Variable Gravitational Mass in the Electromagnetic Field Described with the Complex Quaternion

Zi-Hua Weng
School of Physics and Mechanical & Electrical Engineering
Xiamen University, Xiamen 361005, China

Abstract— J. C. Maxwell represented the physical feature of electromagnetic field with the vector terminology and the quaternion analysis. The method inspires other scholars to apply the complex quaternion depicting the classical field theory. The complex quaternion is able to deduce the field equations and force in the electromagnetic and gravitational fields, and predict some inferences. One of them is that the gravitational mass is variable. The gravitational mass will fluctuate with the alteration of the electromagnetic and gravitational strengths. Further the gradient of the variable part of the gravitational mass may form the gradient force. The gradient force can be used to capture the charged/neutral particles, and even the dark matter particles. And it is capable of explaining the bidirectional jets phenomenon of the accretion disc surrounding certain objects. The results reveal that the feature of the gravitational mass is different to that of the inertial mass. Strictly speaking, only in the case there is neither the electromagnetic strength nor the gravitational strength, the gravitational mass may equal to the inertial mass accurately. Therefore it is necessary to further validate the Eötvös experiment under the uniform and strong electromagnetic strength.

1. INTRODUCTION

In 1843 W. R. Hamilton invented the quaternion. Later J. T. Graves and A. Cayley introduced independently the octonion. In the twentieth century the scholars began to apply the complex quaternion/octonion to depict the electromagnetic theory [1], and even the gravitational theory [2]. In the twenty-first century the researches on these field theories have been inheriting and further developing. In the field theory described with the complex octonion, it is able to deduce directly the Maxwell's equations [3] and the force equilibrium equation, and refine further the inference that the gravitational mass varies with the fluctuation of field strength [4]. Moreover the energy gradient associated with the variable gravitational mass will impact the movement of the particle. The deduction can be applied to capture the low-speed neutral particle.

The field theory described with the complex octonion believes that the comparative strong gravitational and electromagnetic strength will evidently result in the fluctuation of gravitational mass. In the gravitational field, the gravitational acceleration will decrease the gravitational mass. In the electromagnetic field, the electric intensity increases the gravitational mass, while the magnetic flux density decreases that. It is similar to the mass difference amongst some particles in the strong nuclear field, including the proton/neutron, and the charged/neutral pi meson etc..

The gradient of the norm of field strength is one component of force, and may be called as the 'gradient force' for the moment. The gradient force will result in the accretion disc surrounding certain objects to emit jet bidirectionally [5]. For the magnetic flux density, the gradient force is attractive. For the electric intensity, the gradient force is repulsive. In the gravitational and electromagnetic fields, when the gradient force is attractive, the particle will be in repeating motion within one local region, which is similar to the repeating motion of the quark particle with the asymptotic freedom. When the gradient force is repulsive, the particle will be in repeating motion within one local region, which is located in between two face-and-face and collinear gradient forces. And it is similar to the repeating motion of charged particle in the magnetic mirror effect.

Up to now each Eötvös experiment [6] has only been validated in the weak gravitational field, but has never been tested in the strong gravitational field. Further the Eötvös experiment has never been tested in the electromagnetic field, and the high-speed situation. So the scholars never stop doubting the validity of Eötvös experiment. In the force equilibrium equation, the force includes the inertial force, gravity, energy gradient, and Lorentz force etc.. As long as the energy distribution is no uniform, the energy gradient, as one part of force, will impact the equilibrium relation of the inertial force and the gravity. Therefore it is necessary to further validate the Eötvös experiment under an electromagnetic environment.

