Imaging Dielectric Objects by Limited Diversity of Scattering Data

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Abstract— In the reconstruction of dielectric objects by integral equation approach (IEA), the forward scattering integral equation (FSIE) and the inverse scattering integral equation (ISIE) are alternatively solved in the frame of distorted Born iterative method (DBIM) or its variations. In this work, we consider the reconstruction under a limited observation which is more practical. We use a DBIM-like Gauss-Newton minimization approach (GNMA) incorporated with a multiplicative regularization method (MRM) and a line search method (LSM) to solve the ISIE and the inversion quality of scattering data can be enhanced. The numerical example for reconstructing a two-dimensional (2D) dielectric object is presented to illustrate the inversion approach.

1. INTRODUCTION

Solving inverse scattering problems is challenging because the involved governing equations are nonlinear and the solutions are inherently nonunique [1]. This is particularly true when measuring conditions are unfavorable and diverse scattering data are not available. Usually, the inverse problems are solved by linearizing governing equations and gradually minimizing the mismatch between calculated data and measured data in an iterative scheme. In the integral equation approach (IEA) for reconstructing dielectric objects, there are forward scattering integral equation (FSIE) and inverse scattering integral equation (ISIE) and we need to alternatively solve them in the context of Born iterative method (BIM) [2] or distorted Born iterative method (DBIM) [3] although other methods can also be used [4]. The solution of permittivity from the ISIE can reveal the profile of unknown object in the imaging domain.

The ISIE is insolvable by direct inversion because the matrix equation is inherently ill-posed and the diversity of measured data with noise contamination is usually limited. One has to transform the inversion of measured data into an optimization problem and the best solution is chosen by minimizing a cost functional. The optimization problem can be solved by different approaches and we apply the DBIM-like Gauss-Newton minimization approach (GNMA) in this work [5] since it can be conveniently incorporated with the multiplicative regularization method (MRM) [6] and line search method (LSM) [7] to enhance the inversion quality. The MRM was proposed for gradient-type algorithms and was used for the inversion by the finite difference time domain (FDTD) approach [8], but we use it for the IEA in this work. The MRM can adaptively vary the regularization parameter as the iteration proceeds so it is unnecessary to perform a large number of numerical experiments for choosing the regularization parameter. We also incorporate the LSM which can optimize the search of step size in each iteration and accelerate the convergence of inversion process. The use of MRM and LSM is essential when we consider the reconstruction with a poor measurement condition which may be encountered in many applications [9]. A numerical example for reconstructing a two-dimensional (2D) dielectric object with a limited view is presented to illustrate the approach.

2. GOVERNING EQUATIONS

In the IEA, reconstructing an unknown dielectric object is to determine its profile and material property by solving the ISIE and FSIE alternatively in the context of the DBIM or its variations. We only consider the simplified 2D cases in this work and the generalized 3D cases will be addressed in our future work. The ISIE and FSIE in the 2D case with a TM_z incident wave can be written as [1]

$$E_z^{sca}(\boldsymbol{\rho}) = \int_S g(\boldsymbol{\rho}, \boldsymbol{\rho}'; k_b) \cdot \Delta k(\boldsymbol{\rho}') E_z(\boldsymbol{\rho}')\, dS' \tag{1}$$

$$E_z(\boldsymbol{\rho}) = E_z^{inc}(\boldsymbol{\rho}) + \int_S g(\boldsymbol{\rho}, \boldsymbol{\rho}'; k_b) \cdot \Delta k(\boldsymbol{\rho}') E_z(\boldsymbol{\rho}')\, dS' \tag{2}$$

respectively. In the above, $\boldsymbol{\rho}$ is the position vector of an observation point, $\boldsymbol{\rho}'$ is the position vector of a source point, E_z^{inc} is the incident electric field, E_z^{sca} is the scattered electric field obtained by measurement, and E_z is the total electric field inside the imaging domain S. The imaging domain enclosing the unknown object is in the xy plane. Also,

$$\Delta k(\boldsymbol{\rho}') = k^2(\boldsymbol{\rho}') - k_b^2 = k_b^2[\varepsilon_r(\boldsymbol{\rho}') - 1] \tag{3}$$

is the contrast or difference between the squared wavenumber k^2 of the imaging domain and the squared wavenumber k_b^2 of the background. The contrast can determine the distribution of permittivity inside the imaging domain when its material is assumed to be nonmagnetic. In addition,

$$g(\boldsymbol{\rho}, \boldsymbol{\rho}'; k_b) = \frac{i}{4} H_0^{(1)}(k_b|\boldsymbol{\rho} - \boldsymbol{\rho}'|) \tag{4}$$

is the 2D Green's function in which $H_0^{(1)}$ is the first kind of Hankel function with the zeroth order. The above FSIE can be easily solved and we do not address it here.

3. GAUSS-NEWTON MINIMIZATION APPROACH FOR SOLVING THE ISIE

To solve the ISIE by the GNMA, we transform the inversion of model parameters into an optimization problem in which the following cost functional $F(\mathbf{x})$ is minimized [5]

$$F(\mathbf{x}) = \frac{1}{2}\left\{\gamma\left[\left|\left|\overline{\mathbf{V}}_d \cdot \mathbf{e}(\mathbf{x})\right|\right|^2 - \delta^2\right] + \left|\left|\overline{\mathbf{V}}_m \cdot (\mathbf{x} - \mathbf{x}_q)\right|\right|^2\right\} \tag{5}$$

