the source plane, associated with $\boldsymbol{\rho}^2 = x^2 + y^2$, R standing for a positive constant, w_0 being the beam width of the PCELG beam at the source plane. $H(x)$ indicating an Hermite polynomial, m and n meaning the orders of the PCELG beam. As $m = n = 0$, it reduces to the electric field of the well known fundamental Gaussian beam.

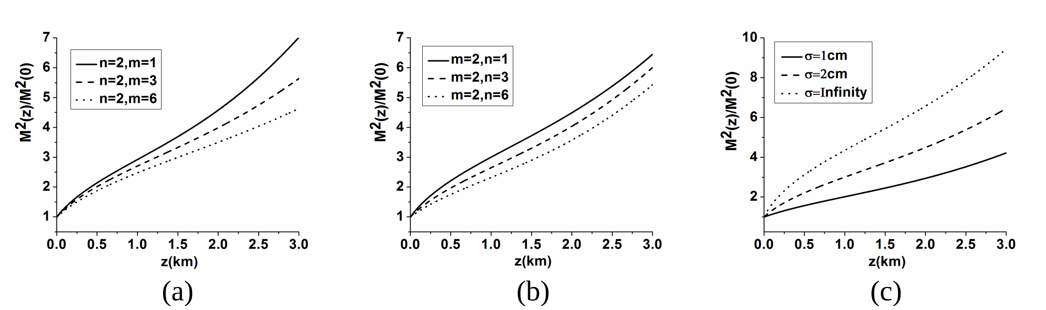

Figure 1: Propagation factor of PCELG beams $M^2(z)/M^2(0)$ versus propagation distance z through the oceanic turbulence for different beam parameters: (a) $\sigma = 2\,\text{cm}$, $m = 2$, $n = 1, 3, 6$; (b) $\sigma = 2\,\text{cm}$, $n = 2$, $m = 1, 3, 6$; (c) $m = 2$, $n = 1$, $\sigma = 1\,\text{cm}$, $2\,\text{cm}$, infinity.

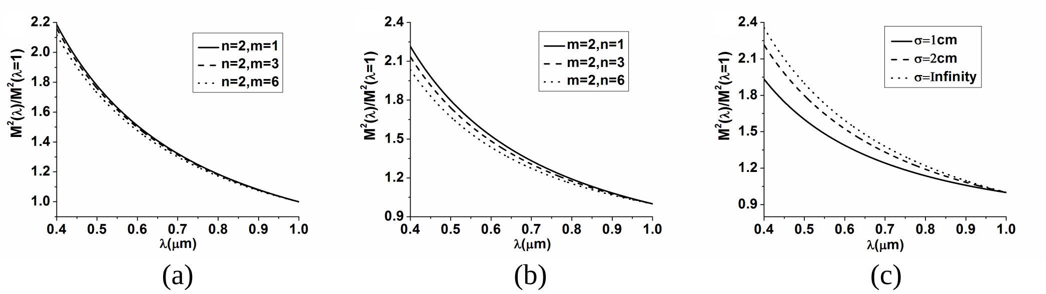

Figure 2: A comparison between propagation factor of PCELG beams $M^2(z)/M^2(0)$ in oceanic turbulence with different parameters and in free space: (a) $\varepsilon = 10^{-4}\,\text{m}^2\,\text{s}^{-3}$, $\zeta = -2.5$, $\chi_T = 10^{-10}$, $10^{-9}\,\text{K}^2\,\text{s}^{-1}$, in free space; (b) $\chi_T = 10^{-10}\,\text{K}^2\,\text{s}^{-1}$, $\zeta = -2.5$, $\varepsilon = 10^{-8}, 10^{-7}\,\text{m}^2\,\text{s}^{-3}$, in free space; (c) $\chi_T = 10^{-9}\,\text{K}^2\,\text{s}^{-1}$, $\varepsilon = 10^{-4}$, $\zeta = -5$, -3, in free space.