2. FIELD EQUATIONS

In the quaternion space $\mathbb{E}_g$ for the gravitational field, the basis vector is $\mathbb{E}_g = (\boldsymbol{i}_0, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3)$, the radius vector is $\mathbb{R}_g = ir_0\boldsymbol{i}_0 + \Sigma r_k\boldsymbol{i}_k$, and the velocity is $\mathbb{V}_g = iv_0\boldsymbol{i}_0 + \Sigma v_k\boldsymbol{i}_k$. The gravitational potential is $\mathbb{A}_g = ia_0\boldsymbol{i}_0 + \Sigma a_k\boldsymbol{i}_k$, the gravitational strength is $\mathbb{B}_g = h_0\boldsymbol{i}_0 + \Sigma h_k\boldsymbol{i}_k$, and the gravitational source is $\mathbb{S}_g = is_0\boldsymbol{i}_0 + \Sigma s_k\boldsymbol{i}_k$. In the S-quaternion space $\mathbb{E}_e$ (short for the second quaternion space) for the electromagnetic field [7], the basis vector is $\mathbb{E}_e = (\boldsymbol{I}_0, \boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3)$, the radius vector is $\mathbb{R}_e = iR_0\boldsymbol{I}_0 + \Sigma R_k\boldsymbol{I}_k$, and the velocity is $\mathbb{V}_e = iV_0\boldsymbol{I}_0 + \Sigma V_k\boldsymbol{I}_k$. The electromagnetic potential is $\mathbb{A}_e = iA_0\boldsymbol{I}_0 + \Sigma A_k\boldsymbol{I}_k$, the electromagnetic strength is $\mathbb{B}_e = H_0\boldsymbol{I}_0 + \Sigma H_k\boldsymbol{I}_k$, and the electromagnetic source is $\mathbb{S}_e = iS_0\boldsymbol{I}_0 + \Sigma S_k\boldsymbol{I}_k$. Herein $\mathbb{E}_e = \mathbb{E}_g \circ \boldsymbol{I}_0$. The symbol $\circ$ denotes the octonion multiplication. r_j, v_j, a_j, s_j, R_j, V_j, A_j, S_j, h_0, and H_0 are all real. h_k and H_k are all complex numbers. i is the imaginary unit. $\boldsymbol{i}_0 = 1$, $\boldsymbol{i}_k^2 = -1$, $\boldsymbol{I}_j^2 = -1$. $j = 0, 1, 2, 3$. $k = 1, 2, 3$.

Two quaternion spaces, $\mathbb{E}_g$ and $\mathbb{E}_e$, may combine together to become one octonion space, $\mathbb{E} = \mathbb{E}_g + \mathbb{E}_e$. In the octonion space $\mathbb{E}$ for the electromagnetic and gravitational fields, the octonion radius vector is $\mathbb{R} = \mathbb{R}_g + k_{eg}\mathbb{R}_e$, the octonion velocity is $\mathbb{V} = \mathbb{V}_g + k_{eg}\mathbb{V}_e$, with k_{eg} being the coefficient. The octonion field potential is $\mathbb{A} = \mathbb{A}_g + k_{eg}\mathbb{A}_e$, the octonion field strength is $\mathbb{B} = \mathbb{B}_g + k_{eg}\mathbb{B}_e$. Apparently $\mathbb{V}$ and $\mathbb{A}$ etc. are all octonion functions of $\mathbb{R}$.

The octonion definition of field strength is,

$$\mathbb{B} = \Box \circ \mathbb{A}$$

where $\mathbb{B}_g = \Box \circ \mathbb{A}_g$, $\mathbb{B}_e = \Box \circ \mathbb{A}_e$. The quaternion operator is $\Box = i\boldsymbol{i}_0\partial_0 + \Sigma\boldsymbol{i}_k\partial_k$. $\nabla = \Sigma\boldsymbol{i}_k\partial_k$, $\partial_j = \partial/\partial r_j$. $v_0 = \partial r_0/\partial t$. v_0 is the speed of light, and t is the time.

