Bandwidth Limitations and Trade-off Relations for Wide- and Multi-band Array Antennas over a Ground Plane

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Abstract— There has been a recent drive to find fundamental limitations of bandwidth performance. Such results include Rozanovs (2000) bandwidth performance of absorbers, and Gustafssons (2007) limitation for single port antennas, similar bounds on transmission coefficients through a frequency selective screen and high-impedance surfaces are also known. In the present work we show a related bound for array antennas over a ground plane. We illustrate how this bound can be formulated as an array figure of merit, weighting impedance bandwidth, return loss, scan range and material and size information against each other.

The derivation of the array figure of merit is based on Bode-Fano theory for a scattering passive object, and is related to the Rozanov bound of absorbers. This result is essentially a sum-rule result that only requires that the array element is scattering passive, time-invariant and linear. The bound is derived for linearly polarized periodic arrays over an infinite planar ground-plane.

The array figure of merit can both be used to evaluate existing antennas. It can also be used as a prediction tool and hence as an a-priori assessment of impedance bandwidth limitations for a given antenna element. The array figure of merit contains return-loss, impedance bandwidth, scan range, and element specific information. We show how to use the array figure of merit to investigate trade-off between, e.g., return-loss and bandwidth or bandwidth versus thickness. We note that the array figure of merit easily extend to multi-band antennas.

1. INTRODUCTION

Physical limitations on impedance bandwidth for array antennas is a fairly recent research area. Initial investigation based on an antenna Q approach are known in [1, 2]. A sum-rule approach to these limitations are derived and analyzed in [3–5]. On the other hand, small single port antennas bandwidth limitations are known starting from [6, 7] for spherical regions, using a spherical mode approach. Antenna Q for small antennas analyzed through the input impedance are given in [8]. Arbitrary shaped small antenna limitations were first derived in [9] through a sum-rule for partial directivity over antenna Q, D/Q. Antenna Q, and D/Q through a current-density approach has been studied in [10–14]. Other approaches to bandwidth through stored energy can be found in [13, 15–18].

The present result is based on sum-rules for array antennas. Such sum-rules follow directly from that a unit-cell array antenna can be seen as a passive system [19–21], see also [22]. This approach is similar to scattering of layered homogeneous absorbers [23]. It was extended to periodic structures above a ground-plane in [24, 25] and applied to arrays over a ground-plane in [3]. In [4] we showed that these bounds are valid for linearly polarized multi-band array antennas. Here we also defined *the array figure of merit*. For a comparison of performance of recent array antennas see [4]. Extension to circular polarization can be found in [5].

In the present paper we illustrate how the array figure of merit can be used as a trade-off tool between scan-range, bandwidth at a particular return loss level and thickness for an array antenna. This enables us to predict what the fundamental limitations of an array are. Furthermore, it can also be used as a thumb-rule based on existing arrays as an a-priori figure of merit for a particular design-specification. The figure of merit is here limited to planar unit-cell arrays over a ground-plane. It is derived under the assumption that the array is passive, loss-less, reciprocal and time-invariant.

The paper is organized as follows, in Section 2 we introduce the array figure of merit. In Section 3 we illustrate how the array figure of merit can be used as a trade-off tool for arrays. We end with a conclusion.

2. ARRAY FIGURE OF MERIT

Consider a planar array antenna over a ground plane, see Fig. 1. We simplify the array to a model with a single unit cell with periodic boundary condition over a ground-plane. The antenna is feed below the ground-plane, and matching is done within the unit-cell above the ground-plane. The unit-cell extend from the ground-plane a distance d upwards and with area A to include one element of the periodic array.

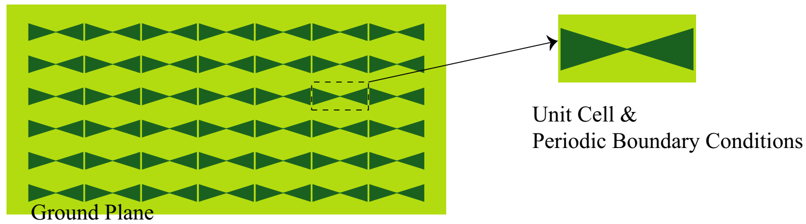

Figure 1: Schematic picture from above of a finite array with a ground-plane. The enlargement shows a unit-cell of the array.

