Effective Implementation of the CFS-PML Using DSP Techniques for Truncating Dispersive Medium FDTD Domains

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Abstract— Efficient and unsplit-field finite-difference time-domain (FDTD) implementation of the complex frequency-shifted perfectly matched layer (CFS-PML), based on the digital signal processing (DSP) techniques and the material independence relations via applying the electric flux density (D) and the magnetic flux density (B), is proposed for truncating three dimensional FDTD computational domain entirely composed of dispersive material realized with a Drude model. The CFS-PML implementation is introduced based on the stretched coordinate PML (SC-PML) and the uniaxial anisotropic PML (UPML), respectively. The implementation of the proposed CFS-PML formulations is based on the SC-PML due to the fact that has advantage of simple implementation in the corners and the edges of the PML regions. Moreover, these proposed formulations are completely independent of the material properties of the FDTD computational domain and hence can be applied to truncate arbitrary media without any modification because of the D-B constitutive relations used in Maxwell's equations. Besides, the DSP techniques include the Bilinear Z-transform (BZT) method and the Matched Z-transform (MZT) method, respectively. However, from the point of view of the Courant-Friedrichs-Levy (CFL) condition, to the best of our knowledge, time step based on the BZT only needs to meet CFL condition, whereas time step based on the MZT method has to make it smaller for retaining stability and desirable accuracy. Consequently, the former one is introduced into the proposed formulations. A numerical example has been carried out in a three dimensional FDTD computational domain to validate the proposed formulations. It is clearly shown that the proposed formulations with CFS scheme are efficient in attenuating evanescent waves and reducing late-time reflections.

1. INTRODUCTION

One of the most widely methods in electromagnetic simulations, finite difference time domain (FDTD) method [1], has been used in various fields. Recently, the Z-transform method has been incorporated with FDTD scheme to model dispersive frequency dependent electromagnetic application [2]. This method takes the advantage of simplicity as it allows direct FDTD implementations of Maxwell's equations.

Meta-materials are specifically referred to as a class of artificial materials that have simultaneous negative electric permittivity and magnetic permeability [3] in a certain frequency band. These meta-materials have drawn considerable attention owing to their unusual electromagnetic properties [4]. To model open region double-negative (DNG) meta-material problems, absorbing boundary conditions (ABCs), such as perfectly matched layer (PML), are needed. New PML formulations have been successfully introduced for modeling DNG meta-material problems [6].

In this paper, an efficient DSP implementation of the CFS-PML is proposed based on stretched coordinate PML (SC-PML) and D-B formulations. In addition, these formulations are implemented using the electric flux density D and magnetic flux density B fields instead of the conventional E and H fields. This method has the advantage of making the PML completely independent of the material properties of the FDTD computational domain.

2. FORMULATIONS

In three-dimensional (3D) SC-PML regions, the frequency domain modified Maxwell's curl equations can be written in terms of D and B as:

$$j\omega\varepsilon_0 D(\omega) = \nabla_s \times H(\omega) \tag{1}$$
$$j\omega\mu_0 B(\omega) = -\nabla_s \times E(\omega) \tag{2}$$

where D and B are given by

$$D(\omega) = \varepsilon_r(\omega)E(\omega) \tag{3}$$
$$B(\omega) = \mu_r(\omega)H(\omega) \tag{4}$$

where $\varepsilon_r(\omega)$ and $\mu_r(\omega)$ are, respectively, the relative permittivity and permeability of the FDTD domain and the operator ∇_s is expressed as

$$\nabla_s = \hat{x}S_x^{-1}\partial_x + \hat{y}S_y^{-1}\partial_y + \hat{z}S_z^{-1}\partial_z \tag{5}$$

and S_η, $(\eta = x,\, y,\, z)$ is the PML stretched coordinate variable given by [6].