In the quaternion space $\mathbb{E}_g$, the gauge condition is, $h_0 = -\partial_0 a_0 + \nabla \cdot \mathbf{a} = 0$. And then the $\mathbb{B}_g$ is reduced to $\mathbf{h} = i\mathbf{g}/v_0 + \mathbf{b}$. The gravitational acceleration is, $\mathbf{g}/v_0 = \partial_0\mathbf{a} + \nabla a_0$, while the other component of gravitational strength is $\mathbf{b} = \nabla \times \mathbf{a}$, which is similar to the magnetic flux density. In the S-quaternion space $\mathbb{E}_e$, the gauge condition is, $\mathbf{H}_0 = -\partial_0\mathbf{A}_0 + \nabla \cdot \mathbf{A} = 0$. Therefore the $\mathbb{B}_e$ is reduced to $\mathbf{H} = i\mathbf{E}/v_0 + \mathbf{B}$. The electric field intensity is $\mathbf{E}/v_0 = \partial_0\mathbf{A} + \nabla \circ \mathbf{A}_0$, while the magnetic flux density is $\mathbf{B} = \nabla \times \mathbf{A}$. Herein $\mathbf{a} = \Sigma a_k\boldsymbol{i}_k$, $\mathbf{h} = \Sigma h_k\boldsymbol{i}_k$. $\mathbf{A}_0 = A_0\boldsymbol{I}_0$, $\mathbf{A} = \Sigma A_k\boldsymbol{I}_k$. $\mathbf{H}_0 = H_0\boldsymbol{I}_0$, $\mathbf{H} = \Sigma H_k\boldsymbol{I}_k$.

The octonion field source $\mathbb{S}$ of the electromagnetic and gravitational fields can be defined as,

$$\mu\mathbb{S} = -(i\mathbb{B}/v_0 + \Box)^* \circ \mathbb{B} = \mu_g\mathbb{S}_g + k_{eg}\mu_e\mathbb{S}_e - (i\mathbb{B}/v_0)^* \circ \mathbb{B}, \tag{1}$$

where μ, μ_g, and μ_e are coefficients. $\mu_g < 0$, and $\mu_e > 0$. $*$ denotes the conjugation of octonion. In the case for single one particle, a comparison with the classical field theory reveals that, $\mathbb{S}_g = m\mathbb{V}_g$, and $\mathbb{S}_e = q\mathbb{V}_e$. m is the mass density, while q is the density of electric charge. For the charged particle, there is $\mathbb{V}_e = \mathbb{V}_g \circ \mathbf{I}(\boldsymbol{I}_j)$. The unit $\mathbf{I}(\boldsymbol{I}_j)$ is one function of $\boldsymbol{I}_j$, with $\mathbf{I}(\boldsymbol{I}_j)^* \circ \mathbf{I}(\boldsymbol{I}_j) = 1$.

Expanding of Eq. (1) is capable of deducing the Maxwell's equations in the classical electromagnetic theory, and the Newton's law of universal gravitation in the classical gravitational theory when $\mathbf{b} = 0$ and $\mathbf{a} = 0$.

Table 1: The multiplication of the operator and octonion physics quantity.

definition	expression meaning
$\nabla \cdot \mathbf{a}$	$-(\partial_1 a_1 + \partial_2 a_2 + \partial_3 a_3)$
$\nabla \times \mathbf{a}$	$\boldsymbol{i}_1(\partial_2 a_3 - \partial_3 a_2) + \boldsymbol{i}_2(\partial_3 a_1 - \partial_1 a_3) + \boldsymbol{i}_3(\partial_1 a_2 - \partial_2 a_1)$
∇a_0	$\boldsymbol{i}_1\partial_1 a_0 + \boldsymbol{i}_2\partial_2 a_0 + \boldsymbol{i}_3\partial_3 a_0$
$\partial_0\mathbf{a}$	$\boldsymbol{i}_1\partial_0 a_1 + \boldsymbol{i}_2\partial_0 a_2 + \boldsymbol{i}_3\partial_0 a_3$
$\nabla \cdot \mathbf{A}$	$-(\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3)\boldsymbol{I}_0$
$\nabla \times \mathbf{A}$	$-\boldsymbol{I}_1(\partial_2 A_3 - \partial_3 A_2) - \boldsymbol{I}_2(\partial_3 A_1 - \partial_1 A_3) - \boldsymbol{I}_3(\partial_1 A_2 - \partial_2 A_1)$
$\nabla \circ \mathbf{A}_0$	$\boldsymbol{I}_1\partial_1 A_0 + \boldsymbol{I}_2\partial_2 A_0 + \boldsymbol{I}_3\partial_3 A_0$
$\partial_0\mathbf{A}$	$\boldsymbol{I}_1\partial_0 A_1 + \boldsymbol{I}_2\partial_0 A_2 + \boldsymbol{I}_3\partial_0 A_3$

3. OCTONION FORCE

In the octonion space, the octonion linear momentum density $\mathbb{P}$ is defined from the octonion field source $\mathbb{S}$, and can be written as follows,

$$\mathbb{P} = \mu\mathbb{S}/\mu_g,$$

where $\mathbb{P} = \mathbb{P}_g + k_{eg}\mathbb{P}_e$. $\mathbb{P}_g = \{\mu_g \mathbb{S}_g - (i\mathbb{B}/v_0)^* \circ \mathbb{B}\}/\mu_g$, and $\mathbb{P}_e = \mu_e \mathbb{S}_e/\mu_g$.

