Analysis of Conditions on the Boundary between Layers

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Abstract— The authors report on an analysis of conditions on the boundary between layers having varied electromagnetic properties. The research is performed using consistent theoretical derivation of analytical formulas, and the underlying problem is considered also in view of multiple boundaries, including the effect of the propagation of electromagnetic waves exhibiting different instantaneous speed.

The paper includes a theoretical analysis and references to the generated algorithms. The main algorithm was assembled to enable simple evaluation of all components of the electromagnetic field in relation to the speed of the wave propagation in a heterogeneous environment. The proposed algorithms are compared by means of different numerical methods for the modelling of electromagnetic waves on the boundary between materials; moreover, electromagnetic field components in common points of the model are also subject to comparison.

When in conjunction with tools facilitating the analysis of material response to the source of a continuous signal, the algorithms constitute a supplementary instrument for the design of a layered material. Such design enables the realization of, for example, recoilless plane, recoilless transition between different types of environment, and filters for both optical and radio frequencies. This phenomenon occurs in metamaterials.

1. INTRODUCTION

Inhomogeneities and regions with different parameters generally appear even in the cleanest materials. During the passage of an electromagnetic wave through a material, we can observe amplitude decrease and wave phase shift. These phenomena are due to the concrete material characteristics, such as conductivity, permittivity, or permeability [1]. If a wave impinges on an inhomogeneity, there occurs a change in its propagation. The change manifests itself in two forms, namely in reflection and refraction. In addition to this process, polarisation and interference may appear in these waves [2].

In the Matlab program, algorithms were created that simulate reflection and refraction in a lossy environment on the boundary between two dielectrics. The reflection and refraction are in accordance with Snell's law for electromagnetic waves, as shown in Fig. 1(a).

The interpretation of the propagation of electromagnetic waves on a layered heterogeneous medium is expressed by the formula

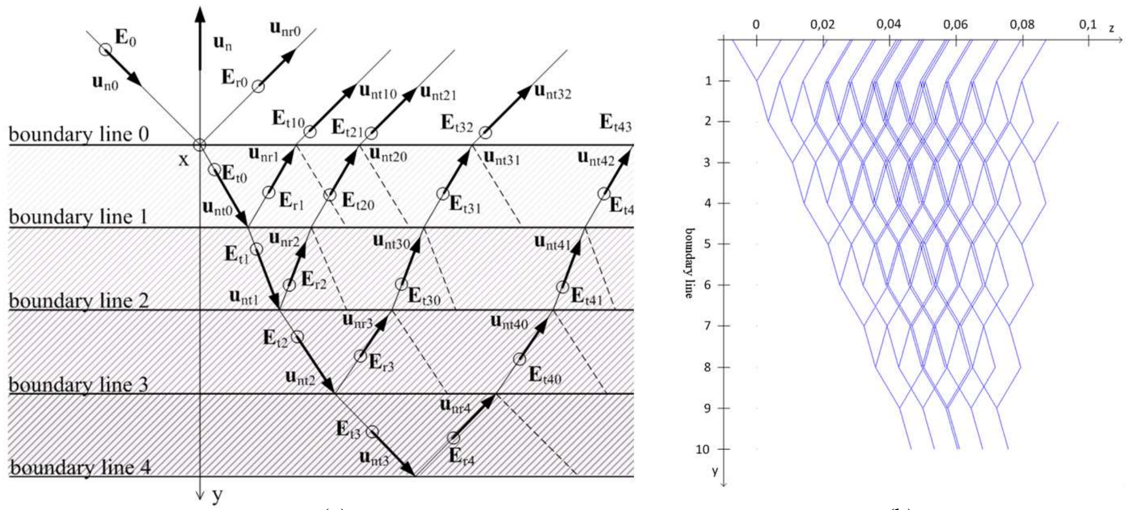
$$\mathbf{E}_{\mathrm{r}l} = \mathbf{E}_{\mathrm{i}l}\rho_{\mathrm{E}l} \cdot \mathrm{e}^{-\mathrm{j}\mathbf{k}_l \mathbf{u}_{\mathrm{nr}l} \times \mathbf{r}_l}, \quad \mathbf{E}_{\mathrm{t}l} = \mathbf{E}_{\mathrm{i}l}\tau_{\mathrm{E}l} \cdot \mathrm{e}^{-\mathrm{j}\mathbf{k}_l \mathbf{u}_{\mathrm{nt}l} \times \mathbf{r}_l}, \tag{1}$$