$$S_\eta = 1 + \frac{\sigma_\eta}{j\omega\varepsilon_0\varepsilon_r(\omega)} \tag{6}$$

with the CFS scheme, S_η was defined as

$$S_\eta = \kappa_\eta + \frac{\sigma_\eta}{\alpha_\eta + j\omega\varepsilon_0\varepsilon_r(\omega)} \tag{7}$$

where σ_η and α_η are assumed to be positive real and κ_η is real and ≥ 1. $\varepsilon_r(\omega)$ and $\mu_r(\omega)$ are, respectively, the DNG meta-material permittivity and permeability which are assumed to be identical and realized by the Drude-type DNG mode [4] as

$$\varepsilon_r(\omega) = \mu_r(\omega) = 1 + \frac{\omega_p^2}{-\omega^2 + j\omega\Gamma} \tag{8}$$

where ω_p and Γ are, respectively, the medium plasma frequency and loss factor.

Now the vector components of the curl operators of (1) and (2) are written out in three dimensions

$$j\omega\varepsilon_0 D_x(\omega) = S_y^{-1}\frac{\partial H_z(\omega)}{\partial y} - S_z^{-1}\frac{\partial H_y(\omega)}{\partial z} \tag{9}$$

$$j\omega\varepsilon_0 D_y(\omega) = S_z^{-1}\frac{\partial H_x(\omega)}{\partial z} - S_x^{-1}\frac{\partial H_z(\omega)}{\partial x} \tag{10}$$

$$j\omega\varepsilon_0 D_z(\omega) = S_x^{-1}\frac{\partial H_y(\omega)}{\partial x} - S_y^{-1}\frac{\partial H_x(\omega)}{\partial y} \tag{11}$$

$$j\omega\mu_0 B_x(\omega) = -S_y^{-1}\frac{\partial E_z(\omega)}{\partial y} + S_z^{-1}\frac{\partial E_y(\omega)}{\partial z} \tag{12}$$

$$j\omega\mu_0 B_y(\omega) = -S_z^{-1}\frac{\partial E_x(\omega)}{\partial z} + S_x^{-1}\frac{\partial E_z(\omega)}{\partial x} \tag{13}$$

$$j\omega\mu_0 B_z(\omega) = -S_x^{-1}\frac{\partial E_y(\omega)}{\partial x} + S_y^{-1}\frac{\partial E_x(\omega)}{\partial y} \tag{14}$$

It is clearly and easily observed that similar form can be found among (9)–(14).

First, let us consider the discretization of (14) taken as an example in the edges of SC-PML regions, which are overlaps of PML's faces that run parallel with the z direction Transforming (14) from the frequency to the Z-domain, Equation (14) can be written as

$$(1 - z^{-1})\mu_0 B_z/\Delta t = -\frac{1}{S_x(z)}\frac{\partial E_y}{\partial x} + \frac{1}{S_y(z)}\frac{\partial E_x}{\partial y} \tag{15}$$

where Δt is time step and $S_\eta(z)$, $(\eta = x,\, y,\, z)$, can be obtained by applying the bilinear transform method [7] using relation $j\omega \to (2/\Delta t)(1 - z^{-1})/(1 + z^{-1})$

$$S_\eta(z) = w_\eta\frac{(1 + u_\eta z^{-1} + \theta_\eta z^{-2})}{(1 + v_\eta z^{-1} + \varphi_\eta z^{-2})} \tag{16}$$

where

$$w_\eta = \kappa_\eta \frac{\left[(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2 + (\sigma_\eta\Gamma)/\kappa_\eta) + (\Delta t\alpha_\eta/2) + \varepsilon_0 + (\Delta t\varepsilon_0\Gamma/2) + (\Delta t\sigma_\eta)/(2\kappa_\eta)\right]}{\left[(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2) + (\Delta t\alpha_\eta/2) + \varepsilon_0 + (\Delta t\varepsilon_0\Gamma/2)\right]}$$

$$u_\eta = \frac{\left[2(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2 + (\sigma_\eta\Gamma)/\kappa_\eta) - 2\varepsilon_0\right]}{\left[(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2 + (\sigma_\eta\Gamma)/\kappa_\eta) + (\Delta t\alpha_\eta/2) + \varepsilon_0 + (\Delta t\varepsilon_0\Gamma/2) + (\Delta t\sigma_\eta)/(2\kappa_\eta)\right]}$$