where $\mathbf{x}$ is a vector describing the model parameters such as the distribution of permittivity in the imaging domain and $\mathbf{e}(\mathbf{x})$ is the error vector reflecting the mismatch between the predicted or calculated data (scattered electric field) and the measured data. Also, γ is a Lagrange multiplier whose inverse is the regularization parameter, δ is *a priori* estimate of noise in the measured data, $\overline{\mathbf{V}}_m^T \cdot \overline{\mathbf{V}}_m$ is the inverse of the model covariance matrix representing the degree of confidence in the prescribed model parameters $\mathbf{x}_q$, and $\overline{\mathbf{V}}_d^T \cdot \overline{\mathbf{V}}_d$ is the inverse of the data covariance matrix describing the estimated uncertainty due to the noise contamination in the measured data. The above cost functional can be minimized with the GNMA whose details can be found in [5]. In the GNMA, we have employed the MRM to select the regularization parameter which is the inverse of Lagrange multiplier γ. The MRM can adaptively vary γ as the iteration proceeds so that a large number of numerical experiments for selecting the parameter can be avoided. The MRM treats the regularization as a multiplicative factor in the cost functional and therefore the regularization parameter is set to be proportional to the original or non-regularized cost functional, i.e.,

$$\frac{1}{\gamma} = \frac{1}{2\eta^2}\left|\left|\overline{\mathbf{V}}_d \cdot \mathbf{e}(\mathbf{x})\right|\right|^2 \tag{6}$$

where η is a constant parameter determined by numerical experiments. Note that the reconstructed results are much more insensitive to η than to γ and η can be fixed in all simulations for one type of measured data [5]. To accelerate the reconstruction, we can use a line search method (LSM) [7] to optimize the step size. The LSM searches a real positive factor ζ_k along the search direction $\mathbf{b}_k$ so that the new step is changed into $\mathbf{x}_{k+1} = \mathbf{x}_k + \zeta_k \mathbf{b}_k$ which can result in a sufficient reduction for the cost functional.

4. NUMERICAL EXAMPLE

To validate the inversion approach, we consider the reconstruction for two 2D dielectric circular cylinders which are contacting and identical in a $1.4\lambda \times 1.4\lambda$ imaging domain where λ is the wavelength in free space. Each cylinder has a radius $a = 0.3\lambda$ and a homogeneous relative permittivity $\varepsilon_r = 4.5$. Unlike the widely-adopted experimental setup of Institut Fresnel which allows a full view to the reconstructed object [10], we consider a limited view with less diverse measurement data in this work. We use 10 TM_z incident waves from the transmitter located along the line on a plane above the imaging domain to illuminate the object and the frequency is $f = 3.0\,\mathrm{GHz}$. The vertical distance between the line and the center of imaging domain is 20λ and the spacing between two neighboring transmitting positions is 0.5λ. We take 50 observation positions which are equally distributed along the same line to measure the scattered electric field (the measured data is replaced with the calculated data in simulations). Due to the lack of *a priori* information, we choose $\delta = 0$ and $\overline{\mathbf{V}}_d = \overline{\mathbf{V}}_m = \bar{\mathbf{I}}$. Figure 1(a) shows the reconstructed permittivity by the Born iterative method

(BIM) at the 40th step while Figure 1(b) and Figure 1(c) show the results by the GNMA at the 30th step and the 40th step, respectively. It can be seen that the GNMA images resemble the original distribution as shown in Figure 1(d) whereas the BIM cannot give a convergent result. Note that the discontinuity of the original distribution can only be reconstructed by using certain edge-preserving technique [5].

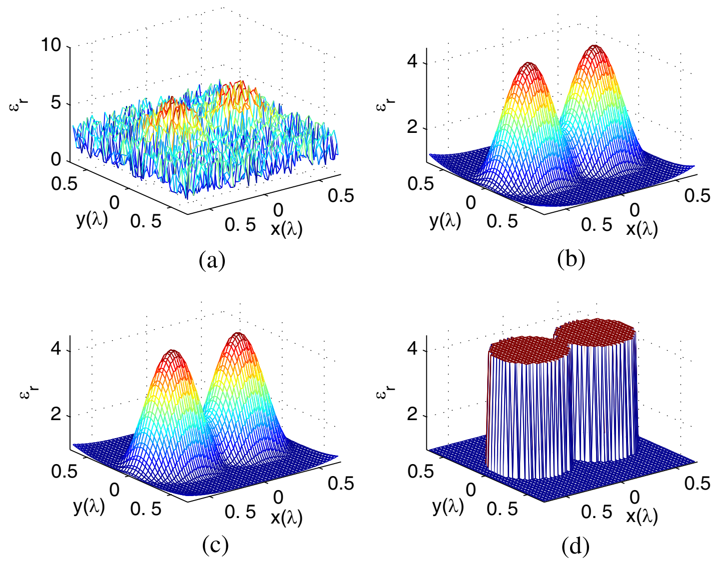

Figure 1: Reconstruction of permittivity for two 2D dielectric circular cylinders which are contacting and identical. Each cylinder has a radius $a = 0.3\lambda$ and a relative permittivity $\varepsilon_r = 4.5$. (a) BIM at the 40th step. (b) GNMA at the 30th step. (c) GNMA at the 40th step. (d) Original distribution.

5. CONCLUSION

Obtaining diverse scattering data could be difficult due to unfavorable measurement conditions in practice. In this work, we consider the inversion of scattering data under a limited observation and employ the GNMA to minimize the cost functional by the IEA. The scheme incorporates the MRM to adaptively select the regularization parameter and the LSM to optimize the step size or search direction. The MRM can avoid a large number of numerical experiments in the selection of regularization parameter while the LSM can accelerate the convergence of inversion process. A numerical example for reconstructing 2D dielectric objects has been presented to demonstrate the inversion approach and good result can be observed.

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