From the above, one can define the octonion angular momentum, $\mathbb{L} = (\mathbb{R} + k_{rx}\mathbb{X})^{\times} \circ \mathbb{P}$, the octonion torque $\mathbb{W}$, and octonion force $\mathbb{N}$ in the Table 2. Herein $\mathbb{X}$ is the integral function of field potential $\mathbb{A}$, and $\times$ denotes the conjugation of complex number, with k_{rx} being the coefficient.

If it is able to neglect the contribution of some tiny terms, the octonion torque will be reduced to, $\mathbb{W} \approx (2\mathbb{P}v_0)$, when $(\Sigma r_k \boldsymbol{i}_k)$ is one three-dimensional vector. And then the octonion force is written as follows,

$$\begin{aligned}\mathbb{N}/2 = & -v_0(i\mathbb{B}_g \circ \mathbb{P}_g/v_0 + ik_{eg}^2 \mathbb{B}_e \circ \mathbb{P}_e/v_0 + \Box \circ \mathbb{P}_g) \\ & -k_{eg}v_0(i\mathbb{B}_g \circ \mathbb{P}_e/v_0 + i\mathbb{B}_e \circ \mathbb{P}_g/v_0 + \Box \circ \mathbb{P}_e),\end{aligned}$$

where $\mathbb{N}_g/2 = -v_0(i\mathbb{B}_g \circ \mathbb{P}_g/v_0 + ik_{eg}^2 \mathbb{B}_e \circ \mathbb{P}_e/v_0 + \Box \circ \mathbb{P}_g)$ stays on the quaternion space $\mathbb{E}_g$, including the inertial force, gravity, and Lorentz force etc.. $\mathbb{N}_e/2 = -v_0(i\mathbb{B}_g \circ \mathbb{P}_e/v_0 + i\mathbb{B}_e \circ \mathbb{P}_g/v_0 + \Box \circ \mathbb{P}_e)$ situates on the S-quaternion space $\mathbb{E}_e$, and is the interacting term between the gravitational field and electromagnetic field, including the current continuity equation etc..

In the quaternion space $\mathbb{E}_g$, when there are the gravitational and electromagnetic fields, the force term $\mathbb{N}_g$ can be expressed as

$$N_{10}^i/2 = -\mathbf{b} \cdot \mathbf{p} - k_{eg}^2(\mathbf{B} \cdot \mathbf{P}), \tag{2}$$

$$N_{10}/2 = v_0\partial_0 p_0 - v_0 \nabla \cdot \mathbf{p} + \mathbf{g} \cdot \mathbf{p}/v_0 + k_{eg}^2(\mathbf{E} \cdot \mathbf{P}/v_0), \tag{3}$$

$$\mathbf{N}_1^i/2 = -v_0\partial_0 \mathbf{p} + p_0\mathbf{g}/v_0 - v_0\nabla p_0 - \mathbf{b} \times \mathbf{p} + k_{eg}^2(\mathbf{E} \circ \mathbf{P}_0/v_0 - \mathbf{B} \times \mathbf{P}), \tag{4}$$

$$\mathbf{N}_1/2 = -v_0 \nabla \times \mathbf{p} + \mathbf{g} \times \mathbf{p}/v_0 + p_0\mathbf{b} + k_{eg}^2(\mathbf{E} \times \mathbf{P}/v_0 + \mathbf{B} \circ \mathbf{P}_0), \tag{5}$$

where $\mathbb{N}_g = iN_{10}^i + N_{10} + i\mathbf{N}_1^i + \mathbf{N}_1$. $\mathbb{P}_g = ip_0 + \mathbf{p}$. $\mathbf{p} = \Sigma p_k \boldsymbol{i}_k$. $\mathbb{P}_e = i\mathbf{P}_0 + \mathbf{P}$. $\mathbf{P} = \Sigma P_k \boldsymbol{I}_k$. $\mathbf{P}_0 = P_0 \boldsymbol{I}_0$. $\mathbf{N}_1 = \Sigma N_{1k} \boldsymbol{i}_k$, $\mathbf{N}_1^i = \Sigma N_{1k}^i \boldsymbol{i}_k$. P_j, p_j, N_{1j}, and N_{1j}^i are all real.

