Kd-tree Based Shooting and Bouncing Ray Method for Fast Computation of Near Field Scattering

P. C. Gao, Z. C. Liang, and W. Gao
Science and Technology on Electromagnetic Scattering Laboratory, China

Abstract— In this paper, we propose an efficient kd-tree based shooting and bouncing ray (SBR) method for solving near field scattering problems. The kd-tree is highly effective in handling the irregularly distribution of triangles of the target. It is utilized to accelerate the ray tube tracing in the SBR, especially for electrically large and complex targets. The SBR is based on the combination of the geometrical optics (GO) and the physical optics (PO). The far field assumption results in the decrease of the accuracy of the standard PO in the near field observation. By applying the locally expanded phase approximations together with surface partitioning, the refined PO can extend the region of validity of the SBR to near field scattering. Numerical examples demonstrate that the proposed method can efficiently handle the near field scattering problems of realistic complex targets with high accuracy.

1. INTRODUCTION

The efficient high frequency asymptotic methods have been widely used in computation of scattering from electrically large targets. The shooting and bouncing ray (SBR) method [1] is one of the most commonly adopted techniques for radar cross section (RCS) prediction of arbitrarily shaped targets. This is because it provides more accurate results by applying the ray-tube tracing to take into account the multiple bounces contribution.

The procedure of the SBR involves two steps: ray tube tracing and electromagnetic computing. In near field analysis, the incident field is approximated as a spherical wave, not a plane wave. Bundles of rays radiating from source instead of a grid of parallel rays are shot toward the target. The density of the rays on the virtual aperture should be greater than about ten rays per wavelength in view of the convergence. Thus, the ray tube tracing in the SBR is very time-consuming for electrically large and complex targets. Jin et al. [2] utilized the octree, recursively subdividing the box into eight equal children boxes using three axis-perpendicular planes, to reduce the number of intersection tests. It is based on the assumption that triangles are uniformly distributed in the target space. However, the hypothesis is hardly satisfied by realistic target. The kd-tree recursively split the target space into uneven axis-aligned boxes according to the ray tracing cost estimation model. It is highly effective in handling the irregularly distribution of triangles of the target and can significantly decrease the number of ray-triangle intersection tests. Tao et al. [3] presented the kd-tree based ray tube tracing to accelerate the SBR.

Most published work on high frequency scattering calculation is related to far field analysis. However, computation of near field scattering is crucial due to its military value, i.e., essential for radar-target end-game electromagnetic modeling. The SBR is based on the combination of the geometrical optics (GO) and the physical optics (PO). The standard PO utilizes the far field assumption to simplify the radiation integral for surface currents and makes possible practical closed-form solutions. Thus, the most straightforward way of refine the PO in the near field is to employ a more accurate representation of the Green's function. Pouliguen et al. [4] as well as Neto [5] applied the exact Green's function. This approach achieves excellent accuracy at the cost of requiring numerical integration of the surface currents, which reduces the performance especially for electrically large objects. Legault [6] overcomes the difficulty and preserves the simplicity of standard PO formulation by introducing a phase approximation with an expansion center used in conjunction with surface partitioning. Sui et al. [7] proposed the concept of distinct wave propagation vector (DWPV) to model incident wave with spherical wavefront for radar signature prediction of near field targets. The aforementioned approaches are based on the similar idea. It is dealing with each small piece of the target surface individually, and the observation point and the source point are in the far field zone of each small piece. Jeng [8] applied the same method to modify the SBR to handle near field scattering. Here, we apply the locally expanded phase approximations to refine the SBR in the near field, and utilize the kd-tree acceleration structure to make it efficient for the scattering from electrically large targets.

This paper is organized as follows. Section 2 introduces the kd-tree acceleration structure. Section 3 presents formulas of the near field SBR. In Section 4, numerical results are given with discussion, followed by the conclusions and future work.

2. KD-TREE CONSTRUCTION AND TRAVERSAL

The kd-tree has been publicly known the best space-partition data structure to accelerate ray tracing of the static scenes in computer graphics [9]. Recently, it has been successfully applied in RCS prediction with the SBR [3]. A kd-tree is constructed by recursively employing the axis-perpendicular plane to split the target space into uneven axis-aligned boxes. At each step of the procedure, a node, which contains a group of triangles that overlap the axis-aligned box of the node, is processed to be an interior node or a leaf node. The optimal splitting plane is searched based on the greedy surface area heuristic (SAH) [10] strategy, which minimizes the traversal time of interior nodes and the ray-triangles intersection time to construct the approximately optimal kd-tree. While the octree simply puts the splitting positions at the middle point of the extend in each axis, the kd-tree can provide faster ray tracing by taking into account the triangle distribution in the target space.