The key parameter for the figure of merit is the array feed reflection coefficient Γ_Z. It is a function of angular frequency ω, note that $|\Gamma_Z| \leq 1$, and that it is analytic in $s = \sigma + j\omega$, $\mathrm{Re}s > 0$ [19, 26]. It also depend on the scan-angle θ. For the TE-radiating case we have the following relation [4]:

$$I(\theta) = \int_0^{\omega_G} \omega^{-2} \ln\left(\left|\Gamma_Z^{\mathrm{TE}}(\omega,\theta)\right|^{-1}\right) d\omega \leq q_{\mathrm{TE}}(\theta), \tag{1}$$

where ω_G is the angular frequency anzats of the grating-lobes and

$$q_{\mathrm{TE}} = \frac{\pi d}{c}\left(1 + \frac{\tilde{\gamma}}{2dA}\right)\cos\theta \leq \frac{\pi d \mu_s}{c}\cos\theta. \tag{2}$$

Here c is the speed of light, μ_s is the maximal static relative permeability, d is the unit-cell thickness, A is the unit-cell area and $\tilde{\gamma}$ is a function of the magnetic polarizability tensor, see [25]. The array figure of merit is derived out (1) through the idea of measuring the maximal value of $I(\theta)/q(\theta)$. Let $\lambda = 2\pi c/\omega$, the we maximize I/q over one or several working bands $[\lambda_{-,m},\, \lambda_{+,m}]$, $m = 1, \ldots, M$ below ω_G and over the scan-range $R = [\theta_0,\, \theta_1]$. Given a maximal reflection coefficient Γ_m over working band m and for all angles in R, we find that the array figure of merit is defined as

$$\eta_M^{\mathrm{TE}} = \frac{\sum_{m=1}^{M} \ln\left(|\Gamma_m|^{-1}\right)(\lambda_{+,m} - \lambda_{-,m})}{2\pi c\tilde{q}} \leq 1. \tag{3}$$

We obtain the highest value of the array figure of merit η_M^{TM}, if we know detailed structure information $\tilde{\gamma}$ about the unit cell, and insert $\tilde{q} = \min_{\theta\in R} q^{\mathrm{TE}}(\theta)$ above. For most cases we only know the array thickness and maximal static permeability μ_s, in this case we use $\tilde{q} = \pi d\mu_s(\cos\theta)/c$. This imply that we get a slightly conservative measure of η^{TE} of the performance corresponding to making full use of the available volume. Note that the above calculation assumes a perfect ground-plane to determine $\tilde{\gamma}$ through a mirroring approach. Possible holes in the ground-plane could improve the possible performance limit.

The simplest case, with scan-range region $[0,\, \theta_1]$, and one working band $[\lambda_-,\, \lambda_+]$, assuming no information about the unit-cell structure we find the array figure of merit as:

$$\eta = \eta_1^{\mathrm{TE}} = \frac{\ln\left(|\Gamma_m|^{-1}\right)(\lambda_{+,m} - \lambda_{-,m})}{2\pi^2 d\mu_s \cos\theta_1} \leq 1. \tag{4}$$

The highest known value of a wide-band and wide-scan array using (4) is $\eta^{\mathrm{TE}} = 0.64$, see [4] and where also the TM-case is discussed. Given a measured or simulated array, it is fairly easy to estimate its array-figure of merit see [4]. Below we illustrate through an example how to use (4) as a tool for trade-off.

3. TRADE-OFF USING THE ARRAY FIGURE OF MERIT

The main point in this paper is to investigate how to use the array figure of merit as a trade-off tool and to evaluate different a-priori design-criteria. To do this, consider an array with a high performance requirement. The desired array specification is:

$$\text{bandwidth ratio: } \frac{\lambda_+}{\lambda_-} = 6 \text{ at } RL = 15\,\text{dB or } RL = 10\,\text{dB} \tag{5}$$

$$\text{scan range: } [0, 60^\circ] \text{ or } [0, 45^\circ] \tag{6}$$

$$\text{thickness ratio: } \frac{d}{\lambda_-} = 0.5 \text{ or } \frac{d}{\lambda_-} = 1. \tag{7}$$

Here, $RL = 20\log_{10}(|\Gamma|^{-1})$ is the return-loss. The smaller scan-range might be possible, but the larger is preferred. Similarly the higher return loss is desired, but if trade-off requires it the lower one is acceptable over the working band. Similarly, an array should clearly be as thin as possible and the 0.5 thickness ratio is the goal. Given such a specification it is often hard to know if its realizable and how it compares with existing arrays. Let's check the hardest requirements with the array figure of merit, $RL = 15\,\text{dB}$, $\theta_1 = 60^\circ$, $d/\lambda_- = 0.5$ and $\lambda_+/\lambda_- = 6$ we find that $\eta = 1.75 > 1$ and thus it is beyond the physically realistic case.