When $\mathbb{N}_g = 0$, the equation for force equilibrium is yielded from $\mathbf{N}_1^i = 0$. The mass continuity equation is derived from $N_{10} = 0$, and it will be reduced to that in the classical field theory when there is no field strength. The strength components $\mathbf{b}$ and $\mathbf{B}$ will impact the current helicity of charged particle according to $N_{10}^i = 0$. While $\mathbf{N}_1 = 0$ can deduce the curl of toque component $\mathbf{p}v_0$. And it is capable to infer the angular velocity of Larmor precession with respect to the orbital angular momentum of charged particle. The equation $\mathbf{N}_1^i = 0$ means that the gravitational acceleration $\mathbf{g}$ is the counterpart of the linear acceleration, $v_0\partial_0\mathbf{v}$. The equation $\mathbf{N}_1 = 0$ states that the component $\mathbf{b}$ is the counterpart of the velocity curl, $\nabla \times \mathbf{v}$, which is the double of the precessional angular velocity when the dimension of vector $\mathbf{r}$ is 2.

The force $\mathbf{N}_1^i$ consists of the inertial force, gravity, Lorentz force, and proper energy gradient etc.. Comparing with the force density in the classical field theory states that $k_{eg}^2 = \mu_g/\mu_e < 0$ and $k_{rx} = 1/v_0$. The inertial force $(-v_0\partial_0\mathbf{p})$ claims that the gravitational field must situate on the quaternion space $\mathbb{E}_g$, while the electromagnetic field has to stay on the quaternion space $\mathbb{E}_e$, but not vice versa.

4. VARIABLE GRAVITATIONAL MASS

In the electromagnetic and gravitational fields, the inertial mass density is m, and the gravitational mass density is $m_g = m + m'$, with m' being $\{-\mathbb{B}^* \circ \mathbb{B}/(\mu_g v_0^2)\}$. Obviously the m_g will fluctuate with the alteration of the electromagnetic strength $\mathbb{B}_e$ and of gravitational strength $\mathbb{B}_g$. According to Eq.(4), the variation of gravitational mass density is,

$$m' = -(\mathbf{b}^* \cdot \mathbf{b} - \mathbf{g}^* \cdot \mathbf{g}/v_0^2)/(\mu_g v_0^2) - (\mathbf{B}^* \cdot \mathbf{B} - \mathbf{E}^* \cdot \mathbf{E}/v_0^2)/(\mu_e v_0^2)\ . \tag{6}$$

Comparative strong field strength will impact the fluctuation of gravitational mass. Apparently the magnetic flux density $\mathbf{B}$ or the gravitational acceleration $\mathbf{g}$ will cause the decrease of gravitational mass. While the electric field intensity $\mathbf{E}$ or the gravitational strength component $\mathbf{b}$ will result in the increase of gravitational mass.

On the basis of the existing Eötvös experiments, the application of comparative strong electromagnetic strength will impact the ratio of these two masses to a certain extent. This test scheme can be considered as one direct validation experiment for the variable gravitational mass. One of inferences about the variable gravitational mass is the gradient force, which can be regarded as the indirect evidence for the variable gravitational mass.

According to Eq. (4), when $\nabla m = 0$, the gradient force can be written as,

$$\mathbf{N}_B/2 = -v_0 \nabla p_0 = -v_0^2 \nabla (m + m') = \nabla (\mathbb{B}^* \circ \mathbb{B}/\mu_g) \ . \tag{7}$$

In the electromagnetic and gravitational fields, the gradient force is in a directly proportional to the gradient of the norm of field strength. And the gradient force is independent to not only the mass and electric charge of the test particle, but also the direction of field strength. Either the steep gradient of field strength or the strong field strength can result in the prominent gradient force. It can be used to impel the bulk flow of the plasma, and drive/capture the particles of the normal matter and even of the dark matter.