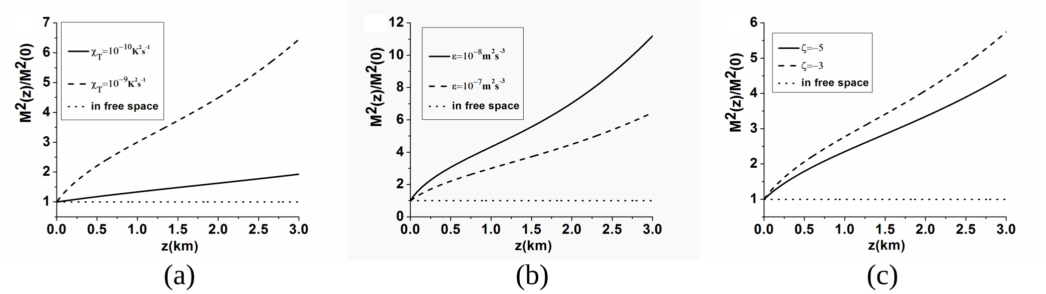

Figure 3: Propagation factor of PCELG beams $M^2(\lambda)/M^2(\lambda = 1)$ versus wavelength λ through the oceanic turbulence for different beam parameters: (a) $\sigma = 2\,\text{cm}$, $m = 2$, $n = 1, 3, 6$; (b) $\sigma = 2\,\text{cm}$, $n = 2$, $m = 1, 3, 6$; (c) $m = 2$, $n = 1$, $\sigma = 1\,\text{cm}$, $2\,\text{cm}$, infinity.

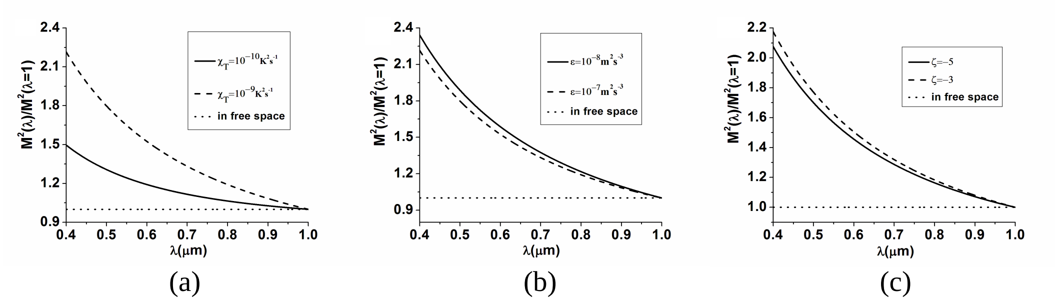

Figure 4: Propagation factor for PCELG beams $M^2(\lambda)/M^2(\lambda = 1)$ in oceanic turbulence with different parameters and in free space: (a) $\varepsilon = 10^{-4}\,\mathrm{m^2\,s^{-3}}$, $\zeta = -2.5$, $\chi_T = 10^{-10}$, $10^{-9}\,\mathrm{K^2\,s^{-1}}$, in free space; (b) $\chi_T = 10^{-10}\,\mathrm{K^2 s^{-1}}$, $\zeta = -2.5$, $\varepsilon = 10^{-8}$, $10^{-7}\,\mathrm{m^2\,s^{-3}}$, in free space; (c) $\chi_T = 10^{-9}\,\mathrm{K^2 s^{-1}}$, $\varepsilon = 10^{-4}$, $\zeta = -5$, -3, in free space.

The M^2-factor for the PCELG beam through the oceanic turbulence takes the following form

$$M^2(z) = k\left[\left(\langle x^2\rangle + \langle y^2\rangle\right)\left(\langle\theta_x^2\rangle + \langle\theta_y^2\rangle\right) - \left(\langle x\theta_x\rangle + \langle y\theta_y\rangle\right)^2\right]^{1/2} \tag{3}$$

3. NUMERICAL CALCULATIONS

Corresponding numerical calculations have been illustrated, for discussing effects of parameters of beam and the oceanic turbulence on the M^2-factor for the PCELG beam in the turbulent ocean. It is of necessity to note that the beam width w_0 is set to be 5 cm.