With one or several of the lower requirements we can enter into the physically possible range. Here it is interesting to compare performance with existing top preforming arrays to understand how these requirements has to be modified in order for the design to be a reasonable input for an array design. We will show how the different parameters can be trade-against each other.

Given the specification of the array, to compare it with existing high-performing arrays we choose the figure of merit as $\eta \in [0.57, 0.63, 0.70]$. This corresponds to slightly below 0.57, at 0.64 and beyond 0.70 the best known array element today. We see that Eq. (4) for the scan-range depend only on θ_1, thus we fix the two different maximal ranges 45° and 60° for our illustration. Similarly we also consider the two cases of return loss 10 dB and 15 dB. Sweeping thicknesses, we find the corresponding bandwidth ratio λ_+/λ_- in Fig. 2.

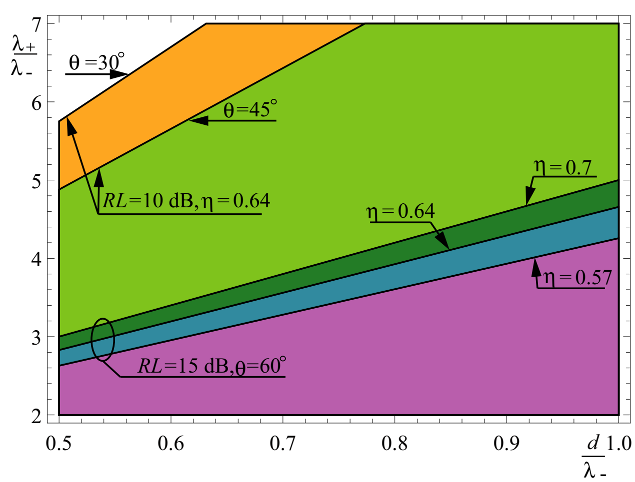

Figure 2: Bandwidth versus relative thickness. We consider two ranges of return loss 10 dB and 15 dB and compare with today's best arrays $\eta \in [0.57, 0.64, 0.70]$ for $[0, \theta_1]$ where $\theta_1 \in [30, 45, 60^\circ]$.

From this figure we see that with present day state-of-the art $\eta = 0.64$, a 6 : 1 bandwidth ratio with 10 dB return loss and 45° is possible only if the relative thickness satisfy $d/\lambda_- > 0.64$. If we desire 15 dB in return loss and $\theta_1 = 60^\circ$ we need a relative thickness ~ 1.4. One can also verify that if we for $RL = 15\,\text{dB}$ lower the max scan-range to $\theta_1 = 45^\circ$, then a 6 : 1 bandwidth and relative thickness 1 correspond to the array figure of merit $\eta = 0.64$, which is the best figure-of-merit preforming wideband array. Thus a fixed array figure of merit gives us a performance surfaces on which the antenna parameters trade-off at the same performance level. It unfortunately does not tell us how to realize these arrays, only that they are physically possible, and that there exists other arrays for a particular combination parameters of this performance level.

We can further illustrate how to trade the parameters, by sweeping return loss and scan-range against bandwidth ratio see Fig. 3. Fix the relative thickness of d/λ_- to 0.5 or 1, for a range of array figure of merit performances. It is clear from Fig. 3(a) that for a relative thickness of 0.5, we have the choice of a lower return loss of $\sim 7.7\,\text{dB}$ to obtain the desired 6 : 1 bandwidth ratio for