The depth-first kd-tree traversal starts with the root node of the kd-tree, and a stack is used as a priority queue of nodes left to visit according to how closer to the ray origin. In order to remove the unnecessary traversal steps of interior nodes, the kd-tree can be augmented with ropes in leaf nodes. The rope on each face of leaf nodes is a pointer to the adjacent leaf node, the smallest interior node or a nil node for leaf nodes on the border. Thus, the ray passing through a leaf node can efficiently continue the traversal with the rope, and it avoids the requirement of the stack to keep to-be-visited nodes. Figure 1 shows the traversal procedure of a 2D kd-tree with ropes.

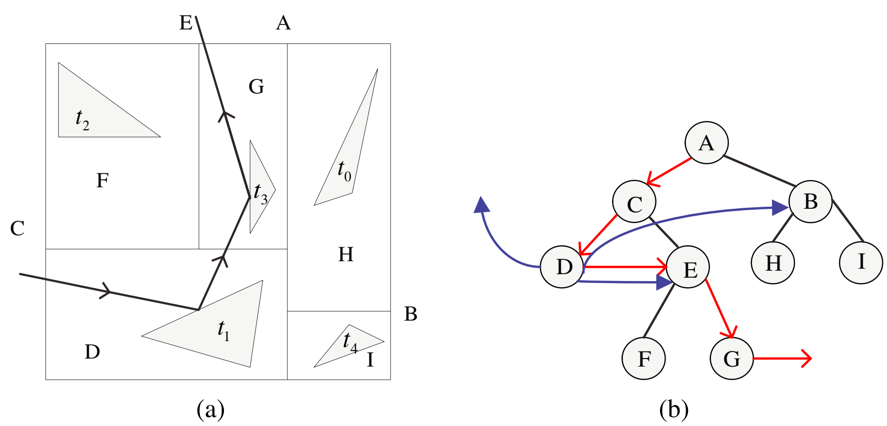

Figure 1: A 2D kd-tree and its traversal. (a) The interior nodes (B, C, E) of the kd-tree are labeled as their splitting planes and leaf nodes (D, F, G, H, I) are labeled in their boxes. The ray is recursively traced and has two intersections with the target. (b) The traversal path is shown as a red bold line. The traversal begins with the root node A, and proceeds down through the interior node C. Then an intersection is found in the leaf node D. With the help of the rope of D (the rope of D is marked with blue bold line), the generated reflected ray directly moves to the smallest interior node E including the leaf node F and G. Ray tracing ends with the second reflected ray exiting the leaf node G.

3. NEAR FIELD SBR

As seen in Figure 2, the incident spherical wave is modeled as a dense grid of ray tubes with distinct incident propagation vector, which radiate from source towards the target. With object geometrical center being the origin of the rectangular coordinates, where $\mathbf{r}'$ is the position vector of a point on the target, and $\mathbf{r}$ denote observation and source points. Given a finite observation range r, the accuracy of Green's function with far field assumption decreases with increasing $\mathbf{r}'$. A good way of removing the difficulty is to apply phase approximations with an arbitrary expansion centre as opposed to one essentially fixed at the origin. When $\mathbf{r}'$ lies in the neighbourhood of the expansion centre $\mathbf{r}_\mathbf{n}^\mathbf{c}$, the Green's function can be approximated as

$$\frac{e^{-jk\hat{\mathbf{R}}\cdot(\mathbf{r}-\mathbf{r}')}}{R} \simeq \frac{e^{-jk\hat{\mathbf{k}}^{\mathbf{sn}}\cdot(\mathbf{r}-\mathbf{r}')}}{r} \tag{1}$$

where $R = |\mathbf{R}| = |\mathbf{r} - \mathbf{r}'|$, and the unit vector

$$\hat{\mathbf{k}}^{\mathbf{sn}} = \frac{\mathbf{R_n}}{|\mathbf{R}_\mathrm{n}|} = \frac{\mathbf{r} - \mathbf{r}^\mathbf{c}_\mathbf{n}}{|\mathbf{r} - \mathbf{r}^\mathbf{c}_\mathbf{n}|} \tag{2}$$