4. CONCLUSIONS

In this paper, the analytical formula for the M^2-factor of the PCELG beam in oceanic turbulence is theoretically deduced and corresponding numerical calculations and analysis have been accomplished. The obtained results indicate that the M^2-factor of the PCELG beam with lower beam orders, larger initial coherence length, bigger χ_T and ζ or smaller ε is more influenced by turbulent ocean and the poorer beam quality is acquired whether for increasing in propagation distance or the wave length. Particularly, in free space, as the propagation distance or wave length varies, the M^2-factor always stay the same. Above all, the results may be of some significance for underwater laser tracking, remote sensing and optical communication.

ACKNOWLEDGMENT

Jun Qu acknowledges the support by the National Natural Science Foundation of China (NSFC) under Grant No. 11374015, Yangsheng Yuan acknowledges the support by Anhui Provincial Natural Science Foundation of China under Grant No. 1408085QF112.

REFERENCES

1. Eyyuboglu, H. T. and Y. K. Baykal, "Flat topped beams and their characteristics in turbulent media," *Optics Express*, Vol. 14, No. 10, 4196–4207, 2006.
2. Ponomarenko, S. A., J. J. Greffet, and E. Wolf, "The diffusion of partially coherent beams in turbulent media," *Optics Communications*, Vol. 208, No. 1, 1–8, 2002.
3. Roychowdhury, H., S. A. Ponomarenko, and E. Wolf, "Change in the polarization of partially coherent electromagnetic beams propagating through the turbulent atmosphere," *Journal of Modern Optics*, Vol. 52, No. 11, 1611–1618, 2005.
4. Ghafary, B. and M. Alavinejad, "Changes in the state of polarization of partially coherent flat-topped beam in turbulent atmosphere for different source conditions," *Applied Physics B*, Vol. 102, No. 4, 945–952, 2011.
5. Taherabadi, G., M. Alavynejad, F. D. Kashani, et al., "Changes in the spectral degree of polarization of a partially coherent dark hollow beam in the turbulent atmosphere for on-axis and off-axis propagation point," *Optics Communications*, Vol. 285, No. 8, 2017–2021, 2012.
6. Takenaka, T., M. Yokota, and O. Fukumitsu, "Propagation of light beams beyond the paraxial approximation," *JOSA A*, Vol. 2, No. 6, 826–829, 1985.

7. Mei, Z. and J. Gu, "Comparative studies of paraxial and nonparaxial vectorial elegant Laguerre-Gaussian beams," *Optics Express*, Vol. 17, No. 17, 14865–14871, 2009.
8. Wang, F., Y. Cai, and O. Korotkova, "Partially coherent standard and elegant Laguerre-Gaussian beams of all orders," *Optics Express*, Vol. 17, No. 25, 22366–22379, 2009.
9. Farwell, N. and O. Korotkova, "Intensity and coherence properties of light in oceanic turbulence," *Optics Communications*, Vol. 285, No. 6, 872–875, 2012.
10. Chen, F., Y. Chen, and Q. Zhao, "Change of propagation quality factor of partially coherence hermite-gaussian beams traveling through oceanic turbulence," *Chinese Journal of Lasers*, Vol. 40, No. 4, 0413002-1–0413002-7, 2013.
11. Tang, M. and D. Zhao, "Spectral changes in stochastic anisotropic electromagnetic beams propagating through turbulent ocean," *Optics Communications*, Vol. 312, 89–93, 2014.
12. Fu, W. and H. Zhang, "Propagation properties of partially coherent radially polarized doughnut beam in turbulent ocean," *Optics Communications*, Vol. 304, 11–18, 2013.