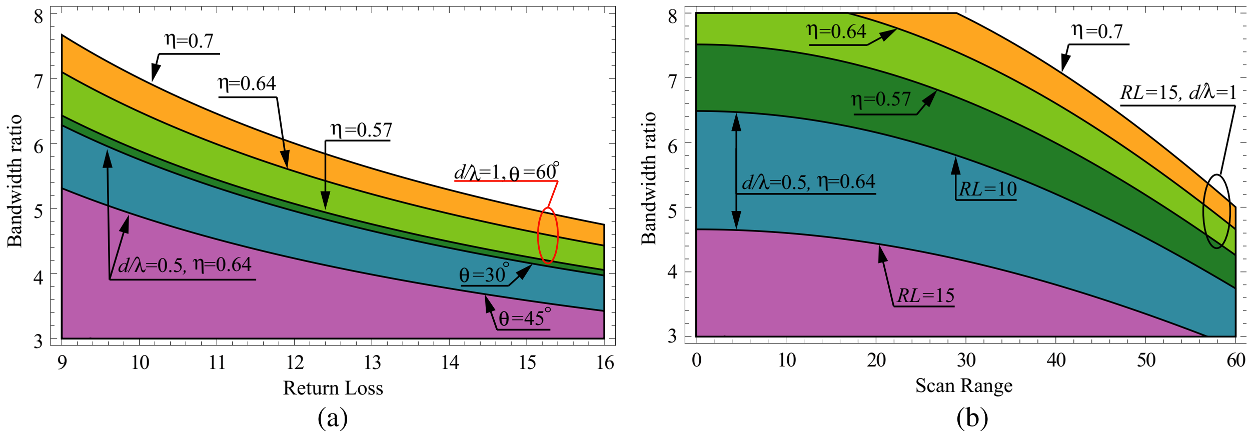

Figure 3: Bandwidth versus (a) return loss and (b) scan-range $[0, \theta_1]$.

45° or reducing the max scan-range to 30° and live with a return-loss of 9.4 dB. If we instead use, e.g., a WAIM layer and have a thick array with relative thickness 1, we see that a 6 : 1 bandwidth at 10 dB return-loss and maximal scan-range 60°, can be obtained for array figure of merits in $0.57 < \eta < 0.64$.

In Fig. 3(b) of trade-off is between bandwidth ratio and scan-range. Here we once again consider the two cases of $RL = 10$ dB and $RL = 15$ dB, and the two relative thicknesses 0.5 and 1. It is clear that the $RL = 10$ dB with relative thickness 0.5 reaches a 6 : 1 bandwidth if we accept a scan-range of [0, 20°] for $\eta = 0.64$. However $RL = 15$ and $d/\lambda = 1$ reaches 6 : 1 bandwidth ratio in the interval [0, 40] degrees for $\eta = 0.57$, whereas if we can improve the design to beyond the state-of-the art $\eta = 0.7$, we can reach a scan-range of [0, 52°].

From the above considerations we see that there are several combination of the specified parameters that are in pair with today's state-of-the art array. With the above illustrations we have shown how return-loss, scan-range and thickness can be traded, in order to reach a good array specification. In the end, which of these parameters that should be weakened depend on the application.

Note that there are currently few known arrays with performance above $\eta > 0.45$, and unfortunately that the waist majority of arrays are below 0.30. A careful choice of antenna design and its technology is required to reaching towards $\eta = 0.64$. A survey of the best arrays in a desired array technology will easily yield a technology-specific figure of merit, which hence can be used as a design-tool and goal setting of such arrays.

4. CONCLUSION

In the present paper, we illustrate how the array figure of merit can be used to trade different array performance parameters against each other. We consider a high-performance array specification, and compare with the array figure of merit corresponding to a state-of-the art array of today. The array figure of merit enables us to realize how the array parameters depend on each-other. We can thus compare with the physical limitation $\eta = 1$, but also against top-performing published arrays. Through the array figure of merit, it is clear how the specification can be adjusted, in order to get an array that is comparable with existing arrays. Realizing a physical array design that have this performance remain an antenna engineering challenge, however we know that there exists realizations on this performance level.

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REFERENCES

1. Tomasic, B. and H. Steyskal, "Minimum Q of the element in an infinite phased array — TM^z case," *19th International Conference on Applied Electromagnetics and Communications, 2007, ICECom 2007*, 1–8, 2007.