In the electromagnetic and gravitational fields, four components of field strength are capable of impacting the gradient force.

(1) For the magnetic flux density **B** or the gravitational acceleration **g**, the gradient force has the attraction effect. And it will arouse the repeating motion of the particle within one local region, which is similar to the reciprocating motion of the small object fixed on the end of the spring.

(2) For the electric field intensity **E** or the gravitational strength component **b**, the gradient force has the repulsion effect. Also it will cause repeating motion of the particle within one local region, which is similar to the repeating motion of charged particle in the magnetic mirror effect.

Specially the distribution of the gravitational strength component **b** is similar to that of the magnetic flux density **B**. Because of the closing feature of **b** force-line, the magnitude of **b** within the central region of galaxy will increase drastically. According to the above analysis, the galaxy center may emit the jet and counter-jet along the precessional axis associated with **b**. When the axis of total angular velocity keeps still, the precessional axis will circle around the axis of total angular velocity. Such as the Cygnus X-1, Messier 87, SS433, and neutron star etc..

Table 2: Some definitions of the physics quantity in the gravitational and electromagnetic fields described with the complex quaternion/octonion spaces.

physics quantity	definition	note
radius vector	$\mathbb{R} = \mathbb{R}_g + k_{eg}\mathbb{R}_e$	$k_{eg}^2 = \mu_g/\mu_e < 0$
integral function	$\mathbb{X} = \mathbb{X}_g + k_{eg}\mathbb{X}_e$	
field potential	$\mathbb{A} = i\square^\times \circ \mathbb{X}$	
field strength	$\mathbb{B} = \square \circ \mathbb{A}$	gauge condition
field source	$\mu\mathbb{S} = -(i\mathbb{B}/v_0 + \square)^* \circ \mathbb{B}$	field equations
linear momentum	$\mathbb{P} = \mu\mathbb{S}/\mu_g$	
angular momentum	$\mathbb{L} = (\mathbb{R} + k_{rx}\mathbb{X})^\times \circ \mathbb{P}$	$k_{rx} = 1/v_0$
octonion torque	$\mathbb{W} = -v_0(i\mathbb{B}/v_0 + \square) \circ \mathbb{L}$	torque, work, energy
octonion force	$\mathbb{N} = -(i\mathbb{B}/v_0 + \square) \circ \mathbb{W}$	power, force

5. CONCLUSIONS

For a long time, the scholars never stop doubting the validity of Eötvös experiment. Up to now each Eötvös experiment has only been validated in the weak gravitational field. And it has never been tested in the strong gravitational field, or in the strong electromagnetic field, or in the high-speed situation. Apparently the Eötvös experiment should consider more influence factors, including the uniform and comparative strong electromagnetic strength etc..

The application of the complex quaternion approach can depict the classical electromagnetic and gravitational theories. Further it is able to predict some inferences, including the variable gravitational mass, gradient force, and repeating motion within one local region etc.. In the description method of complex quaternion, the variation of gravitational mass is related with both of the electromagnetic strength and the gravitational strength. And that the gradient of the norm of electromagnetic strength and of gravitational strength yield the gradient force. The force is independent to not only the direction of field strength, but also the mass or electric charge of the test particle. The gradient force can drive/capture several kinds of particles, and confine the motion of test particles within the local region. And it is capable of explaining the jet phenomenon of accretion disc surrounding certain objects along the precessional axis.

The paper discussed the variability of gravitational mass in the electromagnetic and gravitational fields, and some simple applications of gradient force. And it expatiated on distinctly some major influence factors to alter the gravitational mass, and the application range of the variable gravitational mass etc.. It set the foundation for the further research about the gravitational mass. In the following study, we plan, (1) to validate the Eötvös experiment within some comparative strong uniform magnetic fields; (2) to observe the repeating motion of the escaping particles near the ends of magnetic mirror, based on the existing experiments of magnetic mirror; (3) to apply the gradient force to capture particles of the normal matter, and even of the dark matter.

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