where $\mathbf{E}_{\mathrm{r}l}$ and $\mathbf{E}_{\mathrm{t}l}$ are the reflection and refraction of the electromagnetic waves on the boundary line ($l = 1, \ldots, \max$) according to Fig. 1(a), $\mathbf{E}_{\mathrm{i}l}$ is the amplitude electric field strength on the boundary line l, $\rho_{\mathrm{E}l}$ and $\tau_{\mathrm{E}l}$ are the reflection coefficient and transmission factor on the boundary line l, $\mathbf{k}_l$ is the wave number of the layer, $\mathbf{r}_l$ is the electromagnetic wave positional vector on the boundary line l, $\mathbf{u}_{\mathrm{nt}l}$ and $\mathbf{u}_{\mathrm{nr}l}$ are the unit vectors of the propagation direction.

2. OBLIQUE INCIDENT WAVE ON A LAYERED MEDIUM

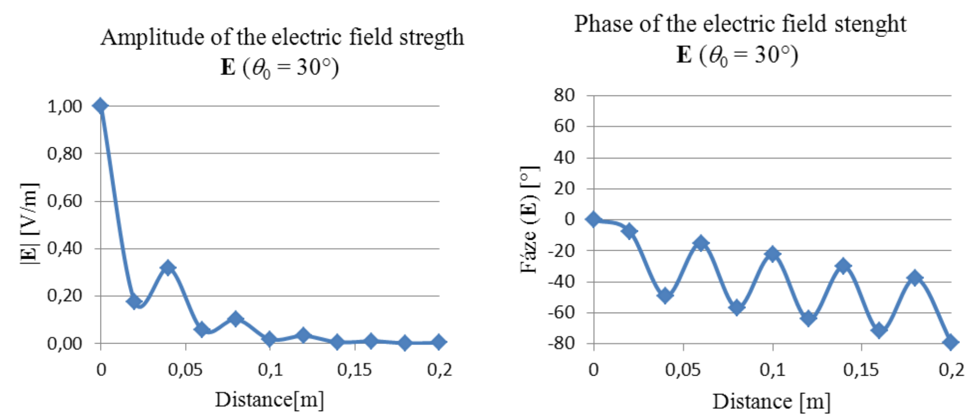
The Matlab-based analysis using Equation (1) was performed in the planar layers. Figs. 2 and 3 show the response of the environment of 5 layers with the same thickness $d = 20$ mm. During the transmission, the wave at the frequency of 1.5 GHz passes through a material 1 with parameters $\varepsilon_{r1} = 1$, $\mu_{r1} = 1$ and $\gamma_1 = 1 \cdot 10^{-9}$ S/m, where the wave is reflected and refracted. Furthermore, the wave propagates through material 2 with parameters $\varepsilon_{r2} = 81.6$, $\mu_{r2} = 0.999991$, and $\gamma_2 = 4.405 \cdot 10^{-9}$ S/m. The incidence of the wave on boundary 1 is at the angle of θ_0. The described configuration is according to Fig. 1(a). The selection of the material parameters is only of testing character, and thus it does not have any special meaning.

Figure 2 presents the response of the separate layers to the primary refraction electromagnetic wave. The response of the material is the response to an electromagnetic wave of the pulse source.



(a) (b)

Figure 1: The reflection and refraction of the electric component of an electromagnetic wave on a layered medium: (a) layout, (b) Matlab for 2000 cycles.



(a) (b)

Figure 2: The strength of the electric component of the TE wave on a layered medium at the angle $\theta_0 = 30°$.

Fig. 2, however, does not show the interferences for a more descriptive interpretation of the attenuation of the electromagnetic waves. However, the algorithm evaluates all of the reflected and refracted waves.

The results of the analysis shown in the images consist in the modules of the electric field strength **E** along the boundary between the first layer and the external environment. To a certain extent, these values can be compared with already known methods for the analysis of the electromagnetic field. The analysis of the response to the incidence of an EMG wave on the surface of a multilayered environment within the given time interval is presented in Fig. 3; here, the module of the electric field strength **E** is introduced depending on the plane leading from the center of the model towards its edge. The distribution of the modules of the electric field strength **E** on the surface of the material is obvious from the waveform of the EMG waves propagating through and reflecting from a multilayered material, as shown in Fig. 1(b).