2. Kwon, D. and D. Pozar, "Energy storage and radiation Q of infinite planar dipole phased arrays," *IEEE Trans. Antennas Propagat.*, Vol. 62, No. 1, 2014.
3. Doane, J. P., K. Sertel, and J. L. Volakis, "Bandwidth limits for arrays backed by a conducting ground plane," *IEEE Trans. Antennas Propagat.*, 2511–2518, 2013.
4. Jonsson, B. L. G., C. I. Kolitsidas, and N. Hussain, "Array antenna limitations," *IEEE Antenn. Wireless Propag. Lett.*, Vol. 12, 1539–1542, 2013.
5. Doane, J., K. Sertel, and J. L. Volakis, "Bandwidth limits for lossless, reciprocal PEC-backed arrays of arbitrary polarization," *IEEE Trans. Antennas Propagat.*, Vol. 62, No. 5, 2531–2542, 2014.
6. Chu, L. J., "Physical limitations of omni-directional antennas," *J. Appl. Phys.*, Vol. 19, 1163–1175, 1948.
7. Collin, R. E. and S. Rothschild, "Evaluation of antenna Q," *IEEE Trans. Antennas Propagat.*, Vol. 12, 23–27, Jan. 1964.
8. Yaghjian, A. D. and S. R. Best, "Impedance, bandwidth, and Q of antennas," *IEEE Trans. Antennas Propagat.*, Vol. 53, No. 4, 1298–1324, 2005.
9. Gustafsson, M., C. Sohl, and G. Kristensson, "Physical limitations on antennas of arbitrary shape," *Proc. R. Soc. A*, Vol. 463, 2589–2607, 2007.
10. Vandenbosch, G. A. E., "Reactive energies, impedance, and Q factor of radiating structures," *IEEE Trans. Antennas Propagat.*, Vol. 58, No. 4, 1112–1127, 2010.
11. Vandenbosch, G. A. E., "Simple procedure to derive lower bounds for radiation Q of electrically small devices of arbitrary topology," *IEEE Trans. Antennas Propagat.*, Vol. 59, No. 6, 2217–2225, 2011.
12. Gustafsson, M., M. Cismasu, and B. L. G. Jonsson, "Physical bounds and optimal currents on antennas," *IEEE Trans. Antennas Propagat.*, Vol. 60, No. 6, 2672–2681, 2012.
13. Capek, M., P. Hazdra, and J. Eichler, "A method for the evaluation of radiation Q based on modal approach," *IEEE Trans. Antennas Propagat.*, Vol. 60, No. 10, 2012.
14. Gustafsson, M. and S. Nordebo, "Optimal antenna for Q, superdirectivity, and radiation patterns using convex optimization," *IEEE Trans. Antennas Propagat.*, Vol. 61, No. 3, 1109–1118, 2013.
15. Mikki, S. M. and Y. M. Antar, "A theory of antenna electromagnetic near field — Part I & II," *IEEE Trans. Antennas Propagat.*, Vol. 60, No. 12, 4691–4724, 2011.
16. Gustafsson, M. and B. L. G. Jonsson, "Stored electromagnetic energy and antenna Q," ArXiv Physics e-prints, arXiv:1211.5521, 2013.
17. Geyi, W., "On stored energies and radiation Q," ArXiv Physics e-prints, arXiv:1403.3129, 2014.
18. Yaghjian, A. D., M. Gustafsson, and B. L. G. Jonsson, "Minimum Q for lossy and lossless electrically small dipole antennas," *Progress In Electromagnetic Research*, Vol. 143, 641–673, 2013.
19. Youla, D., L. Castriota, and H. Carlin, "Bounded real scattering matrices and the foundations of linear passive network theory," *IRE Transactions on Circuit Theory*, Vol. 6, No. 1, 102–124, 1959.
20. Zemanian, A. H., "An n-port realizability theory based on the theory of distributions," *IEEE Transactions on Circuit Theory*, Vol. 10, No. 2, 265–274, 1963.
21. Nussenzveig, H. M., *Causality and Dispersion Relations*, Academic Press, London, 1972.
22. Bernland, A., A. Luger, and M. Gustafsson, "Sum rules and constraints on passive systems," *J. Phys. A: Math. Theor.*, Vol. 44, No. 14, 145205, 2011.
23. Rozanov, K. N., "Ultimate thickness to bandwidth ratio of radar absorbers," *IEEE Trans. Antennas Propagat.*, Vol. 48, No. 8, 1230–1234, Aug. 2000.
24. Sjöberg, D., "Low frequency scattering by passive periodic structures for oblique incidence: Low pass case," *J. Phys. A: Math. Theor.*, Vol. 42, 385402, 2009.
25. Gustafsson, M. and D. Sjöberg, "Physical bounds and sum rules for high-impedance surfaces," *IEEE Trans. Antennas Propagat.*, Vol. 59, No. 6, 2196–2204, 2011.
26. Wohlers, M. and E. Beltrami, "Distribution theory as the basis of generalized passive-network analysis," *IEEE Transactions on Circuit Theory*, Vol. 12, No. 2, 164–170, 1965.