where $\mathbf{r}^\mathbf{c}_\mathbf{n}$ is the center of the projected footprint of nth ray tube, and $\mathbf{R_n}$ is the position vector from $\mathbf{r}^\mathbf{c}_\mathbf{n}$ to the observation point $\mathbf{r}$. The incident propagation vector of the nth ray tube is defined as $\hat{\mathbf{k}}^{\mathbf{in}} = -\hat{\mathbf{k}}^{\mathbf{sn}}$ in monostatic cases. The incident electric field $\mathbf{E}^{\mathbf{in}}$ for the nth ray tube is

$$\mathbf{E}^{\mathbf{in}} = \left(E^{i\theta}\hat{\boldsymbol{\theta}} + E^{i\varphi}\hat{\boldsymbol{\varphi}}\right) e^{-jk\hat{\mathbf{k}}^{\mathbf{in}}\cdot\mathbf{r}'_\mathbf{n}} \tag{3}$$

where $\mathbf{r}'_\mathbf{n}$ is the position vector of a point on the footprint of the nth ray tube with footprint center being the origin. In respective, $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\varphi}}$ are the unit elevation vector and unit azimuth vector, $E^{i\theta}$ and $E^{i\varphi}$ are the scalar component of $\mathbf{E}^{\mathbf{in}}$ in $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\varphi}}$.

The near field PO integral is carried out on the four-sided polygon modeled by the exit position of ray tube. At an observation point, the formula of the scattered field of the nth ray tube is given

$$\mathbf{E}^{\mathbf{sn}} = -\frac{j}{\lambda}\frac{e^{-jkR_n}}{R_n}\hat{\mathbf{n}} \times \left(E^{i\varphi}\hat{\boldsymbol{\theta}} - E^{i\theta}\hat{\boldsymbol{\varphi}}\right) \iint_{S'} e^{-jk\left(\hat{\mathbf{k}}^{\mathbf{in}} - \hat{\mathbf{k}}^{\mathbf{sn}}\right)\cdot\mathbf{r}'_\mathbf{n}} ds'. \tag{4}$$

The integral $\iint_{S'} e^{-jk\left(\hat{\mathbf{k}}^{\mathbf{in}} - \hat{\mathbf{k}}^{\mathbf{sn}}\right)\cdot\mathbf{r}'_\mathbf{n}} ds'$ can be calculated with the well-known Gordon's contour integration [11].

4. NUMERICAL RESULT

Several numerical results are presented to evaluate the proposed near field SBR with the kd-tree acceleration structure. The results are derived on an Intel Core i5 2.6 GHz CPU without special declaration. The targets are perfect electric conductor (PEC) in this paper.

A square plate with dimension of 10 m, which is illuminated by a Hertzian dipole of 0.3 GHz located on x-axis at 33 m, lying in the y-z plane is used to verify the accuracy of the proposed method. For the plate, the corresponding maximum dimension is $D{=}10\sqrt{2}$ m, so that the far field limit is $\mathrm{r}_{ff}(D) = 2D^2/\lambda = 400$ m. The scattering parameters are: $r = 33$ m, $\theta = 90°$, $\varphi = 0° \sim 60°$. As shown in Figure 3, the result of the proposed kd-tree based near field SBR agrees well with FEKO's MLFMA result over angles of 0° to 30°, and the deviation is seen in the observation angles after 30° both for the proposed approach and FEKO's PO. This is in concordance with the widely accepted rule of thumb of restricting observation angles within 30° of boresight for accurate backscattering results using PO with planar structures.

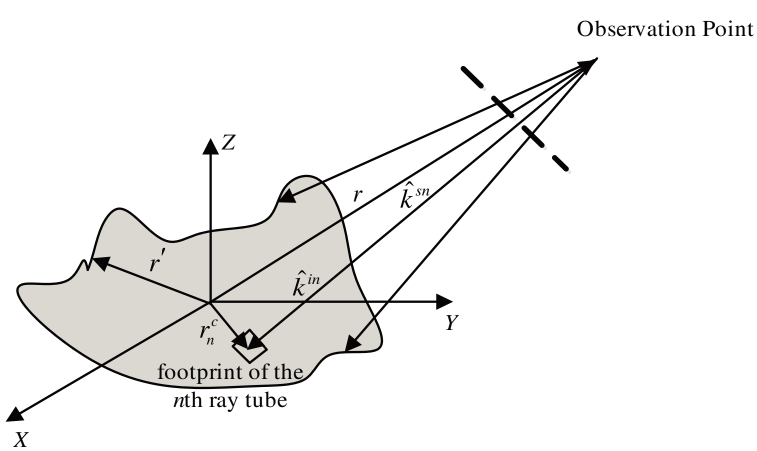

Figure 2: Geometry of near field scattering.