$$v_\eta = \frac{\left[2(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2) - 2\varepsilon_0\right]}{\left[(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2) + (\Delta t\alpha_\eta/2) + \varepsilon_0 + (\Delta t\varepsilon_0\Gamma/2)\right]}$$

$$\theta_\eta = \frac{\left[(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2 + (\sigma_\eta\Gamma)/\kappa_\eta) - (\Delta t\alpha_\eta/2) + \varepsilon_0 - (\Delta t\varepsilon_0\Gamma/2) - (\Delta t\sigma_\eta)/(2\kappa_\eta)\right]}{\left[(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2 + (\sigma_\eta\Gamma)/\kappa_\eta) + (\Delta t\alpha_\eta/2) + \varepsilon_0 + (\Delta t\varepsilon_0\Gamma/2) + (\Delta t\sigma_\eta)/(2\kappa_\eta)\right]}$$

$$\varphi_\eta = \frac{\left[(\Delta t/2)^2(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2) - (\Delta t\alpha_\eta/2) + \varepsilon_0 - (\Delta t\varepsilon_0\Gamma/2)\right]}{\left[(\Delta t/2)^2\left(\alpha_\eta\Gamma + \varepsilon_0\omega_p^2\right) + (\Delta t\alpha_\eta/2) + \varepsilon_0 + (\Delta t\varepsilon_0\Gamma/2)\right]}$$

substituting (16) into (15), we obtain

$$\left(1 - z^{-1}\right)\mu_0 B_z/\Delta t = -\frac{1}{w_x}\frac{\left(1 + v_x z^{-1} + \varphi_x z^{-2}\right)}{\left(1 + u_x z^{-1} + \theta_x z^{-2}\right)}\frac{\partial E_y}{\partial x} + \frac{1}{w_y}\frac{\left(1 + v_y z^{-1} + \varphi_y z^{-2}\right)}{\left(1 + u_y z^{-1} + \theta_y z^{-2}\right)}\frac{\partial E_x}{\partial y} \tag{17}$$

Introducing the following auxiliary variables:

$$\phi_{zx} = \frac{1}{w_x}\frac{1}{\left(1 + u_x z^{-1} + \theta_x z^{-2}\right)}\frac{\partial E_y}{\partial x} = -u_x z^{-1}\phi_{zx} - \theta_x z^{-2}\phi_{zx} + \frac{1}{w_x}\frac{\partial E_y}{\partial x} \tag{18}$$

$$\phi_{zy} = \frac{1}{w_y}\frac{1}{\left(1 + u_y z^{-1} + \theta_y z^{-2}\right)}\frac{\partial E_x}{\partial y} = -u_y z^{-1}\phi_{zy} - \theta_y z^{-2}\phi_{zy} + \frac{1}{w_y}\frac{\partial E_x}{\partial y} \tag{19}$$

(18) and (19) can be written in simple FDTD form, respectively, as

$$\phi_{zx}^{n+1} = -u_x\phi_{zx}^n + Q_{zx}^n + \frac{1}{w_x}\frac{\partial E_y^{n+1/2}}{\partial x} \tag{20}$$

$$Q_{zx}^{n+1} = -\theta_x\phi_{zx}^n \tag{21}$$

$$\phi_{zy}^{n+1} = -u_y\phi_{zy}^n + Q_{zy}^n + \frac{1}{w_y}\frac{\partial E_x^{n+1/2}}{\partial y} \tag{22}$$

$$Q_{zy}^{n+1} = -\theta_y\phi_{zy}^n \tag{23}$$

where Q_{zx}^{n+1} and Q_{zy}^{n+1} are auxiliary variables. Substituting (20) and (22) into (17), we obtain

$$B_z^{n+1} = B_z^n + \frac{\Delta t}{\mu_0}\left[-\left(\phi_{zx}^{n+1} + v_x\phi_{zx}^n + m_{zx}^n\right) + \left(\phi_{zy}^{n+1} + v_y\phi_{zy}^n + m_{zy}^n\right)\right] \tag{24}$$

$$m_{zx}^{n+1} = \varphi_x\phi_{zx}^n \tag{25}$$

$$m_{zy}^{n+1} = \varphi_y\phi_{zy}^n \tag{26}$$

where m_{zx}^{n+1} and m_{zy}^{n+1} are auxiliary variables. Similar manipulations can be obtained for the FDTD implementations of (9)–(13).