3. FEM MODEL OF A LAYERED MEDIUM

In order to verify the properties of the analytical model of an accurate evaluation of wave propagation in a layered environment, we used another model. This numerical approach utilised the finite element method (FEM) and exhibited the same properties as the previous model. As the mathematical expression, we applied the enhanced wave equation for a lossy environment:

$$\nabla^2 u + f\frac{\partial u}{\partial t} + g\frac{\partial^2 u}{\partial t^2} - f_c(x,y,z,t) = 0 \ \forall g(x,y,z) \neq 0,\ \forall f(x,y,z) \neq 0,\ \text{in}\,\Omega \qquad (2)$$

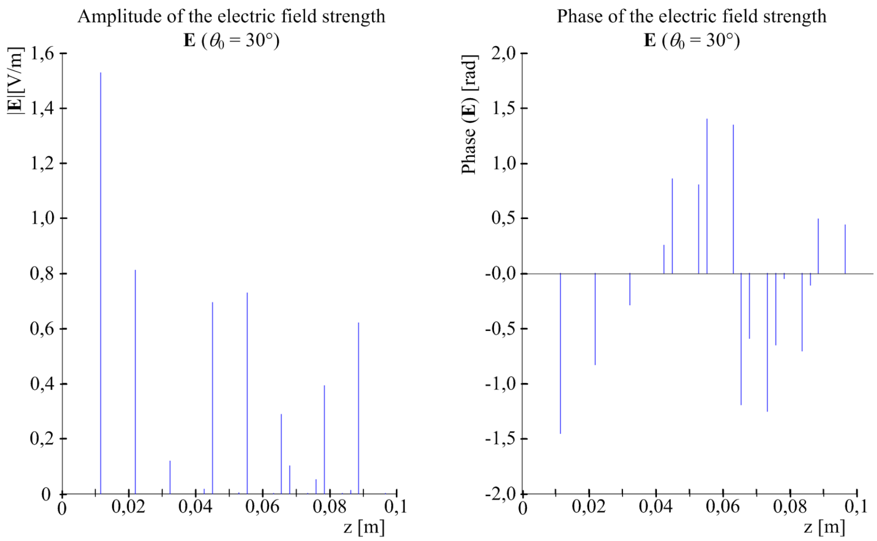
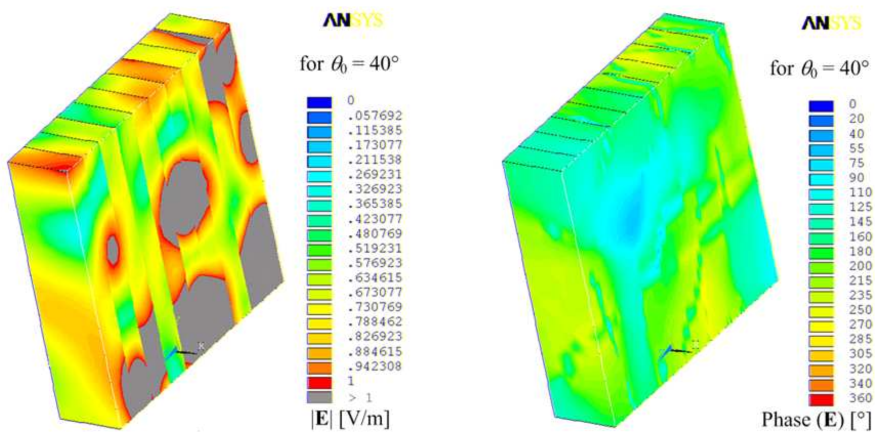


Figure 3: The strength of the electric component of the TE wave on a layered medium at the angle $\theta_0 = 30°$.



(a) (b)

Figure 4: The distribution of the electric field strength **E**: (a) amplitude, and (b) phase for $\theta_0 = 40°$.

where u is the searched functional, f a function of the electromagnetic wave damping, g a function of the electromagnetic wave excitation, f_c a function of the lossy environment, Ω the defining domain of variables and functions.

The distribution of the electric field **E** and its phase ϕ are both shown in Fig. 4. The characteristics in Fig. 4 correspond to the axis direction perpendicular to the layer plane.