In order to demonstrate the efficiency of the proposed approach for scattering from electrically large targets, the RCS of a missile illustrated in Figure 4 at 10 GHz is calculated. The missile is $33\lambda \times 21\lambda \times 8\lambda$, and the corresponding far field limit is about 96 m. The Hertzian dipole is located at the position (10 m, 70°, 30°), and the scattering parameter is: φ from 0° to 90° on the $\theta = 70°$ plane

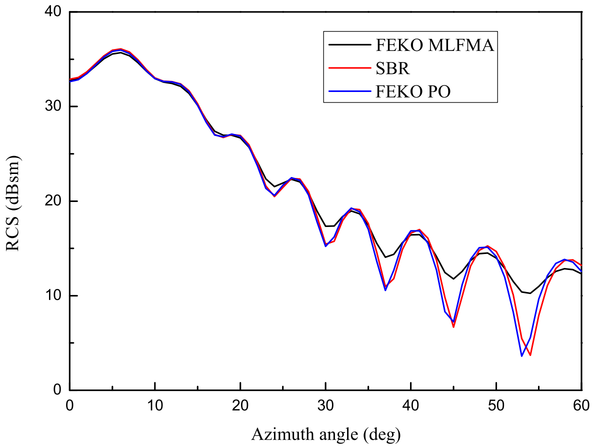

Figure 3: The near field RCS of a square plane with 10 m length as a function of azimuth angle in θ-polarization.

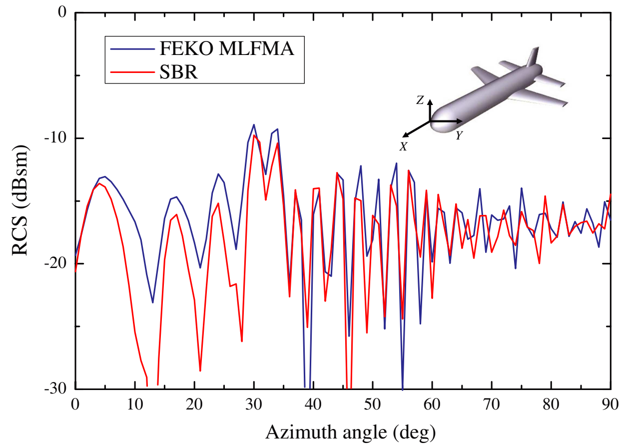

Figure 4: The near field RCS of a missile-like target as a function of azimuth angle in θ-polarization.

with an angular resolution 1° at $r = 10\,\mathrm{m}$. As can be seen clearly from Figure 4, a good agreement is observed between the proposed method result and the FEKO's MLFMA result, and the deviation may be partly due to the lack of the edge-diffraction effect. It has been proven that the kd-tree based SBR is more efficient than the one with other acceleration structures (e.g., octree) [3]. Thus, no comparison of efficiency is presented in this paper. The missile is composed of 11404 triangles, and 7037640 rays are shot toward it to ensure the convergence of result. The computation time for one observation point is 6.3 seconds, i.e., the kd-tree based near field SBR can process over a million rays per second including both ray tube tracing and electromagnetic computing with only one core. Parallel MLFMA of FEKO takes 132 seconds with 22 cores to solve the same problem. For the similar computational resource, the proposed algorithm can be parallelized to efficiently handle challenging realistic problems with a dimension of several hundreds wavelengths.

5. CONCLUSION

It has been shown that the SBR can efficiently handle the scattering from electrically large targets by employing the kd-tree acceleration structure constructed based on the distribution of triangles of the target. Additionally, the standard PO formulation is adapted to near field computations using locally expanded approximation in conjunction with surface partitioning. Numerical results show excellent agreement with the well-known commercial EM software FEKO, and demonstrate the efficiency of the kd-tree based near field SBR.

ACKNOWLEDGMENT

This work was supported in part by National Natural Science Foundation of China (No. 61302036).

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