3. NUMERICAL STUDY

To validate the proposed methods, a 3D numerical test was carried out for validating the proposed formulations. In this numerical example, a Gaussian pulse was excited at the center of $50\Delta x \times 50\Delta y \times 50\Delta z$ 3D domain composed of DNG meta-material realized by a Drude model [4] with the following parameters: $\omega_p = 2.665 \times 10^{11}$ rad/s and $\Gamma = 1 \times 10^8$ rad/s. Under these parameters, both $\varepsilon_r(\omega)$ and $\mu_r(\omega)$ are negative in the frequency range 0–42.5 GHz. The excited Gaussian pulse was given by $H_z = \sin(2\pi f_c t)\exp(-((t - T_0)/\tau)^2)$, where $f_c = 30$ GHz, $\tau = 26$ ps and $T_0 = 4\tau$. The computational domain was truncated by 10 cells thickness PML, as shown in Fig. 1. The

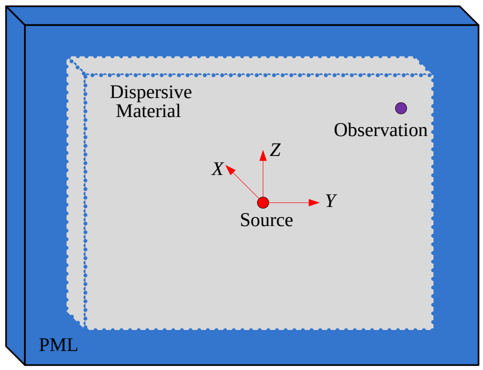

Figure 1: The FDTD grid geometry in this simulation.

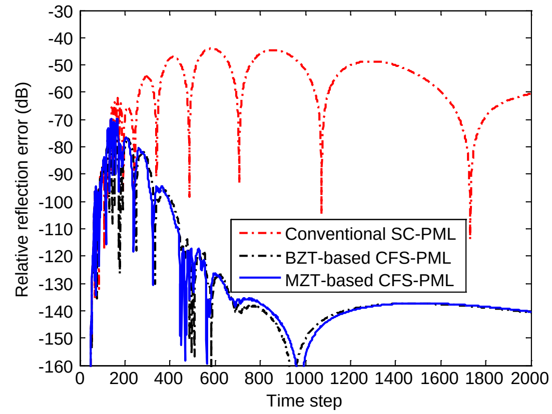

Figure 2: Relative reflection error versus time step.

observation point was chosen to be in the diagonal direction of the source and located one space cell from the PML corner region. The space cell size was taken as $\Delta x = \Delta y = \Delta z = 0.5\,\text{mm}$. The time step size was chosen to satisfy the Courant-Friedrichs-Lewy (CFL) stability limit [1] for the 3D case.

The results are shown in Fig. 2. The maximum relative errors of the conventional SC-PML, the BZT-based CFS-PML, the MZT-based CFS-PML are -45 dB, -70 dB, and -70 dB, respectively. It can be concluded from Fig. 2 that the absorbing performance of the proposed BZT-based and MZT-based CFS-PMLs have 25 dB improvement in terms of the maximum relative reflection error as compared with the conventional SC-PML, and also holds better absorbing performance as compared with the conventional SC-PML in terms of reducing late-time reflections.

4. CONCLUSION

The CFS-PML formulations have been presented for truncating dispersive material FDTD computational domains. These formulations are independent of the material properties of the FDTD computational domains due to their features of the *D-B* constitutive relations. A numerical test carried out in a 3D FDTD computational domain shows that the proposed formulations provide better absorbing performance as compared with the conventional SC-PML without CFS scheme.

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