4. COMPARISON OF BOTH MODELS

Direct comparison of the different analysis results obtained via the applied methods can be performed only with substantial difficulty. For this reason, we designed algorithms to evaluate the selected time intervals in the ray-tracing model. The evaluation of the module of the electric field strength **E** on the surface of the material at these time intervals is indicated in Fig. 3. A continuous electromagnetic wave was generated by the source. A suitably selected time interval of the medium response expressed by the maximum values of the electromagnetic field strength facilitates verification by the instantaneous values obtained via the applied finite element method. Fig. 5 shows that the results acquired through both analyses are comparable.

The EMAG module of ANSYS is not a convenient tool for the evaluation of either the pulse process or the maximum values of an electromagnetic wave in a heterogeneous environment. This drawback stems from the fact that the EMAG module is characterised by multiple interferences and various wave propagation velocities; consequently, the behaviour of the electromagnetic field strength phase is not uniform. And if already used parameters are set, the number of the discretisa-

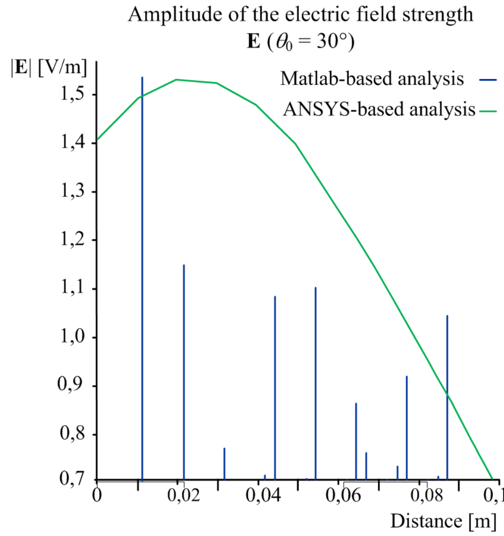


Figure 5: The diagram of the distribution of the electric field strength $\mathbf{E}$: for $\theta_0 = 30°$, on the surface of the material, and in the plane of incidence.

tion mesh divisions in the ANSYS-based model will be unfavourable with respect to the wavelength of the propagating wave. For comparison purposes, however, this negative effect is still acceptable. Consequently, the resulting values (and thus also comparison possibilities) are restricted.

5. CONCLUSION

The article presents a comparison of two approaches to the analysis of wave propagation in a layered material structure. The first of these techniques is based on of the propagation of electromagnetic waves as defined by analytical formulas (1) and was solved in the Matlab program. The second approach exploits the FEM applied to the wave equation. The FEM-based solution was conducted using the ANSYS system. The actual comparison of the results provided by both approaches cannot be performed directly.

Numerical modelling carried out by means of the wave equation and ANSYS produces a continuous source of electromagnetic waves. Interference effects between the reflected and refracted waves arise on the boundary between the layers. Moreover, the interference process is also entered by the time-delayed waves from the source and by interface reflections [4].

A complex form of the angle of reflection and refraction is considered in the case of the analytical approach, which allows accurate determination of the size and intensity of the reflected and refracted waves. In order to determine the direction of the constant phase propagation in a lossy medium, we consider only real parts of the wave number. The analytical solution and its algorithms process the time-varying phenomena of the pulsed source. The analytical solution includes the effect of time-dependent propagation of electromagnetic waves in a heterogeneous medium, and it results in the distribution of the electromagnetic field at the boundaries during certain moments of time.

The results of the ANSYS-based analysis of the propagation of electromagnetic waves in a material in the time-domain corresponds to the resulting distribution of the superimposed field intensities on individual boundaries of the analytical model. The approach using the finite element method in ANSYS is limited to spatial distribution of quantities with distributed parameters. This solution, despite being very robust in the time domain, is not suitable for multiple layers due to the method of the element mesh division. In FEM-based numerical models, the number of divisions of the discretised mesh can be determined only with difficulty (considering the wavelength of the propagating wave), and the solution of a large model by currently available means is almost impossible. Importantly, this drawback can be eliminated via the proposed method. The analysis of the pulsed source of the electromagnetic wave by means of an analytical model provides expectable results in heterogeneous structures since the behavior of phase change is uniform. The analytical solution allows us to analyze the response of the material in detail for the individual parts.

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